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History Remembered: Optimal Sovereign Default on Domestic and External Debt

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Abstract

Infrequent but turbulent overt sovereign defaults on domestic creditors are a "forgotten history" in Macroeconomics. We propose a heterogeneous-agents model in which the government chooses optimal debt and default on domestic and foreign creditors by balancing distributional incentives v. the social value of debt for self-insurance, liquidity, and risk-sharing. A rich feedback mechanism links debt issuance, the distribution of debt holdings, the default decision, and risk premia. Calibrated to Eurozone data, the model is consistent with key long-run and debt-crisis statistics. Defaults are rare (1.2 percent frequency), and preceded by surging debt and spreads. Debt sells at the risk-free price most of the time, but the government's lack of commitment reduces sustainable debt sharply.

Keywords: public debt, sovereign default, debt crisis, European crisis

JEL Classifications: E6, E44, F34, H63

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In loving memory of Dave Backus

1 Introduction

The central finding of the seminal cross-country analysis of the history of public debt by Reinhart and Rogoff [48] is that governments defaulted outright on their *domestic* debt 68 times over the past 250 years. The United States is no exception. Hall and Sargent [34] document in detail the domestic default that followed the American Revolutionary War. These are *de jure* defaults in which governments reneged on the contractual terms of domestic debt via forcible conversions, lower coupon rates, reductions of principal and suspension of payments, separate from *de facto* defaults due to inflation or currency devaluation. Overt domestic defaults are rare, with an unconditional frequency of about 1.1 percent in the Reinhart-Rogoff dataset (68 events for 64 countries, with data for most covering the 1914-2007 period and for some since 1750), but they are turbulent episodes in terms of financial instability and macroeconomic performance. Also, all of the domestic defaults triggered external defaults, in some cases even at low external debt ratios.[1] Despite these striking facts, Reinhart and Rogoff found that domestic defaults represent a "forgotten history" in the Macroeconomics literature.

Recent events raising the prospect of domestic defaults in advanced economies make this history much harder to forget. The European debt crisis, historically high public debt ratios in other advanced economies (e.g.the U.S., Japan), and large unfunded liabilities in the entitlement programs of many governments, demonstrate that the conventional wisdom treating domestic public debt as a risk-free asset is flawed and that there is a critical need to understand its riskiness and the dynamics of domestic defaults. The relevance of these issues is emphasized further by the sheer size of domestic public debt markets: The global market of local currency government bonds is worth roughly half of the world's GDP and is six times larger than the market for investment-grade sovereign debt denominated in foreign currencies. Domestic debt also accounts for a large fraction of total public debt in most countries, almost two-thirds on average.[2]

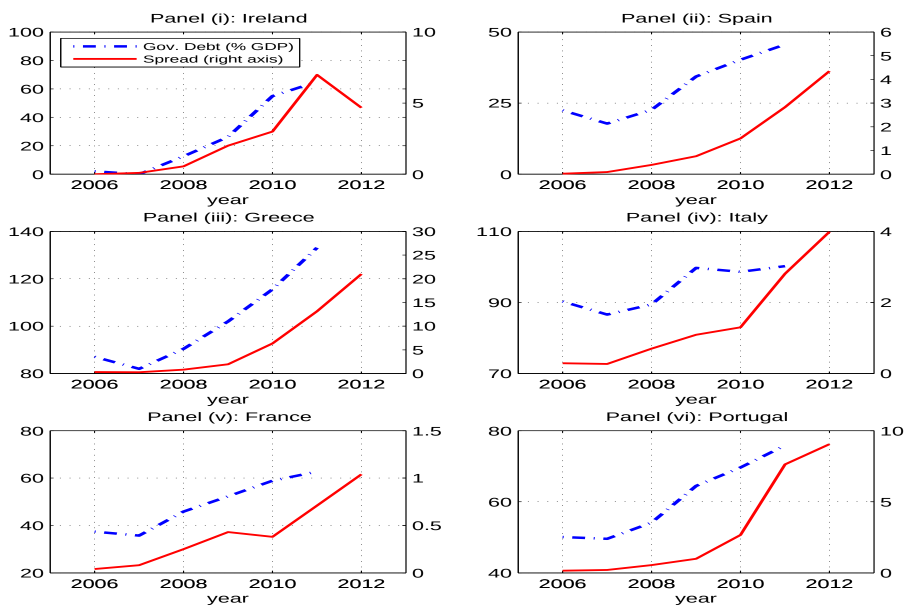
The European debt crisis is often, but mistakenly, viewed as a set of conventional external sovereign debt crises. This view ignores three features of the Eurozone that make a sovereign

[1] Reinhart and Rogoff also noted that decomposing public debt into domestic and external is difficult. Several studies, including this paper, define domestic debt as that held by domestic residents, for which data are available for a limited number of countries in international databases (e.g., OECD Statistics and more recently Arslanalp and Tsuda [10], both of which are used in this paper). Other studies define domestic debt as debt issued under domestic jurisdiction. The two definitions are correlated, but not perfectly, and in some episodes have differed significantly (e.g. most of the bonds involved in the debt crises in Mexico, 1994 and Argentina, 2002 were issued domestically but with large holdings abroad).

[2] Estimates of the global government bond market values and debt ratios are from *The Economist,* Feb. 11, 2012, and from the International Monetary Fund.

default by one member more akin to a domestic default: First, a large fraction of Eurozone public debt is held within Europe, so default by one member can be treated as a (partial) domestic default from the point of view of the Eurozone as a whole. Second, the Eurozone's common currency prevents individual countries from unilaterally reducing the real value of their debt through inflation (i.e., implementing country-specific de facto defaults). Lojsch et al. [40] report that about half of the public debt issued by Eurozone countries was held by Eurozone residents as of 2010, and 99.1 percent of this debt was denominated in euros.[3] Third, and most important from the standpoint of the model proposed in this paper, policy discussions and strategies for dealing with the crisis emphasized the distributional implications of a default by one member country on all the Eurozone, and the costly implications of impairing public debt markets for financial systems across the Eurozone. This is a critical difference relative to external defaults because it shows the concern of the parties pondering default decisions for the adverse effects of a default on the governments' *creditors*, in terms of both their balance sheets and their use of public debt instruments as a core financial asset.[4]

Figure 1: Eurozone Debt Ratios and Spreads



During the European debt crisis, net public debt of countries at the epicenter of the crisis (Greece, Ireland, Italy, Portugal, and Spain) ranged from 45.6 to 133.1 percent of GDP, and their spreads v. Germany were large, ranging from 280 to 1,300 basis points (see Appendix

[3]Adding European public debt holdings of European countries outside the Eurozone (particularly Denmark, Sweden, Switzerland, Norway, and the UK), 85 percent of European public debt is held in Europe.

[4]Still, the analogy with a domestic default is imperfect, because the Eurozone lacks a fiscal authority with taxation powers across all its members, except for seigniorage collected by the European Central Bank.

A-1). Debt ratios in the core countries, France and Germany, were also relatively high at 62.7 percent and 51.5 percent, respectively. Figure 1 shows that both debt ratios and spreads were stable before 2008 but grew rapidly afterward (except in Italy, where debt was already high but spreads widened also after 2008). The fractions of each country's debt held by residents of the same country ranged from 27 percent in Greece to 64 percent in Spain.

This paper proposes a model with heterogeneous agents and incomplete financial markets in which domestic default can be optimal for a government that uses debt and default to redistribute resources across agents and through time in response to idiosyncratic income shocks and aggregate government expenditure shocks. Issuing new debt causes "progressive redistribution" favoring agents with below-average bond holdings, while debt repayment causes "regressive redistribution" in the opposite direction. Default prevents the latter ex-post, but the ex-ante probability that this can happen lowers bond prices at which new debt can be issued and thus hampers the government's borrowing capacity and its ability to engage in progressive redistribution. Default is optimal when the aggregation of individual utility gains from default across agents that differ in bond holdings and income using a social welfare function is positive (i.e., when the social payoff of default exceeds that of repayment).

The above distributional default incentives are tempered by endogenous default costs that result from the role of public debt as a vehicle for self-insurance, liquidity-provision, and risk-sharing. Public debt is the asset agents use to build precautionary savings against uninsurable shocks, provides liquidity (i.e., resources) to a fraction of agents who are endogenously credit-constrained, and facilitates risk-sharing as agents that draw high (low) income buy (sell) debt. Default wipes out the debt holdings of all agents, forcing them to restart the costly process of deferring consumption to rebuild their buffer stock of savings. Agents who have a stronger need to either draw from this buffer stock or to buy bonds to build them up incur a larger utility cost if the government defaults. Moreover, the utility cost of default is also large for poor agents with low income and no bond holdings, because they face binding borrowing limits and thus value highly the liquidity that public debt provides.

Since the distribution of bond holdings evolves endogenously over time and the government cannot discriminate among its creditors (in line with the *pari passu* clause typical of sovereigh debt), repayment and default affect the cross section of agents differently and these differences evolve over time.[5] Each period, the social welfare gain of default summarizes the government's trade-off between distributional default incentives and default costs.

The government also levies a proportional income tax as an alternative vehicle for re-

[5]Government bonds generally rank *pari passu* with each other and with other unsecured government obligations. The meaning and enforceability of this clause had been subject of debate, but its enforcement in a 2000 case involving Peru and a more recent case involving Argentina solidified its legal standing (see Olivares-Caminal [43]). This treatment of domestic debt is also consistent with the evidence presented by Reinhart and Rogoff [48] showing that all domestic defaults in their sample were also external defaults.

distribution that operates in the usual way to improve risk-sharing of *idiosyncratic* income shocks. A 100 percent tax on individual income to finance a uniform lump-sum transfer provides perfect risk-sharing of these shocks but still does not provide insurance against the aggregate shocks. We study equilibria in which the income tax rate matches actual tax rate estimates, which are well below 100 percent.

Foreign creditors also participate in the public debt market, so that we can study the distribution of debt across domestic v. foreign creditors. These creditors are modeled as the risk-neutral investors typical of the Eaton-Gersovitz [26] (EG hereafter) class of external default models, which yields the standard condition equating expected returns in government debt with the world opportunity cost of funds, linking default risk premia to default probabilities. As in EG models, we allow for the possibility that default imposes an exogenous income cost. Default, debt, and risk premia dynamics, however, respond to very different forces from those at work in EG models, because the government's payoff function factors in the utility of all domestic agents, including its creditors, and default has the endogenous costs that result from impairing the use of debt for self-insurance, liquidity and risk-sharing.

A rich feedback mechanism connects the government's debt issuance and default choices, the price of government bonds, the optimal plans of individual agents, and the dynamics of the distribution of bonds across agents. The latter are driven by the agents' optimal plans and determine the evolution of individual utility gains of default across the cross section of agents. In turn, a key determinant of the agents' plans is the default risk premium reflected in the price of public debt, which is determined by the probability of default, which is itself determined by the governments aggregation of the individual default gains.

Public debt, spreads, and the social welfare gain of default evolve over time driven by this feedback mechanism as the exogenous shocks hit the economy. With low debt and/or low government expenditures, repayment incentives are stronger producing "more negative" welfare gains of default, which in turn make repayment and increased debt issuance optimal. The balance changes at higher debt and/or higher government expenditures, and as the dispersion of individual gains from default widens and the social welfare gain from default rises, debt can reach levels at which default is optimal. In the baseline case, default wipes out the debt and sets the economy back to a state in which repayment incentives are strong because with zero debt the social value of debt is high. These dynamics also affect the holdings of public debt by domestic and external agents. After a default, external debt rises faster at first, as domestic demand grows gradually because of the utility cost of postponing consumption to rebuild the buffer stock of savings, but over time, as domestic demand continues to rise, domestic agents hold a larger share of public debt than foreign agents.

The optimal debt moves across three zones. First, a zone in which repayment incentives are strong (i.e., the social gain of default is "very negative"), the optimal debt is sold at zero default risk, and that debt is lower than that which maximizes the resources that can be gained by borrowing. Second, a zone in which the optimal debt is still offered default-risk-free but equals the amount of debt that yields the most resources possible. Here, weaker repayment incentives result in bond prices that fall sharply if debt exceeds this amount, so debt is sold at the risk-free price but constrained by the government's inability to commit to repay. Third, a zone in which repayment incentives are in between the first two cases, so that the optimal debt carries default risk but still generates more resources than risk-free debt and less than the maximum that could be gained with risky borrowing. In the first zone, debt increases with government expenditures so as to serve the standard role of smoothing lump-sum taxation, while in the other two the stronger default incentives make debt fall as government expenditures rise.

We study the model's quantitative predictions by solving numerically the recursive Markov equilibrium without commitment using parameter values calibrated to the Eurozone. The model supports equilibria with debt and default, and dynamics both over the long-run and around default events are qualitatively in line with key features of the data. Comparing peak values for high-default-risk events excluding default (since most Eurozone countries did not default), the model approximates well the average total, domestic and external debt ratios, and produces spreads even higher than in the data. In the long-run, the model matches the ranking of the correlations of government expenditures with spreads, consumption, and net exports. Matching these correlations is important because government expenditure shocks (the model's only aggregate shock) are central to the model's feedback mechanism, since these shocks weaken (strengthen) repayment incentives when they are high (low). The model also nearly matches the relative variability of consumption and net exports, and produces correlations with disposable income that have the same signs as in the data.

Defaults in the model have a low long-run frequency of 1.2 percent, very near the 1.1 percent unconditional frequency of domestic defaults in the Reinhart-Rogoff database. As in the data, debt and spreads rise rapidly and suddenly in the periods close to a default, while in earlier periods, debt is stable and sold at the risk-free price. External debt falls relative to domestic debt as a default approaches, and is about 55 percent of total debt when default hits. Thus, to an observer of the model's time series, a debt crisis looks like a sudden shock following a period of stability and with small variations in external debt. The debt buildup coincides with relatively low government expenditures, which at first strengthen repayment incentives and reduce the social welfare gain of default, but then as debt rises have the opposite effects and yield rapidly rising spreads. Default occurs with a modest increase in government purchases, which at the higher debt is enough to shift the distribution of individual default gains to yield a positive social gain of default.

We use the model's equilibrium recursive functions to show that there is significant dispersion in the effects of changes in debt and government expenditures on individual gains from default across agents with different bond holdings and income. This dispersion reflects differences in the agents' valuation of the self-insurance, liquidity, and risk-sharing benefits of debt, and also in the effect of the exogenous income shock of default. As a result of these differences, the social distribution of default gains shifts markedly across states of debt and government purchases, producing large shifts in the social welfare gain of default. The bond pricing function has a shape similar to that in EG models: starting at the risk-free price when debt is low and falling sharply as debt starts to carry default risk. The associated debt Laffer curves shift downward and to the left at higher realizations of government expenditures and display the three zones across which the optimal debt moves.

We conduct a sensitivity analysis to study the effects of changes in the social welfare weights, the parameters that drive self-insurance incentives, the income tax rate, the exogenous cost of default, and allowing for partial default. The quantitative results hinge on how default incentives vary with each scenario, but in all scenarios the model sustains average debt ratios comparable to those in the data and at a low but positive default frequency. Spreads are large in most cases, except when the exogenous default cost is removed completely, but in this scenario the debt that can be sustained is still significantly constrained by the government's inability to commit. In this case, debt is optimally chosen to be risk-free because otherwise bond prices drop too much, so that choosing risky debt generates fewer borrowed resources.

This paper is part of the growing research programs on optimal debt and taxation in incomplete markets models with heterogeneous-agents and on external sovereign default. Well-known papers in the heterogeneous-agents literature explore the implications of public debt in models in which debt provides similar benefits as in our model (e.g., Aiyagari and McGrattan [6], Azzimonti et al. [11], Floden [27] and Heathcote [36]). Aiyagari and McGrattan [6] quantify the welfare effect of debt in a setup with capital and labor, distortionary taxes, and an exogenous supply of debt. Calibrating the model to U.S. data and solving it for a range of debt ratios, they found a maximum welfare gain of 0.1 percent. In contrast, a variant of our model without default risk predicts that the gain of avoiding an unanticipated, once-and-for-all default can reach 1.35 percent. Azzimonti et al. [11] link wealth inequality and financial integration with the demand and supply for public debt to explain growing debt ratios in the last decade. Heathcote [36] derives non-Ricardian implications from stochastic proportional tax changes because of borrowing constraints. Floden [27] shows that transfers rebating distortionary tax revenue dominate debt for risk-sharing of idiosyncratic risk. As in this paper, these papers embody a mechanism that hinges on the variation across agents in the benefits of public debt, but they differ from this paper in that they abstract from sovereign default.

The recent literature on external default has made important contributions to the classic EG model, following the early work by Aguiar and Gopinath [5], Arellano [8] and Yue [51].[6] Of particular relevence for our analysis are studies dealing with tax and expenditure policies, external debt denominated in domestic currency, and models of international coordination (e.g., Cuadra et al. [20]), Dias et al. [22], and Du and Schreger [25]). The key difference relative to our setup is that these studies assume a representative agent and do not focus on default on domestic debt-holders. Other studies in this literature related to our work include those focusing on the effects of default on domestic agents, foreign and domestic lenders, optimal taxation, the role of secondary markets, discriminatory versus nondiscriminatory default and bailouts (e.g., Guembel and Sussman [30]; Broner et al. [15]; Gennaioli et al.[29]; Aguiar and Amador [1]; Mengus [42]; Di Casola and Sichlimiris [23]; Perez [46]; Bocola [14]; Sosa Padilla [50]; and Paczos and Shakhnov [44]). As in some of these studies, default in our setup is non discriminatory, but in general, these studies abstract from distributional default incentives and social benefits of debt for self-insurance, liquidity and risk-sharing.

There is also a more recent literature in the intersection of heterogeneous-agents and external default models. Bai and Zhang [12] study a model with a continuum of heterogeneous countries each facing a participation constraint. Our model differs in that we look at a continuum of domestic heterogeneous agents with a single government, and we model the default decision instead of a participation constraint. Dovis et al. [24] study distributional incentives to default on external debt in a model with heterogeneous agents. Our work is similar in that both models produce debt dynamics characterized by periods of sustained increases followed by large reductions, and in both default has distributional incentives. The two differ in that they focus on external default, and when they introduce uncertainty they study equilibria without default spreads and assume complete markets, which alters the social value of debt. In addition, we conduct a quantitative analysis exploring the model's ability to explain the observed dynamics of debt and spreads. Aguiar et al. [3] study a setup in which the heterogeneity is across country members of a monetary union, instead of across agents inside a country. They show how lack of commitment and fiscal policy coordination leads countries to overborrow due to a fiscal externality, focusing on public debt traded across countries by risk-neutral investors, instead of default on risk-averse domestic debt holders. Andreasen et al. [7] and Jeon and Kabukcuoglu [33] study models in which domestic income heterogeneity plays a role in the determination of external defaults, and Arellano et.al. [9]

[6] Panizza et al. [45]; Aguiar and Amador [2]; and Aguiar et al. [4] survey the literature in detail.

and Rojas [49] study sovereign risk in models with heterogeneous firms.

The rest of this paper is organized as follows: Section 2 describes the model and defines the recursive Markov equilibrium. Section 3 examines two variants of the model simplified to highlight distributional default incentives (in a one-period setup without uncertainty) and the social value of public debt (as the welfare cost of a surprise once-and-for-all default). Section 4 discusses the calibration procedure and examines the model's quantitative implications. Section 5 provides conclusions. An online Appendix provides details on the data, solution method and additional features of the quantitative results.

2 A Bewley Model of Domestic & External Default

Consider an economy inhabited by a continuum of private agents with aggregate unit measure and a benevolent government. There is also a pool of risk-neutral international investors that face an opportunity cost of funds equal to an exogenous, world-determined real interest rate. Domestic agents face two types of non insurable shocks: idiosyncratic income fluctuations and aggregate shocks in the form of fluctuations in government expenditures and the possibility of sovereign default. Asset markets are incomplete because the only available vehicle of savings are one-period, non-state-contingent government bonds, which both domestic agents and international investors can buy. The government also levies proportional income taxes, pays lump-sum transfers, and chooses whether to repay its debt or not.

2.1 Private Agents

Agents have a standard constant relative risk aversion (CRRA) utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad u(c_t) = c_t^{1-\sigma}/(1-\sigma), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, c_t is individual consumption, and σ is the coefficient of relative risk aversion.

Each period, an agent draws an idiosyncratic income realization from a discrete Markov process with a bounded, non-negative set of realizations $y_t = \{\underline{y}, \ldots, \overline{y}\} \in \mathcal{Y}$, and a transition probability matrix defined as $\pi(y_{t+1}, y_t)$, with stationary distribution $\pi^*(y)$. These shocks have zero mean across agents so that aggregate income is non-stochastic.

Agents can buy government bonds in the amount $b_{t+1} \in \mathcal{B} \equiv [0, \infty)$. They cannot take short positions, so they face the standard no-borrowing constraint $b_{t+1} \geq 0$. The distribution of agents over debt and income at date t is denoted as $\Gamma_t(b, y)$.

If the government repays its outstanding debt, an individual agent's budget constraint at date t is:

$$c_t + q_t b_{t+1} = y_t(1 - \tau^y) + b_t + \tau_t. \tag{2}$$

The right-hand side of this expression shows the after-tax resources available to the agent: the agent's income realization y_t net of a proportional income tax levied at rate τ^y, income from the payout on individual debt holdings b_t, and lump-sum transfers τ_t. This disposable income pays for consumption and purchases of new government bonds b_{t+1} at the price q_t.

Before writing the individual budget constraint for default states, we note two important assumptions about default costs. First, the government re-enters the bond market in the following period after a default. Hence, we relax the standard assumption of EG models according to which one cost of default is exclusion from credit markets either forever or for a stochastic number of periods. Second, we allow for the possibility that default imposes an exogenous income cost akin to those widely used in the sovereign default literature. We use it to construct a calibration comparable to those in the literature, and show later that the model sustains debt even without it. In EG models, this cost is modeled as a function of the realization of a stochastic endowment and designed so that default costs are higher at higher income. Since aggregate income is constant in our setup, we model the cost as a function of the realization of g instead. Aggregate income when a default occurs falls by the amount $\phi(g)$, which is decreasing in g, so that that the default cost is higher when income is higher.

If the government defaults, an individual agent's budget constraint is:

$$c_t = y_t(1 - \tau^y) - \phi(g) + \tau_t. \tag{3}$$

Three important effects of government default on households are implicit in this constraint: (a) Bond holdings of all agents are wiped out, which hurts more agents with large bond holdings; (b) the public debt market freezes, so that agents drawing high (low) income realizations cannot buy (sell) bonds for self-insurance, and public debt cannot provide liquidity to credit-constrained agents; and (c) everyone's income falls by $\phi(g)$.

2.2 Government

Each period, the government collects $\tau^y Y$ in income taxes, pays for g_t, and, if it repays outstanding debt B_t, it chooses the amount of new bonds to sell B_{t+1} from the non-negative set $B_{t+1} \in \mathcal{B} \equiv [0, \infty)$. The tax rate τ^y is exogenous, constant, and strictly positive. Government expenditures evolve according to a discrete Markov process with realization set $\mathcal{G} \equiv \{\underline{g}, \ldots, \overline{g}\}$ and associated transition probability matrix $F(g_{t+1}, g_t)$.[7] The processes for

[7]Nothing prevents consumption for agents with sufficiently low income from becoming nonpositive in default states (i.e., $c_t = y_t(1 - \tau^y) - \phi(g) - g_t + \tau^y Y \leq 0$), although this does not happen in our quantitative exercises. Ruling this out would require a restriction on the y and g processes to ensure positive consumption

y and g are assumed to be independent for simplicity.[8] Lump-sum transfers are determined endogenously as explained below, and their sign is not restricted, so $\tau_t < 0$ represents lump-sum taxes. As discussed in Heathcote et al. [38], affine tax functions, like the one used here, approximate well actual tax and transfer programs. Notice also that $\tau^y Y$ is constant (since both τ^y and Y are constant), but *individual* income tax bills fluctuate with y.

The government has the option to default on B_t at each date t. The default choice is denoted by the binary variable d_t, with $d_t = 1$ indicating default. The government's preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \int_{\mathcal{B}\times\mathcal{Y}} \sum_{y_i \leq y} u(c_t(b_t, y_t)) d\omega(b_t, y_t). \tag{4}$$

Hence, the government is a benevolent planner who maximizes a social welfare function that aggregates the utility of agents who own bonds b and draw income y using the welfare weights given by the function $\omega(b, y)$, which is defined as follows:

$$\omega(b, y) = \sum_{y_i \leq y} \pi^*(y_i) \left(1 - e^{-\frac{b}{\bar{\omega}}}\right). \tag{5}$$

In the y dimension, the weights match the long-run distribution of individual income $\pi^*(y)$. In the b dimension, the weights are given by an exponential function with scale parameter $\bar{\omega}$, which we label "creditor bias," because with a higher $\bar{\omega}$ the government weights more the utility of agents who hold larger bond positions. In the quantitative exercise, we first calibrate these weights to match the mean spreads observed in the data, and then consider variations of the $\omega(b, y)$ function, including a case in which we replace it with the average distribution of bonds and income in the economy. Bhandari et. al. [13] and Chang et al. [16] follow similar approaches of calibrating welfare weights to match data targets and using also weights given by the long-run average of the model's wealth distribution.

B_{t+1} and τ_t are determined after the default decision. Lump-sum transfers are set as needed to satisfy the government budget constraint. If the government repays, once the debt is chosen, the government budget constraint implies:

$$\tau_t^{d=0} = \tau^y Y - g_t - B_t + q_t B_{t+1}. \tag{6}$$

If the government defaults, the current repayment is not made and new bonds cannot be issued. Thus, default entails a one-period freeze of the public debt market. The government

at the lowest y for all g: $g + \tau^y Y < (1 - \tau^y)\underline{y} - \phi(g)$.

[8] The independence assumed here is between *individual* income and *aggregate* government expenditures.

budget constraint implies then:

$$\tau_t^{d=1} = \tau^y Y - g_t. \tag{7}$$

The above treatment of transfers is analogous to that in the EG models, except that in EG models the resources generated by government debt (plus the primary surplus if any) are transferred to a representative agent, instead of to a continuum of heterogeneous agents. In the calibration, these transfers approximate a data average on welfare and entitlement payments to individuals net of capital tax revenues, which are not modeled.

