The Ronald O. Perelman Center for Political Science and Economics (PCPSE)
133 South 36th Street
Philadelphia, PA 19104-6297

pier@econ.upenn.edu
http://economics.sas.upenn.edu/pier

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A New Test for Market Efficiency and Uncovered Interest Parity

RICHARD T. BAILLIE
Michigan State University
University of London

FRANCIS X. DIEBOLD
University of Pennsylvania

GEORGE KAPETANIOS
University of London

KUN HO KIM
Concordia University

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Richard T. Baillie
Michigan State University, USA
King's College, University of London, UK

Francis X. Diebold
University of Pennsylvania, USA

George Kapetanios
King's College, University of London, UK

Kun Ho Kim
Concordia University, Canada

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Abstract: We suggest a new single-equation test for Uncovered Interest Parity (UIP) based on a dynamic regression approach. The method provides consistent and asymptotically efficient parameter estimates, and is not dependent on assumptions of strict exogeneity. This new approach is asymptotically more efficient than the common approach of using OLS with HAC robust standard errors in the static forward premium regression. The coefficient estimates when spot return changes are regressed on the forward premium are all positive and remarkably stable across currencies. These estimates are considerably larger than those of previous studies, which frequently find negative coefficients. The method also has the advantage of showing dynamic effects of risk premia, or other events that may lead to rejection of UIP or the efficient markets hypothesis.

JEL Classification: C22, C31.

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1 Introduction

Considerable past research in international finance tests the major parity conditions and/or models the role of risk premia and informational inefficiency in currency markets. This paper introduces a new and simple single-equation approach for testing uncovered interest parity (UIP), which also allows for the inclusion of other variables that could represent time-varying risk premia. Our approach is based on a single-equation dynamic regression model and is widely applicable to situations where the maturity time of a forward contract exceeds the sampling period of the data. It (1) avoids the need to use inefficient HAC robust inference and automatically delivers consistent and asymptotically efficient estimates of the dynamic regression parameters, (2) provides evidence on both short- and long-run adjustments to the UIP condition, and (3) facilitates incorporating additional restrictions on the error process implied by the efficient markets hypothesis (EMH) and rational expectations in foreign exchange spot and forward markets.

UIP asserts that the interest rate differential between two countries, or equivalently the forward premium, is an efficient predictor of spot exchange rate returns. This is requires the existence of rational expectations and a constant risk premium. A widespread and important situation occurs when the sampling frequency of the data exceeds the maturity time of the forward contract. In this case the forward rate is a multi-step prediction of the future spot rate, so the errors in (UIP) regressions of spot returns on forward premia are generally serially correlated.

Hansen and Hodrick (1980) noted that the consistency of the GLS estimator commonly invoked to correct for serial correlation requires strictly exogenous regressors, which is unlikely to hold in *UIP* regressions, and they therefore recommended using OLS with a HAC robust estimated covariance matrix. The OLS estimator would then be a consistent, albeit inefficient, estimator of the regression parameters, and this has led to a plethora of HAC regression methods (e.g., Newey and West, 1987).

An alternative approach has been to estimate a vector autoregression, (VAR) and then to test cross-equation restrictions that correspond to the EMH; see Hakkio (1981), Baillie, Lippens and McMahon (1983) and Levy and Nobay (1986). Some comparisons between the different methodologies are given in Hodrick (1987) and Baillie (1989). The potential advantage with the

VAR approach is that it generally provides increased asymptotic efficiency compared with the single equation approach. The disadvantage is that it requires the specification and estimation of a full multi-equation VAR.

In this paper we propose a different approach, based on a single dynamic regression, which we call $DynReg$. It requires only OLS estimation and does not require strict exogeneity, yet we show both theoretically and in simulations that it is consistent and asymptotically efficient, and that associated hypothesis tests have good finite-sample size and power.

We apply the $DynReg$ method to 32 years of weekly data, regressing spot returns on the lagged forward premium. We find clear rejections of the UIP hypothesis (a UIP regression parameter, β, of unity), consistent with the presence of time-varying risk premia, yet our coefficient estimates are more reasonable than those of many earlier studies. In particular, they are remarkably stable across currencies and all positive, whereas previous studies often found large negative β's. We also provide rolling $DynReg$ estimates, which indicate quite stable and relatively similar estimated β coefficients across time and currencies. Similar analysis is also provided for the forward rate forecast error regressed on past errors. These results are more volatile over time and include periods when the UIP condition cannot be rejected.

The plan of the rest of the paper is as follows. Section 2 describes the formulations of the UIP hypothesis, reviews previous econometric tests, describes the $DynReg$ procedure, and shows to implement it in the context of UIP tests. Section 3 presents simulation evidence documenting the fine performance of $DynReg$ estimates of UIP regressions compared to OLS/HAC. Section 4 describes the results of a $DynReg$ UIP analysis of several floating exchange rates. Section 5 provides a brief conclusion.

2 UIP and EMH

Here we develop both economic and econometric aspects of uncovered interest parity and the efficient markets hypothesis.

2.1 Conceptual Formulations

The natural logarithm of the spot exchange rate at time t is denoted by s_t,which is denominated in terms of the amount of foreign currency per one numeraire US dollar. While f_t is the natural logarithm of the corresponding forward exchange rate at time t with maturity time, or forecast horizon, of $k \geq 1$. On denoting the domestic nominal interest rate as i_t and the corresponding foreign interest rate as i_t^*, then the theory of Uncovered Interest Parity (UIP) implies that

$$E_t(s_{t+k} - s_t) = (i_t - i_t^*)\,, \tag{1}$$

where E_t represents the conditional expectation based on a sigma field of information available at time t. Hence UIP requires the twin assumptions of rational expectations and a constant or zero risk premium. Given the no arbitrage condition, Covered Interest Parity (CIP) condition implies that $(i_t - i_t^*) = (f_t - s_t)$ and will hold as an identity, and as an empirical matter CIP does indeed hold almost exactly (Frenkel and Levich, 1975; Taylor, 1987). Hence the UIP condition in equation (1) is also frequently expressed as

$$E_t(s_{t+k} - s_t) = (f_t - s_t)\,. \tag{2}$$

The condition can be tested from the regression

$$(s_{t+k} - s_t) = \alpha + \beta\,(f_t - s_t) + u_{t+k}, \tag{3}$$

so that the k period rate of appreciation of the spot rate is predictable from the forward premium. A test of UIP or the EMH, is that $H_0 : \alpha = 0$ and $\beta = 1$ and the error process is subject to the restriction

$$Cov\,(u_{t+k}u_{t+k-j}) = 0 \text{ for } j > k. \tag{4}$$

Bilson (1981) and Fama (1984) analyzed the $k = 1$ case with the sampling frequency matching the maturity time of the forward contract, so that a natural test of UIP and EMH was to estimate the regression

$$\Delta s_{t+1} = \alpha + \beta\,(f_t - s_t) + u_{t+1}, \tag{5}$$

where UIP implies that that $H_0 : \alpha = 0$ and $\beta = 1$ and u_{t+1} is a serially uncorrelated white noise process. It has been noted by Fama (1984) and many subsequent studies that the estimated slope coefficient is frequently $\beta < 0$. This implies a violation of UIP with the country with the higher rate of interest having an appreciating currency rather than a depreciating currency;

which is known as the Forward Premium Anomaly.

Another way of testing UIP is to express the condition as the forward rate forecast error being unpredictable and to estimate the model

$$(s_{t+k} - f_t) = \alpha + \beta (s_t - f_{t-k}) + u_{t+k}, \tag{6}$$

and to test $H_0 : \alpha = 0$ and $\beta = 0$ and was tested by Hansen and Hodrick (1980).

