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The Wealth of Working Nations

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Abstract

Due to population aging, GDP growth per capita and GDP growth per working-age adult have become quite different among many advanced economies over the last several decades. Countries whose GDP growth per capita performance has been lackluster, like Japan, have done surprisingly well in terms of GDP growth per working-age adult. Indeed, from 1998 to 2019, Japan has grown *slightly faster* than the U.S. in terms of per working-age adult: an accumulated 31.9% vs. 29.5%. Furthermore, many advanced economies appear to be on parallel balanced growth trajectories in terms of working-age adults despite important differences in levels. Motivated by this observation, we calibrate a standard neoclassical growth model in which the growth of the working-age adult population varies in line with the data for each economy. Despite the underlying demographic differences, the calibrated model tracks output per working-age adult in most economies of our sample. Our results imply that the growth behavior of mature, aging economies is not puzzling from a theoretical perspective.

Keywords: Demographics, Growth, Developed Economies

JEL codes: E2, J1

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1 Introduction

As the populations of advanced economies age, output growth per capita is becoming an increasingly misleading indicator for growth theory. Changes in the working-age population have become so large that output growth per capita can hide important movements in output per working-age adult, a more natural object to focus on from the perspective of theory.[1]

The paradigmatic case for how misleading the notion of output growth per capita has become is Japan. Between 1990 and 2019, GDP in Japan grew at an annual rate of 0.93%, much lower than the 2.49% of the U.S. This seemingly disappointing performance motivated a myriad of books and academic papers analyzing the origins of Japan's lackluster growth and presenting a multitude of policy remedies. Just as one example among many, Pesek (2014), who popularized the term "Japanization," writes:

> ...few lessons are more timely or critical than those offered by Japan, a once-vibrant model for developing economies that joined the world's richest nations, lost its way, and has been struggling to relocate it ever since.
>
> In this book I explore what the world can learn from a Japanese economic funk that began more than 20 years ago and has never really ended. That means exploring where Japan went wrong, how it sank under the weight of hubris and political atrophy, and missed opportunity after opportunity to scrap an insular model based on overinvestment, export-led growth, and excessive debt.

However, the outlook is dramatically different if we look at GDP per working-age adult. Japan has grown at an annual rate of 1.44%, while the U.S. has grown at a very similar 1.56%. Indeed, from 1998 to 2019, Japan has grown *slightly faster* than the U.S. in terms of per working-age adult: an accumulated 31.9% vs. 29.5%. Even more strikingly, if we focus on the period 2008-2019 (i.e., after the outbreak of the financial crisis), Japan has the highest growth rate in terms of per working-age adult among our sample of G7 countries plus Spain. We argue below that there

[1] As is common in the literature and statistical surveys, we define the working-age population as adults between 15 and 64 years old. Child labor is minimal in the advanced economies we consider (the G7 plus Spain). On the other hand, participation rates for adults 65 and older are low but, in some cases, significant. In 2019 (the last year of our dataset), participation rates of adults 65 and older ranged from 2.5% in Spain to 25.3% in Japan. We will revisit later the possible need to redefine the working-age population.

is nothing particularly mysterious about Japan's total GDP growth: it is a mere consequence of an annual *fall* in the working-age population of about 0.5%.[2]

To document the previous observation more systematically, we report basic facts on GDP growth and population for a set of mature economies over the last several decades. In particular, we concentrate on the G-7 countries plus Spain. In doing so, we provide a big picture of the growth process accompanying changing demographics.

We find that while the adult population of Canada and the United States grew by about 30% between 1990 and 2019, the working-age adult population of Italy and Germany marginally declined by about 2%. Meanwhile, the working-age adult population of Japan fell by about 14%. Clearly, these differences among countries are large.

Yet, output per working-age adult behaved quite similarly over time for all economies except Italy. Between 1990 and 2019, the annual growth rate in output per working-age adult averaged between 1.3% (France) and 1.56% (United States). For Italy, it averaged 0.8%. From these observations, all economies except Italy appear to be on parallel trends, resembling the balanced growth paths of textbook growth models. Furthermore, these trajectories are independent of diverging trajectories in the size of the working-age adult population.

We then develop and calibrate a standard one-sector growth model with exogenous technological growth. The size of the working-age population varies in line with the data for each economy. In this context, we ask the extent to which the observed growth patterns of developed economies are consistent with the predictions of basic theory.

The logic of the model is straightforward. The economy travels along a growth path mainly determined by the exogenous growth of technology, the discount factor, and total population growth, as in the textbook neoclassical growth model with exogenous population growth and technological progress. However, since the production function depends on labor, not total population, variations in the ratio of labor over total population shift the slope of the growth path up or down and induce transitional dynamics. In terms of the Euler equation characterizing the equilibrium

[2]The participation rate of adults in Japan 65 and over has not changed much over time. It was 24.3% in 1990 and 25.3% in 2019, with a low point of 19.8% in 2006. In comparison, the participation rate in the U.S. of adults 65 and over has grown from 11.8% in 1990 to 20.2% in 2019. In other words, if we were to redefine working-age adults to include those 65 and over willing and able to work (or some other suitable redefinition), Japan's difference in output growth per working-age adult between 1990 and 2019 with respect to the U.S. would be even smaller. Conversely, the median worker in the working-age population is older than before, which leads to a drop in participation rates because of a composition effect (i.e., 60-year-old workers participate less in the labor force than 40-year-old workers). Eppsteiner et al. (2017) estimate that population aging accounts for 79% of the decline in the labor force participation rate in the U.S. from 2007 to 2017.

behavior of the economy, having a lower labor/population ratio is equivalent to having a negative technological shock in a standard real business cycle model (and with the same persistent and propagation effects).

This intuition illustrates that the key insight of our paper is that aging changes the ratio of working-age adults over the total population. In the very long run, as the consequences of lower fertility rates and longer life expectancy are worked out through the population pyramid, we might return to a situation where the labor/population ratio stabilizes. At that moment, output growth per capita and output per working-age adult will again become roughly the same.

We find that our model does a very good job of tracking the observed paths of output per working adult in all cases, again, except for Italy. Our interpretation of this result is *not* that our model fits the data perfectly. For example, one could consider a model with overlapping generations that captures the granularity of demographic change. Our interpretation of our quantitative results is, instead, that they strongly support the argument that output growth per working-age adult is a *key* measure from the perspective of growth theory.

We subsequently discuss the predictions in different ways. Our conclusion from the analysis is straightforward: the observed behavior of mature, aging economies is not puzzling. Rather, the observed growth paths agree with the simple prediction of standard theory under a slow-moving change in the size of its labor force.

In one of these exercises, we extend our sample and discuss the cases of two large growing economies in the midst of a structural transformation: China and India. In the case of China, the combination of very fast economic growth and a moderate growth rate of population and working-age population between 1980 and 2019 means that looking at total GDP growth, GDP growth per capita, or GDP growth per working-age adult gives us roughly the same conclusions. Therefore, China is a modern incarnation of the type of behavior common in more advanced economies in the 1960s or 1970s, when modern growth theory was developed. Nonetheless, as China ages over the next two decades, a gap between per capita and working-age adult GDP growth is bound to appear. In contrast, India is the mirror image of Japan: very fast growth of the working-age adult population means that high total GDP growth rates look less impressive in terms of working-age adults.[3]

[3] In a classic paper, Bloom and Williamson (1998) argued that much of the success of East Asian countries between 1965 and 1990 came from a fast demographic transition that allowed for a quick growth of the ratio of working-age population over total population. Thus, a possible reason for India's slower growth than its East Asian

Policy implications. We are cautious about how our results should inform economic policy. The first fundamental caveat is that the working-age adult population of a country is not an exogenous process. It could be affected by migration and fertility policies (direct, e.g., a child tax credit, and indirect, e.g., low fertility rates caused by high youth unemployment). In fact, immigration into several G7 countries and Spain is a first-order mechanism in the data to account for the relatively fast population growth in countries like Canada and the U.S. Our analysis is silent about the economic impact of immigration except for one point. At first sight, there seems to be little correlation between immigration and output growth per working-age adult, with Japan, a low-immigration country, outperforming Canada, a high-immigration country. While this observation does not rule out a positive effect of immigration on output growth per working-age adult, it shows that such a positive effect can be harder to document than is sometimes hypothesized.

In contrast, we are less concerned with endogenous fertility. Changes in the native-born working-age population, for instance in Japan, in the late 2010s, were determined by fertility choices in previous decades, with the effect of low birth rates in the 1990s having a minimal impact (a child born in 1992, when Japanese asset prices started a long decline, only entered the working-age population in 2007). Japan's fertility rate fell below the replacement rate (2.1 children per woman) in 1974 (2.05), when the economy was still booming.

The second caveat is that our argument for focusing on output growth per working-age adult as a *key* object of interest for growth theory does not imply that other objects, such as total output growth or output growth per capita, are not relevant.[4] For example, total output growth matters for public debt and social security sustainability (Faruqee and Mühleisen, 2003; Kitao, 2015). Broadly, the output growth per capita rate gives us a sense of how fast the average resources available to each inhabitant of an economy are changing. Similarly, Klenow et al. (2017) argue for the importance of considering total population to evaluate social welfare growth. We are deliberately silent about social welfare.

Our argument, more pointedly, is that if a researcher wants to evaluate the impact of fiscal and

countries is its slower demographic transition.

[4]We are less convinced of the usefulness of other measures, such as output growth per employed worker or hours worked. Those measures get contaminated by many confounding factors. For example, it is often argued that Western Europe's performance in terms of output per worker is much better than its performance in terms of output per capita. But if restrictive labor regulations expel from the labor force the less productive workers (as one would expect), average labor productivity growth would increase through a composition effect. This is a case where high labor productivity growth is a symptom of malaise, not a bright spot.

monetary policies (or other economic policies), one must frame them within the context of the demographic evolution of an economy, especially after the 1990s, when population aging started to be keenly felt in many mature economies. Judging Japanese monetary policy from 1990 to 2019 as a failure because it could not deliver faster output growth per capita faces the fundamental challenge that monetary policy can do next to nothing about long-run demographic forces. Given that Japan's and the U.S.' output growth per working-age adult was roughly the same between 1990 and 2019, it is hard to see what else the Bank of Japan could have done.