2.3 Foreign Investors

Foreign investors are modeled in the same way as in EG models: risk-neutral investors with "deep pockets" who face an opportunity cost of funds $\bar{r}$. Their holdings of domestic government debt are denoted $\hat{B}_{t+1}$, which is also the economy's net foreign asset position (NFA), and their expected profits are: $\Omega_t = -q_t\hat{B}_{t+1} + \frac{(1-p_t)}{(1+\bar{r})}\hat{B}_{t+1}$. In this expression, p_t is the probability of default at $t+1$ perceived as of date t, $-q_t\hat{B}_{t+1}$ is the value of bond purchases in real terms (i.e., the real resources lent out to the government at date t), and $\frac{(1-p_t)}{(1+\bar{r})}\hat{B}_{t+1}$ is the expected present value of the debt payout at $t+1$, which occurs with probability $(1-p_t)$. Arbitrage implies that $\Omega_t = 0$, which yields this well-known no-arbitrage condition:

$$q_t = \frac{(1-p_t)}{(1+\bar{r})}. \tag{8}$$

2.4 Timing of transactions

The timing of decisions and market participation at any date t is as follows:

1. Date t begins. The values of y and g are realized.
2. Individual states $\{b, y\}$, the distribution $\Gamma_t(b, y)$, and aggregate states $\{B, g\}$ are known, and income taxes are paid.
3. The government makes its optimal debt and default decisions, and agents make their optimal plans.
    - If the government repays, $d_t = 0$, B_t is paid, the market of government bonds opens, new debt B_{t+1} is issued, lump-sum transfers are set according to equation (6), private agents choose b_{t+1}, and q_t is determined.
    - If the government defaults, $d_t = 1$, B_t and all domestic and foreign holdings of government bonds are written off, the debt market closes, and lump-sum transfers are set according to equation (7).
4. Agents consume, and date t ends.

2.5 Recursive Markov Equilibrium

We study a Recursive Markov Equilibrium (RME) in which the government chooses debt and default optimally from a set of Conditional Recursive Markov Equilibria (CRME) that represent optimal allocations and prices conditional on given debt and default choices. To characterize these equilibria, we first rewrite the optimization problem of domestic agents and the no-arbitrage condition of foreign investors in recursive form.

The aggregate state variables are B and g.[9] The optimal debt issuance and default decision rules are characterized by the recursive functions $B'(B,g)$ and $d(B,g) \in \{0,1\}$, respectively.[10] The probability of default at $t+1$ evaluated as of t, denoted $p(B',g)$, is:

$$p(B',g) = \sum_{g'} d(B',g')F(g',g). \tag{9}$$

The probability of defaulting at t+1 on an amount of debt B' conditional on information available at t is formed by adding up the transitional probabilities from g to g' for which, at the corresponding g', the government would choose to default.

The state variables for an individual agent are the agent's bond holdings and income (b,y) and the aggregate states (B,g). Agents take as given $d(B,g)$, $B'(B,g)$, $\tau^{d=0}(B,g)$, and $\tau^{d=1}(g)$, a bond pricing function $q(B',g)$, and the Markov processes of y and g. These functions allow agents to project the evolution of aggregate states and bond prices, so that an agent's continuation value if the government *has chosen* to repay ($d(B,g)=0$) and issued $B'(B,g)$ bonds can be represented as the solution to the following problem:

$$V^{d=0}(b,y,B,g) = \max_{\{c\geq 0, b'\geq 0\}} \left\{u(c) + \beta E_{(y',g')|(y,g)}[V(b',y',B',g')]\right\} \tag{10}$$

s.t.

$$c + q(B'(B,g),g)b' = b + y(1-\tau^y) + \tau^{d=0}(B,g), \tag{11}$$

where $V(b',y',B',g')$ (without superscript) is the next period's continuation value for the agent *before* the default decision has been made that period.

Similarly, the continuation value if the government *has chosen* to default is:

$$V^{d=1}(y,g) = u(y(1-\tau^y) - \phi(g) + \tau^{d=1}(g)) + \beta E_{(y',g')|(y,g)}[V^{d=0}(0,y',0,g')]. \tag{12}$$

[9]It is critical to note that $\Gamma_t(b,y)$ is not a state variable despite the presence of aggregate risk. This is because it does not affect bond prices directly, since q_t satisfies the foreign investors' no-arbitrage condition, and the welfare weights are set by $\omega(b,y)$.

[10]In the recursive notation, variables x_t and x_{t+1} are denoted as x and x', respectively.

Finally, the continuation value at date t *before* the default decision is:

$$V(b,y,B,g) = (1-d(B,g))V^{d=0}(b,y,B,g) + d(B,g)V^{d=1}(y,g). \tag{13}$$

The solution to this problem yields the individual decision rule $b' = h(b,y,B,g)$ and the associated value functions $V(b,y,B,g), V^{d=0}(b,y,B,g)$ and $V^{d=1}(y,g)$. By combining the agents' bond decision rule, the Markov transition matrices of y and g, and the government's default decision, we can obtain expressions that characterize the evolution of the distribution of bonds and income under repayment and default. The distribution at the beginning of $t+1$ is denoted $\Gamma' = H^{d' \in \{0,1\}}(\Gamma, B, g, g')$. If $d(B', g') = 0$, for $\mathcal{B}_0 \subset \mathcal{B}$, $\mathcal{Y}_0 \subset \mathcal{Y}$, Γ' is:

$$\Gamma'(\mathcal{B}_0, \mathcal{Y}_0) = \int_{\mathcal{Y}_0, \mathcal{B}_0} \Big\{ \int_{\mathcal{Y}, \mathcal{B}} I_{\{b'=h(b,y,B,g) \in \mathcal{B}_0\}} \pi(y', y) d\Gamma(b,y) \Big\} db' dy', \tag{14}$$

where $I_{\{\cdot\}}$ is an indicator function that equals 1 if $b' = h(b,y,B,g)$ and zero otherwise. Note that g' is an argument of $H^{d' \in \{0,1\}}$ because Γ' is formed after d' is known, and d' depends on g'. If $d(B', g') = 1$, for $\mathcal{Y}_0 \subset \mathcal{Y}$, Γ' is given by:

$$\Gamma'(\{0\}, \mathcal{Y}_0) = \int_{\mathcal{Y}_0} \Big\{ \int_{\mathcal{Y}, \mathcal{B}} \pi(y', y) d\Gamma(b,y) \Big\} db' dy', \tag{15}$$

and zero otherwise. This is because at default all households' bond positions are set to zero, and hence Γ' is determined only by the evolution of the income process (i.e., if the government defaults, $\Gamma'(b,y) = \pi^*(y)$ for $b = 0$ and zero for any other value of b).

The recursive form of the foreign investors' no-arbitrage condition is:

$$q(B', g) = \frac{(1 - p(B', g))}{(1 + \bar{r})}. \tag{16}$$

The wedge between the price at which foreign investors are willing to buy sovereign debt ($q(\cdot)$) and the price of international bonds ($1/(1+\bar{r})$) compensates them for the risk of default measured by the default probability. At equilibrium, bond prices and risk premia are formed by a combination of exogenous factors (the Markov process of g) and the endogenous government decision rules $B'(B,g)$ and $d(B,g)$. Despite the similarity with the debt pricing condition of EG models, however, the mechanism determining default probabilities and default risk is very different. In EG models, these probabilities follow from the values of continuation v. default of a representative agent, which exclude the welfare of the government's creditors. In this model, default probabilities are determined by comparing continuation v. default values for the social welfare function, which include the welfare of domestic creditors and depend on the dispersion of individual payoffs of default v. repayment. Hence, inequality affects default risk via changes in the dispersion of these payoffs. Later in this Section, we characterize the mechanism driving the dispersion in payoffs, and in Section 4 we illustrate it quantitatively.

Using the foreign lenders' no-arbitrage condition to price the debt implies that they are the marginal buyer. This assumption is valid if at that price $\hat{B}' \geq 0$, indicating that domestic demand for public debt is smaller than the supply the government issues, which makes NFA negative (i.e. the country is a net external borrower). Hence, at the no-arbitrage price foreign lenders are indifferent between the sovereign bond and the world asset that pays $\bar{r}$. Relative to that price, for a given (B', g) and abstracting from changes in future default incentives, a lower price results in excess demand for sovereign debt (since foreign and domestic demand increase) and a higher price results in excess supply (since foreign and domestic demand decrease). This assumption is validated quantitatively, because in the experiments with the baseline calibration and several variants $\hat{B}' \geq 0$ in all periods along the equilibrium path.[11]

We now define the CRME for given debt and default decision rules. The definition includes the following three aggregate variables. First, aggregate consumption is given by:

$$C = \int_{\mathcal{Y}\times\mathcal{B}} c(b, y, B, g) \;\; d\Gamma(b, y), \tag{17}$$

where $c(b, y, B, g)$ corresponds to individual consumption by each agent. Second, aggregate (nonstochastic) income is:

$$Y = \int_{\mathcal{Y}\times\mathcal{B}} y \; d\Gamma(b, y). \tag{18}$$

Third, aggregate domestic demand for newly issued bonds is:

$$B^{d'} = \int_{\mathcal{Y}\times\mathcal{B}} h(b, y, B, g) \; d\Gamma(b, y). \tag{19}$$

Definition: Given an initial distribution $\Gamma_0(b, y)$, a default decision rule $d(B, g)$, a government debt decision rule $B'(B, g)$, an income tax rate τ^y, and lump-sum transfers $\tau^{d\in\{0,1\}}$ defined by (6) and (7), a **CRME** is defined by a value function $V(b, y, B, g)$ with associated household decision rule $b' = h(b, y, B, g)$, a transition function for the distribution of bonds and income $H^{d'\in\{0,1\}}(B, g, g')$, a default probability function $p(B', g)$, and a bond pricing function $q(B', g)$ such that:

1. Given the $q(B', g)$ and government policies, $V(b, y, B, g)$ and $h(b, y, B, g)$ solve the individual agents' optimization problem.

[11] The solution algorithm assumes that if $\hat{B}' < 0$ domestic agents buy foreign bonds at the price $1/(1 + \bar{r})$ for the amount by which their demand exceeds the bonds sold by the government, and NFA becomes positive. Quantitatively, this only happens in one of the sensitivity experiments with a high CRRA coefficient.

2. The foreign investors' arbitrage condition (equation (16)) holds.
3. The transition function of the distribution of bonds and income satisfies conditions (14) and (15) in states with repayment and default, respectively.
4. The government budget constraints (6) and (7) hold.
5. The market of government bonds clears: $\hat{B}' + B^{d'} = B'$.
6. The aggregate resource constraint of the economy is satisfied. If the government repays: $C + g = Y + \hat{B} - q(B', g)\hat{B}'$, and if the government defaults: $C + g = Y - \phi(g)$.

The model's RME is defined as a CRME in which $B'(B, g)$ and $d(B, g)$ are optimal government choices. If $B > 0$ at the beginning of period t, the government sets its optimal $d(B, g)$ as the solution to the following problem:

$$\max_{d \in \{0,1\}} \left\{ W^{d=0}(B, g), W^{d=1}(g) \right\}, \tag{20}$$

where the social value of continuation is:

$$W^{d=0}(B, g) = \int_{\mathcal{Y} \times \mathcal{B}} V^{d=0}(b, y, B, g) d\omega(b, y),$$

and the social value of default is:

$$W^{d=1}(g) = \int_{\mathcal{Y} \times \mathcal{B}} V^{d=1}(y, g) d\omega(b, y).$$

If the government chooses to repay, it also chooses an optimal amount of new debt to issue. To characterize this choice, assume that the government first considers an intermediate step in which it evaluates how any arbitrary debt level (denoted $\tilde{B}'$) affects individual agents. The corresponding value for an agent with a (b, y) pair is the solution to the following problem:

$$\tilde{V}(b, y, B, g, \tilde{B}') = \max_{\{c \geq 0, b' \geq 0\}} u(c) + \beta E_{(y',g')|(y,g)}[V(b', y', \tilde{B}', g')] \tag{21}$$

$$\text{s.t.} \begin{cases} c + q(\tilde{B}', g)b' = y(1 - \tau^y) + b + \tau \\ \tau = \tau^y Y - g - B + q(\tilde{B}', g)\tilde{B}'. \end{cases}$$

Note that $V(\cdot)$ in the right-hand side of this problem is given by the solution to the agents' problem (10), which implies that the government is assessing the lifetime utility effect of deviating from the optimal policy only in the current period. The optimal debt issuance decision rule then solves this problem:

$$\max_{\tilde{B}'} \int_{\mathcal{Y} \times \mathcal{B}} \tilde{V}(b, y, B, g, \tilde{B}') d\omega(b, y). \tag{22}$$

Now we can define the model's RME:

Definition: A **RME** is a CRME in which the default decision rule $d(B, g)$ solves problem (20) and the debt decision rule $B'(B, g)$ solves problem (22).

2.6 Optimality Conditions & Feedback Mechanism

We discuss next key features of the model's optimality conditions that illustrate the feedback mechanism linking default incentives, default risk, the distribution of bond holding, and the dispersion of individual gains from a default. This material will also be used for the analysis of the quantitative results in Section 4.

(a) Default risk and demand for government bonds.

Assuming the agents' value functions are differentiable, the first-order condition for b' in a state in which the government has repaid is:

$$-u'(c)q(B', g) + \beta E_{(y',g')|(y,g)} \left[V_1(b', y', B', g')\right] \leq 0, \ = 0 \text{ if } \ b' > 0, \tag{23}$$

where $V_1(\cdot)$ denotes the derivative of $V(\cdot)$ with respect to its first argument. Using the envelope theorem, this condition can be rewritten as:

$$u'(c) \leq \beta E_{(y',g')|(y,g)} \left[(1 - d(B', g'))\frac{u'(c')}{q(B', g)}\right], \tag{24}$$

which holds with equality if $b' > 0$. The right-hand-side of this expression shows that, in assessing the marginal benefit of buying an extra unit of b', agents take into account the possibility of a future default. In states in which a default is expected, $d(B', g') = 1$ and agents assign zero marginal benefit to buying bonds.[12] In states in which repayment is expected, the marginal benefit of buying bonds is $\frac{u'(c')}{q(B',g)}$, which includes the default risk premium embedded in the price paid for newly issued bonds.

These results imply that, conditional on B', a larger default set (i.e., a larger set of values of g' for which the government defaults) reduces the expected marginal benefit of an extra unit of b'. In turn, this implies that, everything else equal, a higher default probability reduces individual domestic demand for government bonds unless an agent has high enough (b, y) to be willing to take the risk of demanding more bonds at higher risk premia (lower bond prices) and expect future adjustments in τ. This has important distributional implications because, as we explain below, the government internalizes when making the default decision how it affects the probability of default and bond prices. Notice also that future default risk at any date later than t, not just $t+1$, influences the agents' demand for b_{t+1} because of the

[12] In the next Section, we solve also variants of the model that allow for partial defaults, in which the marginal benefit of buying bonds in the default state is positive, instead of zero.

time-recursive structure of Euler equation (24). Hence, even if debt is offered at the risk-free price at t, bond demand still responds negatively to default risk if default has positive probability beyond $t+1$ (i.e., agents factor in the risk of a future default wiping out their wealth as they build their individual stock of savings).

(b) Self-insurance, liquidity, and risk-sharing roles of public debt

The roles of public debt as a vehicle for self-insurance, liquidity, and risk-sharing can be illustrated by combining the agents' budget constraint with the government budget constraint using the variable transformation $\tilde{b} = (b - B)$ to obtain:

$$c = y + \tilde{b} - q(B', g)\tilde{b}' - \tau^y(y - Y) - g \tag{25}$$

$$\tilde{b}' \geq -B' \tag{26}$$

The liquidity benefit of public debt is evident in condition (26): Selling new debt (B') relaxes the borrowing constraint for agents for whom it is binding. That is, it provides them with liquidity in the form of extra resources for consumption. The self-insurance role can be inferred from condition (25): Agents who draw sufficiently high y, regardless of their existing holdings of b, want to buy more debt, and agents who draw sufficiently low y want to draw from their accumulated precautionary savings. The risk-sharing benefit is also reflected in condition (25): by buying (selling) debt, agents drawing high (low) income share the risk of idiosyncratic income fluctuations with each other, albeit imperfectly.

The role of income taxation as an alternative means to improve risk-sharing of idiosyncratic income shocks is also evident in condition (25): The term $-\tau^y(y - Y)$ implies that agents with below (above) average income effectively receive (pay) a subsidy (tax). If income is taxed 100 percent, full social insurance against these shocks is provided (i.e. perfect risk sharing), and all agents after-tax income equals Y. But this still would not remove the need for precautionary savings, because aggregate shocks to government expenditures as well as government defaults cannot be insured away.

In making its debt and default choices, the government trades off the above social benefits of debt v. the distributional implications of debt repayment and issuance. At any date t, repayment of B results in regressive redistribution in favor of agents with sufficiently large holdings of the outstanding public debt (i.e., agents with $\tilde{b} > 0$, or "above average" holdings relative to B). In contrast, issuing new debt B' causes progressive redistribution in favor of agents who buy sufficiently little or no new debt (i.e., agents with $\tilde{b}' < 0$, or below average holdings relative to B'). The magnitude and cross-sectional dispersion of these effects changes over time as the endogenous distribution of bond holdings evolves.

The two forms of redistribution are connected inter-temporally. Under repayment, more progressive redistribution at t implies more regressive redistribution in the future. Because of the government's inability to commit to repay, however, the extent to which progressive redistribution can be implemented at t is inversely related to the expectation that in the future the planner may wish to avoid regressive redistribution by defaulting. This is because the price at which new debt is sold at t depends negatively on the probability of a default at $t+1$. This weakens the government's ability to produce progressive redistribution, because $q(B', g)$ falls as B' rises, since the default probability is nondecreasing in B'. Hence, the resources generated by debt, $q(B', g)B'$, follow a Laffer curve similar to the familiar one from EG models, because in those models bond prices also fall and default probabilities also rise as debt rises. In EG models, however, the resources generated by debt are transferred to a representative agent, while here they are transferred to heterogeneous agents, and although τ is uniform across agents, the heterogeneity in bond holdings makes the transfers generated by debt vary across agents (inversely with the value of $\tilde{b}'$).

(c) Feedback mechanism

The feedback mechanism driving the model follows from the features highlighted in *(a)* and *(b)*. In particular, it is critical to note that the probability of default and the price of debt at t depend on the dispersion of payoffs of default versus repayment across agents at $t+1$, because the government's social welfare function aggregates these payoffs to make the default decision. This is a feedback mechanism because the debt issued at t becomes the initial debt outstanding at $t+1$ and this matters for the dispersion of the agents' payoffs, affecting agents with different (b, y) differently, as we illustrate quantitatively in Section 4. Thus, the debt issued at t affects the default decision at $t+1$, which affects default probabilities and bond prices at t, which in turn affects the agents' date-t demand for bonds and the government's debt choice. The links of this chain are connected via the distributional effects of debt issuance, the social benefits of debt, and the dispersion of payoffs of default versus repayment across agents.

This feedback mechanism cannot be fully characterized analytically in closed form, but we can gain further intuition about it as follows. Define $\Delta c \equiv c^{d=0} - c^{d=1}$ as the difference in consumption across repayment and default in a given period for an agent who has a particular $\tilde{b}$ when the aggregate states are (B, g). Δc can be expressed as:

$$\Delta c \;=\; \tilde{b} - q(B', g)\tilde{b}' + \phi(g) \tag{27}$$

The right-hand-side of this expression includes the distributional effects noted in *(b)*. If inequality in the initial distribution of government debt is high, so that a larger fraction of agents have $\tilde{b} < 0$, and strong default incentives make default risk high, so that $q(B', g)$ is low, a larger fraction of agents have $\Delta c < 0$ and are more likely to be better off with a default, which in turn justifies the distributional incentives to default. The opposite is true if initial inequality in bond holdings and default risk are low. Moreover, given initial inequality and bond prices, higher inequality in the end-of-period distribution of government debt (i.e., a larger fraction of agents with $\tilde{b}' < 0$) reduces the fraction of agents with $\Delta c < 0$. Hence, changes in the distribution of public debt, default incentives, and default risk interact in determining the dispersion of $\Delta c < 0$ across agents. The interaction does not follow a monotonic pattern, however, because Δc can be negative also for agents with sufficiently high (b, y) who buy more risky debt attracted by the higher risk premia. Thus, as we look across agents with different bond holdings, $\frac{db'}{dB}$ changes sign and, for some large bond holders it can even be the case that Δc decreases with B.

It is also important to note that Δc alone does not determine individual payoffs of default or repayment. These depend on both date-t differences in consumption (or utility) and differences in the continuation values $V^{d=0}(b', y', B', g')$ and $V^{d=0}(0, y', 0, g')$. Still, the interaction between the distribution of government debt, consumption differentials across default and repayment states, and default risk discussed previously is illustrative of the feedback mechanism driving the model. Moreover, we can also establish that, since $V^{d=0}$ is increasing in b as in standard heterogeneous-agents models, there is a threshold value of bond holdings $\hat{b}(y, B, g)$, for given (y, B, g), such that agents with $b \geq \hat{b}$ prefer repayment (since $V^{d=0}(b, y, B, g) \geq V^{d=1}(y, g)$), and those with $b < \hat{b}(y, B, g)$ prefer default. That is,

$$\hat{b}(y, B, g) = \{b \in \mathcal{B} : V^{d=0}(b, y, B, g) = V^{d=1}(y, g)\}. \tag{28}$$

We can conjecture that $\hat{b}(y, B, g)$ is increasing in B because the difference in τ under repayment v. default widens at higher debt: Higher debt reduces transfers both because of the higher repayment on B even without default risk and because higher risk premia reduces the price at which B_{t+1} is sold, causing a debt-overhang effect (i.e., additional borrowing is used to service debt). As a result, agents need to have higher individual wealth in order to prefer repayment as B rises. This conjecture was verified numerically in Appendix A-6.

3 Distributional Incentives & Social Value of Debt

This Section examines two simplified versions of the model. First, a one-period variant designed to isolate the distributional default incentives. By construction, this setup abstracts from the social benefits of debt for self-insurance, liquidity, and risk-sharing. The second variant isolates these social benefits by quantifying the welfare costs of a once-and-for-all default. The quantitative analysis of the full model in the next Section combines the elements isolated in these exercises.

3.1 Distributional default incentives

Consider a one-period variant of the model without uncertainty and a predetermined distribution of debt ownership. There are two types of agents: A fraction γ are $L-$type agents with low bond holdings denoted b^L, and the complement $(1-\gamma)$ are $H-$type agents with high bond holdings b^H. The government has an exogenous stock of debt B, which is deciding whether to repay or not, and default may entail an exogenous cost that reduces income by a fraction $\phi \geq 0$. We include this cost because, as we show below, distributional incentives alone cannot sustain debt in this simple model unless the social welfare function weights L types by less than γ. This cost can proxy for the endogenous default costs driven by the social value of debt in the full model.

The budget constraints of the government and households under repayment are $\tau^{d=0} = B-g$ and $c^i = y+\tau^{d=0}+b^i$ (for $i = L, H$), respectively, and under default are $\tau^{d=1} = -g$ and $c^i = (1-\phi)y + \tau^{d=1}$ (for $i = L, H$), respectively. The utility function can be as in Section 2, but what is necessary for the results derived here is that it be increasing and strictly concave.

Since there is only one period, the agents' choices of b^L and b^H (or equivalently their consumption allocations) are predetermined. In particular, we consider a given exogenous "decentralized" distribution of debt holdings characterized by a parameter ϵ, so that the bond holdings of L-types are $b^L = B - \epsilon$ and then market-clearing in the bond market requires $b^H = B + \frac{\gamma}{1-\gamma}\epsilon$. We are still assuming agents cannot borrow, so it must be that $\epsilon \leq B$, and since by definition $b^H \geq b^L$, it must be that $\epsilon \geq 0$. Using the budget constraints, the decentralized consumption allocations under repayment and default are $c^L(\epsilon) = y-g-\epsilon$ and $c^H(\gamma,\epsilon) = y - g + \frac{\gamma}{1-\gamma}\epsilon$ and $c^L = c^H = y(1-\phi) - g$, respectively. Notice that under repayment, ϵ determines also the dispersion of consumption across agents, which increases with ϵ, and under default there is zero consumption dispersion.