Early work by Frenkel (1977, 1979) tested the hypothesis $f_t = E_t s_{t+1}$ by estimating the regression

$$s_{t+1} = \alpha + \beta f_t + u_{t+1}, \tag{7}$$

and testing that $\alpha = 0, \beta = 1$ and u_{t+1} serially uncorrelated. These early studies which used monthly data with 30 day forward rates so that the maturity time of the forward contract exactly matched the sampling interval of the data, generally found that the EMH could not be rejected. However, equation (3) is complicated by the fact that the variables in question are non stationary. In particular, see Baillie and Bollerslev (1989), Husted and Rush (1990) and Corbae and Ouliaris (1988) who all found strong evidence that nominal spot and forward rates are well represented as $I(1)$ processes, which also appear to be cointegrated with the forward premium $(s_t - f_t)$ being stationary. Hence either equations (1) or (2) provide the natural economic theory to be tested.

It was also realized that more powerful tests of UIP and the EMH could be obtained by using higher-frequency data where the maturity time of the forward contract exceeds the sampling interval of the data; so that $k > 1$. This initially led to weekly data being used by Hansen and Hodrick (1980), Hakkio (1981), Baillie, Lippens and McMahon (1983), bi weekly data in Hansen and Hodrick (1983); and daily data in Baillie and Osterberg (1997). The availability of higher frequency data then led to the development of a variety of other testing procedures. Both the specifications of the tests for UIP and EMH in equations (3) and (6) provides the interesting complication, given in equation (4) that a valid linear model for u_{t+k} would be an $MA(k-1)$ process, with the possibility of additional forms of non-linearity. The question now arises as how equations (3) and (6) should be estimated.

2.2 Econometric Tests

Both equations (3) and (6) can be expressed as linear regressions,

$$y_{t+k} = \alpha + \beta x_t + u_{t+k}. \tag{8}$$

The estimation of equation (3) proceeds by setting $y_{t+k} = (s_{t+k} - s_t)$ and $x_t = (f_t - s_t)$, and the estimation of equation (6) has $y_{t+k} = (s_{t+k} - f_t)$ and $x_t = y_t = (s_t - f_{t-k})$. In both cases the error process is defined in equation (4), with the precise MA representation to be given later. Both models have overlapping data with $k > 1$, and both have error processes where weak exogeneity is not in doubt, because $E\{x_t u_{t+k}\} = 0$.

However, as noted by Hansen and Hodrick (1980), consistency of time series versions of GLS techniques require the strict econometric exogeneity of the x process in equation (8), in the sense that $E(u_{t+k} \mid x_t\ , x_{t-1}, x_{t+1}, ...) = 0,$, so that x is uncorrelated with all past and future values of u. GLS estimation of β implicitly filters the data, which distorts orthogonality conditions and renders GLS inconsistent in the absence of strict exogeneity.

Because of the possible lack of strict exogeneity in equation (8), producing inconsistency of GLS, Hansen and Hodrick (1980) recommend the use of OLS rather than GLS. OLS is consistent but inefficient when disturbances are serially correlated, and the usual OLS standard errors are inconsistent. One can, however, work out the correct standard error. In particular, the consistent but asymptotically inefficient OLS estimator is

$$\widehat{\beta_{OLS}} = \left(\sum_{t=1}^{T} x_t x_t^{/}\right)^{-1} \left(\sum_{t=1}^{T} x_t y_{t+k}\right),$$

with limiting distribution

$$T^{1/2}(\widehat{\beta_{OLS}} - \beta) \to N(0, M), \tag{9}$$

where β is the true value of β and $M = Q^{-1}\Omega Q^{-1}$, with

$$Q = p\lim\left(T^{-1}\sum_{t=1}^{k} x_t x_t^{/}\right) = p\lim\left(T^{-1}X^{/}X\right). \tag{10}$$

The practical use of the above result depends on the estimated covariance matrix of the error process, so that

$$\widehat{M} = Q^{-1}\widehat{\Omega}Q^{-1} \tag{11}$$

Hansen and Hodrick (1980) recommended estimating Ω by a k-dimensional band diagonal matrix, which would allow for an $MA(k-1)$ error process. Subsequently there has been a vast literature focusing on the estimation of Ω, which then leads to the use of robust ("HAC") standard errors with OLS-estimated regression parameters. The method of Newey and West (1987) has become particularly influential.

It is worth noting that the above complications and considerations do not arise in the VAR approach where the hypothesis that one variable is a k-step-ahead prediction of another variable can be handed by a set of non-linear restrictions on the VAR parameters. However, we will not pursue this issue here since this has been previously discussed by Baillie (1989) and our aim in this paper is to focus on an alternative to the above robustness approach in single equation estimation.

Before explaining an alternative single equation procedure that delivers asymptotically efficient parameter estimates and tests (unlike OLS/HAC), we first note some additional restrictions to the theory of EMH. Due to the k-1 period overlap in sequential k-step-ahead forecasts, u_{t+k} can be expected to be an $MA(k-1)$ process,

$$u_{t+k} = \varepsilon_{t+k} - \theta_1\, \varepsilon_{t+k-1} - .... - \theta_{k-1}\varepsilon_{t+1} = \theta(L)\varepsilon_{t+k}, \tag{12}$$

where ε_{t+k} are white noise and $\theta(L) = \left(1 - \theta_1 L - .... - \theta_{k-1}L^{k-1}\right)$. Following the standard approach of previous literature in using weekly data, the forward rate is generally measured on the Tuesday of each week and the spot rate on the Thursday. This method of defining the data produces an average of 22 days in the forward contract, which implies a maturity time of four weeks and two days, or $(22/5)$, or 4.40 weeks. On assuming $k = 4$ in equation (1) then y_{t+k} in equation (7) would have an autocorrelation pattern of $\rho_1 = 17/22 = 0.77$, and $\rho_2 = 12/22 = 0.55$, $\rho_3 = 7/22 = 0.32$, $\rho_4 = 2/22 = 0.09$ and $\rho_k = 0$, for $k \geq 5$. These population autocorrelations imply a unique, invertible, $MA(4)$ process:

$$\theta(L) = \left(1 + 0.8366L + 0.7728L^2 + 0.6863L^3 + 0.2577L^4\right), \tag{13}$$

with roots of $(0.1909 \pm 1.1724i)$ and $(-1.5266 \pm 0.6575i)$, and autoregressive representation $\pi(L) = \theta(L)^{-1}$.

2.3 The DynReg Approach

An attractive alternative to the Hansen-Hodrick OLS/HAC approach – in part because it delivers efficient as opposed to merely consistent parameter estimates – is a single-equation Dynamic Regression ("$DynReg$") approach.[1] Consider the UIP equation (3) from the perspective of a vector process $\mathbf{z}_t = \{y_t, x_t\}$. z is assumed to be covariance stationary with a Wold Decomposition of

$$\mathbf{z}_t = \sum_{k=0}^{\infty} \Psi_k \mathbf{w}_{t-k} \tag{14}$$

and a corresponding VAR representation of

$$\mathbf{z}_t = \sum_{k=1}^{\infty} \Pi_k \mathbf{z}_{t-k} + \mathbf{w}_t, \tag{15}$$

where $\mathbf{\Psi}_k$ and $\mathbf{\Pi}_k$ are absolutely summable sequences of non-stochastic $2x2$ matrices with $\mathbf{\Psi}_0 = \mathbf{I}$. It is further assumed that $E\left(\mathbf{w}_t \mid Z_{t-1}^w\right) = 0$ a.s. and $E\left(\mathbf{w}_t \mathbf{w}_t^{/} \mid Z_{t-1}^w\right) = \mathbf{\Omega}_w$ a.s. with $|\mathbf{\Omega}_w| > 0$ and $||\mathbf{\Omega}_w|| < \infty$ and $sup_t(||\mathbf{w}_t||^4) < \infty$ with Z_{t-1}^w being the σ sigma field generated by $\{\mathbf{w}_s; s \leq t\}$.