Importantly, as Japan's demographic present is the future of many other advanced (and not-so-advanced) economies (Delventhal et al., 2021), economists must learn to judge growth experiences using the right units of measurement. We argue for the importance of output per working-age adult (perhaps with a redefinition of working age to adapt it to changing retirement patterns) as a measure of growth performance.

Related literature. In terms of the related literature, we highlight that we are not the first to report data or calibrate models in terms of per working-age adult. While this practice is perhaps less common than using per capita terms, many papers have followed it (see Klein and Ventura, 2021, or several of the chapters in Kehoe and Nicolini, 2022).

More in general, a very large literature explores the links between economic growth and population aging, going back to the pioneering work of Auerbach and Kotlikoff (1990), Cutler et al. (1990), and Weil (1997). Instead of reviewing each aspect of this literature in detail, it is more fruitful to highlight a few recent contributions that are particularly related to our investigation. Kotschy and Bloom (2023) have already pointed out the link between population aging and economic growth. The main difference between that paper and ours is our emphasis on determining the right measurements of GDP growth performance from the perspective of growth theory and our use of a standard neoclassical growth model to analyze the data instead of empirical regressions. Jones (2022) has used endogenous growth models in which people discover new ideas to study the possible stagnation of living standards as the population shrinks. In comparison, we take technological progress as given.[5] While we are sympathetic to the endogeneity of growth rates, our point is different: researchers need to be careful about the object of measurement in the context of growth theory when working-age populations are falling or stagnating, regardless

[5]See also Sasaki and Hoshida (2017). Sasaki (2019) explores ideas similar to those of Sasaki and Hoshida (2017) and Jones (2022) in the context of a neoclassical growth model with a falling population.

of our views on the production function for ideas.

Jaimovich and Siu (2009) show that demographic change accounts for approximately one-fifth to one-third of the decline in output volatility in the U.S. Ferraro and Fiori (2020) make a similar point about how the aging of the baby boomers considerably reduces the effects of tax cuts on aggregate unemployment. Cravino et al. (2022) quantify how population aging increases the share of services in total consumption. Acemoglu and Restrepo (2021) link automation and population aging. Maestas et al. (2023) present evidence that population aging reduced the U.S. GDP growth per capita rate by 0.3 percentage point per year from 1980 to 2010. Hopenhayn et al. (2022) use population aging to account for the recent evolution of firm concentration, entrepreneurship, and the labor share. Similarly, Karahan et al. (2019) argue that a slowdown in labor supply growth due to demographics caused two-thirds of the decline in the start-up rate in the U.S. since the late 1970s. Finally, Aksoy et al. (2019) estimate a panel VAR to understand how demographic structures affect macroeconomic trends, and they predict, with the help of an endogenous growth model, that the current trends are projected to reduce output growth, investment, and interest rates across OECD countries.

The paper is organized as follows. In Section 2, we document the facts of interest for the G-7 economies plus Spain. In Section 3, we present the standard neoclassical model we use in our quantitative analysis. In Section 4, we present the calibration and solution of the model. Our quantitative results are in Section 5, while Section 6 discusses them. We conclude in Section 7. An appendix reports some further quantitative exercises.

2 Data

This section summarizes the data used in our study. Our main data source is the World Bank's World Development Indicators (WDI) database, which compiles comparable statistics for a large number of countries and territories.[6] Real GDP is the GDP in national constant prices. The working-age population is the population between 15 and 64 years old. For our set of mature economies, we consider the G7 countries (United States, Canada, United Kingdom, Germany, France, Italy and Japan). We also include Spain, the largest Western European economy that is not a member of the G7 and one that has undergone a swift demographic transformation.

[6] For the strength of the WDI vs. alternative databases such as the Penn World Tables (PWT), see Pinkovskiy and Sala-i-Martin (2016).

2.1 Growth Facts I: 1981-2019

We compute basic growth facts for the countries in our sample for 1981-2019. The WDI does not include all the relevant data before 1981 (GDP data in the WDI starts from 1981 for Canada, 1970 for Germany, and 1960 for the other six countries). In any case, the main point of this paper is that relating population aging and GDP growth only became relevant in the 1980s.

Table 1: G7 plus Spain: Basic Growth and Population Facts, 1981-2019

Statistic	Canada	France	Germany	Italy	Japan	Spain	UK	U.S.
GDP	2.37	1.84	1.75	1.17	1.78	2.35	2.30	2.71
GDP per Capita	1.26	1.31	1.60	1.03	1.58	1.76	1.84	1.74
Population	1.10	0.52	0.15	0.15	0.19	0.59	0.45	0.95
GDP per Working-age Adult	1.33	1.42	1.69	1.07	1.96	1.65	1.88	1.78
Working-age Population	1.03	0.41	0.07	0.10	-0.18	0.70	0.42	0.91
Working-age Pop. Ratio	0.68	0.65	0.67	0.67	0.66	0.67	0.65	0.66

Table 1 reports output and population facts for 1981-2019. In this table and the following tables in the paper, we highlight a few numbers in red because they are salient to our argument. All the variables are annual growth rates and expressed in percentage points, except for the working-age population ratio, defined as the ratio of the 15-64 population to the total population averaged across years.

In the first row of Table 1, we see large differences in yearly GDP growth. While Italy (the worst performer in terms of GDP growth) has only grown 1.17% a year for four decades, the U.S. (the best performer) has grown 2.71%. In accumulated terms, the Italian economy has grown 125% since 1981, while the U.S. economy has grown 261%. This is a huge difference: the ratio between the U.S. and Italian economies has grown by 61%.

The second row of Table 1 starts showing our main argument. If we look at per capita terms, the differences in GDP growth become much smaller. Now Italy, still the worst performer, has grown at an annual rate of 1.03%, while the U.S. has only grown at a rate of 1.74%. A difference in total GDP growth of 1.54% implies a difference of only 0.71% in per capita terms. The third row, population growth, accounts for these differences: while Italy's population has grown at 0.15% a year, the U.S.' population has been growing at 0.95%.

But even more interesting is the fourth row of Table 1, where we report GDP growth per working-age adult. The relative performance of Italy vs. the U.S. does not change much, but this is not the case for other countries. For example, let us focus on the case of Japan vs. the U.S. (our numbers in red). Japan's GDP growth is nearly 1% a year lower than the U.S. (1.78% vs. 2.71%). However, in terms of per working-age adult, Japan outperforms the U.S. (1.96% vs. 1.78%). In fact, Japan becomes the top performer in terms of GDP growth per working-age adult! The mechanism behind this difference is the fifth row of Table 1: while the working-age population has fallen in Japan (-0.18% vs. a total annual population growth of 0.19%) due to population aging, it has grown in the U.S. at 0.91% (roughly the same speed as the total population).

The last row of Table 1 documents the working-age population ratio, which is approximately equal across all countries on average over the sample.

To understand the forces behind the numbers in Table 1 better, Figure 2.1 plots the underlying time series. The reader should look, in particular, at the bottom right panel, with the evolution of the population between ages 15 and 64 in each of the eight countries. Compare the evolution of the working-age population in the U.S. (dashed blue line) at the top of the panel with the evolution in Japan (dashed green line at the bottom).

2.2 Growth Facts II: 1990-2019

As mentioned above, the effects of population aging started to be felt in the advanced economies in the 1980s but became much more acute in the 1990s. For instance, the working-age population peaked in Japan in 1994 and started falling afterward. Consequently, if we drop the 1980s and focus on the more recent period 1990-2019, our results become more striking.

Table 2: G7 plus Spain: Basic Growth and Population Facts, 1990-2019

Statistics	Canada	France	Germany	Italy	Japan	Spain	UK	U.S.
GDP	2.31	1.59	1.51	0.73	0.93	2.06	1.97	2.49
GDP per Capita	1.24	1.07	1.35	0.56	0.84	1.39	1.43	1.52
Population	1.06	0.52	0.16	0.18	0.09	0.67	0.54	0.95
GDP per Working-age adult	1.32	1.30	1.58	0.80	1.44	1.41	1.52	1.56
Working-age Population	0.98	0.29	-0.07	-0.06	-0.51	0.64	0.45	0.91
Working-age Pop. Ratio	0.68	0.65	0.67	0.66	0.65	0.68	0.65	0.66

Table 2 replicates the format of Table 1 for the shorter sample (and Figure 2.2 replicates Figure 2.1). Let us zoom in again on the comparison between Japan and the U.S. Japan's GDP growth has been a rather lackluster 0.93% (the second worst performance ahead only of Italy).