The main question to understand distributional incentives to default is: How does an arbitrary distribution of bond holdings (or dispersion of consumption) defined by ϵ differ from the one that is optimal for a government with the option to default? To answer this question, we solve the optimization problem of the social planner with the default option. The planner's welfare weight on L-type agents is ω. The optimal default decision solves:

$$\max_{d\in\{0,1\}} \left\{W_1^{d=0}(\epsilon), W_1^{d=1}(\phi)\right\}, \tag{29}$$

where social welfare under repayment is:

$$W^{d=0}(\epsilon) = \omega u(y-g+\epsilon) + (1-\omega)u\left(y-g+\frac{\gamma}{1-\gamma}\epsilon\right) \tag{30}$$

and under default is:

$$W_1^{d=1}(\phi) = u(y(1-\phi) - g). \tag{31}$$

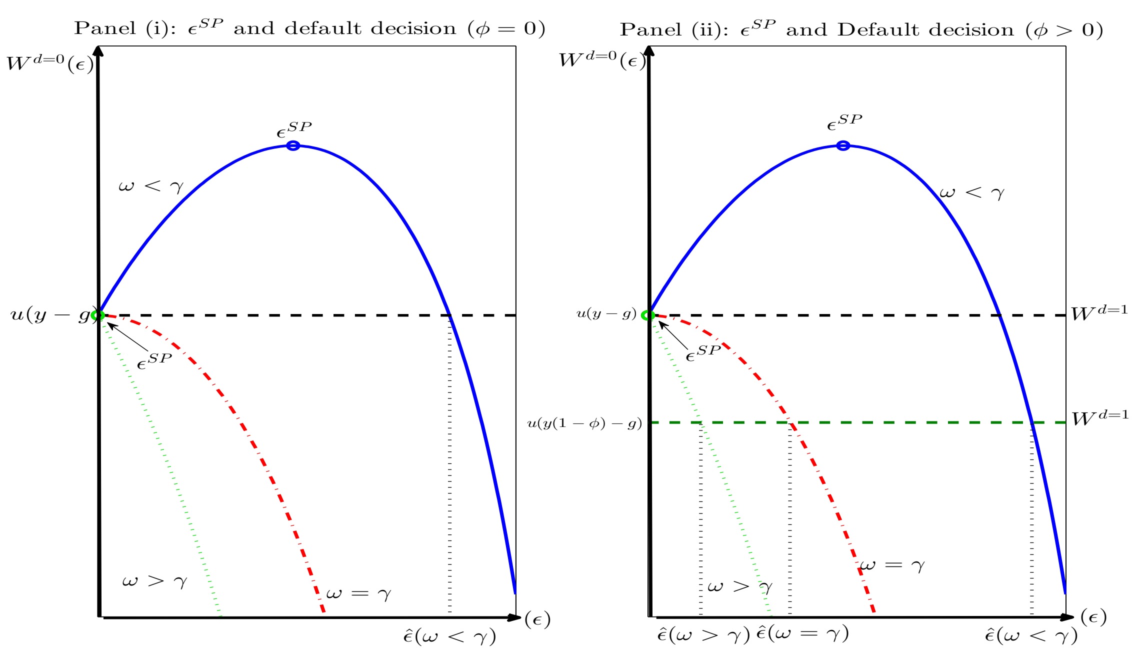
We denote the solution to the above problem as a choice of the socially optimal consumption dispersion ϵ^{SP}, which is the value of ϵ that maximizes $W^{d=0}(\epsilon)$. Since default is the only instrument available to the government to improve consumption dispersion relative to what decentralized allocations for some ϵ support, the planner repays only if doing so allows it to either attain ϵ^{SP} or get closer to it than by defaulting.

The optimality condition for the choice of ϵ^{SP} reduces to:

$$\frac{u'(c^H)}{u'(c^L)} = \frac{u'\left(y - g + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right)}{u'\left(y - g - \epsilon^{SP}\right)} = \left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right). \tag{32}$$

This condition implies that the socially optimal ratio of c^L to c^H increases as ω/γ rises (i.e., as the ratio of the planner's weight on L types to the actual existing mass of L types rises). If $\omega/\gamma = 1$, the planner desires zero consumption dispersion. For $\omega/\gamma > 1$, the planner likes consumption dispersion to favor L types, and the opposite holds for $\omega/\gamma < 1$.

Figure 2: Default Decision with and without Default Costs



The choice of ϵ^{SP} and the default decision in the absence of default costs (i.e., $\phi = 0$) are illustrated in Panel (i) of Figure 2. This figure plots the functions $W^{d=0}(\epsilon)$ for $\omega \gtreqless \gamma$. The value of social welfare at default and the values of ϵ^{SP} for $\omega \gtreqless \gamma$ are also identified in the plot. Notice that the vertical intercept of $W^{d=0}(\epsilon)$ is always $W^{d=1}$ for any values of ω and γ because, when $\epsilon = 0$, there is zero consumption dispersion and that is also the outcome under default. In addition, the bell-shaped form of $W^{d=0}(\epsilon)$ follows from $u'(\cdot) > 0, u''(\cdot) < 0$.[13]

Assume first that $\omega > \gamma$. In this case, ϵ^{SP} would be negative because condition (32) implies that the planner's optimal choice features $c^L > c^H$. However, these consumption allocations are not feasible (since they imply $\epsilon < 0$), and by choosing default the government attains $W^{d=1}$, which is the highest feasible social welfare for $\epsilon \geq 0$. Assuming instead $\omega = \gamma$, it follows that $\epsilon^{SP} = 0$ and default attains exactly that same level of welfare, so default is chosen and it also delivers the efficient level of consumption dispersion. In short, if $\omega \geq \gamma$, the government always defaults for any $\epsilon > 0$, and thus equilibria with debt cannot be supported. Intuitively, the consumption allocations feature $c^H > c^L$ for any $\epsilon > 0$, while the socially efficient consumption dispersion requires $c^H \leq c^L$, and thus default is a second-best policy that brings the planner the closest it can get to ϵ^{SP} (since the only instrument for redistribution is the default choice).

Equilibria with debt can be supported when $\omega < \gamma$. The intersection of the downward-sloping segment of $W^{d=0}(\epsilon)$ with $W^{d=1}$ determines a threshold value $\hat{\epsilon}$ such that default is optimal only for $\epsilon \geq \hat{\epsilon}$. Default is still a second-best policy because with it the planner cannot attain $W^{d=0}(\epsilon^{SP})$, it just gets the closest it can get. As Figure 2 shows, for $\epsilon < \hat{\epsilon}$, repayment is preferable because $W^{d=0}(\epsilon) > W^{d=1}$. Thus, in this simple setup and when default is costless, equilibria with repayment require two conditions: (a) that the government weights H types by more than their share of the government bond holdings *and* (b) that the debt holdings of private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Add now the exogenous cost of default. The solutions are shown in Panel (*ii*) of Figure 2. The key difference is that now it is possible to support repayment equilibria even when $\omega \geq \gamma$. There is a threshold value of consumption dispersion, $\hat{\epsilon}$, separating repayment from default decisions for all values of ω and γ. The government chooses to repay whenever ϵ exceeds $\hat{\epsilon}$ for the corresponding values of ω and γ. It is also evident that the range of values of ϵ for which repayment is chosen widens as γ rises relative to ω. Thus, when default is costly, equilibria with repayment require only that the debt holdings of private agents implicit in ϵ do not produce consumption dispersion in excess of the value of $\hat{\epsilon}$ associated with given values of ω and γ. Intuitively, the consumption of H type agents must not exceed that of L type agents by more than what $\hat{\epsilon}$ allows. If it does, default is optimal.

D'Erasmo and Mendoza [21] extend this analysis to a two-period model with shocks

[13]Note in particular that $\frac{\partial W^{d=0}(\epsilon)}{\partial \epsilon} \gtreqless 0 \iff \frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))} \gtreqless (\frac{\omega}{\gamma})(\frac{1-\gamma}{1-\omega})$. Hence, social welfare is increasing (decreasing) at values of ϵ that support sufficiently low (high) consumption dispersion so that $\frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))}$ is above (below) $(\frac{\omega}{\gamma})(\frac{1-\gamma}{1-\omega})$.

to government expenditures, optimal bond demand choices by private agents, and optimal bond supply and default choices by the government. The results for the distributional default incentives derived above still hold. In addition, we show that the optimal debt and default choices are characterized by a socially optimal deviation from the equalization of marginal utilities across agents, which calls for higher debt the higher the liquidity benefit of debt in the first period (i.e., the tighter the credit constraint on L-types) and the higher the marginal distributional benefit of a default in the second period. We also show that the model still sustains debt with default risk if we introduce a consumption tax as a second tool for redistribution, an alternative asset for savings, and foreign creditors.

3.2 Social Value of Debt

We conduct now a quantitative exercise that measures the endogenous costs of default captured by the social value of debt. In particular, we compute the social cost of a once-and-for-all, unanticipated default, which captures the costs of wiping the buffer stock of savings of private agents, preventing debt issuance from providing liquidity to credit-constrained agents, and precluding purchases (sales) of government bonds from improving risk-sharing. The goal is to show that default in the model of Section 2 can yield large endogenous costs.

To quantify the social cost of a once-and-for-all, unanticipated default, we compare social welfare across two economies. As in the full model, these economies are inhabited by a continuum of heterogeneous agents facing idiosyncratic (income) and aggregate (government expenditure) shocks. In the first economy, the government is fully committed to repay, while in the second there is an exogenous once-and-for-all, unanticipated default in the first period (i.e., a "surprise" default). After that, the government is committed to repay. We perform the experiment for different initial levels of government debt. Since there is no default risk, bond prices are always equal to $1/(1+\bar{r})$ and the domestic aggregate demand for bonds is the same for the different values of B (what changes is the amount traded abroad).

This experiment is related to the one conducted by Aiyagari and McGrattan [6], but with important differences. First, we compute the cost of a surprise default relative to an economy with full commitment, whereas they calculate the cost of changing the debt ratio under full commitment. Second, their model features production and capital accumulation with distortionary taxes, which we abstract from, but considers only idiosyncratic shocks, while we incorporate aggregate shocks. Third, in our setup, the interest rate is always $1/(1+\bar{r})$, whereas they study a closed economy with an endogenous interest rate.

We quantify the social value of public debt as the welfare cost computed as follows: Define $\alpha(b, y, B, g)$ as the individual welfare effect of the surprise default. This corresponds to a compensating variation in consumption such that, at a given aggregate state (B, g), an agent with a (b, y) pair is indifferent between living in the economy in which the government always repays and the one with the surprise default.[14] Formally, $\alpha(b, y, B, g)$ is given by:

$$\alpha(b, y, B, g) = \left[\frac{V^{d=1}(y, g)}{V^c(b, y, B, g)} \right]^{\frac{1}{1-\sigma}} - 1,$$

where $V^{d=1}(y, g)$ represents the value of the surprise default, and $V^c(b, y, B, g)$ is the value under full commitment. For a given (B, g), there is a distribution of these individual welfare measures across all the agents defined by all (b, y) pairs in the state space. The social value of public debt is then computed by aggregating these individual welfare measures using the social welfare function defined in Section 2:

$$\bar{\alpha}(B, g) = \int \alpha(b, y, B, g) d\omega(b, y). \tag{33}$$

Table 1 shows results for four scenarios corresponding to surprise defaults with debt ratios ranging from 5 to 20 percent of GDP.[15]

Table 1: Social Value of Public Debt

Panel (a): Calibrated welfare weights						
B/GDP	B^d/GDP	$\tau(B, \mu_g)/GDP$	$\bar{\alpha}(B, \mu_g)\%$	$\bar{\alpha}(B, \underline{g})$	$\bar{\alpha}(B, \overline{g})$	hh's $\alpha(b, y, B, \mu_g) > 0$
5.0	4.25	25.96	-1.87	-4.66	-1.13	0.9
10.0	4.25	23.87	-0.90	-3.76	-0.12	29.1
15.0	4.25	20.83	0.04	-2.88	0.89	66.0
20.0	4.25	17.29	1.00	-1.99	1.90	83.9
Panel (b): Welfare weights set to mean wealth distribution						
B/GDP	B^d/GDP	$\tau(B, \mu_g)/GDP$	$\bar{\alpha}(B, \mu_g)\%$	$\bar{\alpha}(B, \underline{g})$	$\bar{\alpha}(B, \overline{g})$	hh's $\alpha(b, y, B, \mu_g) > 0$
5.0	4.25	27.16	-1.75	-4.56	-1.00	0.0
10.0	4.25	25.22	-0.95	-3.81	-0.15	9.2
15.0	4.25	22.74	0.00	-2.93	0.85	75.8
20.0	4.25	19.73	1.07	-1.92	1.99	86.9

Note: Values are reported in percentage. B^d/GDP corresponds to the average of 10,000-period simulations with the first 2,000 periods truncated. Positive values of $\bar{\alpha}(B, g)$ denote that social welfare is higher in the once-and-for-all default scenario than under full repayment commitment. "hh's" denotes households.

For each scenario, Table 1 shows GDP ratios of total public debt, B/GDP, domestic debt, B^d/GDP, transfers τ (evaluated at average $g = \mu_g$ and the corresponding level of

[14] We measure welfare relative to this scenario, instead of permanent financial autarky, because it is in line with the one-period debt-market freeze when default occurs in our model. The costs relative to full financial autarky would be larger but less representative of the model's endogenous default costs.

[15] We use the parameters from the calibration described in the next Section and listed in Table 2.

B), as well as $\bar{\alpha}(B, g)$ for different values of g (average μ_g, minimum, $\underline{g}$, and maximum, $\overline{g}$). We also report the fraction of agents with $\alpha(b, y, B, \mu_g) > 0$ (i.e., the fraction of agents benefiting from a default). Since computing B^d requires the distribution $\Gamma(b, y)$, we report B^d for a "panel average," calculated by first averaging over the cross-section of (b, y) pairs within each period, and then averaging across a long time-series simulation. We show results for two sets of welfare weights. Panel (a) uses the $\omega(b, y)$ function defined earlier using the parameterization from the baseline calibration of the next Section. Panel (b) defines $\omega(b, y)$ as the long-run average of the distributions of bond and income obtained by solving the model without default risk, denoted $\bar{\Gamma}^{rf}(b, y)$.

The results show that the social value of debt is large and decreases monotonically as debt rises. For $g = \mu_g$, the results range from a social cost of -1.87 percent for defaulting on a 5 percent debt ratio to a gain of 1.00 for defaulting on a 20 percent debt ratio (i.e. the social value of debt ranges from 1.87 to -1.00 percent). Surprise defaults are very costly for debt ratios of 10 percent or less, while they yield welfare gains at debt ratios of 15 percent or higher. For the low value of g, default remains significantly costly even at a 20 percent debt ratio. Interestingly, at the high value of g the welfare costs are smaller and the gains larger than for average g, and they change from costs to gains at a debt ratio between 10 and 15 percent. These estimates of the social value of debt are significantly larger than those obtained by Aiyagari and McGrattan [6], who reported the largest estimate of the social value of debt at about 0.1 percent, while we obtain 1.87 percent (for $g = \mu_g$). Moreover, our estimates do not vary much if we change the calibrated welfare weights for weights that match the long-run endogenous distribution of bonds and income of the economy without default risk (see Panel (b)).

The smaller social value of debt (higher social value of default) at higher debt ratios follows from the fact that higher debt reduces transfers (τ decreases monotonically) and thus limits the extent to which the government can redistribute resources across agents by repaying, while the benefits of debt for self-insurance, liquidity, and risk-sharing fall. Accordingly, the fraction of agents that favor a default on average increases monotonically with the debt ratio. At relatively low debt (below 10 percent of GDP) only up to 30 percent of the population favors a default. These are agents with relatively low bond holdings who benefit from a smaller cut in transfers after a government default. The larger cut in transfers due to higher debt service when debt increases beyond 10 percent of GDP induces even agents with sizable bond holdings to favor default. For instance, with a 20 percent debt ratio, the average fraction of agents in favor of default is roughly 83.9 percent.

4 Quantitative Analysis

In this Section, we study the model's quantitative predictions. The Section begins with the calibration to Eurozone data, followed by an analysis of time-series properties, equilibrium recursive functions, and a sensitivity analysis. The solution algorithm solves the RME using a backward-recursive strategy over a finite horizon of arbitrary length until the value functions, decision rules, and bond pricing function converge (see Appendix A-3 for details).[16]

4.1 Calibration

We calibrate the model to the Eurozone following our motivation to view the European debt crisis as a domestic debt crisis, in which European institutions internalized the tradeoffs between distributional effects of individual country defaults and their implications for financial markets across the region.[17] During this crisis, default risk rose sharply for several Eurozone countries and one them defaulted (Greece). The model is calibrated at an annual frequency targeting GDP-weighted averages of country-specific moments. Hence, the calibration targets combine countries at the center of the crisis (e.g. Spain, Greece) with others that were less affected (e.g. France, Netherlands).[18] The parameter values that need to be set are the subjective discount factor (β), the coefficient of relative risk aversion (σ), the moments of the processes of individual income (μ_y, ρ_y, σ_u) and government expenditures (μ_g, ρ_g, σ_e), the income tax rate (τ^y), the opportunity cost of foreign investors ($\bar{r}$), the parameters that define $\phi(g)$, and the creditor bias parameter ($\overline{\omega}$).

The Markov processes of y and g approximate log-AR(1) time-series processes:

$$\log(y_{t+1}) = (1-\rho_y)\log(\mu_y) + \rho_y \log(y_t) + u_t, \tag{34}$$

$$\log(g_{t+1}) = (1-\rho_g)\log(\mu_g) + \rho_g \log(g_t) + e_t, \tag{35}$$

where $|\rho_y| < 1$, $|\rho_g| < 1$ and u_t and e_t are i.i.d. normal errors with zero means and standard deviations σ_u and σ_e, respectively. These moments are calibrated to data following the procedure we describe below. The Markov representation is constructed using Tauchen's quadrature method, set to produce grids with 5 evenly-spaced nodes for y and 25 for g, centered at the means, and with the lowest and highest nodes set at plus and minus 2.5 standard deviations from the mean in logs. The variances of the Markov processes are

[16]The algorithm is similar to the one in Corbae et al. [19], who examined a heterogeneous-agents model with a feedback mechanism connecting wealth dynamics and optimal policies but without debt and default.

[17]We use data for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain. See Appendix A-2 for a description of the data, sources, and country-specific moments.

[18]We also conducted the same set of experiments documented in this Section using a calibration based on data for Spain only. The results are available in Appendix A-8.

within 1 percent of their AR(1) counterparts.

The parameter values are assigned in two steps. First, the values of all parameters except β, $\overline{\omega}$ and the function $\phi(g)$ are set to values commonly used in the literature or to estimates obtained from the data. Second, β, $\overline{\omega}$ and $\phi(g)$ are calibrated using the Simulated Method of Moments (SMM) to minimize the distance between target moments from the data and their model counterparts. Thus, these parameters are set by solving the model repeatedly until the SMM converges, conditional on the parameter values set in the first step. We use data from several sources. The sample period for most variables is 1981−2015. Appendix A-2 provides a detailed description of the data and related transformations.

The first step of the calibration proceeds as follows: We set $\sigma = 1$ (i.e. log utility), which is in the range commonly used in macro models. The risk-free interest rate is set to $\bar{r} = 0.013$, which is the average annual real return on German EMU-convergence criterion government bonds in the European Commission's Eurostat database for the period 2002−2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). We start in 2002, the year the euro was introduced, to isolate spreads from currency risk.

Comprehensive disaggregated panel datasets on individual earnings with sufficient detail to estimate the persistence of the y process are unavailable, so we set $\rho_y = 0.85$ which is a standard value in the heterogeneous-agents literature derived from U.S. data (e.g., Guvenen [31]).[19] For the variance of earnings, we use $\text{Var}(\log(y)) = 0.30$, which is close to the midpoint of estimates of the residual cross-sectional variance of log-earnings for Italy, Germany, and Spain, which range from 0.2 to 0.55 (see Fuchs-Schundeln et al. [28] for Germany, Japelli and Pistaferri [32] for Italy, and Pijoan-Mas and Sanchez Marcos [47] for Spain).[20] Using these estimates of the variance and persistence of y, it follows that $\sigma_u^2 = \text{Var}(\log(y))(1 - \rho_y^2) = 0.3116$. Average income is calibrated so that the aggregate resource constraint is consistent with national accounts data, with GDP normalized to one. This implies that Y in the model must equal GDP net of fixed investment because the latter is not explicitly modeled. The GDP-weighted European average of the investment-output ratio was 22.26 percent for the 1981-2015 period, which implies that $Y = \mu_y = 0.7774$.

The g process is calibrated using data on government final consumption expenditures for the period 1981−2015 from the World Bank's World Development Indicators, and fitting an AR(1) process to the logged government expenditures-GDP ratio. The GDP-weighted average of the country-specific estimated parameters yields: $\rho_g = 0.8604$, $\sigma_e = 0.024$ and $\mu_g = 0.1998$.

[19] The data available for the countries in our sample correspond mostly to a set of cross-sections of individuals or short panels. Panel data do not exist or have a short timespan.

[20] The residual cross-sectional variance of log-earnings corresponds to the variance of the residuals of a regression of log-earnings on education, gender, and experience.

The value of τ^y is set to 38.59 percent, matching the average revenue-to-GDP share of effective labor taxes levied on individuals, including both individual labor income and consumption taxes, and excluding all forms of capital income taxation (which yield about 30 percent of GDP in tax revenue). Consumption tax revenues and the split of labor v. capital individual income taxes are obtained using the effective tax rates constructed by Mendoza, Tesar, and Zhang [41].

The exogenous default cost function is:

$$\phi(g) = \phi_1 \max\{0, (\mu_g - g)^{1/2}\}. \tag{36}$$

The cost is decreasing in g above a threshold level set at μ_g, so that the cost rises with income after a threshold, as in EG models.

In the second calibration step, we use the SMM algorithm to set the values of β, $\overline{\omega}$, and ϕ_1 targeting these three data moments: the 1981−2015 average ratio of domestic public debt to total public debt (55.53 percent), the 2002-2015 average bond spread relative to German bonds (0.92 percent), and the 1981-2015 average of the maturity-adjusted public debt-GDP ratio (7.45 percent).[21] The maturity adjustment is necessary because the model considers only one-period debt while actual debt data include multiple maturities. To make the adjustment, we follow the approach of Hatchondo and Martinez [35] and Chatterjee and Eiyigungor [17], which captures the maturity structure of debt by expressing the observed debt as a consol issued in year t that pays one unit of consumption goods in $t+1$ and $(1-\delta)^{s-1}$ units in year $t+s$ for $s > 1$. An observed debt, $\overline{B}$, with a given mean duration, D, has an equivalent one-period representation (i.e., the maturity-adjusted debt) given by $B = \frac{\overline{B}}{D}$, where D is the Macaulay duration rate of the consol (see Appendix A-2 for details). The GDP-weighted average debt-GDP ratio was 0.48 for the 1981-2015 period, and the average maturity was 6.35 years, yielding a maturity-adjusted debt ratio of 7.45 percent.[22] The SMM algorithm minimizes the loss function $J(\Theta) = [M^d - M^m(\Theta)]' \left[M^d - M^m(\Theta)\right]$, where $M^m(\Theta)$ and M^d are 3×1 vectors with model- and data-target moments, respectively.[23] Model moments are averages from 160 repetitions of 10,000-period simulations, with the first 2,000 periods truncated to avoid dependency on initial conditions, and excluding default periods

[21] Total public debt refers to total general government net financial liabilities as a fraction of GDP. The ratio of domestic to total debt corresponds to the fraction of general government gross debt held by domestic investors from Arslanalp and Tsuda [10], extended with the ratio of marketable debt held by residents to total marketable central government debt from OECD Statistics. See Appendix A-2 for further details.

[22] Duration is measured as the average maturity of total central government debt, which is available in OECD Statistics until 2010.

[23] The model moments depend on all parameter values, but our choice of parameters to align with moment targets is based on the fact that, everything else equal, β affects the domestic demand for assets, $\overline{\omega}$ affects the social welfare function and thus the optimal debt choice, and ϕ_1 affects the default frequency, which is informative about debt prices and spreads.

because all Eurozone countries but Greece did not default in the sample period.

Table 2 shows the calibrated parameter values.

Table 2: Model Parameters and Targets

Calibrated from data or values in the literature			
Risk-Free Rate (%)	$\bar{r}$	0.013	Real Return German Bonds
Risk Aversion	σ	1.00	Standard Value
Autocorrel. Income	ρ_y	0.85	Guvenen (2009)
Std. Dev. Error	σ_u	0.31	Std. Dev. Residual Log-Earnings
Avg. Income	μ_y	0.78	GDP Net of Fixed Capital Investment
Autocorrel. G	ρ_g	0.86	Autocorrel. Government Consumption
Std Dev Error	σ_e	0.02	Std. Dev. Government Consumption
Avg. Gov. Consumption	μ_g	0.20	Avg. G/Y
Proportional Income Tax	τ^y	0.39	Marginal Labor Income Tax
Estimated using SMM to match target moments			
Discount Factor	β	0.871	Avg. Ratio Domestic Debt
Welfare Weights	$\bar{\omega}$	0.065	Avg Spread (vs Germany)
Default Cost	ϕ_1	0.793	Avg. Debt to GDP (maturity adjusted)

Table 3 shows the target data moments and the model's corresponding moments in the SMM calibration.

Table 3: Results of SMM Calibration

Moments (%)	Data	Model
Avg. Ratio Domestic Debt	55.53	55.47
Avg. Spread Eurozone	0.92	1.22
Avg. Debt to GDP (maturity adjusted)	7.45	7.87

4.2 Equilibrium Time-Series Properties

The analysis of the model's time-series properties aims to answer two main questions. First, as an assessment of the theory, can the model support an equilibrium in which debt exposed to default risk can be sustained and default occurs along the equilibrium path? Second, to what extent are the model's time-series properties in line with those observed in the data?

We study the model's time-series properties using a simulation with 10,000 periods, truncating the first 2,000 to generate a sample with 8,000 periods, large enough to capture long-run properties. We observe 97 defaults, which implies a default frequency of 1.21

percent, compared with 1.1 percent in the Reinhart-Rogoff data set. Moreover, public debt is issued at zero spread (i.e. with full certainty of repayment) 75 percent of the time. Thus, the model is in line with the data in producing infrequent domestic (and external, since the government defaults on all of its debt) defaults, and in predicting that public debt pays the riskless rate most of the time. In contrast with typical results from EG models, these results rely on the tradeoff between distributional default incentives and endogenous costs that reflect the social value of debt, and do not require continued exclusion from credit markets, permanently or for a random number of periods.

Table 4 compares model and data averages. Since most Eurozone countries did not default, we compare model averages excluding default years v. data averages, and model averages for the years before defaults occur ("Prior Default" column) v. crisis peaks in the data ("Crisis Peak" column), defined as the highest values in the 2008-2012 period.

Table 4: Long-run and Pre-Crisis Moments: Data versus Model

	Data		Model	
Moment (%)	Avg.	Crisis Peak	Average	Prior Default
Gov. Debt B	7.45*	10.94	7.87	10.82
Domestic Debt B^d	4.14	5.92	4.37	4.87
Foreign Debt $\widehat{B}$	3.31	5.02	3.50	5.95
Ratio B^d/B	55.53*	54.15	55.47	44.97
Tax Revenues $\tau^y Y$	30.01*	29.20	30.01	30.01
Gov. Expenditure g	19.98*	21.34	19.99	19.15
Transfers τ	8.15	16.78	9.90	10.35
Spread (%)	0.92*	3.34	1.22	9.53

Note: * identifies moments used as calibration targets. See Appendix A-2 for details on data sources, definitions, and sample periods. Since GDP was normalized to 1, all variables in levels are also GDP ratios.