A single equation of the VAR in equation (15) can be conveniently expressed as

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \sum_{i=1}^{k} \sum_{j=0}^{q} \beta_{i,j} x_{i,t-j} + \varepsilon_t, \tag{16}$$

or

$$\phi(L) y_t = \sum_{i=1}^{k} \beta_i(L) x_{i,t} + \varepsilon_t. \tag{17}$$

This single equation is the dynamic regression ($DynReg$) of interest. Its parameters are $\phi(L) = (1 - \phi_1 L - ... - \phi_p L^p)$ and $\beta_i(L) = (\beta_{i,0} - \beta_{i,1} L - ... - \beta_{i,q} L^q)$, and there are $(k + p + kp)$ parameters in total[2] The full set of parameters are denoted by $\theta^{/} = (\phi_1, ..\phi_p, \beta_{1,0}..\beta_{1,q}, \beta_{2,0}, ...\beta_{k,q})$.

The OLS estimates of the $DynReg$ parameters are denoted by $\widehat{\theta}$, and following the standard

[1] Here we give a basic sketch; for a more complete treatment see Baillie, Diebold, Kapetanios and Kim (2022).

[2] In the simulations and empirical applications of subsequent sections we set $p = q$ and select p using the the Schwarz (1978) (BIC) criterion, $ln(\widehat{\sigma}_{u,h}^2) + \{k(1+p) + p\}\, T^{-1} \ln(T)$,

assumptions in Grenander (1981) and Hannan and Deistler (1988), then as $T \to \infty$, we have

$$\widehat{\theta} = \theta + O_p(T^{-1/2}),$$

and moreover that

$$T^{1/2}(\widehat{\theta} - \theta) \to N(0, Q^{-1}),$$

where θ is the true value of the parameters and Q is analogous to the definition in equation (10) and is

$$Q = p \lim T^{-1} \sum_{t=1}^{k} z_t z_t^{/}, \tag{18}$$

where $z_t^{/} = (y_{t-1}, ..y_{t-p}, x_{1,t}, ...x_{k,t}, x_{1,t-1}, ...x_{k,t-p})$.

Under the null hypothesis of EMH with Rational Expectations and constant risk premium, we wish to estimate the model in equation (8), subject to the restriction of having the $MA(k)$ error process defined in equation (12); namely $u_t = \theta(L)\varepsilon_t$. The estimation of the general model in equation (8) can be specialized to either the Fama regression in equation (3), or the forward rate forecast error model in equation (6). On premultipying through equation (8) by the filter $\theta(L)^{-1}$, we obtain

$$\left\{\theta(L)^{-1} y_{t+k}\right\} = \alpha^* + \beta \left\{\theta(L)^{-1} x_t\right\} + \varepsilon_{t+k} \tag{19}$$

where the new intercept is $\alpha^* = \alpha\theta(1)^{-1}$. The filtered explanatory variable is uncorrelated with current and future innovations, ε_{t+k}, so that strict exogeneity is satisfied. Then estimation of equation (19) by OLS will produce consistent and asymptotically efficient estimates of the regression parameters. In practice it is convenient to use the approximation $\theta(L)^{-1} \approx \pi(L)$ where $\pi(L) = (1 - \pi_1 L - ... - \pi_p L^p)$ and is a polynomial in the lag operator of order p and has all its roots lying outside the unit circle.[3] The $DynReg$ will then be

$$\pi(L) y_{t+k} = \alpha^* + \beta\pi(L) x_t + \varepsilon_{t+k} \tag{20}$$

which is a restricted version of the general dynamic regression in equation (16) and can also be estimated by restricted OLS and now contains $(k+1)p$ parameters.

[3] Again, the choice of p can based on BIC.

For the case of weekly data, with $k = 4$, and from equation (13), then

$$\theta(L) = \left(1 + 0.8366L + 0.7728L^2 + 0.6863L^3 + 0.2577L^4\right)$$

and on inverting $\theta(L)$ we find $\pi(L)$ such that $\pi_1 = -0.84$, $\pi_2 = -0.07$, $\pi_3 = 0.02$, $\pi_4 = 0.38$, $\pi_5 = 0.08$, etc. and the weights quickly decay to zero after eleven lags. The above restricted $DynReg$ model or $RDynReg$ model, can be contrasted with the unrestricted $DynReg$ in equation (16). Both the restricted and unrestricted dynamic regressions are reported in the following simulation results and also the tests of the EMH based on the estimated models. We also report Likelihood Ratio (LR) tests to compare the OLS model with the $DynReg$ and also to compare the $DynReg$ model to the $RDynReg$ model, which is based on the full set of EMH restrictions.

3 Simulation Results

The simulation work was based on observed weekly spot exchange rates, and an artificially generated error process from equation (13). Hence the artificially generated forward rate is

$$f_t = s_{t+4} - u_{t+4}$$

and is generated to satisfy the null hypothesis of rational expectations and a time invariant risk premium. The innovations ε_t are generated from an assumed $NID(0, \sigma^2)$ process, where from equation (13) it can be seen that $2.8345\sigma^2 = Var(u_t)$, where the $Var(u_t)$ is calculated for each currency from an initial forward premium regression. The artificial forward rates are then used to construct y_{t+k} and x_t in equation (8). The weekly spot exchange rates were from January 1989 through April 2021, for the six major currencies of Australia, Canada, Japan, New Zealand. Switzerland and UK against the numeraire US dollar. The spot rates were recorded on the Thursday of each week and realized $T = 1,941$ observations and were obtained from Bloomberg. Monte Carlo results for the unrestricted and restricted $DynReg$ are presented in Table 1. The first six rows are from estimation by OLS of the traditional static regression in equation (6), with the first row reporting conventional OLS robust standard errors; while the next five rows use different HAC covariance matrices. In order, the methods are: Hansen and Hodrick (1980), Newey West (1987), Andrews (1991), Kiefer-Vogelsang (2001) and finally the

Equally Weighted Cosine (EWC) method of Lazarus et al (2018).

The seventh and eighth rows of Table 1, in contrast, provide results from using the $DynReg$ and $RDynReg$ approaches. The $DynReg$ method has an unrestricted parameterization as in equation (16), while the $RDynReg$ method imposes the restrictions associated with UIP In all the estimated models the lag order, p, is selected by BIC for each simulation replication.

The $DynReg$ and $RDynReg$ estimators of β clearly have substantially reduced biases and MSEs. This result holds for all six simulation designs, corresponding to the six different spot rates. Hence the inclusion of lagged information in estimation makes a large difference compared with static HAC estimation of equation (6).

Table 1 also presents estimates of the empirical test size, which is the probability of rejecting the null hypothesis when it is true. OLS clearly has poor size properties, and all other test statistics offer massive improvement, with $DynReg$ and $RDynReg$ faring slightly better than the HAC alternatives.

Finally, the simulation results of Figures 1–3 we show size-corrected power curves for sample sizes $T = 250$, $T = 500$ and $T = 1,000$, respectively, for $\beta \in [-0.3, 0.3]$. $DynReg$ clearly dominates, for all six currencies. The high $DynReg$ test power is a natural consequence of its higher estimation efficiency.

In summary, Table 1 and Figures 1–3 clearly indicate that the $DynReg$ and $RDynReg$ methods improve on all competitors in all dimensions.

4 Empirical Results for Six Currencies

The above methodology was also implemented on the same weekly spot exchange rate data between January 1989 through April 2021 and were complemented with the *actual* 30 day forward rate data which was recorded on the Tuesday of each week. This provides $T = 1,941$ observations for each bi-variate system for each of the six currencies. In practice, due to the occurrence of holidays, religious festivals, and weekends, all of which produce market closures, the length of time between a forward rate and its corresponding spot rate in the data set is between 19 and 25 days.

There are several sets of results; each of which includes both OLS/HAC and $DynReg$ estimation. Table 2 presents results for the model in equation (6), where the forward rate forecast error is regressed on its lagged value. The OLS/HAC results have positive but small values for the estimated β with significant rejections of the $\beta = 0$ null for all countries apart from Switzerland. The $DynReg$ results uniformly do not reject the null.