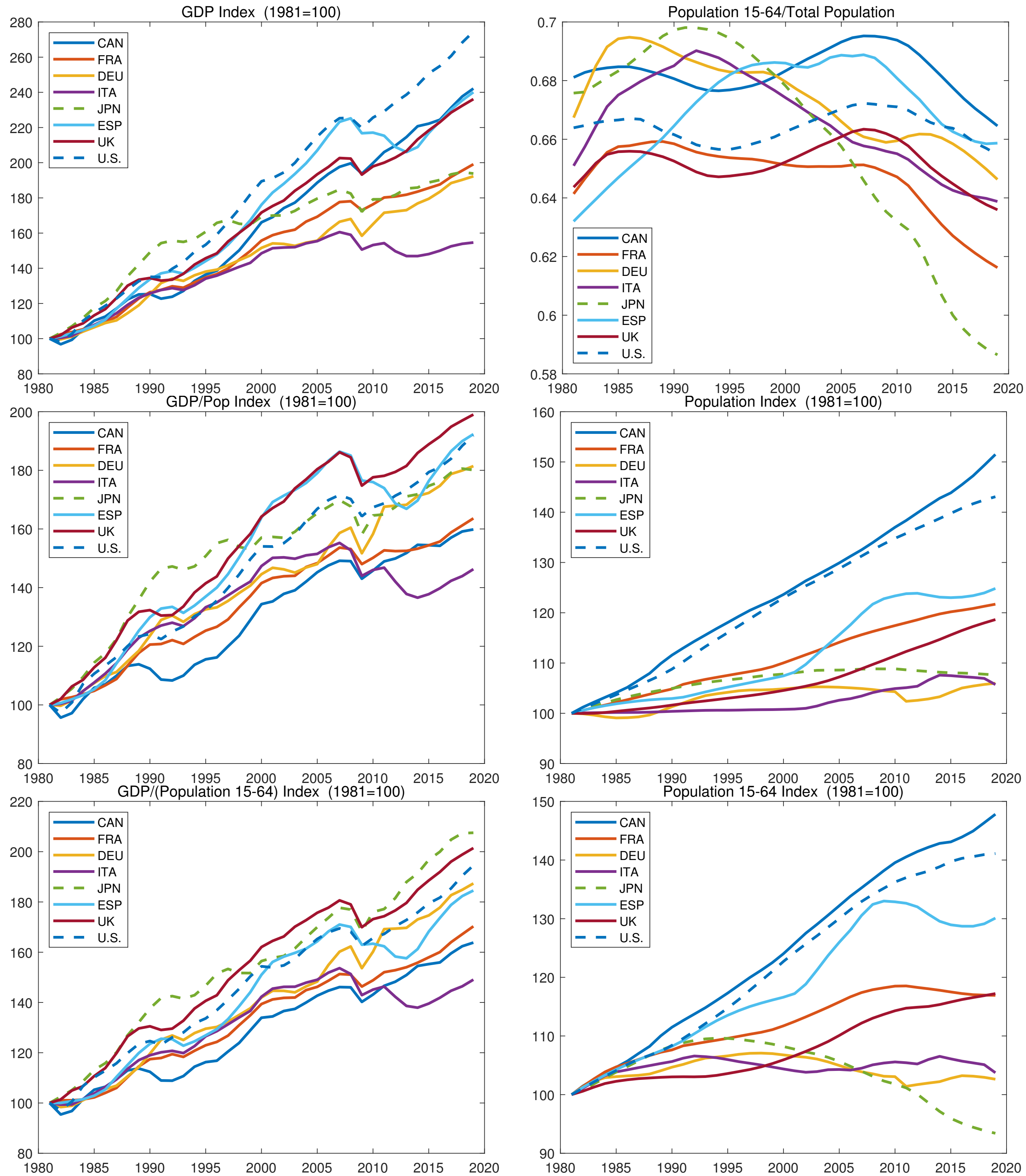

Figure 2.1: G7 and Spain: 1981 - 2019

As discussed in the introduction, one could fill a small library with all the books and papers diagnosing the forces behind this performance, from the aftermath of the asset bubble in the late 1980s to too timid monetary and fiscal policy.

Growth of GDP per working-age adult. However, when we look at GDP per working-age adult, Japan has grown at a rate of 1.44%, only slightly worse than the U.S. (1.56%) and ahead of Canada, France, Germany, Italy, and Spain. Indeed, if one further drops the early 1990s from the sample (the years of the asset price collapse), Japan was growing faster than the U.S. in terms of per-working-age adults from 1998 to 2019. In that period, Japan has grown *slightly faster* than the U.S. in terms of per terms of working-age adults: an accumulated 31.9% vs. 29.5%. The working-age population declined in Japan by about half a percentage point per year, while, in the U.S., it grew at 0.91%, a total difference of nearly 1.5% a year (the right bottom panel of Figure 2.2, shows the evolution of the population between ages 15 and 64 in each of the eight countries). Suddenly, there is nothing much to explain about Japan's economic performance: there are fewer Japanese of working age, and a smaller labor input leads to lower total GDP growth.

Or, to put it another way, for Japan to match the GDP growth of the U.S., its GDP growth per working-age adult would need to have grown at nearly 3% a year, an outstanding feat once Japan had completed its neoclassical growth transition by the late 1980s.

In fact, returning to Figure 2.2, the bottom left panel shows that seven out of our eight economies (the exception being Italy) move in roughly the same way from 1990 to 2019. This is just the explicit time series behind the fourth row of Table 2, where GDP growth per working-age adult except in Italy ranges from 1.30 (France) to 1.58 (Germany), a relatively narrow range: we have much less dispersion in growth rates than in the sample 1981-2019.

Levels vs. growth rates. The low dispersion in growth rates mentioned above is most interesting because the *level* of the growth path of these economies is quite different (recall that in Figure 2.2 we have normalized output per working-age adult to 100 in 1990). For example, Spain's output per capita in PPP terms is around one-third lower than that of the U.S. And yet, there is no evidence of convergence toward the U.S., the highest-income country in the sample, and some clear evidence of divergence in the case of Italy.

The case of Italy is most intriguing. Until the mid-2000s, Italy tracks the other countries in the sample (although closer to the bottom of the pack). However, after the financial crisis, Italy falls behind and cannot recover the level of output from before the crisis. Interestingly, Italy did not have a financial collapse (like Ireland) nor was it under a memorandum of understanding with the European Union (like Spain in July 2012). Italy's problems seem deeper than digesting the

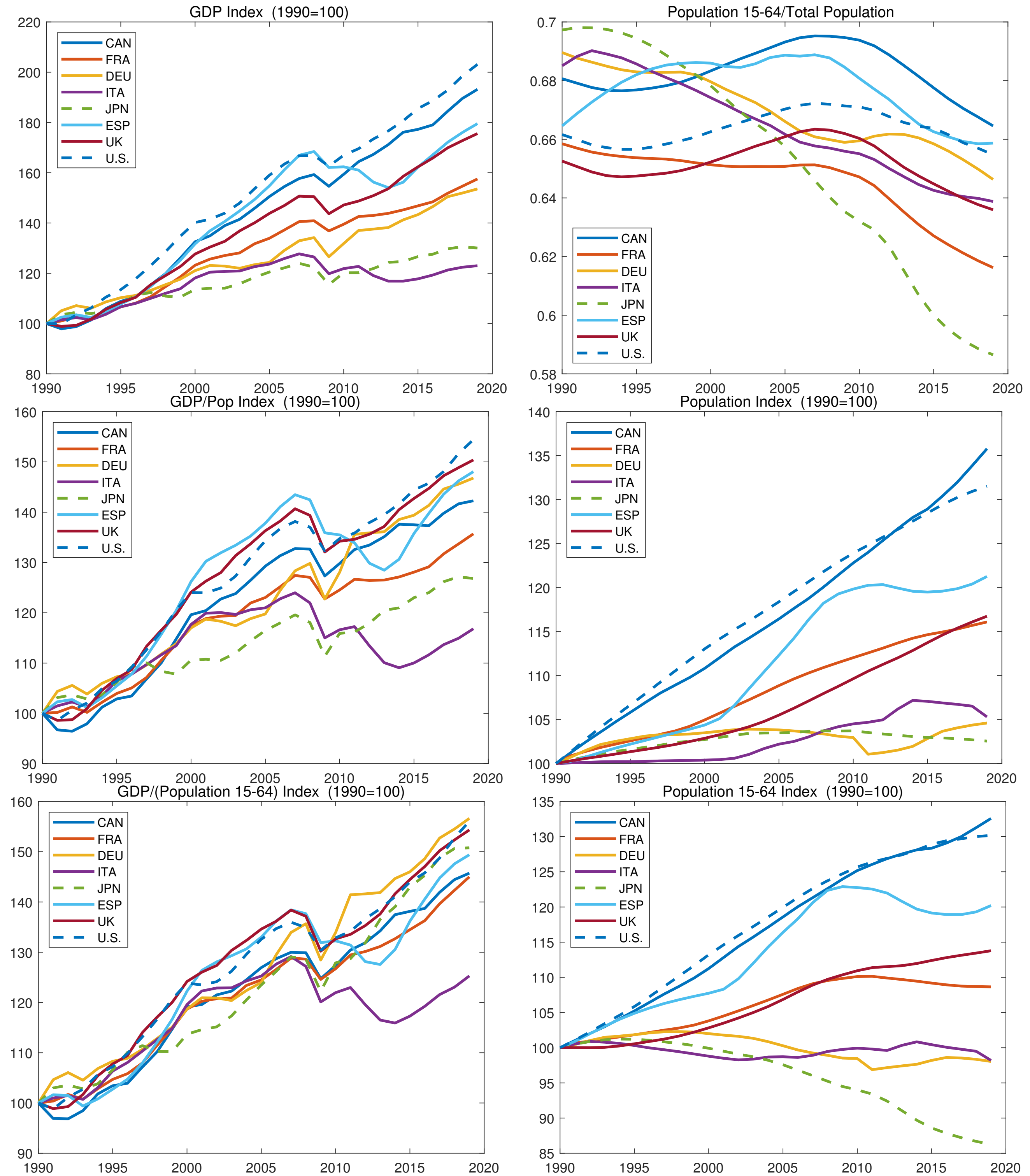

Figure 2.2: G7 and Spain: 1990 - 2019

aftermath of a financial meltdown.

The data we have presented suggest that a common model can account for the experience of all these countries, delivering roughly equivalent growth experiences in terms of per working-age adult but different output growth path levels. This motivates us to postulate, in the next section, a simple neoclassical growth model with country-specific technological processes.

3 A Standard Neoclassical Growth Model

We formulate a standard neoclassical growth model with exogenous technological change and demographics. Each country is modeled as a different economy, without any other interaction except a possible common technology trend. This simple growth model will help us determine how much the growth experience of the eight countries in our sample fits standard theory. To ease notation, we will work directly with the social planner's problem formulation of the model. Since both welfare theorems hold in our model, the solution to the social planner's problem is also the market allocation.

3.1 Preferences and Technology

The economy is populated by an infinitely lived representative household of varying size N_t. Later, when we take the model to the data, we will equate N_t with the total population.