Given the stylized structure of the model, it is noteworthy that it approximates reasonably well key moments of the data. The averages of total debt, the ratio of domestic to total debt, government expenditures, tax revenue and spreads were calibration targets, so these moments align with the data by construction. The rest of the model averages approximate well the data averages. Regarding crisis peaks, the model is close to the data for total, domestic and external debt, tax revenues and government expenditures. The model underestimates the ratio of domestic to total debt because it overestimates (underestimates) external (domestic) debt by about 1 percentage point. The model yields larger spikes in spreads than the European average (9.5 v. 3.3 percent), but note that in Portugal and Greece spreads peaked at 9 and 21 percent, respectively (see Table A.3 in Appendix A-2). Note also that these results contrast sharply with the difficulties that external default models often have in producing large spreads at reasonable debt ratios.

Table 5 compares standard deviations (relative to the standard deviation of income) and correlations with disposable income and government expenditures. We use disposable income because in the model it fluctuates with τ, while GDP is constant, and we report correlations with government expenditures because g is the model's exogenous aggregate shock. We explain important effects of default risk in the model comparing results v. a version of the model in which the government is committed to repay.

Table 5: Cyclical Moments: Data versus Model

	Standard Dev.			Correl($x, hhdi$)			Correl($x, g/GDP$)		
Variable x	Data	Baseline Model	Model w.o. default	Data	Baseline Model	Model w.o. default	Data	Baseline Model	Model w.o. default
Disp. Inc. (hhdi)	1.05	1.75	1.64	1.00	1.00	1.00	-0.09	-0.58	-0.49
Consumption	0.89	0.84	0.70	0.60	0.98	0.95	-0.31	-0.71	-0.60
TB/GDP	0.68	0.60	0.57	-0.21	-0.86	-0.81	0.03	0.10	-0.18
Spreads	0.35	1.79	-	-0.33	-0.02	-	-0.07	-0.26	-
Gov. Debt / GDP	2.82	1.31	1.40	-0.26	-0.08	-0.65	0.40	-0.62	0.60
Dom. Debt / GDP	2.15	0.24	0.39	-0.27	-0.31	-0.55	0.32	-0.30	0.43

Note: *hhdi* denotes household disposable income and TB denotes trade balance. In the model, $hhdi = (1-\tau^y)Y + \tau$ and $TB = Y - C - g$. *hhdi* and C are logged and HP filtered with the smoothing parameter set to 6.25 (annual data). GDP ratios are also HP filtered with the same smoothing parameter. Standard deviations of all variables except *hhdi* correspond to ratios to the standard deviations of *hhdi*. Since the sample for spreads is short (2002-2015) and for a period characterized by a sustained rise in spreads since 2008, we generate comparable model data by isolating events spanning 10 years before spikes in spreads, defining spikes as observations in the 95 percentile. The standard deviation of spreads is demeaned to provide a comparable variability ratio. See Appendix A-2 for details on data sources.

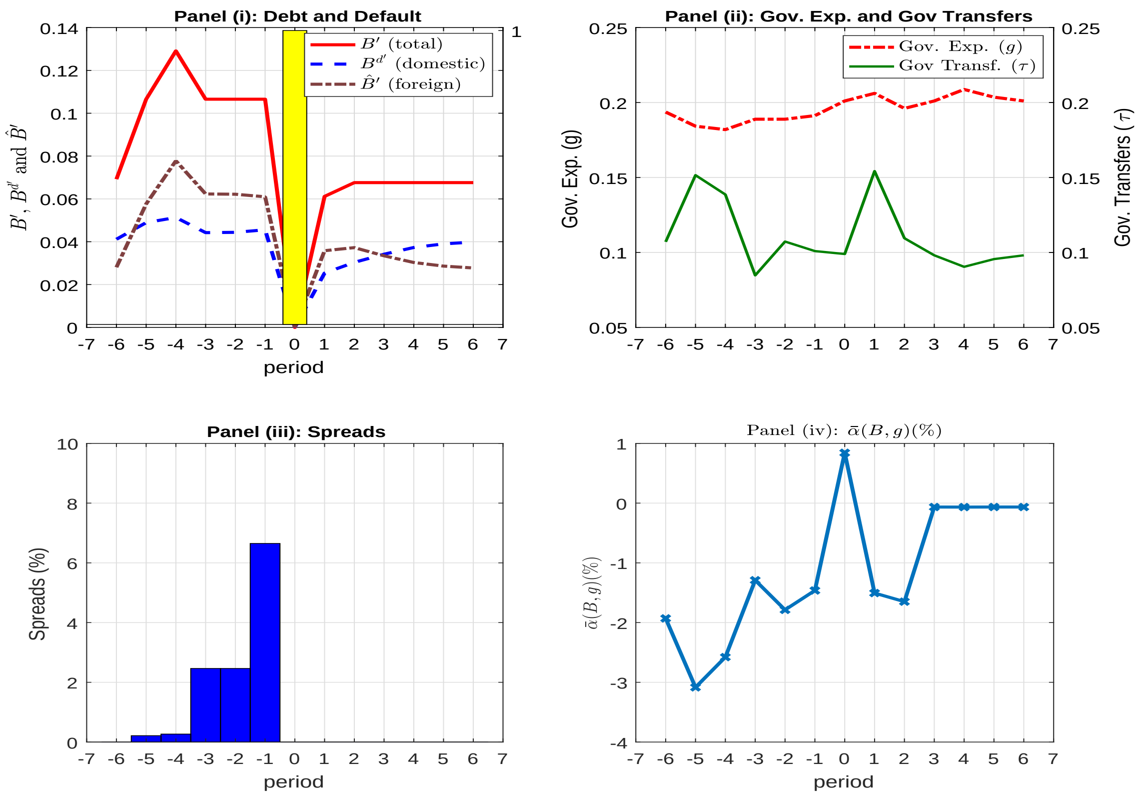
The model approximates closely the variability measures for consumption and net exports, and less so the ones for disposable income and total debt. It also matches the ranking of variability of income, consumption and net exports. On the other hand, the model overestimates the variability of spreads and underestimates that of domestic debt. Regarding income correlations, the model matches the data in producing a positive correlation with consumption and negative correlations with net exports, spreads, total debt and domestic debt. In terms of magnitudes, the correlations with spreads, total debt and domestic debt are comparable with the data, but the consumption correlation is too high and the one with net exports is too low. The model also matches closely the correlation between net exports and spreads (0.03 in the data v. 0.10 in the model, not shown in the Table). This suggests that foreign debt also has a weak correlation with spreads, since trade deficits are financed with public debt sold abroad. Looking at correlations with government expenditures, the model also does well at approximating those with consumption, net exports and spreads, but it deviates sharply from the data in producing negative correlations of government expenditures with total debt and domestic debt (-0.62 and -0.3), whereas the data show the opposite (0.4 and 0.32), and it underestimates the correlation with income (-0.58 v. -0.09).

Comparing the baseline model results v. those for the model without default risk shows that the negative correlations of total and domestic debt with government expenditures in the former are due to default risk. Most moments do not differ much across the two cases, but removing default risk shifts markedly the correlations between g and debt from negative to positive (-0.62 to 0.6 for total debt and -0.3 to 0.43 for domestic debt), the correlations between income and the two debt measures fall markedly (-0.08 to -0.65 for total debt and -0.31 to -0.55 for domestic debt), and the correlation between g and income rises (from -0.58 to -0.49). All these changes occur because without default risk the optimal public debt choice smooths the effects of fluctuations in g on τ and income. Higher g_t is less likely to reduce disposable income because, following eq. (6), its negative effect on τ_t is hampered by increasing debt so that $(1/(1+\bar{r}))B_{t+1}$ rises. There is no debt Laffer curve because there is no default risk. Hence, g_t and B_{t+1} are positively correlated and the correlation of income with debt (government expenditures) is more (less) negative. With default risk, however, this tax-smoothing objective can be more than offset by how the optimal debt choice responds to the government's reduced borrowing capacity and the Laffer curve of the resources generated by new debt ($q_t B_{t+1}$). On average, increases in g_t are now associated with reductions in B_{t+1}, making these variables negatively correlated. This also sustains more states in which the implied change in τ yields lower disposable income, making the correlation of income with debt (government expenditures) less (more) negative. Notice spreads are also negatively correlated with g_t, indicating that the government chooses debt mindful of how it affects bond prices and the resources new debt can yield.

The debt correlations in Table 5 may look closer to the data in the model without default risk, but note that these correlations vary widely across countries, and for some the correlations are more in line with the model with default risk. For instance, as Table A.2 in the Appendix shows, the GDP-weighted average of the correlation of domestic debt with government expenditures (0.32) is largely driven by the correlation for Germany (0.71), which is by far the highest and the one with the largest GDP weight. In other countries this correlation is zero or even negative (0.02 for Ireland, -0.19 for Spain, -0.03 for Netherlands). It is also interesting that the positive correlation for Germany is in line with the model with commitment and tax smoothing, whereas the negative one for Spain is closer to the predictions of the model with default risk and reduced borrowing capacity.

We study next dynamics around default events. Figure 3 shows a set of 13-year default event windows centered on the year of default at $t = 0$. These windows show averages for defaults that start from the median debt of all default events at $t = -6$.[24] Panel (i) shows total, domestic and foreign debt holdings. Panel (ii) shows g and τ. Panel (iii) shows bond spreads. Panel (iv) shows the social welfare gain of default denoted $\overline{\alpha}$.

Figure 3: Default Event Analysis



We proceed as in Section 3 to compute $\overline{\alpha}$. First, we calculate individual percent compensating variations in consumption $\alpha(b, y, B, g)$ that render agents identified by a (b, y) pair indifferent between the payoffs $V^{d=0}(b, y, B, g)$ and $V^{d=1}(y, g)$ at the aggregate states (B, g):

$$\alpha(b, y, B, g) = \exp\left((V^{d=1}(y, g) - V^{d=0}(b, y, B, g))(1 - \beta)\right) - 1.$$

$\alpha(b, y, B, g) < 0$ implies that agents with (b, y) prefer repayment. The social welfare gain of

[24] Appendix A-4 examines event windows similar to Figure 3 but starting from the lowest and highest debts at $t = -6$ across all default events. Appendix A-5 examines two default events separated by a nondefault phase that matches the mode duration of the nondefault state in the full simulation. These alternative approaches to study default dynamics yield similar qualitative findings as those reported here.

default is then computed as follows:

$$\overline{\alpha}(B,g) = \int_{\mathcal{B}\times\mathcal{Y}} \alpha(b,y,B,g)d\omega(b,y).$$

Note that, since the functions involved are nonlinear, $\overline{\alpha}$ is not the same as the compensating variation that equates $W^{d=0}(B,g)$ and $W^{d=1}(g)$, but both measures are positive only when the government defaults. Quantitatively, the differences between the two are negligible in the calibrated model. We chose $\overline{\alpha}(B,g)$ because it is easier to relate to individual welfare gains. Keep in mind also that these social welfare measures aggregate individual payoffs taken from the agents' *value* functions, which reflect expected lifetime utility valuations, not just comparisons of contemporaneous utility effects.

Panel *(i)* shows that total debt and the holdings of both foreign and domestic agents rise in the first three periods, in tandem with lower g realizations (see Panel *(ii)*) and a slight increase in spreads (see Panel *(iii)*). The three debt aggregates start near their long-run averages at $t = -6$. The increase in B is in line with the argument behind the negative correlation between B and g discussed earlier: With debt already near its average, the default risk mechanism inducing optimal debt issuance to rise when g falls dominates the tax-smoothing mechanism. The higher debt finances higher τ, which allows the government to do more progressive redistribution and provide more liquidity to credit-constrained agents. Domestic agents increase their debt holdings only slightly because they were also near their average holdings at $t = -6$, and these holdings fluctuate only about 1/4th as much as income. Hence, the bulk of the early increase in B is absorbed by foreign creditors. Spreads rise little because the distributional effects of the higher debt actually strengthen repayment incentives (see Panel *(iv)*), with the social welfare gain of default falling from -2 percent at $t = -6$ to close to -3 percent at $t = -4$. The probability of default still rises, because debt is rising, but only slightly because default incentives weaken.

These early periods are followed by increasing realizations of g from $t = -3$ to the default year $t = 0$. The same default risk mechanism now yields a reduction in debt at $t = -3$ followed by constant debt the next two periods, together with rapidly increasing spreads and default incentives. Since debt is very high, the slight increases in g lead the government to borrow less in order to avoid a sharp rise in the default probability and spreads (i.e. to avoid falling into the decreasing segment of the debt Laffer curve). The fall in debt is again absorbed mainly by foreign creditors, as domestic debt holdings decline slightly at $t = -3$ and then rise marginally in the following two periods. Spreads increase first to about 2 percent and then spike to near 7 percent the year before the default occurs.[25]

[25] As spreads increase, they attract domestic agents with sufficiently high (b, y) to buy more debt, which explains the slight increase in domestic debt holdings.

Hence the model is consistent with the empirical observation of rapidly rising spreads during debt crises. The social welfare gain of default also rises sharply, from close to -3 percent to about -1.5 percent. With debt unchanged between $t = -2$ and $t = -1$, g rising slightly, and distributional incentives now weakening sharply, the probability of default jumps causing the jump in spreads.

At $t = 0$ the welfare gain of default jumps, reaching 1 percent, and since now default yields a positive gain the government defaults. g rises slightly again but, at the high debt, this is enough to cause the jump in $\overline{\alpha}$. The surge in spreads at $t = -1$ and the"sudden" default at $t = 0$ occur with a debt ratio unchanged from the previous year. To an observer, the sudden shifts in spreads and market access may suggest that the crisis resulted from multiplicity or self-fulfilling expectations, but this is not the case. In addition, default occurs with relatively low external debt, which is about a half of the total debt.

After the default, g rises again at $t = 1$, but now since debt starts at zero, the tax-smoothing mechanism dominates, so that B rises with g. The social welfare gain of default falls by 250 basis points from 1 to -1.5 percent, because the social value of debt is very high at low debt. Domestic demand for debt rises gradually but sharply, since agents value highly rebuilding precautionary savings but doing so implies sacrificing consumption. Hence, in the early periods after the default foreign debt rises too, but after $t = 2$ the domestic debt ratio grows bigger to about 4 percent and the foreign debt ratio declines to below 3 percent.The total debt ratio stabilizes around 7 percent, and all through the six post-default years it is sold at zero default risk. With debt this high, and g remaining close to 20 percent of GDP, default incentives are strong but default is not optimal ($\overline{\alpha}$ is barely below 0). We show below in the analysis of the decision rules that in this situation (i.e., when domestic agents desire to increase bond holdings but high g realizations weaken repayment incentives), the government optimally chooses to place as much debt as it can at the risk-free price.

4.3 Equilibrium Recursive Functions

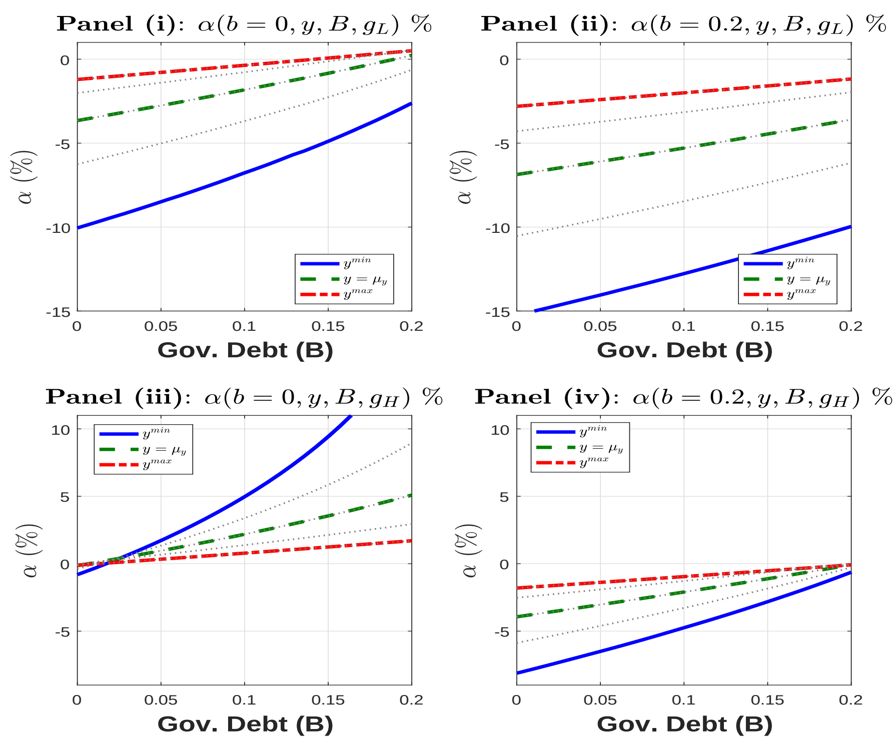
We analyze next the quantitative features of the equilibrium recursive functions, with the aim of illustrating the model's feedback mechanism and adding to the intuition behind the time-series results. First we show that there are significant dispersion and asymmetries in the individual welfare gains of default $\alpha(b, y, B, g)$ as we vary the aggregate states B, g.[26] Figure 4 shows four graphs that plot the gains as a function of B for a range of realizations of y. Panels (i) and (ii) are for $b = 0$ and $b = 0.2$, respectively, both with $g = g_L$. Panels (iii) and (iv) are also for $b = 0$ and $b = 0.2$, respectively, but now for $g = g_H$.

[26] In the charts that follow, B_H and B_L denote 50 percent above and below the long-run average of debt $B_M = 0.079$, y^{max} and y^{min} denote plus and minus 2 standard deviations of mean income $\mu_y = 0.78$, and g_H and g_L denote plus and minus 2 standard deviations of mean government expenditures $\mu_g = 0.20$.

Figure 4 yields three important results about the heterogeneity in default gains:

(1) *Gains differ sharply across those who hold debt and those who do not.* Panel (*iii*) shows that the gains are mostly positive in all the domain of B for agents that do not hold debt and draw any value of y when g is high, as these agents pay the same tax rate as debt holders and do not suffer wealth losses from a default. The gains fall with y because agents drawing higher income would have liked to use the bond market to save. For agents with low y, however, the gains are negative when B is very low because these agents value highly the liquidity and risk-sharing benefits of debt. Panels (*i*), (*ii*) and (*iv*) show a very different pattern: default gains are almost always negative in the domain of B for agents with either low or high b when g is low, and for agents with high b when g is high. The exception are agents who do not hold debt and draw sufficiently high y when g is low and B is large (see Panel (*i*)), because they value much less the benefits of public debt. In contrast, for agents with $b = 0.2$ (Panels (*ii*) and (*iv*)), the gains are always negative and large in absolute value, because a costly loss of wealth is the dominant factor for them.

Figure 4: Individual Gains from Default as a Function of B



(2) *Gains are nonmonotonic in y.* With $b = 0$ and $g = g_h$ (Panel (*iii*)), the gains are higher for agents with lower y (except when B is very low for the reasons explained in (1) above) because high-income agents who do not hold debt value more having access to the

bond market in order to start saving, and τ is smaller when g is high. In contrast, with all the other combinations of b and g (Panels (i), (ii) and (iv)), the gains are smaller for agents with lower y. Low-income agents with high b value more the loss of their assets due to a default precisely when they would like to use their savings for self-insurance (recall that defaults occur in periods of high g, which together with the debt freeze reduce τ sharply).

(3) *Gains are increasing, convex functions of* B *for all* y. This is most evident for agents with $b = 0$ in Panel (iii), as they value increasingly more the progressive redistribution that occurs when a larger B is defaulted on. For low B, default risk is not an issue, and hence gains from default are linearly increasing, simply because of the cut in τ after a default. As B rises, however, default risk starts to affect bond prices and demand for bonds, hampering the ability of using bonds for self-insurance and liquidity-provision, and requiring increasingly larger cuts in τ under repayment (as more resources are devoted to debt service). This happens when the default probability at $t+1$ evaluated at t is positive, which is the case for $B > 0.05$ and it is more evident for agents with low y who rely heavily on public transfers.

Figure 5 shows how $\alpha(b, y, B, g)$ responds to variations in g for different y values. This figure is divided into four plots as the previous Figure, but now for different (b, B) pairs. Panels (i) and (ii) are for $b = 0$ and $b = 0.2$ with a low supply of debt B_L. Panels (iii) and (iv) are for $b = 0$ and $b = 0.2$ with a high supply of debt B_H.

Figure 5: Individual Gains from Default as a Function of g

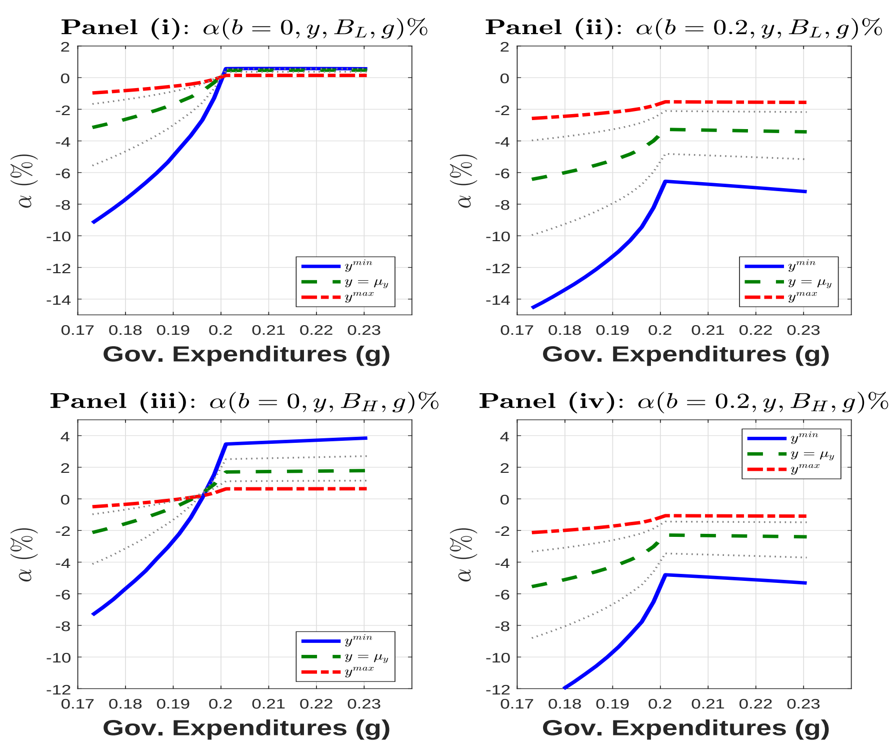


Figure 5 shows similar dispersion and asymmetries in the individual default gains for different g as those shown in Figure 4 for different B. In fact, Results (1)-(3) still hold, except that Result (3) only holds for $g < \mu_g$ instead of for all g. This is because the exogenous income cost making default costlier in "better" states of nature is only present when g is below average. For $g < \mu_g$, the gains are increasing and convex in g as a result of two forces: First, the default cost falls as g rises. Second, default risk increases with g, and this lowers bond prices and affects demand for bonds, resulting in lower transfers that reduce the value of repayment. For $g \geq \mu_g$, the default gains become nearly independent of g, because the adverse income effect of g via the exogenous default cost vanishes, and without it the effects of higher g on repayment and default payoffs are of similar magnitudes. This occurs because the direct effect of g reducing τ is the same for both, and under repayment default risk for $g > \mu_g$ hampers the government's use of debt for tax smoothing. The response of default gains to increases in g is weaker for high-income agents (i.e. α curves are flatter for higher y), because τ and the default cost represent a smaller share of their disposable income.

In addition, Figure 5 shows that default gains for the same y are uniformly higher for agents who do not hold debt than for agents with $b = 0.2$ for all g realizations, just like it was the case for all B in Figure 4. This is because τ is lower and default risk higher under repayment for higher g. Looking now at income variations, the gains are lower (higher) at lower y for agents with (without) bonds for $g \geq \mu_g$. For $g < \mu_g$, however, the gains are for the most part lower for agents with lower y regardless of bond holdings, because in this range of g disposable individual income is reduced by both the lower y and the default cost, which is uniform across agents.

The heterogeneity in default gains as g varies adds another key result:

(4) *Government default incentives are weaker (stronger) when g is below (above) μ_g.* All agents favor repayment and by more at lower y for g slightly below average, but for above-average g the gains differ in sign and in how they vary across agents with different income and bond holdings. Non-bond holders prefer default and those with low income prefer it the most, while bond holders ($b = 0.2$) prefer repayment and those with low income prefer it the most. This result is behind the result from the event analysis showing generally lower $\overline{\alpha}$ and higher B_{t+1} in periods with $g_t < \mu_g$, although the relationship is not monotonic because both B and g are changing every period.

Next we study how the heterogeneity in individual default gains affects the social welfare gains of default and the default decision rule. Figure 6 shows plots of $\overline{\alpha}$ as a function of B (Panel (*i*)) and g (Panel (*ii*)). These plots inherit the properties of the individual default gains: The social gain of default is increasing and convex in B for all g and in g for all B if $g \leq \mu_g$, while for $g > \mu_g$ the social gain of default is nearly independent of g (with the kink at μ_g again deriving from the kink in the default cost). The points at which $\overline{\alpha}$ changes sign identify thresholds above which default is preferable for the government. In Panel (i) $((ii))$, the threshold moves to a lower B (g) for higher g (B) because repayment requires larger transfer cuts. It follows from this result that, if the economy is at any pair (B, g) below the corresponding default threshold, the government repays and issues debt at zero spread. Moreover, for sufficiently low B (g), $\overline{\alpha} < 0$ for all g (B).