The more interesting results appear in Table 3. They are based on the classic Fama forward premium regression (3), which has more economic and financial intuition. The OLS/HAC β estimates are between 0.07 for Canada and -0.11 for Japan. Three of the currencies have an estimated $\beta < 0$, which is the case originally emphasized by Fama (1984). None of these six estimated β coefficients are significantly different from zero at conventional levels. However, the results of $DynReg$ estimation indicate long-run β in the range of 0.24 to 0.31 while the restricted $RDynReg$ are in the range of 0.30 to 0.40 for all six currencies. Hence the $DynReg$ results all indicate significant risk premia but less than those of early studies with monthly data where the $\beta < 0$.

The appropriate Likelihood Ratio (LR) test statistic for the hypothesis of UIP is denoted by λ_{LR} and shows overwhelming rejections of the UIP and EMH for all six currencies. Hence this indicates the importance of information in the lagged forward rate errors, which is likely due to time variation in the risk premium.

Some further insights into testing the UIP condition are obtained by estimating the above models with five years of observations in each rolling sample. The results are reported graphically in Figures 4 through 7. Figures 4 and 5 show the estimates of β from the forward rate forecast error regressions. The OLS estimates in the left hand panel are considerably more jagged and rough than those of the long run beta estimated by $DynReg$ in the right hand set of panels. The estimates of long run β do not significantly depart from zero in any case. The 95% confidence bands for $DynReg$ almost entirely contain the null in the Hansen-Hodrick model (i.e. $\beta = 0$) for Australian Dollar, Canadian Dollar, Japanese Yen and New Zealand Dollar (i.e. Figs 4 and 5). For Swiss Franc and UK Pound, the band mostly covers the null value. This is clearly not the case with OLS.

The results for the Fama regression in (5), shown by Figs 6 and 7, are particularly interesting and indicate considerable stability over the rolling sample from the $DynReg$ estimates. These estimates are all positive and are typically around 0.4 instead of the $\beta = 1$ implied by UIP. Given that the confidence bands for the $DynReg$ estimates in Figs 6 and 7 stay above zero for all six currencies, the estimates are statistically different to zero for virtually all sets of rolling regressions, which is not the case with OLS. Switzerland has slightly increased β during the financial crisis and is otherwise quite stable. New Zealand has a slightly lower β value than the other currencies.

Hence there is considerable evidence that the UIP condition needs to be appended with risk premium terms, or possibly some measure of informational inefficiency. Models with appropriate variables could potentially be included in the modeling framework introduced in this paper.

5 Conclusions

This paper has suggested a new single-equation test for UIP and EMH based on OLS estimation of a dynamic regression. The approach provides consistent and asymptotically efficient parameter estimates, and is not dependent on assumptions of strict exogeneity. This new approach has the advantage of being asymptotically more efficient than the common approach of using HAC robust standard errors in the static forward premium regression. The method also has advantages of showing dynamic effects of risk premia, or other events that may lead to rejection of UIP and EMH. The empirical results when spot returns are regressed on the lagged forward premium are all positive and remarkably stable across currencies.

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UIP

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Table 1: Performance of Tests of $\beta = 0$ in Dynamic Regressions versus OLS with robust standard errors in the Hansen-Hodrick (1980) model:

$$s_{t+k} - f_t = \alpha + \beta(s_t - f_{t-k}) + u_{t+k}.$$

Under EMH that $H_0: \ \alpha = 0$ and $\beta = 0$ and $Cov(u_{t+k},\, u_{t+k-j}) = 0$ for $j > k$,

	Australian Dollar			Canadian Dollar		
	Bias	MSE	Level	Bias	MSE	Level
OLS	-0.0054	0.0019	0.2630	-0.0034	0.0018	0.2620
OLS-HH	–	–	0.0660	–	–	0.0560
OLS-NW	–	–	0.0810	–	–	0.0760
OLS-Andrews	–	–	0.0760	–	–	0.0720
OLS-KV	–	–	0.0520	–	–	0.0540
OLS-EWC	–	–	0.0590	–	–	0.0660
$DynReg$	-0.0004	0.0012	0.0590	0.0010	0.0011	0.0570
$RDynReg$	-0.0018	0.0006	0.0550	-0.0004	0.0006	0.0460
	Japanese Yen			New Zealand Dollar		
	Bias	MSE	Level	Bias	MSE	Level
OLS	-0.0054	0.0017	0.2470	-0.0021	0.0021	0.2880
OLS-HH	–	–	0.0650	–	–	0.0580
OLS-NW	–	–	0.0790	–	–	0.0780
OLS-Andrews	–	–	0.0760	–	–	0.0740
OLS-KV	–	–	0.0540	–	–	0.0620
OLS-EWC	–	–	0.0680	–	–	0.0660
$DynReg$	-0.0017	0.0011	0.0450	-0.0002	0.0012	0.0430
$RDynReg$	-0.0010	0.0006	0.0590	0.0004	0.0007	0.0550

Table 1: (Continued) Performance of Tests of $\beta = 0$ in Dynamic Regressions versus OLS with robust standard errors in the Hansen-Hodrick (1980) model:

$$s_{t+k} - f_t = \alpha + \beta(s_t - f_{t-k}) + u_{t+k}.$$

Under EMH that $H_0: \ \alpha = 0$ and $\beta = 0$ and $Cov(u_{t+k},\, u_{t+k-j}) = 0$ for $j > k$,

	Swiss Franc			UK Pound		
	Bias	MSE	Level	Bias	MSE	Level
OLS	-0.0047	0.0017	0.2420	-0.0029	0.0018	0.2350
OLS-HH	–	–	0.0640	–	–	0.0650
OLS-NW	–	–	0.0720	–	–	0.0740
OLS-Andrews	–	–	0.0710	–	–	0.0740
OLS-KV	–	–	0.0440	–	–	0.0530
OLS-EWC	–	–	0.0610	–	–	0.0660
$DynReg$	-0.0001	0.0011	0.0470	-0.0008	0.0010	0.0390
$RDynReg$	-0.0015	0.0006	0.0480	-0.0003	0.0006	0.0460

Key: The first six test statistics are based on OLS estimation of β with standard errors based on estimated parameters estimated covariance matrix computed by (i) regular OLS, (ii) HH, method of Hansen and Hodrick, (iii) NW, method of Newey and West, (iv) $Andrews$, method of Andrews, (v) KV, method Kiefer and Vogelsang, (vi) EWC, Equally Weighted Cosine. The $DynReg$ statistics are based on estimation of the unrestricted dynamic regression in equation (16) and $RDynReg$ is the restricted dynamic regression in equation (19) that constrains the error to be an MA(k-1) process as defined in equation (12).