The preferences of the representative household over per capita consumption are represented by:

$$\max_{C_t/N_t} \sum_{t=0}^{\infty} \beta^t N_t \log\left(\frac{C_t}{N_t}\right),$$

where β is the discount factor and C_t is aggregate consumption.

Output is given by $Y_t = K_t^{\theta}(A_t L_t)^{1-\theta}$ where K_t is capital, and L_t is the working-age population (15-64 years old). A_t is the level of labor-augmenting technology which grows at a constant rate g, that is, $A_t = A_0(1+g)^t$. Thus, in this economy, total factor productivity (TFP) equals $A_t^{1-\theta}$.

Output is used for consumption or investment I_t. Given a depreciation rate δ, the law of motion for capital is $K_{t+1} = I_t + (1-\delta)K_t$. The resource constraint is given by $C_t + I_t = Y_t$. Finally, N_t grows at an exogenously given time-varying rate n_t, so that $N_t = \prod_{i=1}^{t}(1+n_i)$, given $N_0 = 1$.

We do not have an endogenous labor choice. We abstract from it because we want to focus on the growth properties of our economy, not its business cycle features (the frequency at which most labor fluctuations occur). The textbook formulation of the neoclassical growth model selects utility functions that ensure the working-age population's labor supply is constant along the balanced growth path.[7] Hence, in the interest of simplicity, we can directly drop the labor choice.[8]

Given the growth of technology g, we must normalize the variables. We use the country's technology and population level to make the problem stationary. Specifically, let:

$$c_t = \frac{C_t}{A_t N_t},$$
$$k_t = \frac{K_t}{A_t N_t},$$
$$i_t = \frac{I_t}{A_t N_t},$$
$$y_t = \frac{Y_t}{A_t N_t} = \left(\frac{K_t}{A_t N_t}\right)^{\theta} \left(\frac{A_t L_t}{A_t N_t}\right)^{1-\theta} = k_t^{\theta} l_t^{1-\theta},$$

where l_t denotes the exogenously given working-age population rate L_t/N_t.

With these transformations, we can rewrite the social planner's problem as follows:

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t N_t \log c_t$$
$$\text{s.t. } y_t = k_t^{\theta} l_t^{1-\theta},$$
$$c_t + i_t = y_t,$$
$$i_t = (1+g)(1+n_{t+1})k_{t+1} - (1-\delta)k_t.$$

A standard Euler equation characterizes the solution to this optimization problem:

$$c_t^{-1}(1+g) = \beta c_{t+1}^{-1}\left(\theta(k_{t+1})^{\theta-1}(l_{t+1})^{1-\theta} + 1 - \delta\right).$$

This Euler equation looks like the optimality condition of the textbook neoclassical growth model

[7] One could argue that, as population ages, the balanced growth assumption might not be empirically relevant and that we might observe long-run changes in the labor supply along both the intensive and extensive margins. Kotschy and Bloom (2023) document the health level increases among the elderly, a likely predictor of higher labor supply for those older than 65. And several countries, pushed by fiscal constraints, have increased the retirement age. While this can be an important venue for further research, our aim in this paper is to show that the textbook formulation of the neoclassical growth model rationalizes much of the observed patterns of growth in advanced economies, validating our point about the importance of using working-age adult terms. Nonetheless, statistical surveys might need to reconsider the definition of working-age population to adapt it to later retirement ages.

[8] Similarly, we drop the international aspect of the model. In an international model where the different economies are in their balanced growth path and a common discount factor (as we will use in our calibration), this international aspect would not change any conclusion of importance to us.

with population and trend technological growth except for the presence of a time-varying term l_{t+1}.[9] Imagine, for a second, that $l_{t+1} = \widehat{l}$ is constant, i.e., the working-age population is a constant fraction of the total population. This is equivalent to a constant in front of the (normalized) production function and, hence, irrelevant to the dynamics of the model.

Conversely, consider the case where, as in our calibration below, l_{t+1} changes. This is equivalent to a shift in the level of the (normalized) production function as if we had a technological shock: a raise in l_{t+1} increases total production and hence investment and output; a drop in l_{t+1} lowers total production and thus investment and output. In other words, changes in l_{t+1} have the same effect as technological shocks in a real business cycle model without labor choice (and with the same persistence and propagation).

4 Calibration and Solution

Our model is indexed by the parameters β, θ, and δ plus the exogenous values for g and N. The parameter A_0 is a scaling parameter that we will pick to match the initial value of GDP per working-age adult in 1981. Furthermore, we assume that each economy is at its balanced growth path in terms of per working-age adult at the start of the simulation.

We calibrate β, θ, and δ on an annual basis to match the data for 1981-2019, following commonly used targets. See Table 3 for a summary of the calibration.

Table 3: Calibration

Parameter		Value
Discount factor	β	0.946
Capital share	θ	0.39
Depreciation rate	δ	0.04
Labor augmenting technology growth rate, Canada	g	0.0133
Labor augmenting technology growth rate, France	g	0.0103
Labor augmenting technology growth rate, Germany	g	0.0169
Labor augmenting technology growth rate, Italy	g	0.0107
Labor augmenting technology growth rate, Japan	g	0.0196
Labor augmenting technology growth rate, Spain	g	0.0165
Labor augmenting technology growth rate, U.K.	g	0.0188
Labor augmenting technology growth rate, U.S.	g	0.0178

[9] As such, shocks to N_t and A_t have the usual effects on output and investment.

We select a discount factor β of 0.946 to replicate a 7.6% annual rate of return to capital reported by the PWT 10.0 for the U.S. between 1981 and 2019 (given our model, we want to match the return on all capital goods, not on bonds or other financial assets). We pick the capital share $\theta = 0.39$ to match the average shares between 1981 and 2019 from PWT 10.0. The depreciation rate is the average depreciation rate from PWT 10.0: $\delta = 0.04$ for the U.S. These values imply a capital/output ratio of about 3.36.

These three parameter values are common for all countries, even if our target values come from the U.S. Our presumption is that preferences, capital shares, and depreciation are roughly the same across all G7 countries plus Spain. More importantly, by constraining our degrees of freedom, imposing common parameter values limits the model's flexibility to match each country's observations by varying the parameter values. Nonetheless, we will revisit this calibration in Section 6, where we will show some results when we allow for country-specific parameter values.

We show next how to calibrate the model for the U.S. case. Analogous steps are used for all other countries, and we skip their explanation in the interest of space. First, we select A_0 to match the level of U.S. GDP per capita in 1981. This is just a normalization. The population growth rates match the observed data year by year in the U.S. Finally, we calibrate $g = 0.0178$ to match GDP growth per working-age population from 1981 to 2019 in the U.S.

To find the planner's solution, we take the initial steady state in 1981 and its final steady state in 2019 and compute the transition path between the two using the Euler equation and the investment and resource constraint equations using a nonlinear equation solver. Recall that we assume that the U.S. was on its balanced growth path in 1981 or, in the transformed stationary problem, that the U.S. was at its steady state with capital k_0. More concretely, k_0 satisfies the steady-state Euler equation:

$$1 + g = \beta \left(\theta \left(k_0\right)^{\theta-1} \left(l_0\right)^{1-\theta} + 1 - \delta \right),$$

where l_0 is the working-age population ratio in the U.S. in 1981. To compute $i_0 = (1+g)(1+n_0)k_0 - (1-\delta)k_0$, we take population growth, n_0, to be the population growth observed in 1981.

The final steady-state capital of the U.S. k_T is given by:

$$(1 + g) = \beta \left(\theta \left(k_T\right)^{\theta-1} \left(l_T\right)^{1-\theta} + 1 - \delta \right),$$

where l_T is the U.S. working-age population ratio in 2019. To compute investment at T, n_T, is the population growth observed in 2019.

5 Quantitative Results

In this section, we present the quantitative results of our model. In the interest of space, we will only discuss the most relevant findings.

5.1 1981-2019

Figure 5.1 plots the evolution of income per worker (i.e., the model equivalent of a working-age adult in the data) in each of the eight countries in the sample 1981-2019. In each panel, the dashed blue line represents the model, and the solid red line represents the data for one country (normalized to be 1 in each case in 1981 to ease comparison). While the dashed blue line might appear to be a straight line at first sight, it presents, in fact, small fluctuations due to varying growth rates of the working-age adult population. However, since those variations within one country from year to year are relatively small, they do not change much the slope of the dashed blue line.

The left panel in the top row is the U.S. The model captures well the main evolution of the U.S. economy during the sample, with maximum differences between the model and the data of less than one log point. The most salient divergence is the drop in income per worker after the financial crisis of 2007-2009, with what seems like a permanent change in the trend level of around 0.75 log points. A similar picture of a permanent drop in the output level of around one log point holds for Spain (the left panel in the bottom row) and the U.K. (the right panel in the bottom row). The U.S., Spain, and the U.K. were three economies where many researchers identified a large real estate boom in the early 2000s, which seems to have left long-lasting scars. To explore further the idea that there has been a trend change in income growth per worker, in Subsection 6.2, we explore an alternative specification of the model for the U.S. with two different g's, one before and one after 2007.

The right panel in the top row is Canada. Here, as well as in the case of France (the left panel in the second row) and Germany (the right panel in the second row), the model does surprisingly well, with remarkably minor deviations between the data and the model.