Figure 6: Social Gains of Default

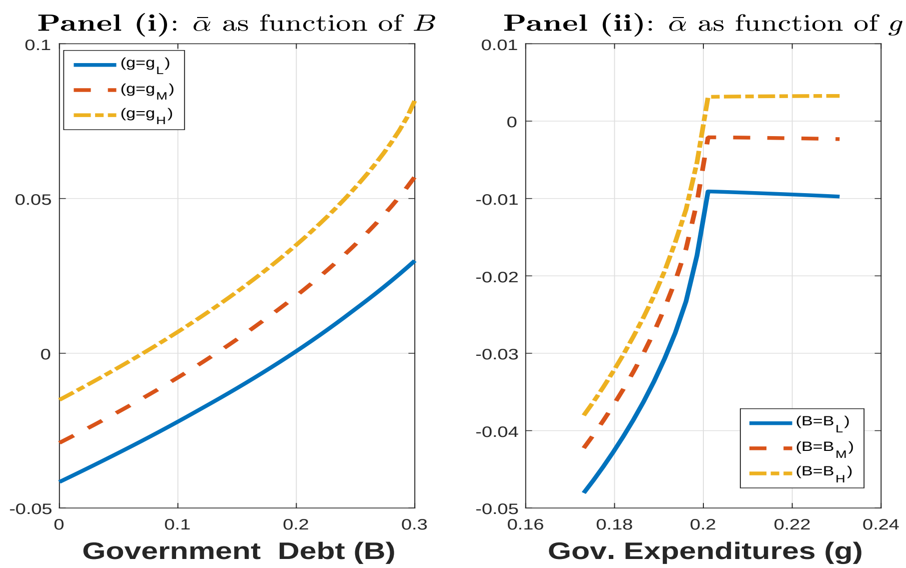
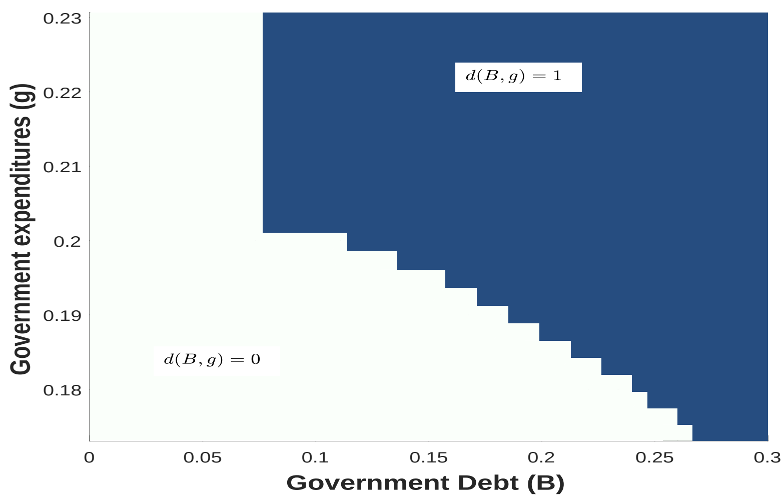


Figure 7 shows the default decision rule $d(B, g)$.

Figure 7: Default Decision Rule $d(B, g)$

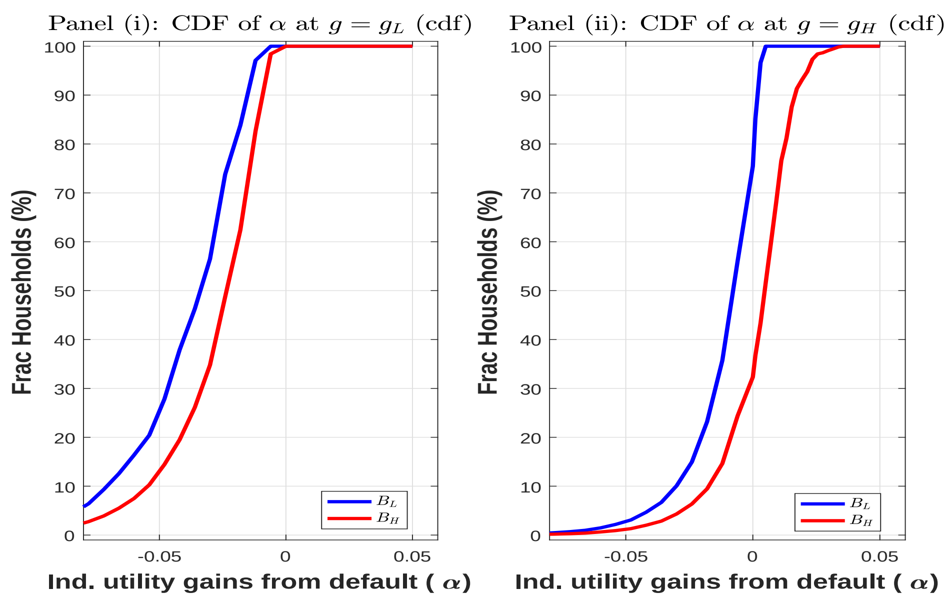


Note: The dark colored area represents $d(B, g) = 1$ and light colored area represents $d(B, g) = 0$.

The default and repayment sets are identified in dark and white colors, respectively. Their features are implied by the shifts in the thresholds of the social welfare gains from default noted above. Since Figure 6 shows that $\bar{\alpha}(B,g) < 0$ for all g when $B < 0.07$, $d(B,g) = 0$ in that same region. If the optimal debt choice falls in this region, that debt is issued with full certainty of repayment. For $0.07 \leq B \leq 0.25$, there is a high enough threshold value of g such that above it the government defaults and below it repays, and the threshold is lower at higher B. For debt to be issued exposed to default risk at equilibrium, the optimal debt choice must fall in this region along the equilibrium path. For $B > 0.27$, the government defaults for all g, and new debt cannot be issued because default would occur with certainty. Note that the default and repayment sets are not symmetric because of the asymmetry in the default cost, which lowers disposable income only if default occurs with $g < \mu_g$.

A limitation of examining the social and individual default gains is that one can either look at $\bar{\alpha}$, which hides the dispersion of the individual gains, or look at the individual αs, which are uninformative about the default choice since it hinges on social valuations. To illustrate the interaction between the two, and their effect on the default decision, Figure 8 shows the *social* distributions of default gains for particular (B,g) pairs.

Figure 8: Social Distributions of Individual Default Gains for Different B and g



These are distributions of the αs induced by the welfare weights $\omega(b,y)$ for four pairs of (B,g) formed by combining B_L, B_H and g_L, g_H.[27] The average of each of these distributions

[27] These plots show cumulative distribution functions (CDF) of $\alpha(b,y,B,g)$ for given (B,g) across all (b,y) pairs. Given a (B,g) pair, each (b,y) maps into a value of $\alpha(b,y,B,g)$ and a welfare weight $\omega(b,y)$. The CDFs are constructed by sorting the $\alpha(b,y,B,g)$ from low to high and integrating over (b,y) using $\omega(b,y)$.

corresponds to a point in the plots of the $\bar{\alpha}$ curves shown in Figure 6 for the corresponding (B, g) pair. These distributions are not the same as $\omega(b, y)$, because the nonlinear, non-monotonic responses of the individual αs to changes in B and g discussed earlier imply that the αs move in different directions across (b, y) pairs when (B, g) changes.[28]

Figure 8 is important because it shows the relevance of agent heterogeneity in the sovereign's aggregation of default gains (even tough the welfare weights $\omega(b, y)$ are fixed), and how it varies with B and g, shifting to the right as B (g) rises for given g (B). Panel (i) shows that, since for either B_L or B_H default is never chosen for $g = g_L < 0.177$ (see Figure 7), the social distributions of default gains have most of their mass in the negative quadrant, which represents agents who prefer repayment. In contrast, the distribution in Panel (ii) for the case with $g = g_H$ and $B = B_H$ has enough mass in the positive quadrant to yield a positive mean, which makes default socially optimal. Even in this case, however, about 32 percent of agents are better off under repayment in the planner's valuation (this is the cumulative social valuation of agents with negative αs for (B_H, g_H)). These distributions also reflect the asymmetric effects of above- v. below-average g shocks on the individual αs: The distributions for g_L in Panel (i) are skewed to the left compared with those for g_H in Panel (ii), even tough the g shocks are symmetric, the two panels use the same two values of B, and the welfare weights are the same.

The welfare weights used to construct Figure 8 differ from the distribution of debt and income across agents, which implies that the social welfare function is not utilitarian. Instead of assigning equal weight to all agents, and thus aggregate individual utilities with $\Gamma_t(b, y)$, the weights determined by $\omega(b, y)$ display the creditor bias implied by $\bar{\omega}$. Comparing these weights with the average distribution of debt and income in the time-series simulation $(\bar{\Gamma}(b, y))$, we find that both mass 90 percent of agents with debt holdings below 0.15 and close to 100 percent below 0.5.[29] They also mass similar fractions of agents with debt holdings below 0.1, at 88 and 78 percent for $\omega(b, y)$ and $\bar{\Gamma}(b, y)$, respectively. The two differ significantly, however, in the weight assigned to agents at or near zero debt holdings (i.e. below 0.01), 81.0 percent for $\bar{\Gamma}(b, y)$ v. 15 percent with $\omega(b, y)$. Hence, while agents with mid to large debt holdings are weighted similarly by $\omega(b, y)$ and $\bar{\Gamma}(b, y)$, calibrating the model to match the observed mean spreads (at the observed mean ratios of total debt to GDP and domestic debt to total debt) requires a value of $\bar{\omega}$ that reduces sharply the weight the sovereign assigns to non-debt-holders relative to their share in the distribution of debt. Without this, default incentives are too strong and the sustainable debt ratios too small.

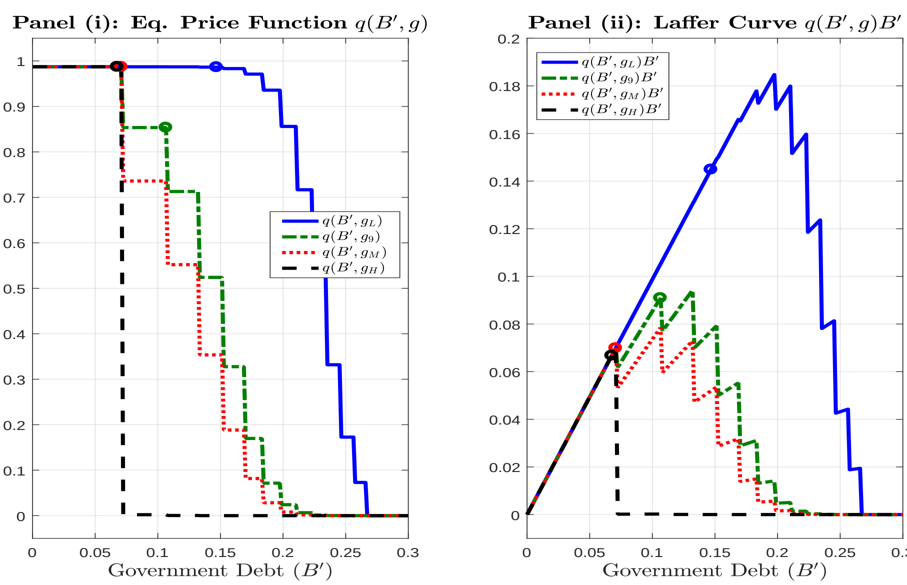
[28]This is also evident in the intensity plots of $\alpha(b, y, B, g)$ in the (b, y) space included in Appendix A-6, which display regions with similar colors (i.e., similar αs) for different (b, y) pairs.

[29]We report the average because $\Gamma_t(b, y)$ is time- and state-contingent. Default episodes are excluded from this calculation. See Appendix A-7 for a detailed comparison of $\omega(b, y)$ and $\bar{\Gamma}(b, y)$.

Despite the much larger weight on non-debt-holders in $\omega(b,y)$ relative to $\bar{\Gamma}(b,y)$, the weights given by $\omega(b,y)$ are actually a much better approximation to the actual distribution of asset holdings in the Eurozone than $\bar{\Gamma}(b,y)$.[30] The ECB's 2016 *Household Finance and Consumption Survey* reports that the shares of net wealth held by the top 10 and 5 percent of agents are 51.2 and 37.8 percent, respectively.[31] In the model, the corresponding weights implied by $\omega(.)$ are 33.1 and 20.3 percent, respectively, while those implied by $\bar{\Gamma}(.)$ are 86.8 percent and 62.7 percent, respectively. The ratios of the median debt holdings to the top 10 percent of the distribution of debt are 20.8 percent in the data v. 29.2 and 0 percent in $\omega(.)$ and $\bar{\Gamma}(.)$, respectively. Hence, $\bar{\Gamma}(.)$ overestimates (underestimates) significantly the fraction of wealth owned by agents at the top (bottom) of the distribution relative to both the data and $\omega(.)$. This also suggests that modeling a social welfare function with creditor bias is a reasonable benchmark.

Consider next the equilibrium pricing function of debt and the debt Laffer curve. Panel (i) of Figure 9 shows the pricing function as a function of new debt issuance B' for four values of g, and Panel (ii) shows the corresponding Laffer curves.

Figure 9: Pricing Function $q(B', g)$ and Debt Laffer Curve



Note: Circles on the curves with $g \in \{g_L, g_M, g_H\}$ mark the optimal debt choice for the corresponding value of g and with $B = B_M$. The circles for curves with $g = g_9$ (the ninth element in the Markov vector of g) denote the optimal debt choice when $B = 0.106$. This (B, g) pair is the one observed at $t = -1$ in Figure 3.

Figure 9 includes results for g_L, μ_g, and g_H, and also for $g = g_9 = 0.193$ (the ninth

[30] The result that $\bar{\Gamma}(.)$ does not match well the actual wealth distribution, particularly its concentration, is a well-known feature of standard heterogeneous-agents models. Extensions with preference heterogeneity and a life-cycle structure perform better (see Heathcote et. al. [37] and Krueger et al [39]).

[31] See Table 4.1 in www.ecb.europa.eu/pub/pdf/scpsps/ecbsp18.en.pdf?d2911394a25c444cd8d3db4b77e8891a.

element in the Markov vector of g), which is the realization observed at $t = -1$, just before the default in Panel (ii) of Figure 3. We marked with circles the optimal choice of B', which are the values implied by the equilibrium decision rule $B'(B, g)$. For $g = \{g_L, g_M, g_H\}$ we use $B = B_M$ and for $g = g_9$ we use $B = 0.106$, which is the value observed at $t = -1$ in Figure 3.

Since bond prices satisfy the same no-arbitrage condition of foreign investors as in EG models, the pricing functions have a similar shape. If B' is low enough for default in the next period to have zero probability, q equals the risk-free price $1/(1+\bar{r})$. Conversely, if B' is high enough for default to be expected with probability 1, the bond market collapses and $q = 0$. In between these two regions, q falls rapidly as B' rises, because the probability of default rises at an increasing rate as more debt is issued. For debt that carries default risk, prices are lower at higher g, because the probability of default is also higher at higher g for given B'.[32] Despite these similarities with EG models, the default probability driving q here is determined in a very different way, with the government taking into account the distribution of gains from default across all domestic agents, including domestic bond holders.

The debt Laffer curves in Panel (ii) show how the resources obtained by issuing debt, $q(B', g)B'$, vary with B'. These Laffer curves first increase linearly when debt is issued at the risk-free price, because q is constant, then they turn increasing but approximately concave as debt rises enough to produce moderate increases in default risk, and then turn decreasing and drop sharply, in line with the steep pricing functions of Panel (i). Note also that the Laffer curves shift down and to the left as g rises (for given B), indicating that the ability to use debt for progressive redistribution weakens considerably as g increases.

The optimal debt choices circled in Panels (i) and (ii) reflect the tradeoffs between distributional incentives and social value of debt faced by the government. There are three regions where the debt choice can be located: First, unconstrained debt at zero spread. For low enough g and/or B, debt is sold at the risk-free price and the optimal debt is located along the upward-sloping segment of the Laffer curve (e.g. the case for $g = g_L$). Second, constrained debt at zero spread. These are states with (B, g) such that new debt is still sold at the risk-free price but the optimal debt is set at the maximum of the Laffer curve, so that it yields the most resources new debt can yield (e.g. the cases for g_M and g_H). Less debt is suboptimal, because it generates fewer resources than desired and the Laffer curve is linearly increasing. More debt is suboptimal, because default risk rises rapidly, making bond prices drop sharply and thus yielding much fewer resources. Hence, in this region, the government's

[32] Notice this is a statement about how the realization g_t affects the probability of a default at $t+1$, whereas what we showed earlier is that, for sufficiently large B_{t+1}, the government optimally chooses to default at $t+1$ if g_{t+1} exceeds a threshold value. However, $p_t(B_{t+1}, g_t)$ rises with g_t because g shocks approximate an AR(1) process with 0.86 autocorrelation.

option to default does not generate a positive spread but limits significantly its borrowing ability. Third, constrained debt with positive spread (i.e. defaultable debt). Debt is sold with a positive spread and the optimal debt choice may be at the maximum of the Laffer curve or less. The case for g_9 yields optimal debt below the maximum of the Laffer curve, but for slightly higher g the maximum is optimal. In this region, the sovereign desires more resources than what debt sold at the risk-free price can generate, but not always as many as the most it can generate with a positive spread. The option to default again restricts the government's ability to sustain debt. Along the equilibrium path, debt is in the third region less frequently than in the second, so that debt has no risk premium most of the time but its amount is still constrained by the government's inability to commit.

The case with $g = g_9$ is also interesting because it is the outcome observed in the model's equilibrium path in the period before the default events studied in Figure 3. The fact that optimal debt sold at $t = -1$ is lower than the maximum value of the Laffer curve indicates that the progressive redistribution attained by selling less debt at a higher price, but smaller than the risk-free price, is socially preferable to selling more debt at a lower price even if it yields more resources, including in both counts the social benefits of debt. The government defaults at $t = 0$ because the regressive redistribution induced by repaying plus the value of the social benefits of debt, is less desirable than the progressive redistribution net of the loss of the social benefits of debt and the exogenous default cost, attained by defaulting.

In terms of the dependency of the debt choice on B, we use the quantitative results to show in Appendix A-6 that, depending on g, the debt choice is either nearly independent of B or increasing in B. It is nearly independent of B for $g \geq \mu_g$, because the optimal debt is the maximum value of the Laffer curve regardless of the value of B, and this maximum does not vary much with B because, as shown earlier, social and individual welfare gains of default are nearly independent of B when $g \geq \mu_g$ due to the asymmetry neutralizing the exogenous default cost (e.g. the optimal debt is 0.107 for g_M and 0.071 for g_H for most of the domain of B). In this interval of g, debt is sold at the risk-free price but as explained earlier it is still constrained by the government's default option. For $g < \mu_g$, the optimal debt rises with B and is always below the maximum of the Laffer curve. Hence, it is at these levels of g that the government can choose debt lower than the maximum value of the Laffer curve, and in some of these states it is optimal to issue debt that carries a default risk premium.

4.4 Sensitivity Analysis & Extensions

To close this Section, we conduct a sensitivity analysis of the quantitative implications of altering the values of key parameters and adding important features to the model.

(a) Welfare Weights

We examine the implications of altering the welfare weights by making two kinds of comparisons. First, we adopt the following more general formulation of $\omega(b,y)$:

$$\omega(b,y)=\sum_{y_i\leq y}\pi^*(y_i)\left(1-e^{-\frac{(b+z)}{\bar{\omega}}}\right). \tag{37}$$

$\bar{\omega}$ still measures creditor bias, but now z controls the weight assigned to agents who do not hold debt (i.e. those hitting the borrowing constraint). These are the agents receiving the liquidity benefit of debt, and the largest redistribution of resources when new debt is issued under repayment or when outstanding debt is wiped out under default. Second, we study a case akin to a utilitarian sovereign by replacing $\omega(b,y)$ with the long-run average of the distribution of (b,y) across agents for the model without default risk, $\overline{\Gamma}^{rf}(b,y)$.[33]

Table 6 compares the results for the Baseline calibration v. three scenarios with different values of $\bar{\omega}$ and z (cases (A)-(C)), and the case with $\omega(b,y)=\overline{\Gamma}^{rf}(b,y)$ (case (D)). We report long-run averages and averages before default events, and three additional sets of statistics that help explain the results. First, the cumulative welfare weights for agents with bond holdings up to a given amount across all income levels, defined as $\Omega(b)=\sum_{y\in\mathcal{Y}}\omega(b,y)$. We consider agents with b up to 0, 0.0004, 0.045 and 0.30, because in the calibrated $\omega(b,y)$ function they yield cumulative weights of 0, 1, 50 and 99 percent respectively. Second, we use equation (28) to report the threshold bond holdings $\hat{b}(\mu_y,\bar{B}^D,\bar{g}^D)$ of an agent who draws income μ_y and is indifferent between repayment and default when B and g are at their averages conditional on the government choosing to default $(\bar{B}^D,\bar{g}^D)$. Agents with $b\geq\hat{b}$ prefer repayment. Third, we report the fraction of agents that favor repayment according to the CDF of the mean distribution of bond and income of each model solution (the mean distribution is denoted $\bar{\Gamma}(b,y)$, and the CDF is $\bar{\gamma}(b,y)$), and the comparable fraction as valued by the government using $\omega(b,y)$.

Comparing the Baseline column with column (A) shows the effects of increasing z from 0 to 0.025. This increases the welfare weight of agents without debt from 0 to 32.06 percent. The cumulative weights of agents with b up to either 0.0004 or 0.045 also rise, to 32.29 and 67.09 percent, respectively v. 1 and 50 percent, respectively in the Baseline. $\hat{b}$ drops from 0.095 to 0.068, and the fraction of agents that the government sees as gaining from repayment drops from 22.4 to 21.9 percent, while the actual fraction of agents that favor repayment *rises* from 4.5 to 5.2 percent. These changes indicate stronger incentives to default, in line with the analysis in the first exercise of Section 3: By assigning positive weight to agents with $b=0$ (and in general higher weight to agents with lower b), the fraction of agents that the government assesses as gaining from a default is closer to the corresponding fraction in the economy's wealth distribution, which reduces incentives to repay. The stronger default

[33] Results using the average distribution of the model with default are quantitatively similar.

incentives result in a lower mean debt ratio and higher mean spreads and default frequency. The mean domestic and external debt ratios also drop, but the ratio of domestic to external rises sharply, from 1.25 to 2.81. Qualitatively similar changes are observed in the averages of these statistics prior to defaults.

Table 6: Sensitivity Analysis: Social Welfare Weights

Moment (%)	Baseline $\bar{\omega} = 0.065$ $z = 0$	(A) $\bar{\omega} = 0.065$ $z = 0.025$	(B) $\bar{\omega} = 0.055$ $z = 0$	(C) $\bar{\omega} = 0.055$ $z = 0.025$	(D) $\omega(b, y)$ $= \bar{\Gamma}^{rf}(b, y)$
Long Run Avg.					
Gov. Debt B	7.87	5.71	6.61	4.93	3.76
Dom. Debt B^d	4.37	4.21	4.27	4.14	3.23
Foreign Debt $\hat{B}$	3.50	1.50	2.35	0.79	0.53
Default Frequency	1.21	2.26	2.10	3.52	4.39
Spreads	1.22	2.32	2.15	3.65	4.592
Transf τ	9.90	9.93	9.92	9.95	10.01
Frac. Hh's $b = 0$	65.58	67.42	67.69	67.33	69.64
$\bar{\alpha}(B, g)$	-0.814	-0.781	-0.862	-0.766	-0.768
Avg. Prior Default					
Gov. Debt B	10.82	7.99	9.26	6.97	5.43
Dom. Debt B^d	4.87	4.79	4.95	4.76	4.28
Foreign Debt $\hat{B}$	5.95	3.20	4.32	2.21	1.15
Spreads	9.53	12.67	12.30	19.78	16.16
Def. Th. $\hat{b}(\mu_y)$	0.095	0.068	0.081	0.060	0.049
%. Favor Repay (1-$\omega(\hat{b}(\mu_y), \mu_y)$)	22.44	21.92	21.59	20.48	4.91
% Favor Repay (1-$\bar{\gamma}(\hat{b}(\mu_y), \mu_y)$)	4.48	5.19	4.97	5.51	5.53
Cumulative Welfare Weights					
$\Omega(b = 0)$	0.00	32.06	0.00	36.59	65.64
$\Omega(b = 0.0004)$	1.00	32.29	0.51	36.84	65.65
$\Omega(b = 0.0447)$	50.00	67.09	57.48	73.01	85.83
$\Omega(b = 0.3025)$	99.00	99.41	99.63	99.77	93.82

Note: All moments reported correspond to averages excluding default periods, except those labeled "Avg. Prior Default" which correspond to the average of observations prior to a default event. The model is simulated 160 times for 10,000 periods, truncating the initial 2,000 periods.

Reducing $\bar{\omega}$ to 0.055 (a 15 percent cut), while keeping $z = 0$, also strengthens default incentives (compare columns (B) v. Baseline). Agents without bond holdings have the same zero welfare weight as in the Baseline, but the weight of agents with relatively small b rises. The cumulative welfare weights of agents with b up to 0.0004 and 0.045 increase, but less so than in the scenario with $z > 0$. Hence, we get the same results qualitatively, but the effects are weaker quantitatively. Introducing both higher z and lower $\bar{\omega}$ (Column (C)) has again similar qualitative effects relative to the Baseline, but quantitatively the effects are now stronger. Agents without bond holdings have a cumulative welfare weight of nearly 36.6 percent, and the weight of agents with b up to 0.045 increases from 50 to 73 percent.

Using $\overline{\Gamma}^{rf}(b, y)$ to define welfare weights also has qualitatively similar effects, but quantitatively the effects are the strongest of all the scenarios. This is because $\overline{\Gamma}^{rf}(b, y)$ assigns more weight to agents with low b than any of the other cases. In fact, the fraction of agents the government sees as in favor of repayment falls sharply to 4.9 percent, and is now about the same as what the mean distribution of bonds and income yields. As before, this results in higher spreads and lower debt. The increase in default risk also reduces the domestic demand for government bonds but by much less than the decline in total debt, resulting in a much higher ratio of domestic to external debt.

Summing up, altering the welfare weights does change the quantitative results, but in all cases the model sustains sizable debt ratios at nontrivial spreads. This is the case for either arbitrary ω functions that assign large welfare weights to agents with little or no debt holdings, or if the welfare weights are set to match the long-run average distribution of bonds and income across agents. The drawback under the alternative formulations is that the predicted spreads and default frequencies are much higher than what is observed in the data. Matching these requires stronger creditor bias on the part of the sovereign.

(b) Preference Parameters and Income Process

Table 7 shows results from solving the model for higher and lower β, σ, and σ_u than in the Baseline. These parameters are key determinants of precautionary savings, and hence are important for driving the model's results. Note that, since bond prices are determined by the no-arbitrage condition of foreign investors, bond prices are affected only indirectly through the default probability determined by the government's debt and default decisions. In particular, changes in σ do not affect bond prices directly via domestic marginal rates of substitution in consumption, although this is still a determinant of domestic bond demand.