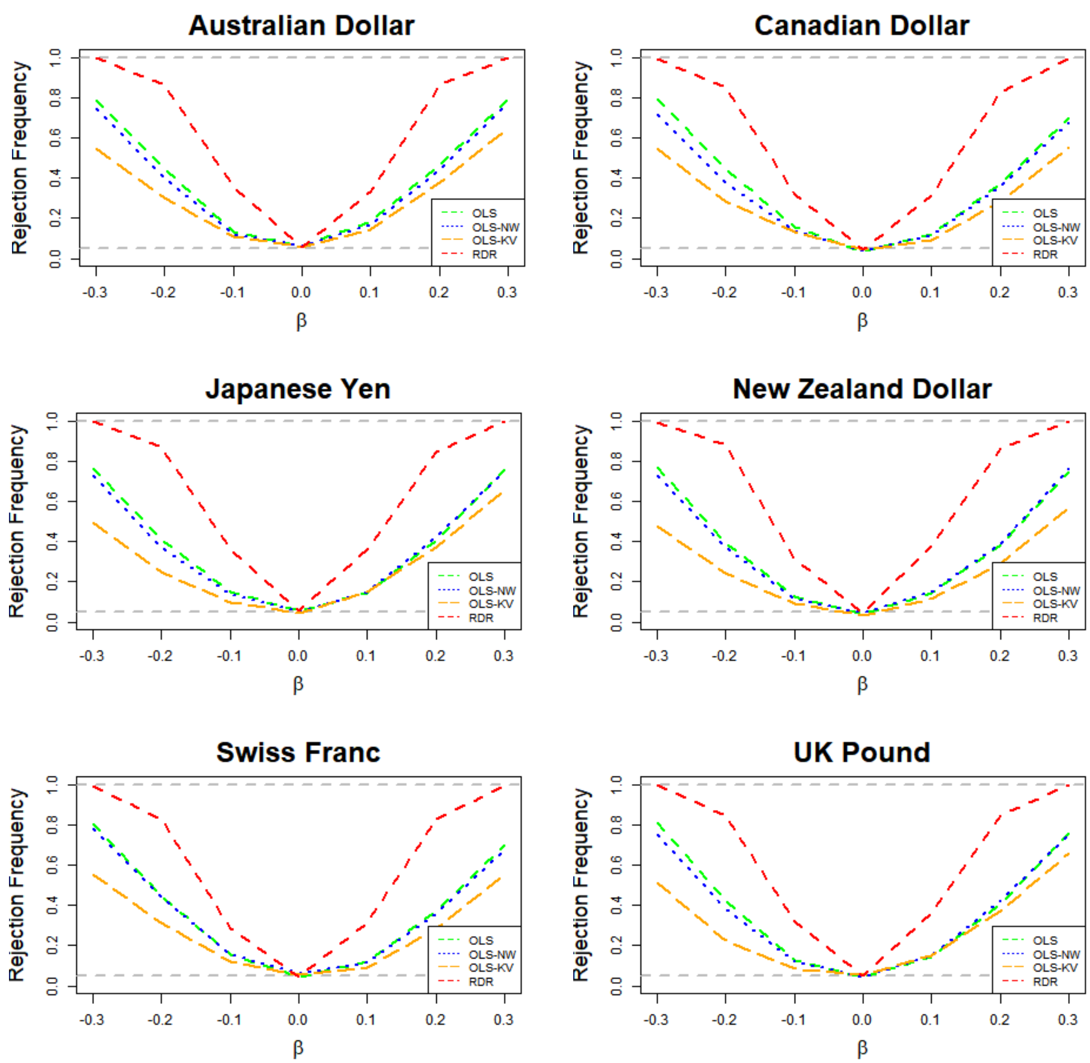


Figure 1: Size-corrected power for OLS (green), OLS-NW (blue), OLS-KV (yellow) and $RDynReg$ (red): A 5% t-test is conducted for the sample size **T=250**. The underlying model is the *Hansen-Hodrick* model (1980). The null value is $\beta = 0$ and the alternatives are $\beta = \pm 0.1, \pm 0.2, \pm 0.3$.

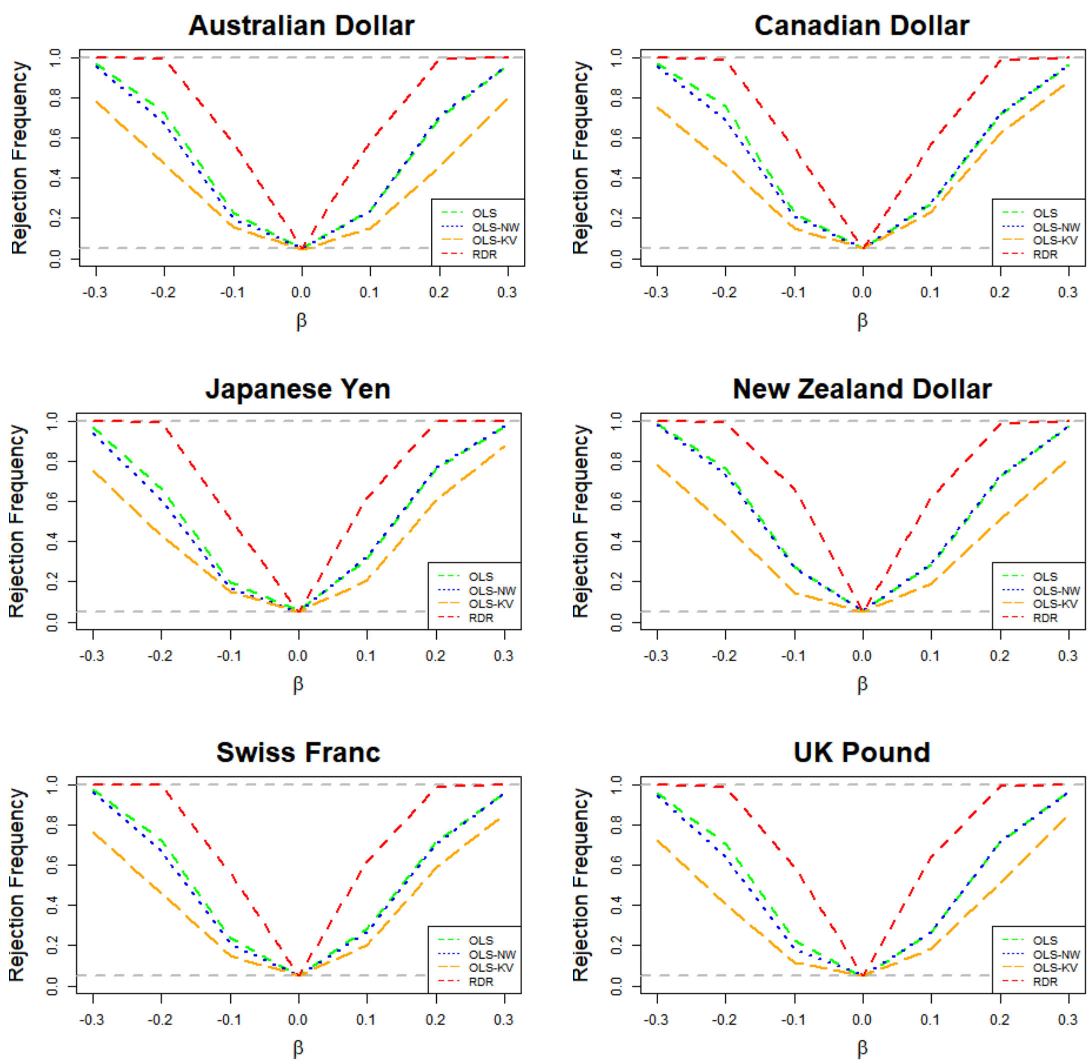


Figure 2: Size-corrected power for OLS (green), OLS-NW (blue), OLS-KV (yellow) and $RDynReg$ (red): A 5% t-test is conducted for the sample size **T=500**. The underlying model is the *Hansen-Hodrick* model (1980). The null value is $\beta = 0$ and the alternatives are $\beta = \pm 0.1, \pm 0.2, \pm 0.3$.