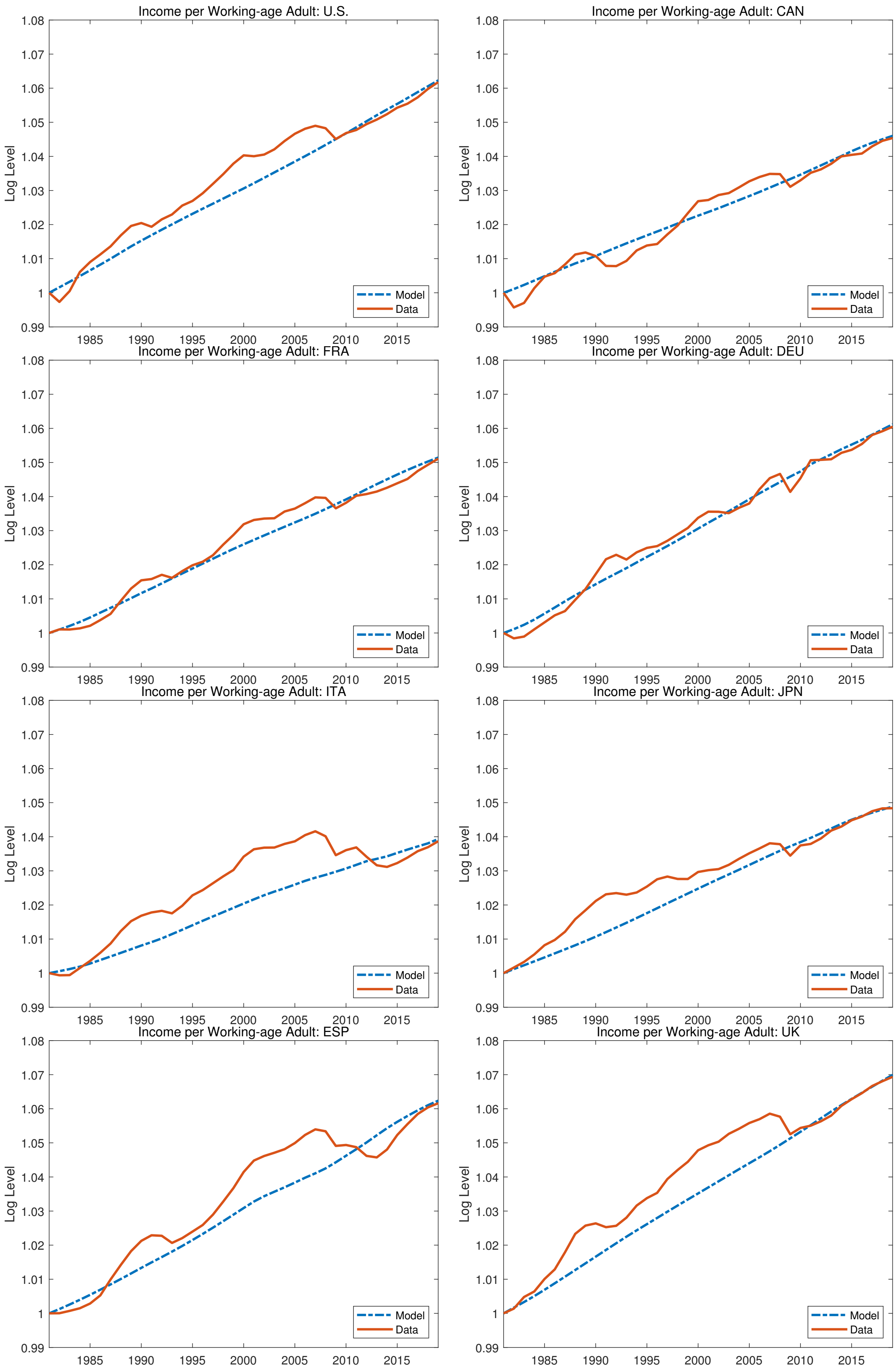

Figure 5.1: Transitional Dynamics: 1981-2019

The left panel in the third row is Italy. This is the country for which our calibration of the model is clearly missing the dynamics of the data. The Italian economy stopped growing in the early 2000s (see Fernández-Villaverde et al. 2023a, for more details on Italy's abysmal performance). The only way the model can capture this observation is by being calibrated to a low level of growth over the whole period 1981-2019, which leads to a large and persistent undershooting of the model with respect to the model between 1981 and 2000. As we mentioned above, for the case of the U.S., a simple solution for this problem of the model would be to introduce two different trends in g for Italy. We will evaluate this modification of the model in Subsection 6.2.

A similar, but less acute, picture holds for Japan, the right panel in the third row. Japan grew at extremely fast rates during the 1980s driven by neoclassical growth convergence (see Fernández-Villaverde et al., 2023b, for an exercise related to the one in this paper that includes Japanese data since 1950). But, next, it suffered a deep crisis after the drop in asset prices after 1992. The model captures the average growth of the Japanese economy during the four decades only by deviating from the data in the 1980s and slowly returning to it after 1992.

5.2 1990-2019

Figure 5.2 repeats the exercise in Figure 5.1 except now with the shorter sample 1990-2019. By eliminating the 1980s, all the discussions in the previous subsection become even sharper. The model accounts very well for the experiences of Canada, France, and Germany, quite well for the experiences of the U.S., Japan, Spain, and the U.K. (except for not capturing the boom of the mid-2000s), and misses aspects of Italy unless we introduce a change in the trend of technology.

As in the data, there is no evidence of convergence of the different countries toward the U.S., the highest-income country in the sample, with all countries traveling along their paths (with their own levels).

The lesson we get from our results is *not* that our simple neoclassical growth model accounts for all features of the data (or that the model is superior to other theoretical frameworks, such as an overlapping generations model) but how, once we look at the data in terms of per working-age adult, there is much more agreement between theory and data than when using total or per capita terms.

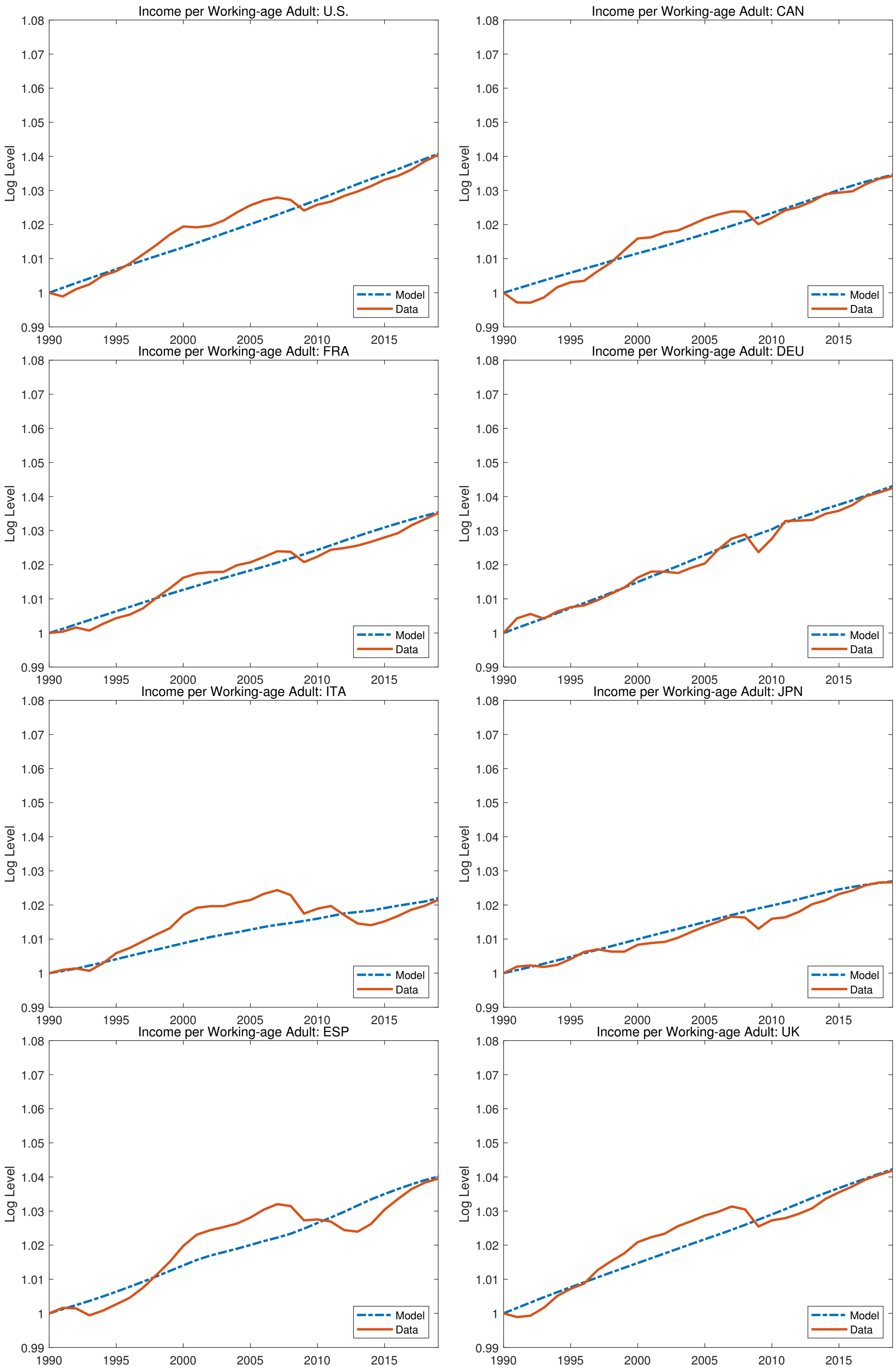

Figure 5.2: Transitional Dynamics: 1990-2019

6 Discussion

In this section, we present several extensions of our study. More concretely, we will analyze variations in different parameter values, having a time-varying trend, and extend our sample to include China and India. Furthermore, Appendix A shows additional exercises where we take TFP growth and we feed it into the model from the PWT.

6.1 Using GDP per Worker Growth Rate from the U.S.

In the baseline calibration, we calibrate a country-specific g. The motivation was that technological progress in each country might be mediated by local institutions and social norms that imply that not all scientific and engineering discoveries and business practice developments are implemented equally across the economies (on the latter point, see Bloom and Van Reenen, 2010).

In our first robustness exercise, we instead impose that each country's GDP growth rate per worker is the same as in the U.S., 0.0178. The time-varying population growth rate and working-age population ratio remain country-specific. This exercise assesses what our model predicts for each country if there are no differences in implementing the new technology.[10] In particular, this exercise controls for the possibility that different aging speeds in each country might lead to different g's (for example, by slower adoption of new technologies by an aged worked forced).