The effects on B^d are standard from the incomplete-markets theory of savings: Increasing (reducing) incentives for self-insurance by rising (lowering) β, σ, or σ_u, increases (reduces) the long-run and before-default averages of domestic bond holdings. The effects on foreign debt are in the opposite direction, so the ratio of domestic to external debt rises (falls) as precautionary savings strengthens (weakens). With higher β, σ, or σ_u, domestic bond demand rises so much that almost all the debt ends up being domestic (with $\sigma = 1.25$, the economy even becomes a net external creditor). The changes in total debt, on the other hand, are nonmonotonic with respect to changes in β and σ_u: Debt is higher in the scenarios in which these parameters are higher or lower than their corresponding values in the Baseline.

Table 7: Sensitivity Analysis: Preference Parameters and Income Process

		β		σ		σ_u		
Moment (%)	Baseline	0.853	0.888	0.75	1.25	$0^{\dagger}$	0.28	0.34
Long Run Avg.								
Gov. Debt B	7.87	7.90	8.03	7.79	7.90	2.82	7.86	7.88
Dom. Debt B^d	4.37	2.45	7.53	1.05	10.07	0.00	2.85	5.92
Foreign Debt $\hat{B}$	3.50	5.46	0.50	6.74	-2.16	2.82	5.01	1.96
Def. Freq.	1.21	1.23	1.19	1.26	1.19	8.92	1.21	1.18
Spreads	1.22	1.24	1.21	1.278	1.202	9.793	1.224	1.199
Transf τ	9.896	9.895	9.897	9.896	9.896	10.35	9.896	9.896
Frac. Hh's $b = 0$	65.58	82.99	60.07	88.75	55.67	99.99	81.30	63.28
$\bar{\alpha}(B, g)$	-0.814	-0.946	-0.698	-0.771	-0.877	-0.666	-0.803	-0.827
Avg. Prior Default								
Gov. Debt B	10.82	10.84	10.74	10.24	10.78	3.73	10.80	10.82
Dom. Debt B^d	4.87	2.78	8.52	1.17	10.60	0.00	3.22	6.74
Foreign Debt $\hat{B}$	5.95	8.06	2.22	9.08	0.17	3.73	7.57	4.08
Spreads	9.530	9.495	9.631	9.299	9.563	14.76	9.562	9.530

Note: Baseline model parameters are $\beta = 0.871$, $\sigma = 1$ and $\sigma_u = 0.31$. † This case has no idiosyncratic uncertainty and income for all agents is set to mean income μ_y. Also, the welfare function is adjusted to coincide with the observed distribution where all agents hold no bonds. All moments reported are averages excluding default, except those labeled "Avg. Prior Default" which are averages of observations prior to a default event. The model is simulated 160 times for 10,000 periods and we drop the initial 2,000 periods.

Higher σ or σ_u reduce default incentives and yield lower spreads and default frequencies, because the social gain of default falls. The benefit of defaulting as a mechanism to substitute for redistribution through risk-sharing with debt decreases, while on the other hand the social value of debt for the provision of liquidity and the accumulation of precautionary savings rises. In the scenario with higher β, in addition to the effects via domestic bond demand, higher discounting makes default costlier, because the government values less the benefit of providing assets for self-insurance for future consumption smoothing. While, as in EG models, the government's incentive to borrow also decreases at a higher β, the effect of the higher endogenous default costs dominates and results in higher debt than in the baseline rendering the debt ratio locally non-monotonic in β.

The case with $\sigma_u = 0$ is of particular interest, because removing individual income shocks renders the model equivalent to an EG model with a representative domestic agent.[34] The volatility of g and the default risk do not generate enough variation in aggregate disposable income to yield a positive net foreign asset (NFA) position (i.e. assets held abroad for self-insurance), so the government issues debt abroad and the NFA position is negative. The

[34]This experiment can be solved with the same algorithm as the others by setting $\bar{\omega} \approx 0$, so that the sovereign does not assign weight to "artificially" heterogeneous agents by weighting the agents' value functions in all the nodes of the grid of bonds.

results are in line with standard EG models: the average debt falls sharply and the default frequency is much higher than in the Baseline.

In summary, in these experiments we find again that the model sustains sizable debt exposed to default risk with infrequent defaults. Domestic debt and the ratio of domestic to foreign debt are more sensitive to the parameter variations we considered than the other model statistics. We also showed that agent heterogeneity is of major relevance for the results, because a comparable EG representative-agent version of the model sustains much less debt and overestimates spreads and the default frequency significantly.

(c) Income Tax Rate and Default Cost

Table 8 reports the effects of changes in the income tax rate (τ^y) and the exogenous default cost function ($\phi(g)$). For the latter, we use the following generalization of $\phi(g)$:

$$\phi(g) = \phi_1 \max\{0, (\hat{g} - g)^{\psi}\}.$$

Here, $\hat{g}$ denotes the threshold realization of g at which the cost vanishes, and ψ controls the curvature of the cost function. In the baseline calibration, $\hat{g} = \mu_g$, $\psi = 1/2$, and ϕ_1 was calibrated to target the Eurozone's mean spread.

Table 8: Sensitivity Analysis: Income Taxes and Default Cost

		τ^y		ϕ_1		ψ		$\hat{g}$	
Moment (%)	Baseline	0.29	0.48	0.59	0.99	0.40	0.60	0.188	0.209
Long Run Avg.									
Gov. Debt B	7.87	7.85	7.87	7.36	8.23	8.26	7.19	6.80	13.52
Dom. Debt B^d	4.37	7.41	2.33	4.22	5.48	4.41	4.24	4.22	4.44
Foreign Debt $\hat{B}$	3.50	0.45	5.54	3.14	2.75	3.85	2.96	2.58	9.09
Def. Freq.	1.21	1.21	1.18	0.42	4.10	1.30	0.31	0.06	2.64
Spreads	1.220	1.223	1.189	0.42	4.28	1.32	0.31	0.06	2.71
Transf τ	9.896	2.40	17.41	9.898	10.91	9.89	9.90	9.90	9.74
Frac. Hh's $b = 0$	65.58	59.94	83.08	66.85	73.63	65.35	66.34	65.68	66.79
$\bar{\alpha}(B, g)$	-0.814	-0.897	-0.766	-0.636	-3.880	-1.046	-0.541	-0.213	-1.963
Avg. Prior Default									
Gov. Debt B	10.82	10.78	10.82	9.40	9.97	12.02	9.03	11.03	18.62
Dom. Debt B^d	4.87	8.31	2.66	4.56	5.85	5.08	4.54	5.37	4.46
Foreign Debt $\hat{B}$	5.95	2.47	8.16	4.85	4.12	6.94	4.49	5.65	14.15
Spreads	9.530	9.560	9.524	4.207	10.33	10.42	2.98	8.46	11.34

Note: Baseline parameters are $\tau^y = 0.386$, $\phi_1 = 0.793$, $\psi = 1/2$ and $\hat{g} = 0.199$. All moments reported are averages excluding default, except those labeled "Avg. Prior Default" which are averages prior to a default event. The model is simulated 160 times for 10,000 periods, dropping the initial 2,000.

Comparing Tables 7 and 8, shows that higher (lower) τ^y has similar qualitative effects as lower (higher) σ_u. This is in part because both parameters affect the variance of idiosyncratic disposable income, which is equal to $(1-\tau^y)^2\sigma_y^2$. In addition, as explained in Section 2, a higher τ_y improves the implicit cross-sectional sharing of idiosyncratic risk provided by government transfers. Hence, these results can be viewed as showing the implications of allowing the government to use means other than debt and default to redistribute resources. The Baseline predictions with a $\tau_y = 0.35$ are not altered much by changing the tax to 0.20 or 0.45, except for the allocation of debt holdings across foreign and domestic agents, with the share of the former being much higher at higher tax rates.

Regarding changes in the default cost function, changes that increase the cost (higher ϕ_1, lower ψ, or higher $\hat{g}$) weaken incentives to default and allow the government to sustain more debt on average. Everything else the same, weaker default incentives should reduce the probability of default and yield lower spreads, but since the weaker incentives also make it optimal for the government to issue more debt (note that the mean social welfare gain of default falls with the higher default costs), the equilibrium default probabilities for the higher debt are higher, resulting in higher spreads. Higher spreads induce an increase in domestic demand for debt. Average debt ratios in the years before defaults occur are also higher with the higher default costs (except in the case of $\phi_1 = 0.99$), and in the three cases the average spreads before defaults are higher.

These results are important because they show the extent to which the model's predictions hinge on the exogenous default cost. The value of ϕ_1 is relevant mainly for the spreads, while the other model moments are less affected. Still, even with a value reduced to three-quarters the size of that in the baseline calibration, the long-run mean spread is about 42 basis points and the average spread before defaults is 421 basis points. The threshold $\hat{g}$ was shown earlier to be important for explaining the dispersion of individual default gains, the government's default incentives, and the association of periods of increasing debt with low g realizations. Here, we showed in addition that lowering $\hat{g}$, so that the exogenous default cost is active for a narrower range of realizations of g, has a small effect on total debt and its domestic and external components. On the other hand, the average social gains of default are significantly higher and spreads are sharply lower. The effects of increasing ψ are similar, since higher ψ lowers the marginal cost of a given reduction of g below the threshold, suggesting that lower $\hat{g}$ could be traded for lower ψ without altering the results significantly.

In all the results shown in Table 8, the model again sustains sizable ratios of total and domestic debt exposed to default risk. Spreads are also non-trivial and default remains an infrequent event preceded by sudden, sharp increases in debt and spreads. The model's ability to produce sizable spreads, however, does depend on the exogenous cost. In light of these findings, we examine the model's predictions if the exogenous cost is removed ($\phi_1 = 0$).

This case still yields sizable debt, with long-run mean debt ratios of total and domestic debt of 5.6 and 4.3 percent respectively, but with a zero mean spread. Debt is optimally chosen to sell at the risk-free price, as incentives to default weaken considerably, resulting in a social gain of default that is still negative but higher than in the Baseline and close to zero, at -0.07 percent. Contrary to the perfect-foresight analysis of Section 3, default *does not* become generally optimal without exogenous costs, because the endogenous costs due to the social value of debt are large. Spreads are zero, however, because the bond pricing function is too steep at debt levels that could be offered with positive spreads, which leads the government to prefer issuing risk-free debt. Hence, as noted earlier, the debt is sold at the risk-free price but the government's borrowing capacity is hampered by its inability to commit to repay.

(d) Partial Default

The last experiment examines the implications of altering the assumption that default requires reneging on all of the debt (i.e. a default rate of $\varphi = 1$). Table 9 presents results for exogenous default rates set to $\varphi = \in \{0.90, 0.80, 0.50\}$ (cases (A), (B), and (C)). The government repays the fraction $(1 - \varphi)$ of its debt during the exclusion period. Case (D) shows results for a setup in which the government chooses $\varphi \in [0, 1]$ optimally. To solve it, we first solve an auxiliary problem to compute how much each agent values an outcome with a given φ (similar to the problem we solve to find the optimal debt issuance). We then integrate the resulting individual indirect utility functions to obtain the associated social welfare value for each φ and let the government choose the one that maximizes welfare. These experiments shed light on the potential effects of renegotiation that yields debt haircuts at the rate φ, the possibility of partial commitment to repay, and the implications of considering partial default mechanisms such as the erosion of nominal debt by inflation, the introduction of wealth or financial taxes, and adjustments in eligibility criteria for entitlement programs (e.g. retirement ages for pensions, needs-based rules for health programs).

The model with endogenous default rate (case (D)) yields the result that along the equilibrium path, if the government chooses to default it always chooses $\varphi = 1$ also. This is not the same as in the original EG model, in which partial default is generally suboptimal. Here, there are regions in the (B, g) space in which the optimal φ is less than 1, but these are never equilibrium outcomes in the time-series simulation of the model. The optimal tradeoff of distributional default incentives v. social value of debt is such that when default is optimal, full default is preferable to partial default. Thus, the baseline and the model with endogenous partial default yield the same results.

In cases (A) through (C), the recursive equilibrium functions show that as the recovery rate $(1 - \varphi)$ increases, the bond pricing schedule shifts up. Hence, for a given debt level and default probability, interest rates are lower. This allows the government to borrow more and results in higher debt levels than in the Baseline. The higher debt is not reflected in higher spreads until the increase in debt is large enough. In cases (A) and (B), spreads are slightly smaller than in the Baseline, while Case (C) shows higher spreads. The demand for domestic debt declines (increases) when spreads decrease (increase). Interestingly, the mean social value of default falls as φ rises, so that debt issued under partial default is endogenously more sustainable because default incentives weaken. This is also reflected in that the fraction of agents who do not hold debt falls, so distributional incentives to default weaken. On the other hand, the default frequency is actually higher with $\varphi = 0.5$ than with $\varphi = 1$, and this is because of the higher debt.

Table 9: Sensitivity Analysis: Partial Default

		(A)	(B)	(C)	(D)
	Baseline	Fixed Default Rate			Endogenous
Moment (%)	$\varphi = 1.0$	$\varphi = 0.90$	$\varphi = 0.80$	$\varphi = 0.50$	Default Rate
Long Run Avg.					
Gov. Debt B	7.87	7.96	8.21	12.62	7.87
Dom. Debt B^d	4.37	4.28	4.30	4.68	4.37
Foreign Debt $\hat{B}$	3.50	3.68	3.91	7.93	3.50
Def. Freq.	1.21	1.10	1.05	1.87	1.21
Spreads	1.220	1.111	1.058	1.902	1.220
Transf τ	9.896	9.910	9.921	10.199	9.896
Frac. Hh's $b = 0$	65.58	67.17	66.43	59.00	65.58
$\bar{\alpha}(B, g)$	-0.814	-0.849	-0.870	-1.112	-0.814
Avg. Prior Default					
Gov. Debt B	10.82	10.94	11.60	20.57	10.82
Dom. Debt B^d	4.87	4.85	4.92	6.12	4.87
Foreign Debt $\hat{B}$	5.95	6.09	6.67	14.45	5.95
Spreads	9.530	8.415	8.098	9.802	9.530
Recovery Rate $(1-\varphi)$	0.00	10.00	20.00	50.00	0.00

Note: All moments are averages excluding default periods, except those labeled "Avg. Prior Default" which are averages prior to default events. The model is simulated 160 times for 10,000 periods, dropping the initial 2,000.

5 Conclusions

This paper proposes a model to explain domestic sovereign defaults in an economy with heterogeneous agents and incomplete markets, in which a government that values the welfare of all domestic agents, including its creditors, chooses debt and default optimally. The government redistributes resources across agents and through time balancing distributional default incentives v. endogenous default costs due to the social benefits of debt for self-insurance, liquidity-provision and risk-sharing, and an exogenous income cost. A rich feedback mechanism links debt issuance and default choices, bond prices, the agents' optimal plans, and the dynamics of the distribution of bond holdings across agents.

A quantitative analysis based on a calibration to Eurozone data yields this key finding: The model sustains sizable public debt ratios exposed to default risk with infrequent defaults. The model is consistent with two important historical facts documented by Reinhart and Rogoff [48]: Domestic defaults are infrequent (1.2 percent frequency in the model v. 1.1 in the data) and defaults occur with relatively low external debt (external debt is roughly two-fifths of the total debt on average). In most periods, debt is sold at the risk-free price, but the amount of debt is always constrained by the government's inability to commit to repay. In addition, pre-default dynamics match typical debt crisis observations: Debt and spreads rise sharply and suddenly in the years before a default. The debt ratio grows 38 percent above its long-run average and spreads reach 953 basis points. The model is also qualitatively consistent with key cyclical moments in the data, particularly correlations of spreads with disposable income and government expenditures, and produces significant, time-varying dispersion in the private valuation of the gains from a default across the cross-section of agents.

The results also show that, because of the risk of default, public debt only serves the conventional role of increasing to smooth taxation when government expenditures rise if default incentives are sufficiently weak. Otherwise, debt falls when government expenditures rise as this strengthens default incentives, reducing the ability of the government to raise resources by borrowing. Along the equilibrium path, the optimal debt moves across three regions: First, for low enough debt and/or government purchases, the optimal debt is sold at the risk-free price and is in the upward-sloping region of the debt Laffer curve. Second, states with high enough debt and/or expenditures such that new debt still sells at the risk-free price but at the maximum of the debt Laffer curve. Third, a region of debt and expenditures in which the optimal debt has a positive spread and may be at the maximum of the Laffer curve or less. The sovereign desires more resources than what debt sold at the risk-free price yields, but not always as many as the most it can generate with a positive spread. Debt is in the third region less frequently than in the others, so that it sells at the risk-free price more often but the option to default always restricts the government's ability to sustain debt.

Our findings are robust to several parameter changes and model extensions. These include changes in relative risk aversion, income variability, subjective discounting, exogenous default costs, and income tax rates, as well as variants of the model with alternative specifications of welfare weights and allowing for exogenous and endogenous partial default rates.

This paper make three main contributions to the literature. First, it addresses Reinhart and Rogoff's "forgotten history of domestic debt" by providing a framework that explains outright defaults on domestic public debt. Second, debt, default and spreads are driven by a feedback mechanism in which social welfare incorporates the utility of domestic bond and non-bond holders, and debt has social value for self-insurance, liquidity, and risk-sharing, instead of by the value of consumption smoothing for a representative agent, as in standard external default models. Third, realistic debt, default, and spread dynamics are obtained, relying on endogenous default costs due to the social value of debt and without exclusion from credit markets beyond the default period, while external default models often rely heavily on exogenous default costs and credit-market exclusions of stochastic length.

The literature on domestic sovereign default is at an early stage. Some important topics to consider for future research include adding a richer structure of saving vehicles in the form of real and financial assets, complementing debt and default choices with an optimal choice of distortionary taxes, and adding secondary debt markets.

References

[1] Aguiar, M., and M. Amador (2016) "Fiscal Policy in Debt Constrained Economies," *Journal of Economic Theory*, 161, pp 37-75.

[2] Aguiar, M., and M. Amador (2014) "Sovereign Debt," Handbook of International Economics, 4, pp 647-87.

[3] Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2015) "Coordination and Crisis in Monetary Unions," *Quarterly Journal of Economics,* 130, pp 1727-1779.

[4] Aguiar M., S. Chatterjee, H. Cole, and Z. Stangebye (2016) "Quantitative Models of Sovereign Debt Crises," forthcoming Handbook of Macroeconomics Vol II.

[5] Aguiar, M., and G. Gopinath (2006) "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 69(1), pp 64-83.

[6] Aiyagari, R., and E. McGrattan (1998) "The Optimum Quantity of Debt, " *Journal of Monetary Economics,* 42, pp 447-469.

[7] Andreasen, E., G. Sandleris, and A. Van der Ghote "The Political Economy of Sovereign Defaults," Universidad Torcuato Di Tella, Business School Working Paper, 2011-07.

[8] Arellano, C. (2008) "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98(3), pp 690-712.

[9] Arellano, C., Y. Bai, and L. Bocola (2017) "Sovereign default risk and firm heterogeneity" *National Bureau of Economic Research*, (No. w23314).

[10] Arslanalp, S. and T. Tsuda (2014) "Tracking Global Demand for Advanced Economy Sovereign Debt" *IMF Economic Review*, 62(3), pp 430-464.

[11] Azzimonti, M. E. de Francisco, and V. Quadrini (2014) "Financial Globalization, Inequality, & the Rising Public Debt," *American Economic Review*, 104(8), pp 2267-2302.

[12] Bai, Y. and J. Zhang (2010), "Solving the Feldstein-Horioka Puzzle with Financial Frictions," *Econometrica*, v. 78, no. 2, pp. 603-632.

[13] Bhandari, A., D. Evans, M. Golosov, and T. Sargent (2017) "Public Debt in Economies with Heterogeneous Agents," *forthcoming Journal of Monetary Economics.*

[14] Bocola, Luigi (2016) "The Pass-Through of Sovereign Risk," *Journal of Political Economy*, Vol 124(4), pp 879-926.

[15] Broner F., A. Martin, and J. Ventura (2010) "Sovereign Risk and Secondary Markets," *American Economic Review*, 100(4), pp 1523-1555.

[16] Chang, B.H., Chang, Y, and S. Kim, (2015), "Pareto Weights in Practice: Income Inequality and Tax Reform," Working Paper No. 590, Rochester Center for Economic Research, University of Rochester.

[17] Chatterjee, S., and B. Eyigungor (2012) "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102(6), pp 2674-99.

[18] Conesa J.C. and T. Kehoe (2005) "Productivity, Taxes and Hours Worked in Spain: 1970−2003," Manuscript, University of Minnesota.

[19] Corbae, D., P. D'Erasmo, and B. Kuruscu (2009) "Politico-Economic consequences of rising wage inequality," *Journal of Monetary Economics,* 56(1), pp 43-61.

[20] Cuadra G., Sanchez J. and Sapriza H. (2010) "Fiscal Policy and Default Risk in Emerging Markets," *Review of Economic Dynamics*, 13(2), pp 452-469.

[21] D'Erasmo P. and E. Mendoza (2016) "Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default," *Journal of the European Ec. Assoc.*, Vol 14 (1), pp 7-44.

[22] Dias D., C. Richmond, and M. Wright (2012) "In for a Penny, in for a 100 Billion Pounds: Quantifying the Welfare Benefits from Debt Relief," mimeo.

[23] Di Casola, P. and S. Sichlimiris (2018) "Domestic and External Sovereign Debt," Sveriges Riksbank Working Paper No. 345.

[24] Dovis, A., M. Golosov and A. Shourideh (2016) "Political Economy of Sovereign Debt: A Theory of Cycles of Populism and Austerity," NBER wp21948.

[25] Du, W. and J. Schreger (2016) "Sovereign Risk, Currency Risk, and Corporate Balance Sheets," mimeo.

[26] Eaton J. and M. Gersovitz (1981) "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48(2), pp 289-309.

[27] Floden M. (2001) "The effectiveness of government debt and transfers as insurance," *Journal of Monetary Economics*, 48, pp 81-108.

[28] Fuchs-Schundeln, N., D. Krueger, and M. Sommer (2010) "Inequality trends for Germany in the last two decades: A tale of two countries" *Review of Economic Dynamics*, 13, pp 103−132.

[29] Gennaioli N., A. Martin and S. Rossi, (2014) "Sovereign default, domestic banks and financial institutions," *Journal of Finance,* 69(2), pp 819-866.

[30] Guembel A. and O. Sussman (2009) "Sovereign Debt without Default Penalties, " *Review of Economic Studies,* 76, pp 1297-1320.

[31] Guvenen, F (2009) "An Empirical Investigation of Labor Income Processes," *Review of Economic Dynamics,* 12(1), 58-79.

[32] Jappelli, T. and L. Pistaferri (2010) "Does consumption inequality track income inequality in Italy?" *Review of Economic Dynamics*, 13, pp 133−153.

[33] Jeon, K. and Z. Kabukcuoglu (2014) "Income Inequality and Sovereign Default", working paper, University of Pittsburgh.

[34] Hall, G., and T. Sargent (2014) "Fiscal Discrimination in Three Wars," *Journal of Monetary Economics*, 61, pp 148−166.

[35] Hatchondo, J.C., and L. Martinez (2009) "Long-duration bonds and sovereign defaults," *Journal of International Economics*, vol. 79(1), pp 117−125.

[36] Heathcote J. (2005) "Fiscal Policy with Heterogeneous Agents," *Review of Economic Studies,* 72, pp 161-188.

[37] Heathcote, J., K. Storesletten, and G. Violante (2009) "Quantitative macroeconomics with heterogeneous households," *Annual Review Economics*, 1, pp. 319354.

[38] Heathcote, J., K. Storesletten, and G. Violante (2016) "Optimal Tax Progressivity: An Analytical Framework," Federal Reserve Bank of Minneapolis, Staff Report 496.

[39] D. Krueger, K. Mitman, and F. Perri (2016) "Macroeconomics and Household Heterogeneity" *Handbook of Macroeconomics. Vol. 2.*, 2016. pp. 843-921.

[40] Lojsch D., M. Rodriguez-Vives and M. Slavik (2011) "The size and composition of government debt in the euro area," Occasional Paper Series 132, *E.C.B.*.

[41] Mendoza, E. G., Tesar, L. L., and Zhang, J. (2014) "Saving europe?: The unpleasant arithmetic of fiscal austerity in integrated economies" *NBER*, wp20200.

[42] Mengus E. (2014) "Honoring Sovereign Debt or Bailing Out Domestic Residents? A Theory of Internal Cost of Default," WP Banque de France 480.

[43] Olivares-Caminal, R. (2013)"The Pari Passu Clause in Sovereign Debt Instruments: developments in recent litigation," Proceedings from Sovereign Risk: A World Without Risk-Free Assets? Bank for International Settlements.

[44] Paczos, W., and K. Shakhnov (2018) "Sovereign debt issuance and selective default." European University Institute, mimeo.

[45] Panizza, U., F. Sturzenegger and J. Zettelmeyer (2009) "The Economics and Law of Sovereign Debt and Default," *Journal of Economic Literature,* 47(3), pp 651-98.

[46] Perez, D. (2015) "Sovereign Debt, Domestic Banks and the Provision of Public Liquidity," Manuscript, New York University.

[47] Pijoan-Mas, J., and V. Sanchez-Marcos (2010) "Spain is Different: Falling Trends of Inequality," *Review of Economic Dynamics*, 13(1), pp 154-178.

[48] Reinhart C., and S. Rogoff (2008) "The Forgotten History of Domestic Debt," *NBER Working Papers 13946*.

[49] Rojas, E. (2018) "Understanding Sovereign Default Costs: The Role of Firm Heterogeneity," mimeo.

[50] Sosa-Padilla, C. (2014) "Sovereign Defaults and Banking Crises," forthcoming *Journal of Monetary Econmics*.

[51] Yue, V. (2010) "Sovereign Default and Debt Renegotiation," *Journal of International Economics*, 80(2), pp 176-87.