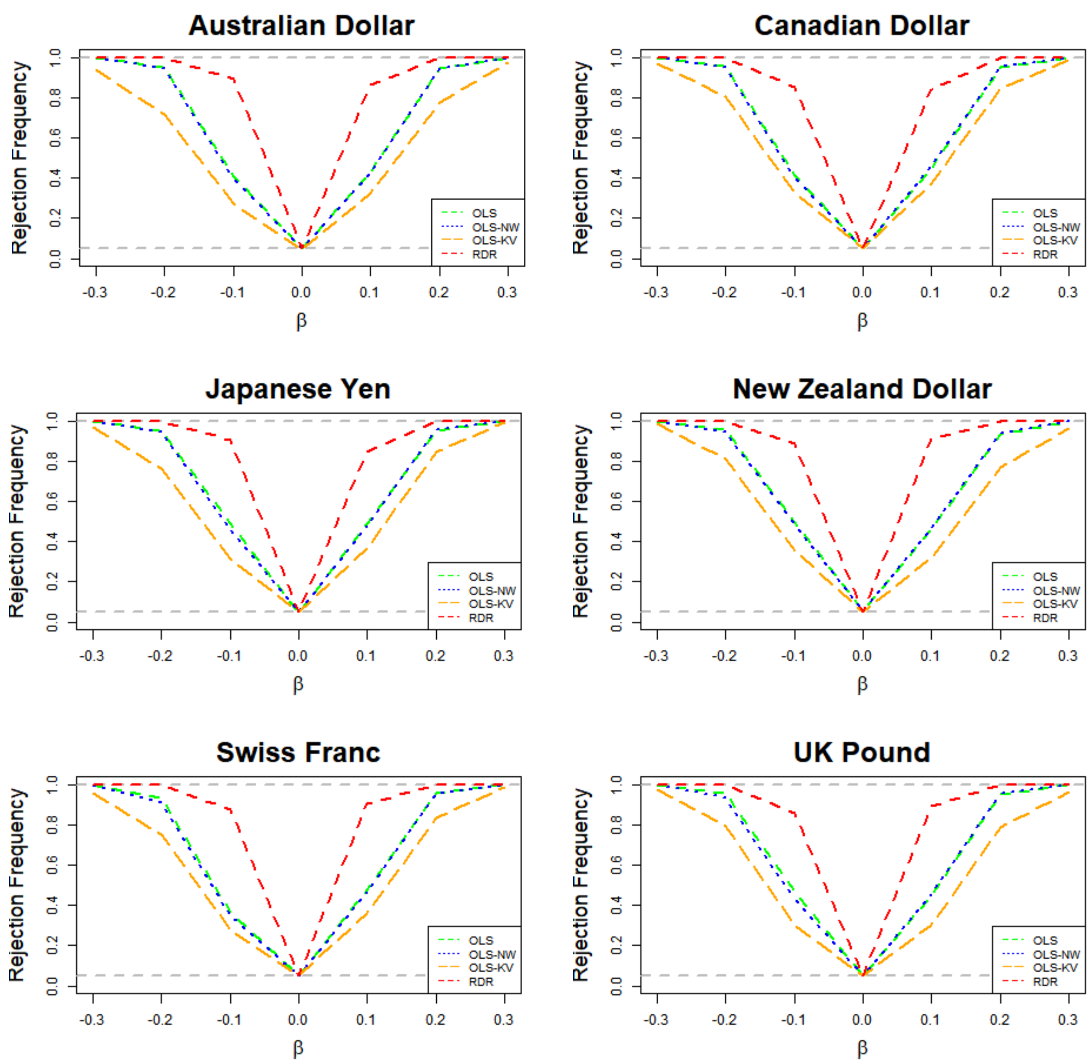


Figure 3: Size-corrected power for OLS (green), OLS-NW (blue), OLS-KV (yellow) and $RDynReg$ (red): A 5% t-test is conducted for the sample size **T=1000**. The underlying model is the *Hansen-Hodrick* (1980) model. The null value is $\beta = 0$ and the alternatives are $\beta = \pm 0.1, \pm 0.2, \pm 0.3$.

Table 2: Estimation of the Hansen-Hodrick (1980) model $s_{t+k} - f_t = \alpha + \beta(s_t - f_{t-k}) + u_{t+k}$ for actual weekly spot and forward exchange rate data from January 1989 through April 2021.

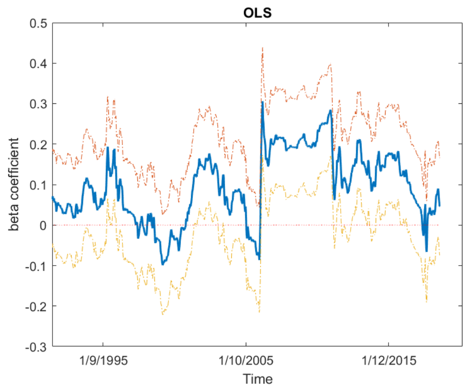
	Australian Dollar			Canadian Dollar		
	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}
OLS	0.1393	0.0241	–	0.1210	0.0243	–
OLS-HH	–	0.0664	–	–	0.0536	–
OLS-NW	–	0.0606	–	–	0.0488	–
OLS-Andrews	–	0.0620	–	–	0.0479	–
OLS-KV	–	0.0158	–	–	0.0069	–
OLS-EWC	–	0.0566	–	–	0.0448	–
$DynReg$	-0.0776	0.0320	2408.6	-0.0167	0.0320	2226.6
$RDynReg$	0.0024	0.0245	–	-0.0003	0.0246	–
	Japanese Yen			New Zealand Dollar		
	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}
OLS	0.1503	0.0242	–	0.0955	0.0255	–
OLS-HH	–	0.0533	–	–	0.0598	–
OLS-NW	–	0.0483	–	–	0.0566	–
OLS-Andrews	–	0.0486	–	–	0.0582	–
OLS-KV	–	0.0098	–	–	0.0089	–
OLS-EWC	–	0.0497	–	–	0.0571	–
$DynReg$	0.0120	0.0324	2479.5	-0.0555	0.0338	2386.3
$RDynReg$	0.0192	0.0245	–	-0.0236	0.0258	–
	Swiss Franc			UK Pound		
	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}
OLS	0.0373	0.0244	–	0.0799	0.0244	–
OLS-HH	–	0.0474	–	–	0.0598	–
OLS-NW	–	0.0446	–	–	0.0540	–
OLS-Andrews	–	0.0455	–	–	0.0542	–
OLS-KV	–	0.0333	–	–	0.0158	–
OLS-EWC	–	0.0438	–	–	0.0576	–
$DynReg$	-0.0415	0.0322	2315.3	-0.0527	0.0321	2391.8
$RDynReg$	0.0280	0.0245	–	-0.0584	0.0245	–

Key: See key to Table 1. The Likelihood Ratio test statistic λ_{LR} is from a test of the static OLS regression model against the DynReg model.

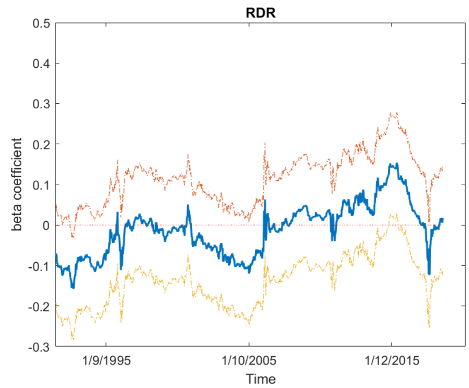
Table 3: Estimation of the Fama (1984) model $s_{t+k} - s_t = \alpha + \beta(f_t - s_t) + u_{t+k}$ for actual weekly spot and forward exchange rate data from January 1989 through April 2021.