Figure 6.1 shows our results. The left panel in the top row, the U.S., is by construction the same as in Figure 5.1. The model accounts well for the behavior of Germany (right panel in the second row) and Spain (left panel in the bottom row). This is not a surprise since we know from Table 1 that Germany's and Spain's output growth rates per working-age adult in 1981-2019 were only slightly below that of the U.S. Thus, substituting their own g with the g of the U.S. makes little difference.

Conversely, the model still does well, except for a lower level of the balanced growth path, for the case of Japan (right panel in the third row) and the U.K. (right panel in the bottom row). Here, the challenge for the model comes from the observation that Japan's and the U.K.'s own g's are a bit higher than that of the U.S.

[10] The U.S. GDP growth rate per working-age adult in 1980-2019 is the highest in our sample behind Japan's (which, as we argued before, was probably still catching up with its balanced growth path in the 1980s) and the U.K. (which in the 1980s was recovering from its turbulent economic maladies of the 1970s). Thus, as a first-order approximation, one can consider the U.S.' g as a measure of the growth of the world's technological frontier.

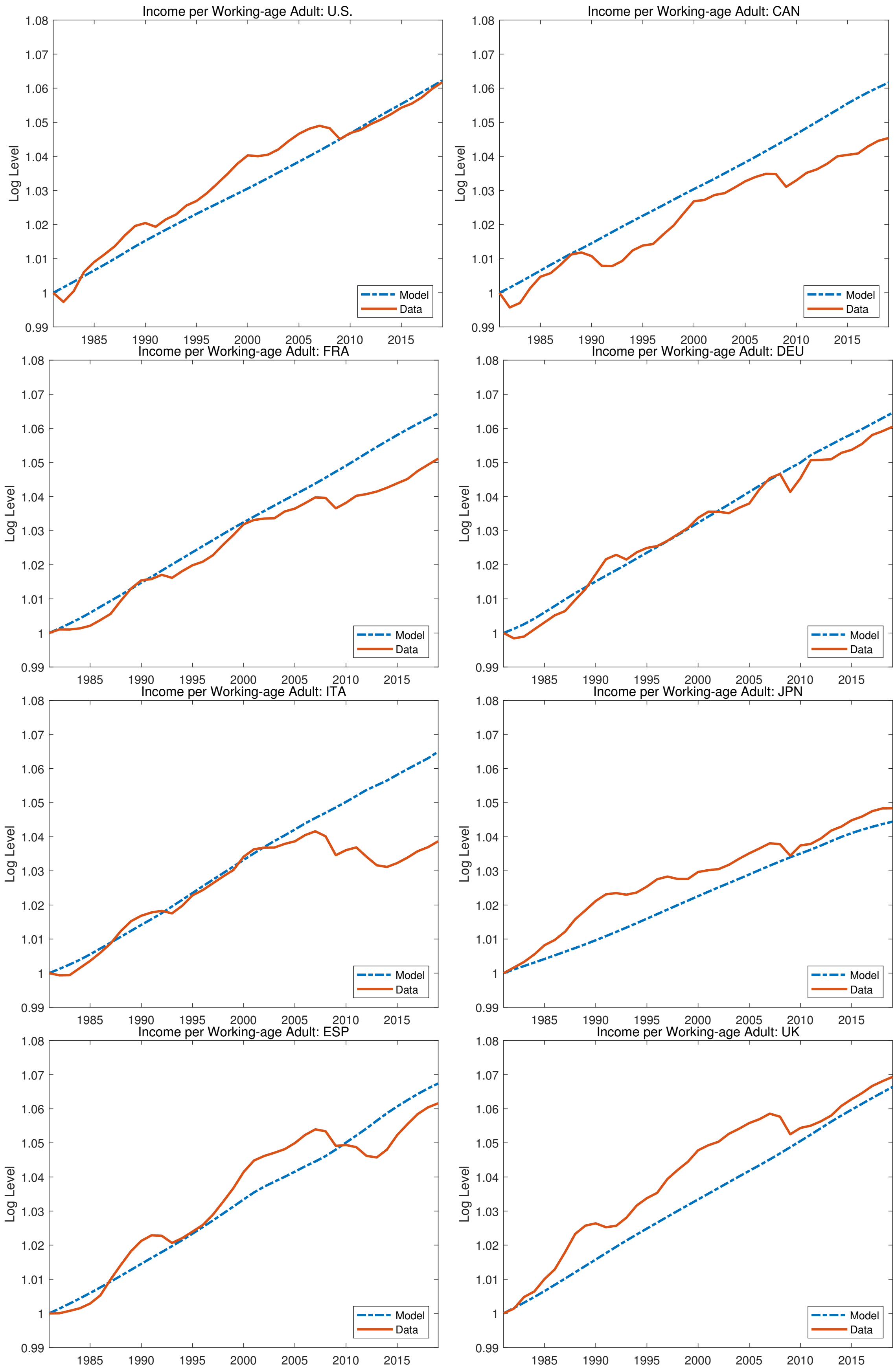

Figure 6.1: Transitional Dynamics: 1981-2019, common g

The interesting observation from this exercise comes from Canada (right panel in the top row) and France (left panel in the second row), which now join Italy in showing a clear underperformance of their economies with respect to the model's prediction. Even after using a sharper measure of labor input (working-age population), Canada, France, and Italy are falling behind with respect to what is achievable.

6.2 Changing Trends

Our findings in Section 5 suggested the importance of considering changes in the growth trend of technology. To explore this possibility in more detail, we split our sample between the periods 1981-2007 and 2008-2019, or before and after the financial crisis.[11]

Table 4: G7 plus Spain: Basic Growth and Population Facts, 1981-2007

1981-2007	Canada	France	Germany	Italy	Japan	Spain	UK	USA
GDP	2.68	2.24	1.99	1.84	2.41	3.15	2.76	3.19
GDP per Capita	1.57	1.67	1.80	1.71	2.08	2.44	2.43	2.11
Population	1.09	0.56	0.19	0.13	0.32	0.70	0.33	1.05
GDP per Working-age Adult	1.49	1.61	1.84	1.67	2.25	2.10	2.31	2.06
Working-age Population	1.17	0.62	0.15	0.17	0.15	1.03	0.44	1.10
Working-age Pop. Ratio	0.68	0.65	0.68	0.67	0.68	0.67	0.65	0.66

Table 5: G7 plus Spain: Basic Growth and Population Facts, 2008-2019

2008-2019	Canada	France	Germany	Italy	Japan	Spain	UK	USA
GDP	1.79	1.03	1.27	-0.23	0.58	0.61	1.43	1.81
GDP per Capita	0.65	0.61	1.16	-0.36	0.68	0.38	0.71	1.11
Population	1.13	0.42	0.11	0.14	-0.10	0.23	0.71	0.70
GDP per Working-age Adult	1.07	1.11	1.35	-0.11	1.49	0.78	1.10	1.34
Working-age Population	0.71	-0.07	-0.08	-0.12	-0.90	-0.16	0.33	0.46
Working-age Pop. Ratio	0.68	0.63	0.66	0.65	0.61	0.67	0.65	0.66

Table 4 replicates Table 1 except for the change in the sample period to 1981-2007. Similarly, Table 5 reports the same statistics for the sample period 2008-2019. As expected, we see large drops in the rates of GDP growth per working-age adult. In the U.S., the rate falls from 2.06% to

[11] Recall that in our original exercise, we look at the data for the periods 1981-2019 and 1990-2019 to gauge more carefully the effect of aging, which started to be felt more prominently after 1990. Here, we switch from looking at the time-varying effects of aging to focusing on the possible long-run consequences of the financial crisis.

1.34%. In the case of Italy, it even goes negative, from 1.67% to -0.11%. Interestingly, the country with the highest growth rate of output per working-age adult in this later period is Japan, with 1.49%.

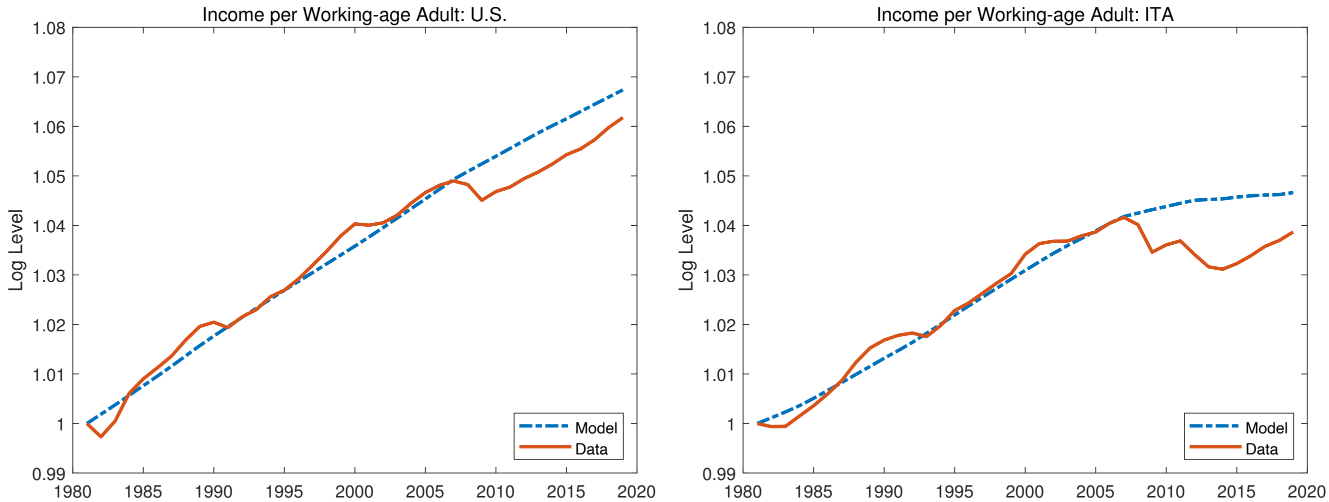

Figure 6.2: U.S. and Italy: 1981 - 2019

We illustrate the effects of a time-varying trend in our neoclassical growth model in Figure 6.2. In the interest of space, we only report the case of the U.S. and Italy (the results for the other countries are roughly similar). The growth of g from 1981 to 2007 is given by its value in Table 4, and the economy is transitioning along its balanced growth path. Then, in 2008 and in an unanticipated way, the growth of g drops to its value in Table 5 from that moment on, and we compute the transition to a new balanced growth path in 2050 (sufficiently far in the future to ensure we have a complete view of the transition).