Online Appendix to History Remembered:
Optimal Sovereign Default on Domestic and External Debt

by

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This Appendix is divided in nine sections. Section A-1 presents a Table with summary indicators of the fiscal situation of the main Eurozone countries in 2011. Section A-2 contains a detailed description of the data sources and transformations for the various macro variables used in the analysis. Section A-3 describes the solution method used to solve for the model's Recursive Markov Equilibrium. Section A-4 offer additional details on the default event analysis. Section A-5 offers an analysis of the model's time-series dynamics between two representative default events. Section A-6 provides further analysis of the recursive equilibrium functions, particularly the individual welfare gains of default and the optimal debt decision rule. Section A-7 contains a more detailed comparison of the welfare weights versus the average bond distribution, looking at marginal distributions over different income levels. Section A-8 discusses the results under a calibration to Spain. Finally, Section A-9 presents the algorithm used to solve the model with endogenous partial default.

A-1 Eurozone Fiscal Situation in 2011

Table A.1: Eurozone Fiscal Situation in 2011

Moment in (%)	Gov. Debt	Gov. Debt Held by Residents	Gov. Exp.	Gov. Rev.	Primary Balance	Sov. Spreads
France	62.73	46.17	24.48	50.60	-2.51	0.71
Germany	51.49	44.47	19.27	44.50	1.69	0.00
Greece	133.10	29.68	17.38	42.40	-2.43	13.14
Ireland	64.97	45.35	18.38	34.90	-9.85	6.99
Italy	100.23	64.33	20.42	46.20	1.22	2.81
Portugal	75.84	37.36	20.05	45.00	-0.29	7.63
Spain	45.60	66.00	20.95	35.70	-7.04	2.83
Austria	52.92	38.03	19.78	48.55	-0.45	0.71
Belgium	83.58	54.73	23.77	50.31	-0.91	1.62
Finland	-48.79	23.90	23.62	53.34	-1.02	0.40
Netherlands	37.19	44.76	25.99	42.68	-3.04	0.38
Avg	59.90	44.98	21.28	44.93	-2.24	3.38
Median	62.73	44.76	20.42	45.00	-1.02	1.62
Avg (GDP w)	62.93	50.14	21.45	45.25	-1.20	2.37

Note: Author's calculations based on OECD Statistics, Eurostat and European Central Bank (ECB)."Gov. Debt" corresponds to total general government net financial liabilities as a fraction of GDP; "Gov. Debt Held by Residents" refers to fraction of gross government debt held by domestic non-financial corporations, financial institutions, other government sectors, households and non-profit institutions; "Gov. Exp." is general government final consumption as a fraction of GDP; "Gov. Rev." corresponds to general government revenues as a fraction of GDP. "Prim. Balance" corresponds to the primary balance (total expenditures net of interest payments minus total revenue) as a fraction of GDP; and "Sov Spreads" correspond to the difference between interest rates of the given country and Germany (for bonds of similar maturity). For a given country i, spreads are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1$. See Appendix A-2 for a detailed explanation of variables and sources.

A-2 Data Description and Sources

This Appendix describes the variables we gathered from the data and the sources. Most data cover the 1981-2015 period, but for some variables the sample starts in 2002. Most of the moments used for the calibration correspond to GDP-weighted averages of country specific moments. The countries we use for this calibration are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain. We calculate the weights using real GDP data from 2007 (the year prior to the start of the crisis). The weights for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain are 0.031, 0.038, 0.021, 0.212, 0.273, 0.026, 0.019, 0.177, 0.067, 0.019, and 0.117, respectively.

The details are as follows:

1. Government debt: total general government net financial liabilities as a fraction of GDP, from OECD Statistics for the period 1981−2015.[35]

2. Fraction of government debt held by residents (also referenced in the paper as fraction of domestic debt): corresponds to fraction of general government gross debt held by domestic investors in the IMF dataset put together by Arslanalp and Tsuda [10]. We extended the data when necessary to complete the 1981−2015 sample using information from OECD Statistics on the fraction of marketable debt held by residents as a fraction of total marketable debt. The correlation between both series when they overlap is equal to 0.84.

3. Government expenditures: general government final consumption as a fraction of GDP from World Development Indicators for the period 1981-2015.

4. Government revenue: Total general government revenue as a fraction of GDP from OECD statistics for the period 1981-2015.

5. Sovereign spreads: constructed using EMU convergence criterion bond yields from Eurostat for the period 2002-2015. For a given country i, spreads are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1.$, where r^{Ger} is the yield on German bonds. Data before 2002, prior to the introduction of the euro, are excluded because spreads were heavily influenced by currency risk, and not just sovereign risk. The GDP-weighted average in this case re-normalizes weights because the average is computed without Germany (the country use as reference for the risk-free rate).

6. Cross sectional variance of log-wages (needed to calibrate the income process) obtained from the cross-sectional variance of residual log-earnings in Germany, Italy and Spain as reported by Fuchs-Schundeln, Krueger and Sommer [28] (Germany), Japelli and Pistaferri [32] (Italy), and Pijoan-Mas and Sanchez Marcos [47] (Spain).

7. Income net of fixed investment (μ_y): constructed as GDP minus gross capital formation (formerly gross domestic investment) as a ratio of GDP, from World Development Indicators for the period 1981-2015.

[35] At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

8. Maturity adjusted debt ratio: computed using the Macaulay duration rate. The Macaulay duration for a consol is $D = \frac{1+r^*}{r^*+\delta}$, where r^* is the consol's constant annual yield. Denoting the observed outstanding debt as $\overline{B}$ and the equivalent one-period debt at the beginning of the period (i.e. the maturity-adjusted debt) as B, we use δ to express $\overline{B}$ as the present value of outstanding coupon claims $\overline{B} = \sum_{s=1}^{\infty} \frac{B(1-\delta)^{s-1}}{(1+r^*)^{s-1}}$, which then reduces to the expression noted in the text:

$$\overline{B} = \frac{B(1+r^*)}{(r^*+\delta)}.$$

   Duration is calibrated to average term to maturity of central government debt. Source: OECD statistics for the period 2002-2010. OECD stopped updating this dataset after 2010.

9. Tax revenue: defined to include only effective labor taxes levied on individuals, accruing to both individual labor income and consumption taxes, and excluding all forms of capital income taxation. Consumption tax revenues and the split of labor and capital components of individual income taxes are obtained using the effective tax rates constructed by Mendoza, Tesar, and Zang [41]) using OECD data for the period 1995-2015.

10. Government transfers: measured as a residual using the government budget constraint. Hence, transfers are equal to transfer and entitlement payments, plus other outlays (total outlays minus current expenditures, debt service and transfers), minus tax revenue other than effective labor taxes, plus the difference between net lending in the general government national accounts and the change in reported net general government financial liabilities. Data from OECD Statistics for the period 1995-2015.

11. Household disposable income: corresponds to gross household disposable income at constant 2010 prices from OECD Statistics (downloaded from Bloomberg) for the period 1981-2015.

12. Trade balance: external balance on goods and services as a fraction of GDP, from World Development Indicators for the period 1981−2015.

Table A.2: Country Specific Moments (averages)

	GDP-weights	G/Y	μ_y	ρ_g	σ_e	B^d/B
France	0.212	22.60	77.96	0.87	0.019	56.45
Germany	0.273	19.24	77.98	0.80	0.021	50.43
Greece	0.026	18.79	76.49	0.88	0.045	25.47
Ireland	0.019	17.72	78.02	0.93	0.061	40.09
Italy	0.177	18.85	79.27	0.82	0.024	62.50
Portugal	0.019	17.46	75.56	0.94	0.033	35.34
Spain	0.117	17.27	75.77	0.94	0.026	73.23
Austria	0.031	18.95	75.07	0.85	0.019	54.30
Belgium	0.038	22.19	77.76	0.92	0.023	52.43
Finland	0.021	21.52	75.98	0.89	0.038	23.33
Netherlands	0.067	22.99	78.27	0.94	0.030	58.39
GDP-weighted avg		19.98	77.74	0.86	0.024	55.53
	Spreads†	B/Y^*	D	$(B/Y)/D$	Tax Rev	τ
France	0.33	31.18	6.52	4.78	33.57	10.02
Germany	0.00	38.47	6.11	6.29	30.33	9.82
Greece	4.88	90.07	7.13	12.63	25.70	10.55
Ireland	1.55	33.04	5.62	5.88	22.78	5.61
Italy	1.21	90.40	6.59	13.72	27.95	4.96
Portugal	2.21	60.52	5.24	11.54	24.41	4.83
Spain	1.15	37.25	6.42	5.80	25.18	6.11
Austria	0.34	43.84	7.46	5.87	35.14	14.23
Belgium	0.53	101.60	6.29	16.14	33.10	4.28
Finland	0.22	-35.31	4.06	-8.71	36.01	12.17
Netherlands	0.21	31.20	6.38	4.89	30.53	6.27
GDP-weighted avg	0.92	48.23	6.35	7.45	30.00	8.15

Note: † GDP-weighted spreads use GDP-weights re-normalized for a sample that excludes Germany. * At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

Table A.3: Country Specific Moments (Peak-Crisis)

	GDP-weights	B/Y^*	D	$(B/Y)/D$	B^d/B	Tax Rev	τ	G/Y	Spreads†
France	0.21	67.44	6.60	10.22	47.59	32.64	15.13	23.93	1.04
Germany	0.27	49.34	5.89	8.38	51.46	29.69	13.86	19.56	0.00
Greece	0.03	101.78	7.10	14.34	29.69	22.80	24.60	23.31	21.00
Ireland	0.02	79.28	4.31	18.40	45.35	21.13	22.62	20.19	6.99
Italy	0.18	111.77	6.80	16.44	67.65	28.19	20.54	20.63	3.99
Portugal	0.02	90.55	5.77	15.69	40.40	23.74	25.26	21.43	9.05
Spain	0.12	59.12	6.40	9.24	73.74	22.93	15.88	20.52	4.35
Austria	0.03	57.90	8.30	6.98	38.30	34.33	19.16	20.58	0.87
Belgium	0.04	92.22	5.94	15.53	55.88	31.99	14.48	24.26	1.62
Finland	0.02	48.79	3.90	12.51	25.41	34.37	28.06	20.70	0.52
Netherlands	0.07	39.69	6.60	6.01	47.32	30.70	15.02	22.06	0.47
GDP-weighted avg		69.36	6.34	10.94	54.15	29.20	16.78	21.34	3.34

Note: † GDP-weighted spreads use GDP-weights re-normalized for a sample that excludes Germany. Peak-Crisis duration refers to the minimum value during 2008-2010. * At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

Table A.4: Country Specific Moments (Business Cycle Correlations)

	France	Ger.	Greece	Ireland	Italy	Port.	Spain	Austria	Belgium	Finland	Neth.	Avg.	Min	Max
Standard Dev.														
$hhdi$	0.85	0.65	n.a.	2.62	1.07	1.57	1.44	0.86	1.03	1.62	1.89	1.05	0.65	2.62
Consumption	0.82	1.12	1.18	1.26	0.98	0.99	0.83	0.36	0.47	0.69	0.37	0.89	0.36	1.26
TB/GDP	0.55	1.05	0.56	0.90	0.55	0.67	0.57	0.51	0.55	0.48	0.29	0.68	0.29	1.05
Spreads	0.16	n.a.	1.85	0.40	0.47	0.92	0.40	0.14	0.21	0.05	0.04	0.35	0.04	1.85
B/GDP	3.42	3.49	4.07	0.06	2.60	2.73	1.60	2.22	2.50	3.57	1.39	2.82	0.06	4.07
B^d/GDP	2.87	2.14	1.11	0.03	2.54	1.13	1.61	1.87	3.14	0.74	1.09	2.15	0.03	3.14
Correl(x,$hhdi$)														
Consumption	0.76	0.56	n.a.	0.32	0.67	0.82	0.46	0.48	0.45	0.49	0.51	0.60	0.32	0.82
TB/GDP	-0.27	-0.01	n.a.	0.36	-0.50	-0.74	-0.33	0.11	-0.09	-0.48	0.08	-0.21	-0.74	0.36
Spreads	-0.24	n.a.	n.a.	-0.23	-0.60	-0.52	-0.61	-0.04	-0.25	-0.20	0.39	-0.33	-0.61	0.39
B/GDP	-0.24	-0.25	n.a.	-0.58	-0.47	-0.18	-0.21	-0.04	-0.18	-0.24	-0.04	-0.26	-0.58	-0.04
B^d/GDP	-0.32	-0.15	n.a.	-0.57	-0.53	0.32	-0.36	0.10	-0.57	0.23	0.02	-0.27	-0.57	0.32
Correl(x,g/GDP)														
$hhdi$	-0.33	-0.19	n.a.	-0.13	0.04	0.05	0.41	-0.10	0.11	0.08	-0.26	-0.09	-0.33	0.41
Consumption	-0.69	-0.31	0.02	-0.47	-0.17	0.18	-0.19	-0.13	-0.15	-0.41	-0.13	-0.31	-0.69	0.18
TB/GDP	0.40	-0.11	0.01	-0.20	-0.11	-0.07	0.27	-0.11	0.15	-0.19	-0.44	0.03	-0.44	0.40
Spreads	0.11	n.a.	0.07	-0.14	-0.21	-0.45	-0.23	0.27	0.20	0.45	-0.34	-0.07	-0.45	0.45
B/GDP	0.47	0.47	0.33	0.31	0.46	0.23	0.20	0.48	0.66	0.01	0.09	0.40	0.01	0.66
B^d/GDP	0.24	0.71	0.25	0.02	0.32	0.33	-0.19	0.38	0.44	0.20	-0.03	0.32	-0.19	0.71

Note: TB denotes trade-balance. $hhdi$ denotes household disposable income. In the model, $hhdi = (1-\tau^y)Y + \tau$ and $TB = Y - C - g$. $hhdi$ and C are logged and HP filtered with the smoothing parameter set to 6.25 (annual data). GDP ratios are also HP filtered with the same smoothing parameter. Standard deviations (except that for $hhdi$) are ratios to the standard deviations of $hhdi$ ($hhdi$ data for Greece is not available, so in this case we provide the ratio to the standard deviation of GDP). GDP-weighted moments for spreads use GDP-weights re-normalized for a sample that excludes Germany.

A-3 Computational Algorithm

This Appendix describes the algorithm we constructed to solve for the model's CRME and RME. The algorithm performs a global solution using value function iteration. We approximate the solution of the infinite horizon economy by solving for the equilibrium of a finite-horizon version of the model for which the finite number of periods (T) is set to a number large enough such that the distance between value functions, government policies and bond prices in the first and second periods are the same up to a convergence criterion. The corresponding first-period functions are then treated as representative of the solution of the infinite-horizon economy.

The algorithm has a backward-recursive structure with the following steps:

1. Define a discrete state space of values for the aggregate states $\{B, g\}$ and individual states $\{b, y\}$
2. Solve for date-T recursive functions for each $\{b, y\}$ and $\{B, g\}$:
   - Government debt choice: $B'_T(B, g) = 0$, because T is the final period of the economy.
   - Price Debt: $q_T(B', g) = 0$, also because T is the final period.
   - The lump-sum tax under repayment follows from the government budget constraint:
     $$\tau_T(B', B, g) = B + g - \tau^y Y.$$
   - Using the agents' budget constraint under repayment, we obtain the agents' value function for arbitrary debt choice (note that at T it is actually independent of $\tilde{B}$ since $q_T(B', g) = 0$)
     $$\tilde{V}_T^{d=0}(\tilde{B}, y, b, B, g) = u((1 - \tau^y)y + b - g - B + \tau^y Y)$$
   - The agents' value functions under repayment and default can then be solved for as:
     $$V_T^{d=0}(y, b, B, g) = \tilde{V}_T^{d=0}(0, y, b, B, g).$$
     $$V_T^{d=1}(y, g) = u((1 - \tau^y)y(1 - \phi(g)) - g + \tau^y Y).$$
   - Given the above, the social welfare functions under repayment and default are:
     $$W_T^{d=0}(B, g) = \int_{Y \times B} V_T^{d=0}(y, b, B, g) d\omega(b, y)$$

$$W_T^{d=1}(g) = \int_{Y \times B} V_T^{d=1}(y,g) d\omega(b,y).$$

- The default decision rule can then be obtained as:

$$d_T(B,g) = \arg \max_{d=\{0,1\}} \{W_T^{d=0}(B,g), W_T^{d=1}(g)\}.$$

- The agents' ex-ante value function (before the default decision is made) is:

$$V_T(y,b,B,g) = (1-d_T)V_T^{d=0}(y,b,B,g) + d_T V_T^{d=1}(y,g).$$

3. Obtain the solution for periods $t = T-1, \ldots, 1$.

   (a) Set $t = T-1$.

   (b) Obtain the default probability for all $\{B', g\}$ as:

$$p_t(B',g) = \sum_{g'} d_{t+1}(B',g')F(g',g).$$

   (c) Solve for the pricing function $q_t(B',g)$:

$$q_t(B',g) = \frac{1-p_t(B',g)}{1+r}.$$

   (d) Given the above, the lump-sum tax under repayment for an initial (B,g) pair and a given B' is:

$$\tau_t(B',B,g) = B + g - q_t(B',g)B' - \tau^y Y.$$

   (e) Solve the agents' optimization problem for each agent with bonds and income b, y and each triple $\{\tilde{B}, B, g\}$:

$$\tilde{V}_t^{d=0}(\tilde{B},y,b,B,g) = \max_{b'} u(c) + \beta E_{g'}[V_{t+1}(b',y',\tilde{B},g')]$$

   s.t.

$$c = (1-\tau^y)y + b - q_t(\tilde{B},g)b' - \tau_t(\tilde{B},B,g).$$

   (f) Given the solution to the above problem, solve for the optimal debt choice of the government:

$$B_t'(B,g) = \arg\max_{\tilde{B}} \int \tilde{V}_t^{d=0}(\tilde{B},y,b,B,g)d\omega(b,y).$$

(g) The agents' continuation value under repayment is:

$$V_t^{d=0}(y,b,B,g) = \tilde{V}_t^{d=0}(B_t'(B,g),y,b,B,g).$$

(h) The agents' continuation value under default is:

$$V_t^{d=1}(y,g) = u((1-\tau^y)y(1-\phi(g)) - g + \tau^y Y) + \beta E_{g'}[V_{t+1}^{d=0}(y',0,0,g')]$$

(i) Given the above, the social welfare functions under repayment and default are:

$$W_t^{d=0}(B,g) = \int_{Y\times B} V_t^{d=0}(y,b,B,g)d\omega(b,y)$$

$$W_t^{d=1}(g) = \int_{Y\times B} V_t^{d=1}(y,g)d\omega(b,y).$$

(j) Compute the government's default decision as:

$$d_t(B,g) = \arg\max_{d=\{0,1\}}\{W_t^{d=0}(B,g), W_t^{d=1}(g)\}.$$

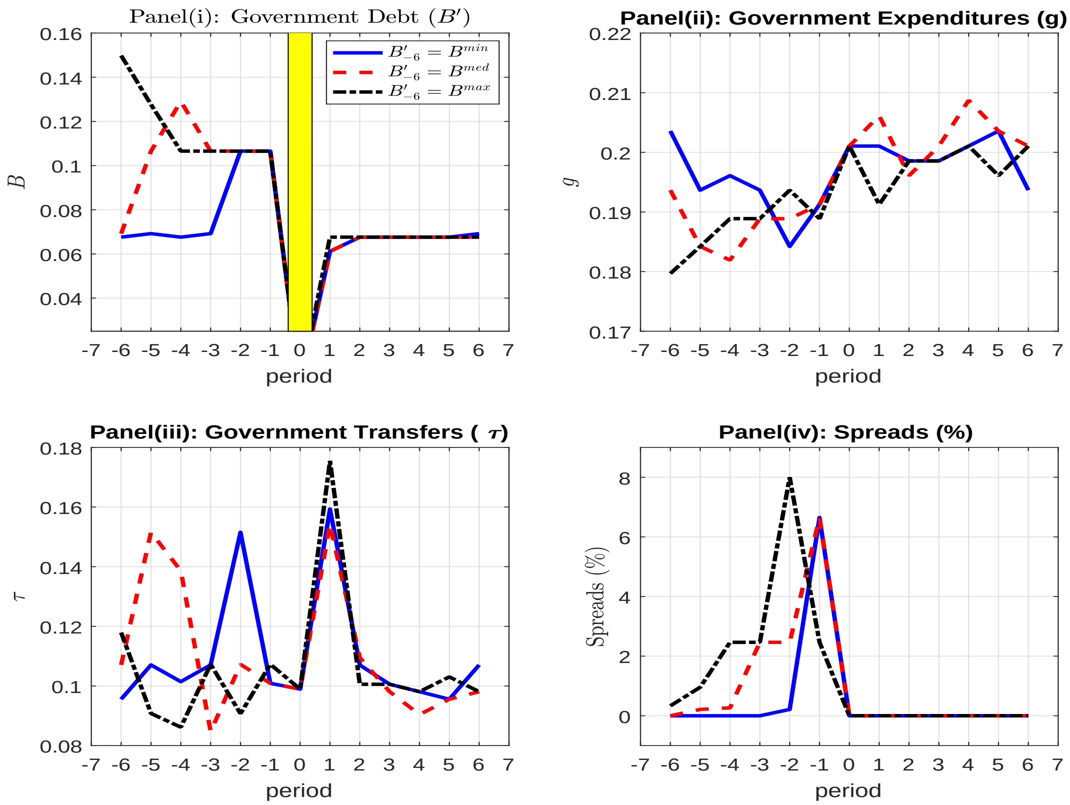
(k) If $t>1$, set $t=t-1$ and return to point 3b. If $t=1$ continue.

4. Check whether value functions, government decision rules, and bond prices in periods $t=1$ and $t=2$ satisfy a convergence criterion. If they do, the functions in period $t=1$ are the solution of the RME and the algorithm stops. If the convergence criterion fails, increase T and return to Step 2.

A-4 Default Event Analysis Extended

Figure A.1 presents the evolution of debt, government expenditures, transfers, and spreads across three different default events: one with the maximum level of debt at the beginning of the default event window (denoted by $B_6 = B^{max}$), other with median level of debt in period $t=-6$ (denoted by $B_6 = B^{med}$ is the same event presented in Figure 3 in the body of the paper), and one with the lowest debt level observed at the beginning of the default window (denoted by $B_5 = B^{min}$).

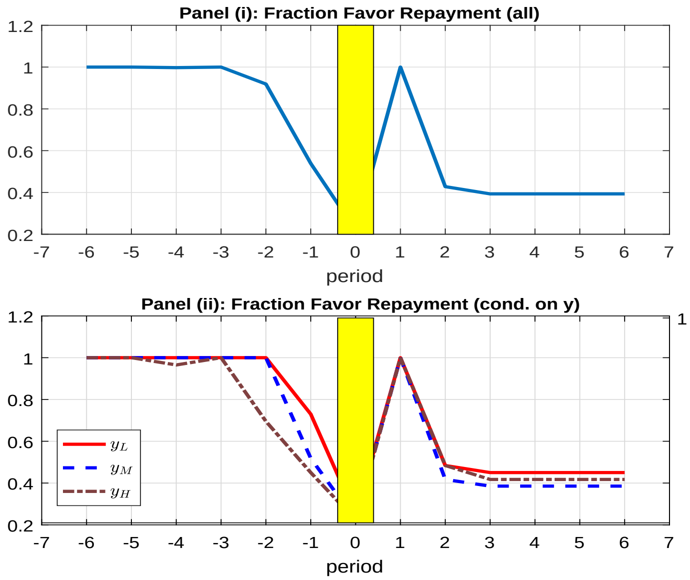
Figure A.1: Default Event Analysis



We observe the same pattern across default events. As government expenditures decrease, the government has more room to redistribute and that results in an increase in the debt level and lump-sum transfer.

Figure A.2 shows event windows for the government's perceived fraction of agents who prefer repayment (i.e., the fraction of agents for whom $\alpha(b, y, B, g) < 0$ obtained by aggregating using the social welfare weights $\omega(b, y)$), again using medians across each of the 121 defaults events for each of the 13 periods in the windows. Panel (i) aggregates across all (b, y) and Panel (ii) splits the results into low, mean and high income levels.

Figure A.2: Preferences over Repayment



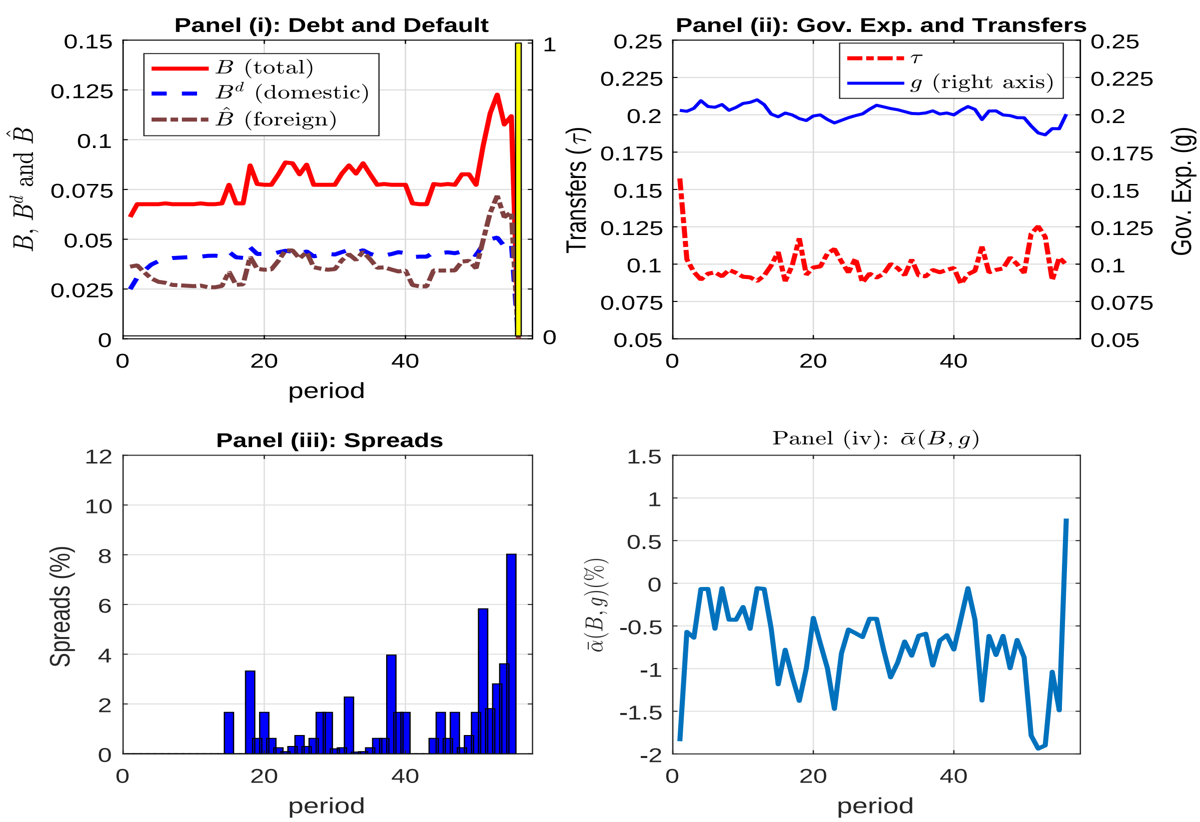
Panel (i) shows that the perceived fraction of agents that prefer repayment remains close to 100 percent until 3 years before the default. It then declines in periods $t = -3, -2, -1$, when Figure 3 shows that default risk rise. Since debt is stable prior to the default, these movements reflect mainly the effects of changes in government expenditures and transfers (a reduction in government redistribution). Then in year 0, the increase in g is sufficient to make default optimal even though debt did not increase in the previous two years.