	Australian Dollar			Canadian Dollar		
	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}
OLS	0.0115	0.0815	–	0.0727	0.0762	–
OLS-HH	–	0.1077	–	–	0.1124	–
OLS-NW	–	0.1055	–	–	0.1141	–
OLS-Andrews	–	0.1043	–	–	0.1129	–
OLS-KV	–	0.0300	–	–	0.0270	–
OLS-EWC	–	0.1068	–	–	0.1057	–
$DynReg$	0.2440	0.0503	2852.4	0.3038	0.0495	2669.7
$RDynReg$	0.3696	0.0340	–	0.3972	0.0325	–
	Japanese Yen			New Zealand Dollar		
	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}
OLS	-0.1148	0.0815	–	-0.0320	0.0842	–
OLS-HH	–	0.0971	–	–	0.0847	–
OLS-NW	–	0.0927	–	–	0.0898	–
OLS-Andrews	–	0.0907	–	–	0.0902	–
OLS-KV	–	0.0171	–	–	0.0639	–
OLS-EWC	–	0.0842	–	–	0.0877	–
$DynReg$	0.2750	0.0503	2847.1	0.2392	0.0514	2845.9
$RDynReg$	0.3462	0.0334	–	0.3398	0.0351	–
	Swiss Franc			UK Pound		
	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}	$\hat{\beta}$	$s.e.(\hat{\beta})$	λ_{LR}
OLS	0.0277	0.0770	–	-0.0737	0.0861	–
OLS-HH	–	0.1545	–	–	0.1066	–
OLS-NW	–	0.1491	–	–	0.1075	–
OLS-Andrews	–	0.1489	–	–	0.1058	–
OLS-KV	–	0.0983	–	–	0.0725	–
OLS-EWC	–	0.1486	–	–	0.1142	–
$DynReg$	0.2427	0.0477	2796.6	0.3054	0.0503	2797.2
$RDynReg$	0.3012	0.0326	–	0.3566	0.0365	–

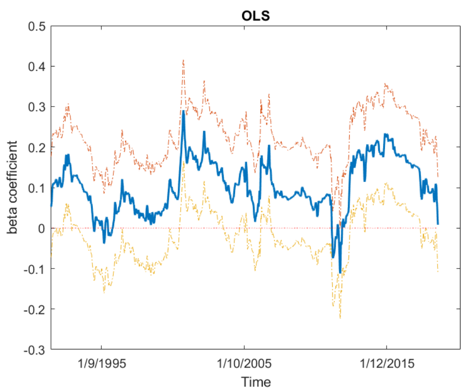
Key: See key to Table 1. The Likelihood Ratio test statistic λ_{LR} is from a test of the static OLS regression model against the $DynReg$ model.



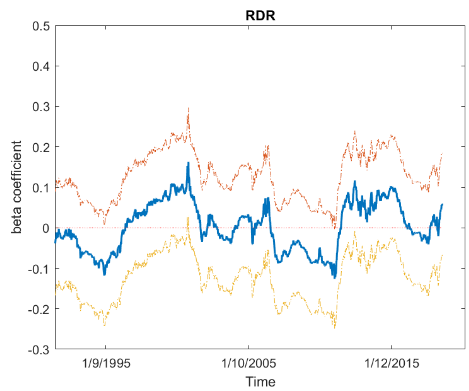
Australian Dollar



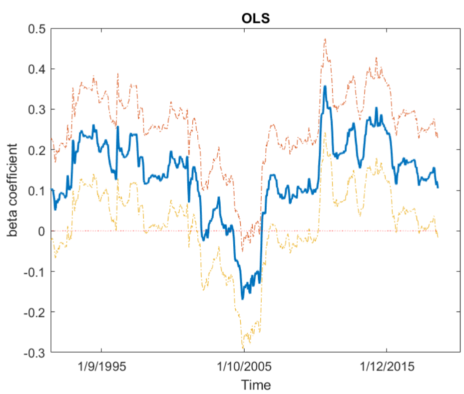
Australian Dollar



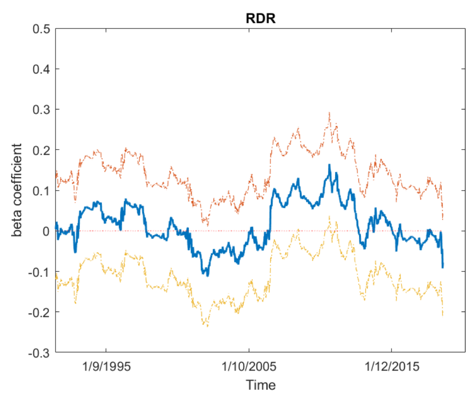
Canadian Dollar



Canadian Dollar

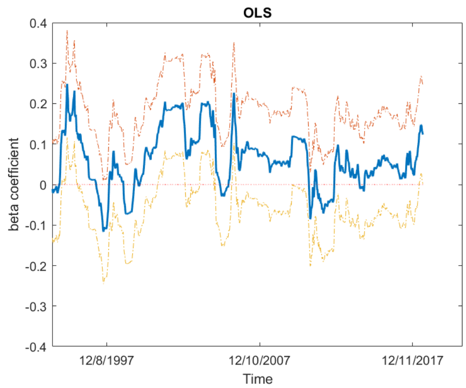


Japanese Yen

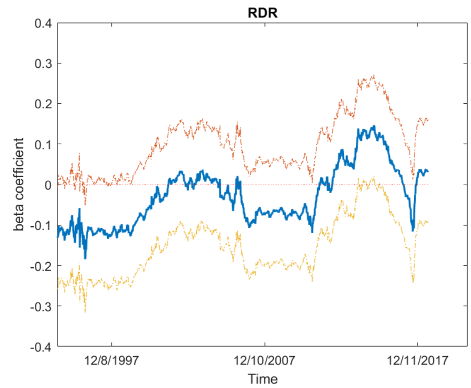


Japanese Yen

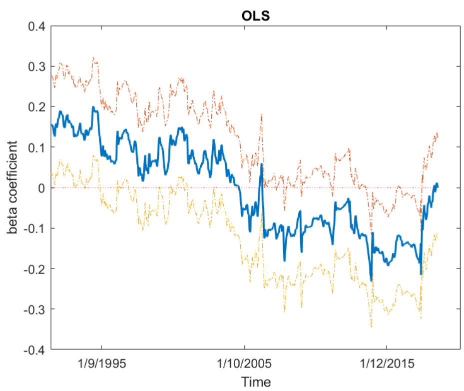
Figure 4: The blue curve is the 5-year rolling OLS (left)/$RDynReg$ (right) estimate of β in the *Hansen-Hodrick* (1980) model; The null is $\beta = 0$ and the dashed ones are the 95% confidence bands.