The model now does a better job of matching the observations for the U.S. and Italy, including the stagnation of the latter. However, our model is completely silent about the sources of the change in g.

The fit is, however, still not perfect. A possible alternative to having two trends would be to have a shock to the level of the growth path. In terms of our model, this would correspond to a sudden drop in A. This is an interesting avenue for further exploration, in particular for the case of Italy.

6.3 China and India

We now extend our analysis to China and India, the two most populated economies in the world. This exercise illustrates when it is and when it is not relevant to distinguish between total, per capita, and per working-age adult output growth rates in economies with behavior different from that of the G7 and Spain.

Table 6: China and India: Basic Growth and Population Facts 1980-2019

1981-2019	China	India
GDP	9.60	6.08
GDP per Capita	8.60	4.25
Population	0.92	1.76
GDP per Working-age Adult	8.18	3.79
Working-age Population	1.31	2.21
Working-age Pop. Ratio	0.68	0.61

Table 6 follows the same format as Table 1. Both China and India have experienced very fast growth since 1980: 9.60% and 6.08%, respectively. The different growth rates for China do not change much whether we look at them in total, per capita, or per working-age adult terms: 9.60%, 8.60%, and 8.18%. This similarity reflects the underlying strong growth of the economy after the start of the economic reforms in 1979 and the relatively moderate growth of the population and working-age population. By 1980, China's fertility rate was already as low as 2.32.

In the case of India, however, we have a mirror image of Japan: the GDP growth rate per working-age adult of 3.79% looks much less impressive than the high rate of total GDP growth, 6.08%, due to the fast growth rate of the working-age population (2.21%).

Table 7: China and India: Basic Growth and Population Facts 1990-2019

1990-2019	China	India
GDP	9.53	6.25
GDP per Capita	8.72	4.57
Population	0.75	1.61
GDP per Working-age Adult	8.50	4.05
Working-age Population	0.95	2.12
Working-age Pop. Ratio	0.70	0.62

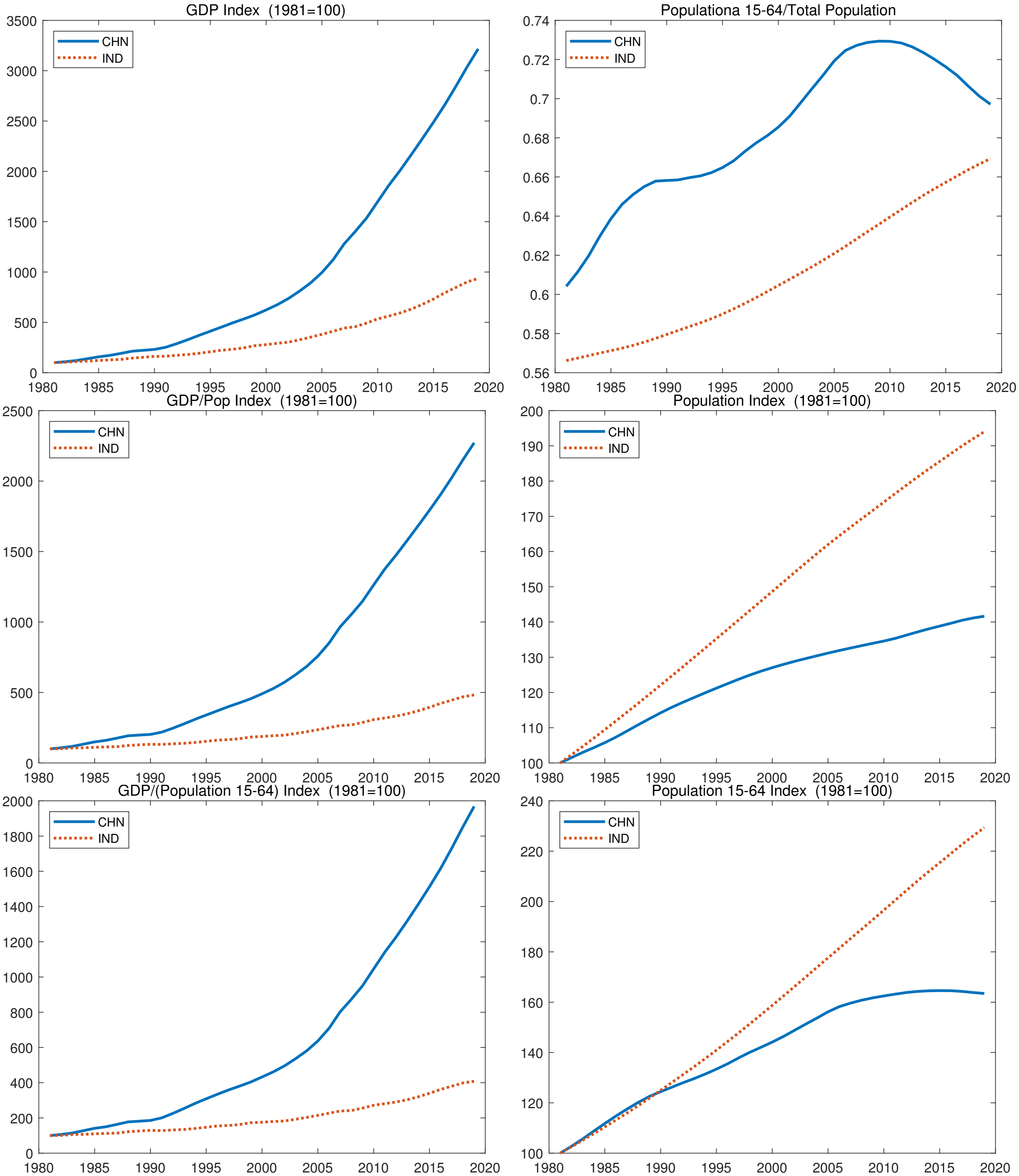

Figure 6.3: China and India: 1981-2019

Table 7 shows the same statistics as Table 6 except with the sample 1990-2019. The interpretation of the results is the same as before: The populations of China and India have not aged enough to make the use of GDP per capita misleading from the perspective of growth theory. However, as China (over the next two decades) and India (starting around 2040) start feeling the effects of much more acute aging, we will see the same mechanisms at work as for the G7 and Spain.[12]

For completeness, Figure 6.3 plots the time series for these two Asian countries. No matter in which terms we compute output growth rates, the difference between China and India is staggering.

We do not include a simulation of our neoclassical growth model for these two countries and compare it to the data. To understand China's and India's growth paths through the lenses of the neoclassical growth model, we would need to consider the whole process of convergence to the advanced economies. This exercise would require the introduction of additional elements in the model. See, for example, Fernández-Villaverde et al. (2023b), where a neoclassical model similar to the one in this paper is calibrated to capture China's technological catch-up with the U.S.

7 Concluding Remarks

As Lucas (1988) famously put it, "Once one starts to think about them [the questions involving economic growth], it is hard to think about anything else." But to do so, we need the right measurements. Historically, economists have looked at total and per capita output growth rates to evaluate an economy's performance and test their theories of growth (and the business cycle). In this paper, we have argued that, as the population ages, total and per capita output growth rates have become increasingly misleading since the early 1990s. The sharpest example is Japan: once we correct for population aging by focusing on output growth rates per working-age adult, Japan appears as a surprisingly robust economy over the last 25 years, outperforming the other G7 countries and Spain.

Admittedly, looking at output growth rates per working-age adult is not without problems: more older individuals are remaining in the labor force, and the trend is likely to continue over the next decades. However, for this paper, this trend is not too important. Given the observed employment rates of individuals 65 and older in the G7 countries and Spain in 1980-2019, any

[12]Recall that, according to China's National Bureau of Statistics, China's total population started falling in 2022.

suitable redefinition of working age would have given us roughly the same results as the ones we report. But the question remains: how do we define (potential) labor inputs in the most fruitful way for theory? While still a good measure of individual welfare, output growth per capita is fast becoming a source of confusion more than a help. Let us look for something better.

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Appendix

A Use TFP growth rate from PWT 10.0

A possible criticism of our exercise in Section 5 is that we take g to match observed growth, not the observed technological change, as proxied by total factor productivity (TFP).

To address this concern, we use the TFP constructed from the PWT 10.0 to pin down the average growth rate of A_t (the WDI does not report TFP). To be specific, we assume the production function has the form:

$$Y_t = (TFP_t)(K_t)^{\theta}(L_t)^{1-\theta}$$

where, to map it into the production function in Section 3, $A_t = TFP_t^{1/(1-\theta)}$. Here, Y_t is real GDP at constant national prices ($RGDP^{NA}$), K_t is capital stock at constant national prices (RK^{NA}), and θ is capital income share ($1 - LABSH$). These data are taken from the PWT 10.0, while L_t (working-age population) is taken from the WDI.

A.1 Same capital income share for all countries

The construction of TFP depends on the choice of capital income share θ. Thus, first, we use the U.S. capital share, $\theta = 0.39$, for all countries in constructing TFP and A_t. Table A.1 reports the average growth rates of labor-augmenting technology A_t and TFP over the sample period for each country.