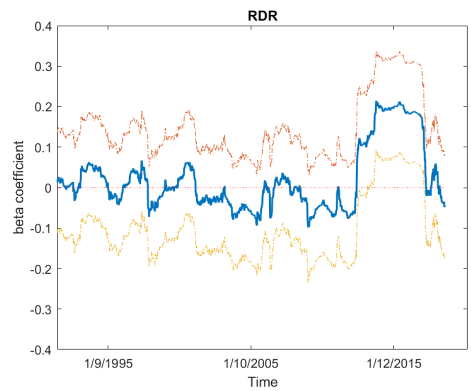
New Zealand Dollar



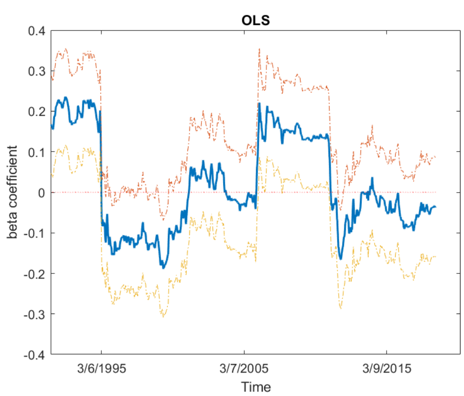
New Zealand Dollar



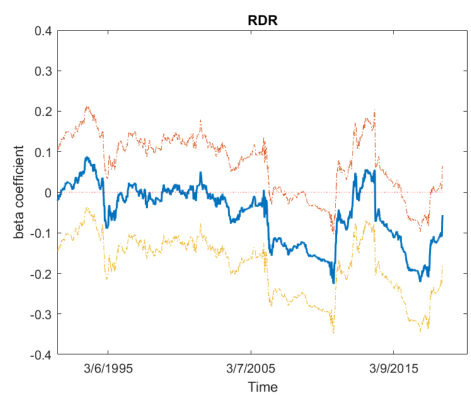
Swiss Franc



Swiss Franc

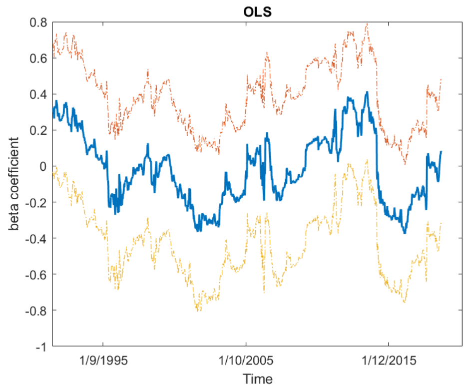


UK Pound

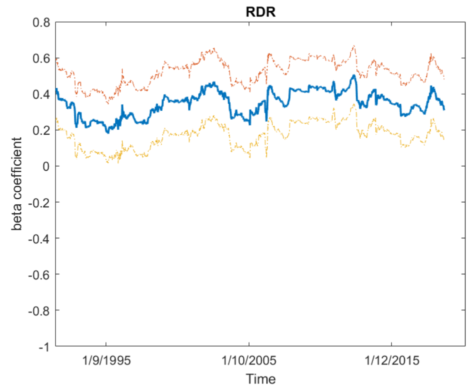


UK Pound

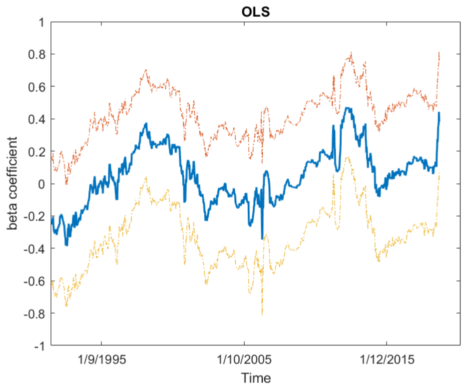
Figure 5: The blue curve is the 5-year rolling OLS (left)/$RDynReg$ (right) estimate of β in the *Hansen-Hodrick* (1980) model; The null is $\beta = 0$ and the dashed ones are the 95% confidence bands.



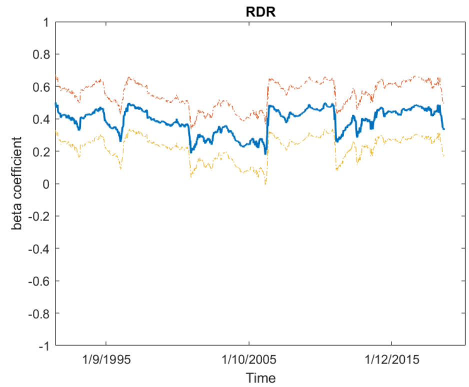
Australian Dollar



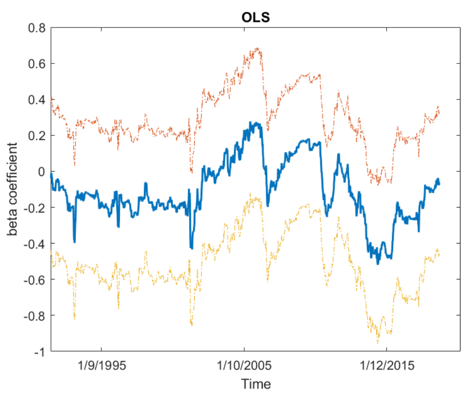
Australian Dollar



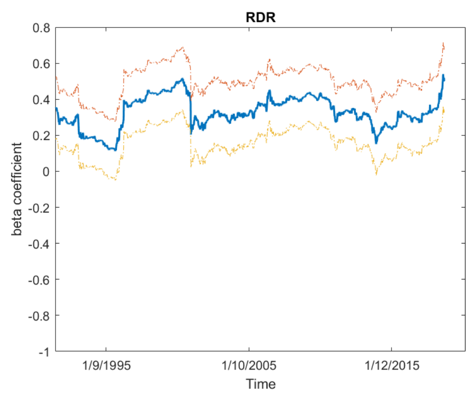
Canadian Dollar



Canadian Dollar

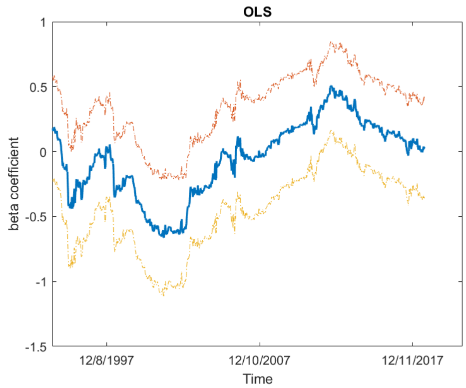


Japanese Yen

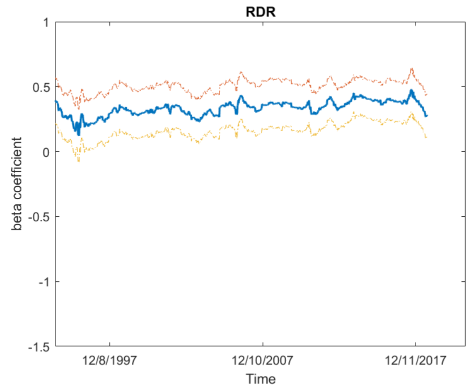


(f) Japanese Yen

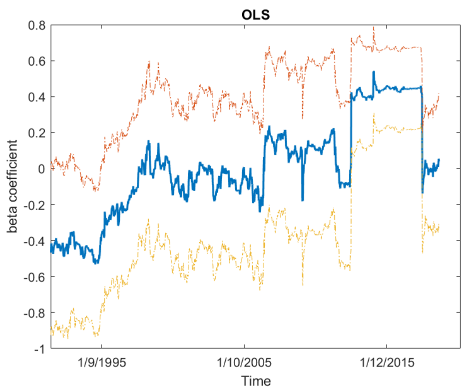
Figure 6: The blue curve is the 5-year rolling OLS (left)/$RDynReg$ (right) estimate of β in the *Fama* (1984) model; The null is $\beta = 1$ and the dashed ones are the 95% confidence bands.



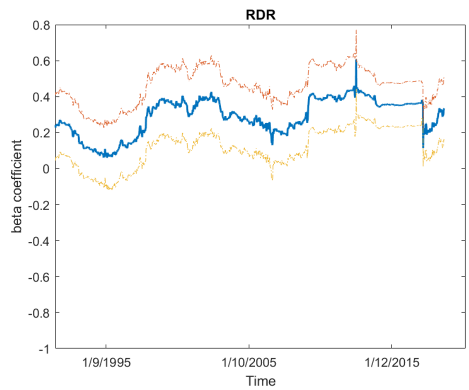
New Zealand Dollar



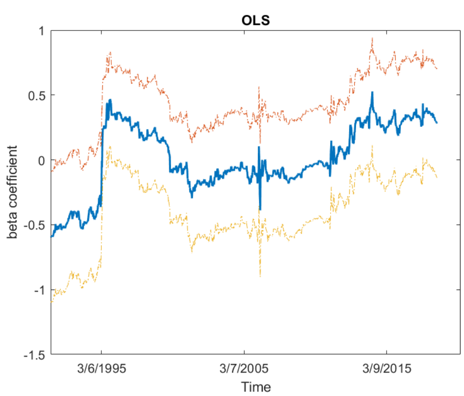
New Zealand Dollar



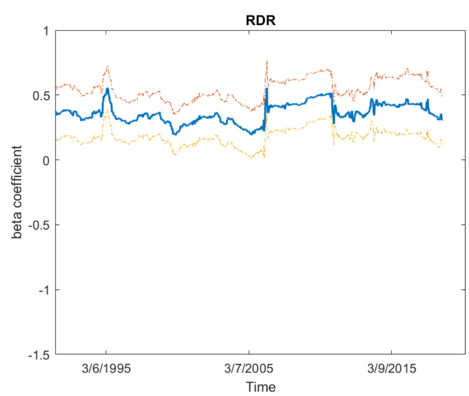
Swiss Franc



Swiss Franc



UK Pound



UK Pound

Figure 7: The blue curve is the 5-year rolling OLS (left)/$RDynReg$ (right) estimate of β in the *Fama* (1984) model; The null is $\beta = 1$ and the dashed ones are the 95% confidence bands.