Table A.1: G7 plus Spain: Growth and TFP, 1981-2019, common θ

1981-2019	Canada	France	Germany	Italy	Japan	Spain	UK	USA
GDP per Working-age Adult	1.33	1.42	1.69	1.07	1.96	1.65	1.88	1.78
TFP	0.49	0.72	0.94	0.36	0.91	0.72	1.16	0.95
$A = TFP^{1/(1-\theta)}$	0.80	1.19	1.54	0.59	1.50	1.18	1.91	1.57
θ	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39

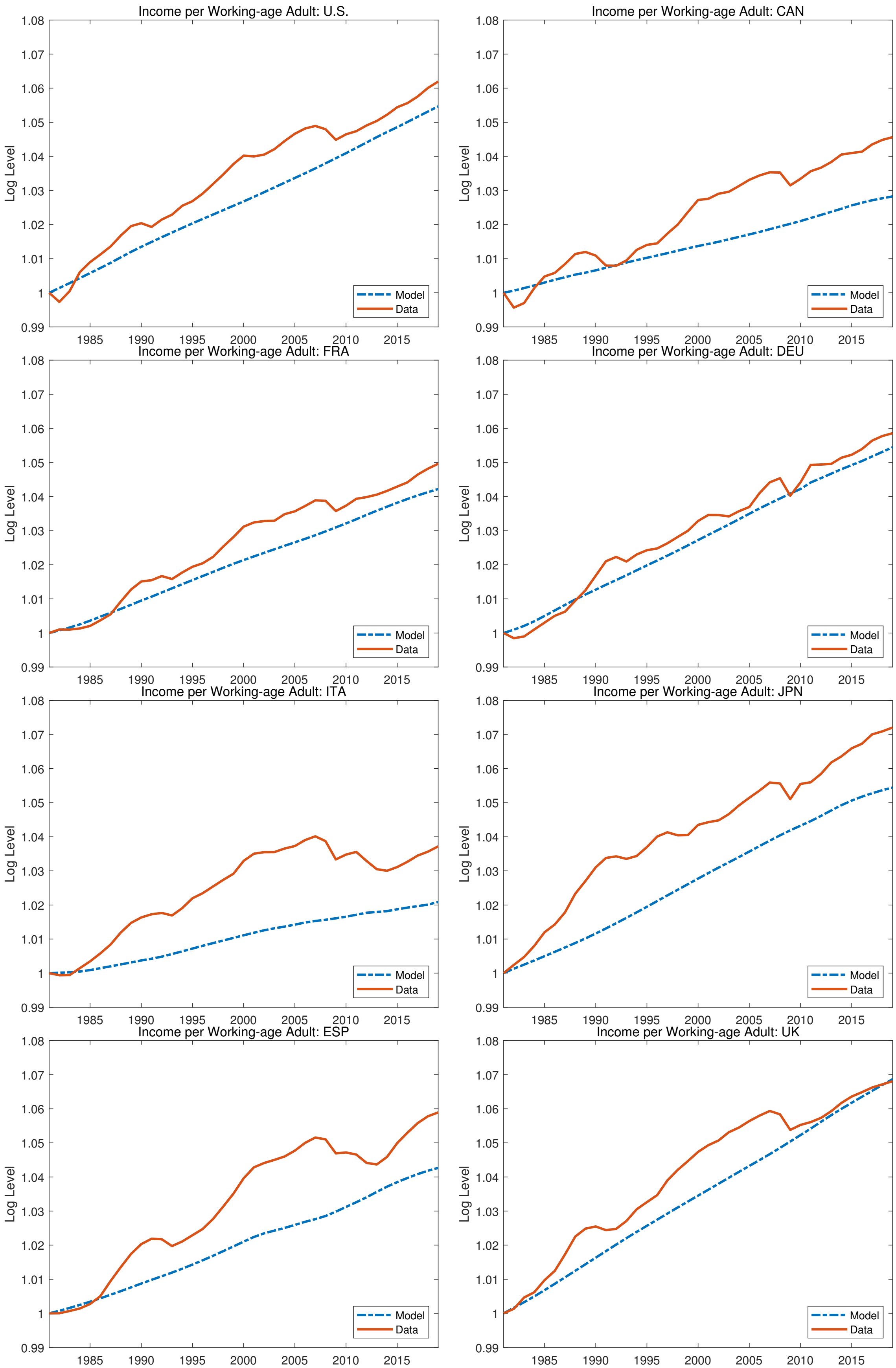

Figure A.1: Transitional Dynamics: 1981-2019, country-specific TFP

Next, we recalibrate our neoclassical growth model, keeping all the parameters, except for g, as in the benchmark. For g, we take the value in the third row of Table A.1 for example, $g = 1.50\%$ for Japan. The new results are reported in Figure A.1.

In this figure, we see much larger divergences between the theory and the model, in particular for Canada, Italy, and Spain. In the case of Italy, the model underperforms by nearly four log points. The reason is that the much lower TFP growth from the PWT also implies much lower investment in capital and, with it, much lower output. In particular, the difference is affected by the time-variation of TFP growth. The model is fed with the average TFP growth, not with the year-specific figure, which leads to missing the level of investment in those years and, hence, capital accumulation. We will return to this point below when we discuss the effects of time-varying trends of technological change.

This divergence between TFP growth in the PWT and the g employed in our baseline calibration might suggest discrepancies between how the PWT builds output and capital data with respect to the data from the WDI.

A.2 Country-specific capital income share

Next, we repeat our exercise by computing A_t and TFP over the sample period for each country as before but now using country-specific capital shares, θ. Table A.2 reports the new results. Except for Canada and Italy, the growth rates of A_t and TFP are roughly the same as in Table A.1.

Table A.2: G7 plus Spain: Growth and TFP, 1981-2019, country-specific θ

1981-2019	Canada	France	Germany	Italy	Japan	Spain	UK	USA
GDP per Working-age Adult	1.33	1.42	1.69	1.07	1.96	1.65	1.88	1.78
TFP	0.61	0.75	1.01	0.23	0.85	0.74	1.08	0.95
$A = TFP^{1/(1-\theta)}$	0.91	1.20	1.57	0.44	1.47	1.21	1.89	1.57
θ	0.33	0.37	0.36	0.46	0.42	0.39	0.42	0.39

Then, we recompute our model with the same parameter values as in the benchmark case, except for g and θ, which are taken from Table A.2. For example, $g = 1.47\%$ for Japan in this exercise.

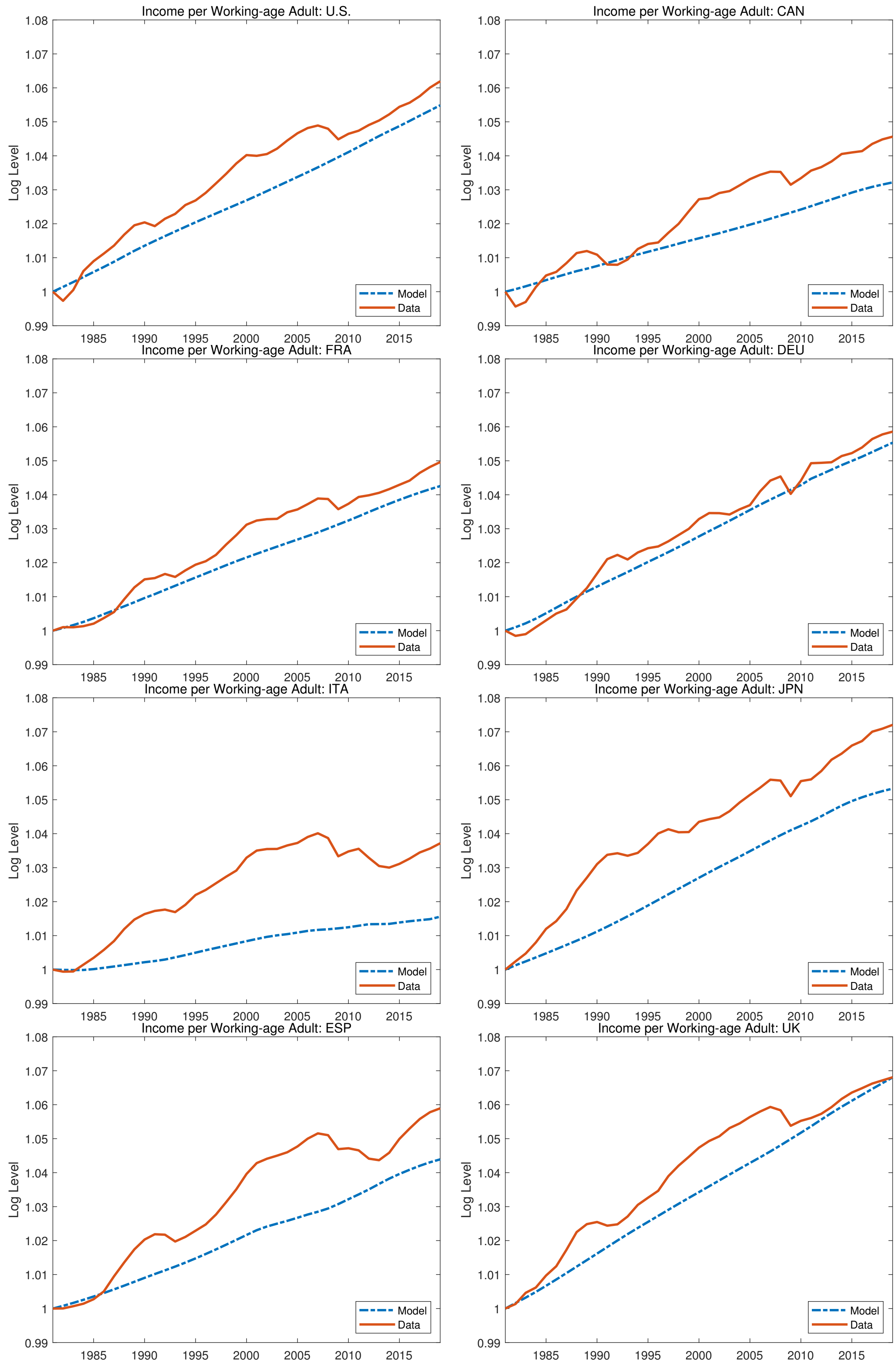

Figure A.2: Transitional Dynamics: 1981-2019, country-specific TFP and θ

The results of this new computation are plotted in Figure A.2, which, as in the case of Figure A.1, shows that now the model deviates from the data for several countries.