Panel (ii) shows interesting dynamics in the perceived fractions of agents who prefer repayment across income levels. The fraction is highest for low-income agents who value lump-sum transfers and the liquidity benefits of debt the most. The fraction of low-income agents who favor repayment drops only in years $t = -2, -1$. The fraction of mid-income and high-income agents who prefer repayment follows a similar pattern, but the decline starts a year earlier in the case of high income households. Mid-income and high-income agents value the liquidity services of debt but rely less on lump-sum transfers that can be sustained with debt. Interestingly, the fraction of agents who favor repayment is above zero in all years before and after the default and for all income levels. This is because there are sufficiently wealthy individuals with very low income that still favor repayment.

A-5 Dynamics Between Default Events

In the text, we illustrated the time series dynamics of the model using an event analysis with 13-year event windows centered on default events. In this appendix, we follow an alternative approach by studying time series dynamics across two default events. Figure A.3 shows the time-series dynamics between two defaults that are separated by a number of years equal to the mode duration of the non default or repayment period in the simulated data set, which is 57 years (the mode of the distribution of periods between default events). This long mode repayment period is in line with the result that defaults occur with a long-run frequency of only 1.2 percent. The figure is divided in the same four panels as the event analysis plots in the text. Panel (i) shows total government bonds (B) and their aggregate domestic and foreign holdings (B^d and $\hat{B}$ respectively). Panel (ii) shows g and transfers (τ). Panel (iii) shows the bond spreads and Panel (iv), displays the social welfare gain of default $\overline{\alpha}$ (in %). These charts start just after the first of the two defaults occurred, and end right when the next default occurs, 57 years later.

Figure A.3: Time-Series Dynamics between Default Events



Panel (i) of Figure A.3 shows that public debt grows rapidly after the initial default but stays close to its mean (the value that maximizes the "Debt Laffer" curve) for a large portion of the sample, and then (around period 50) starts to grow at a faster pace, until it reaches about 12.5 percent of GDP and the second default occurs. In line with what we found in the event analysis, the initial rise in debt occurs with declining g, which makes default more costly due to the exogenous income cost of default, thus strengthening repayment incentives and allowing the government to sustain more debt. Also in line with what the event analysis showed, taxes are generally lower than government purchases when the debt is rising, generating a primary deficit(see Panel (ii)). Spreads are generally small (Panel (iii)), and the social welfare gain of default is negative and relatively large (Panel (iv)).

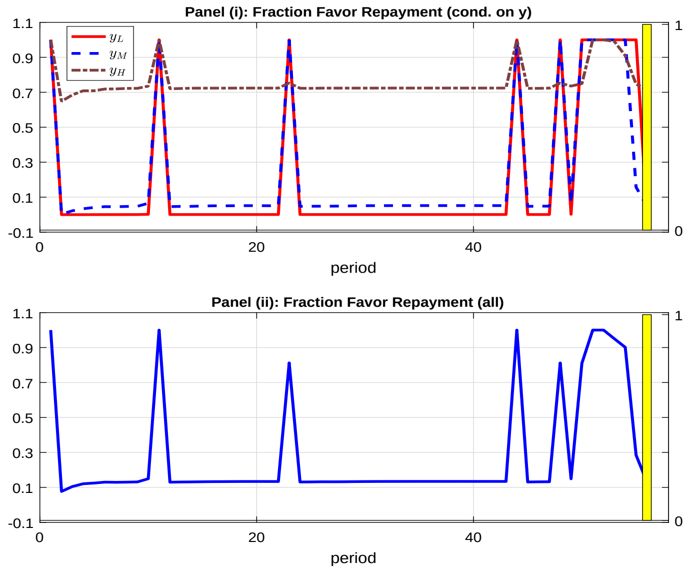
Panel (i) also shows that in the early years after the initial default, when the supply of public debt is increasing, domestic demand for risk-free assets is also rising, as the government is lowering taxes (which increases disposable income) and agents with relatively high-income realizations seek to replenish their buffer stock of savings. Domestic debt remains a higher fraction of total debt in most periods, as well as on average over the 57 years plotted. The ratio of domestic to external debt holdings, however, fluctuates, being smaller in the initial and final years than in the prolonged period in between.

In the last 10 years before the second default, domestic demand for risk-free assets increases but not as fast as total debt, which implies that the bulk of the new debt is placed abroad. With this creditor mix, and since foreign creditors do not enter in the social welfare function, default risk and spreads increase significantly. This pattern of spreads shifting suddenly from, on average, 1 percent to high levels is qualitatively consistent with standard predictions of external default models and with the stylized facts of debt crises. Still, default does not occur because the social welfare gain of default remains negative, until the 57th year arrives and the realization of g is sufficiently high to make default optimal at the existing outstanding debt since the relatively high level of debt in combination with the increase in expenditures forces the government to reduce lump-sum transfers.

The dynamics of the social gain of default in panel (iv) also capture the previous result showing that, even tough the welfare weights given by $\omega(b, y)$ are exogenous, the heterogeneity of agents plays a central role. The fraction of agents that the planner sees as benefiting from a default changes endogenously over time as debt, taxes, and spreads change, and the associated changes in the dispersion of individual gains of default affect the social welfare function, the default decision, and spreads.

We examine next the evolution of the fraction of agents in the economy who value repayment (i.e., those with $\alpha(b, y, B, g) < 0$ in the actual wealth distribution $\Gamma_t(b, y)$). Figure A.4 plots the evolution of this fraction for three income levels in Panel (i) and across all (b, y) in Panel (ii).

Figure A.4: Preferences over Repayment



With sufficiently large fraction of agents close to the borrowing limit, the faction of agents who favors repayment remains relatively low for most of the period. In fact, only close to the default event, the fraction that favors repayment reaches 1 for more than one period. This is due to the fact that as g declines the government issues more debt and increases transfers. As time goes by, the government starts to reduce the level of debt but a new g shock (period 55) results in a reduction in the fraction of agents in favor of repayment, since the government does not have room for further redistribution via debt at a relatively high initial debt and needs to cut transfers to pay, which induces a government default.

In line with the discussion of default payoffs in the text, the fraction of low-income agents who prefer repayment increases faster than the fraction of high-income agents who prefer repayment when confronted with government spending shocks. Interestingly, the fraction of agents with all levels of income, including the lowest, who favor repayment remains positive throughout. This is because, as we also noted in the text, there are sufficiently wealthy individuals with very low income that still favor repayment.

A-6 Details on Recursive Equilibrium Functions

This section of the Appendix provides further details on some of the implications of the recursive equilibrium functions. First we give a broader perspective on the cross-sectional properties of the individual welfare gains of default, which were examined in the paper using two-dimensional charts. Here we show that those properties are more general using intensity plots to illustrate three-dimensional variations. Figure A.5 shows two intensity plots of how $\alpha(b, y, B, g)$ varies over b and y with $g = \mu_g$. Panel (i) is for $B = B_L$ and Panel (ii) is for $B = B_H$.

The intuition for the features of these plots follows from the discussion of the threshold wealth that separates favoring repayment from favoring default, $\hat{b}(y, B, g)$, near the end of Section 2 in the main text. Comparing across panels (i) and (ii), $\alpha(b, y, B, g)$ is higher with the higher B for a given (b, y) pair, because $\hat{b}(y, B, g)$ is increasing in B. Consider next the variations along the b dimension. With $g = \mu_g$, only agents with very low b prefer default at both values of B. These agents benefit from the lower taxes associated with default and suffer negligible wealth losses. As b rises agents value increasingly more repayment for the opposite reason.

Explaining the variations along the y dimension is less straightforward, because both the repayment and default payoffs depend on y. $V^{d=1}(y, g)$ is increasing in y. $V^{d=0}(b, y, B, g)$ is increasing in "total resources," $y + b$, but is non-monotonic on b and y individually. In particular, while for a given b, $\alpha(b, y, B, g)$ is generally increasing in y, it decreases in y for high B and very low b. The reason for this follows from the discussion around Figure 4 in the paper.

Figure A.5: $\alpha(b, y, B, g)$ (for different B at $g = \mu_g$)

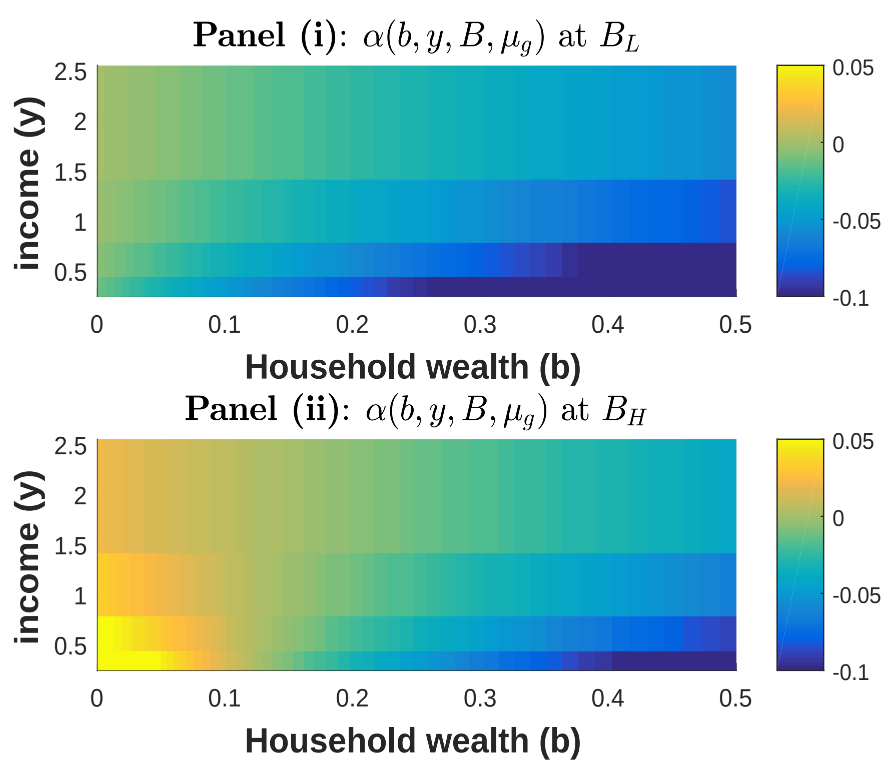


Figure A.6 presents $\hat{b}(y,B,g)$ for different values of y (Panel (i) for $y=y_L$, Panel (ii) for $y=y_M$, and Panel (iii) for $y=y_H$) and different values of g (lines within each panel) as a function of B.

Figure A.6: $\hat{b}(y,B,g)$ (for different g and y as a function of B)

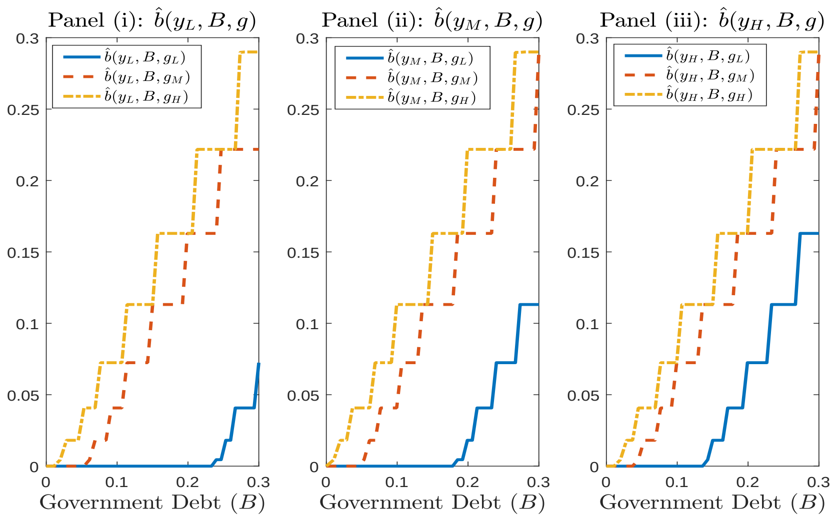


Figure A.6 corroborates than in our calibrated model $\hat{b}(y,B,g)$ is increasing in B for different values of y and g. Higher debt level reduces the level of transfers and limits the amount of redistribution that the government can implement.

Figure A.7: Optimal Debt $B'(B,g)$

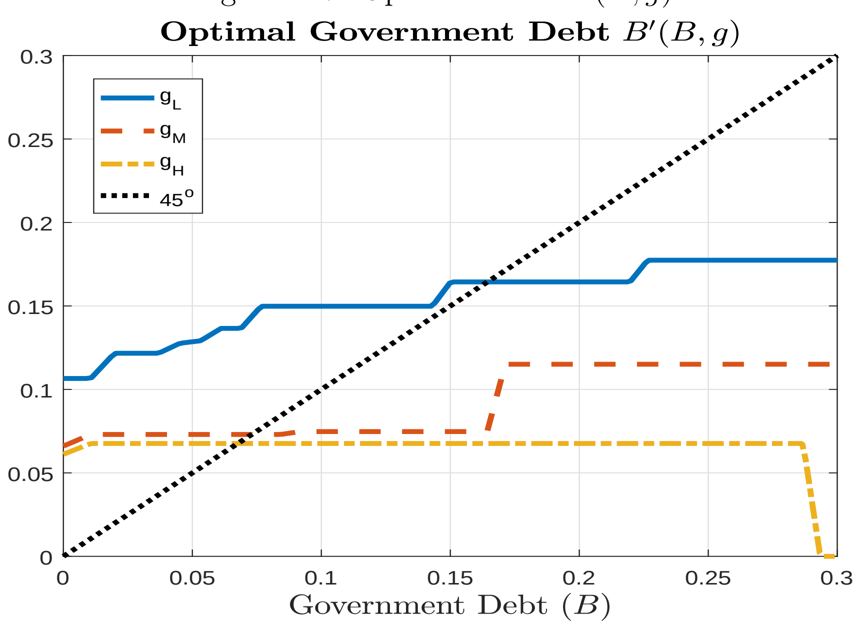


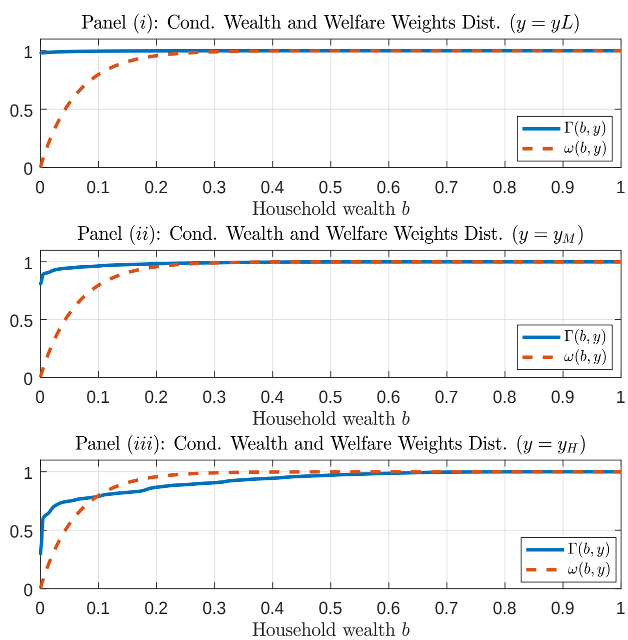
Figure A.7 shows that for high or average g, the optimal debt choice is independent of

B. In both cases, the government chooses the amount of debt that maximizes the Laffer curve regardless of the value of B (0.0106 for g_M and 0.0708 for g_H). Debt is risk-free but effectively "constrained" by the inability to commit to repay. For low g, the optimal debt rises with B and is always below the maximum of the Laffer curve (0.139).

A-7 Welfare Weights versus Wealth Distribution

Figure A.8 compares the weights of the social welfare function $\omega(b, y)$ with the distribution of wealth in the economy $\Gamma(b, y)$. The comparison is useful because, as explained in Section 3 of the main text, the distributional incentives to default are weaker the higher the relative weight of bond holders creditors in $\omega(b, y)$ v. $\Gamma(b, y)$. Since $\Gamma(b, y)$ is time- and state-contingent, we show the average $\bar{\Gamma}$ over the full time series simulation excluding default episodes. The plots show conditional distributions as functions of b for low, average, and high values of y in Panels (i), (ii), and (iii), respectively.

Figure A.8: "Average" Wealth Distribution $\bar{\Gamma}(b, y)$ and Welfare Weights $\omega(b, y)$



This figure shows the extent to which the fraction of agents with low b in the model economy exceeds their welfare weights. The differences are driven solely by differences in b because, by construction, $\bar{\Gamma}$ and ω have the same income distribution conditional on wealth ($\omega(b, y)$ was calibrated using $\pi^*(y)$ along the y dimension). Panels (i) and (ii) show that the majority of agents with income at the mean or lower are at the borrowing constraint or close to it (i.e., their bond holdings are zero or nearly 0), while bond holdings need to be equal to 0.30 and 0.10 to obtain the same fraction of agents using $\omega(b, y)$ for low income and mean income, respectively. For agents with high income, Panel (*iii*) shows that the fraction of agents with $b < 0.1$ is about the same under both distributions.

A-8 Calibration to Spain

This Appendix describes the calibration approach and the results of the model when calibrated to Spain. The first step of the calibration proceeds as follows: We set $\sigma = 1$ (i.e. log utility), which is in the range commonly used in macro models. The interest rate is set to $\bar{r} = 0.021$, which is the average annual return on German EMU-convergence criterion government bonds in the European Commission's Eurostat database for the period 2002−2012 (these are secondary market returns, gross of tax, with around 10 years' residual maturity).

To calibrate the individual income process, we set $\rho_y = 0.85$, which is a standard value in the heterogeneous-agents literature (e.g., Guvenen [31]). Then, we set σ_u to match Spain's cross-sectional variance of log-wages, which Pijoan-Mas and Sanchez Marcos [47] estimated at $\mathrm{Var}(\log(y)) = 0.225$ on average for the period 1994−2001. Hence, $\sigma_u^2 = \mathrm{Var}(\log(y))(1-\rho_y^2)$, which yields $\sigma_u = 0.2498$.[36] Average income is calibrated such that the aggregate resource constraint is consistent with national accounts data with GDP normalized to one. This implies that Y in the model must equal GDP net of fixed investment because the latter is not explicitly modeled. Investment averaged 24 percent of GDP during the period 1981-2012, which implies that $Y = \mu_y = 0.76$.

The g process is calibrated using data on government final consumption expenditures from National Accounts for the period 1981−2012 from the World Bank's World Development Indicators, and fitting an AR(1) process to the logged government expenditures-GDP ratio (controlling for a linear time trend). The results yield: $\rho_g = 0.88$, $\sigma_e = 0.017$ and $\mu_g = 0.18$. The value of τ^y is set to 35 percent following the estimates of the marginal labor tax in Spain (average for 2000-2002) reported by Conesa and Kehoe [18]. They studied the evolution of taxes in Spain from 1970 to 2002.

In the second calibration step, we use the SMM algorithm to set the values of β, $\overline{\omega}$, and ϕ_1 targeting these three data moments: the 1981−2012 average ratio of domestic public debt holdings to total public debt (74.43 percent), the 2002-2012 average bond spread relative to German bonds (0.94 percent), and the 1981-2012 average, maturity-adjusted public debt-GDP ratio (5.56 percent).[37]

[36] The data available for Spain consist of a sequence of cross sections, which prevented Pijoan-Mas and Sanchez-Marcos from estimating the autocorrelation of the income process.

[37] Total public debt refers to total general government net financial liabilities as a fraction of GDP. The

Table A.5 presents the targets and the parameter values.

Table A.5: Model Parameters and Targets

Calibrated from data or values in the literature			
Risk free rate (%)	$\bar{r}$	2.07	Real return German bonds
Risk aversion	σ	1.00	Standard value
Autocorrel. income	ρ_y	0.85	Guvenen [31]
Std. dev. error	σ_u	0.25	Spain wage data
Avg. income	μ_y	0.76	GDP net of fixed capital investment
Autocorrel. G	ρ_g	0.88	Autocorrel. government consumption
Std. dev. error	σ_e	0.02	Std. dev. government consumption
Avg. gov. consumption	μ_g	0.18	Avg. G/Y Spain
Proportional income tax	τ^y	0.35	Marginal labor income tax
Estimated using SMM to match target moments			
Discount factor	β	0.885	Avg. ratio domestic to total debt
Welfare weights	ω	0.051	Avg spread v. Germany
Default cost	ϕ_1	0.603	Avg. debt-GDP ratio (maturity adjusted)

Table A.6 shows the target data moments and the model's corresponding moments in the SMM calibration.

Table A.6: Results of SMM Calibration

Moments (%)	Model	Data
Avg. ratio domestic debt	74.31	74.43
Avg. spread Spain	0.94	0.94
Avg. debt to GDP Spain (maturity adjusted)	5.88	5.56

A-8.1 Equilibrium Time Series Properties

The quantitative analysis aims to answer two main questions. First, from the perspective of the theory, does the calibrated model support an equilibrium in which debt exposed to default risk can be sustained and default occurs along the equilibrium path? Second, from an empirical standpoint, to what extent are the model's time series properties in line with those observed in the data?

ratio of domestic to total debt corresponds to the fraction of general government gross debt held by domestic investors from Arslanalp and Tsuda [10], extended with the ratio of marketable debt held by residents to total marketable central government debt from Organization for Economic Co-operation and Development Statistics. See Appendix A-2 for further details.

To answer these questions, we study the model's dynamics using a time series simulation for 10,000 periods, truncating the first 2,000 to generate a sample of 8,000 years, large enough to capture the long-run properties of the model. This sample yields 73 default events, which implies an unconditional default probability of 0.91 percent. Thus, the model produces optimal domestic (and external, since the government cannot discriminate debtors) sovereign defaults as a low-probability equilibrium outcome, although still roughly twice Spain's historical domestic default frequency of 0.4 percent (Reinhart and Rogoff [48] show only one default episode in 216 years). In contrast with typical results from external default models, these defaults do not require costs of default in terms of exclusion from credit markets, permanently or for a random number of periods, and rely in part on endogenous default costs that reflect the social value of debt for self-insurance, liquidity, and risk-sharing.

Table A.7 compares moments from the model's simulation with data counterparts. Since Spain has not defaulted in the data sample period but its default risk spiked during the European debt crisis, we show model averages excluding default years to compare with data averages, and averages for the years before defaults occur ("prior default") to compare with the crisis peaks in the data (the "peak crisis" column, which shows the highest values observed during the 2008-2012 period).

Table A.7: Long-run and Pre-Crisis Moments: Data versus Model

	Data		Model	
Moment (%)	Avg.	Peak Crisis	Average	Prior Default
Gov. debt B	5.43*	7.43	5.88	7.95
Domestic debt B^d	4.04	4.85	4.29	4.84
Foreign debt $\widehat{B}$	1.39	2.58	1.59	3.11
Ratio B^d/B	74.34*	65.28	74.31	60.94
Tax revenues $\tau^y Y$	25.24	24.85	26.60	26.60
Gov. expenditure g	18.12*	20.50	18.13	18.18
Transfers τ	7.04	7.06	8.35	8.73
Spread	0.94*	4.35	0.94	7.22

Note: * identifies moments used as calibration targets. See Appendix A-2 for details on sources, definitions, and sample periods for data moments. Since GDP was normalized to 1, all variables in levels are also GDP ratios.

Table 4 shows that the model does well at matching several key features of the data. The averages of total debt, the ratio of domestic to total debt, and spreads were calibration targets, so these moments in the model are close to the data by construction. The rest of the model averages (domestic and external debt, tax revenue, transfers, and government expenditures) approximate well the data averages. Taxes and transfers do not match more accurately because, with the Conesa-Kehoe labor tax rate of $\tau^y = 0.35$ and with GDP net of investment at $Y = 0.76$, the model generates 26.6 percent of GDP in taxes, which is 140 basis points more than in the data and results in average transfers exceeding the data average by the same amount. The model is within a 10-percent margin at matching the crisis peaks of total debt, domestic debt, and the ratio of domestic to total debt.

Table A.8 compares an additional set of model and data moments, including standard deviations (relative to the standard deviation of income), income correlations, and correlations with government expenditures.

Table A.8: Cyclical Moments: Data versus Model

	Standard Deviation		Correl($x, hhdi$)		Correl($x, g/GDP$)	
Variable x	Data	Model	Data	Model	Data	Model
Consumption	0.85	0.84	0.43	0.97	-0.32	-0.76
Trade Balance/GDP	0.63	0.55	-0.31	-0.82	0.15	0.08
Spreads	1.04	2.46	-0.44	-0.004	-0.22	-0.23
Gov. Debt / GDP	1.58	1.23	-0.18	-0.07	0.06	-0.07
Dom. Debt / GDP	1.68	0.32	-0.32	-0.34	-0.10	-0.22

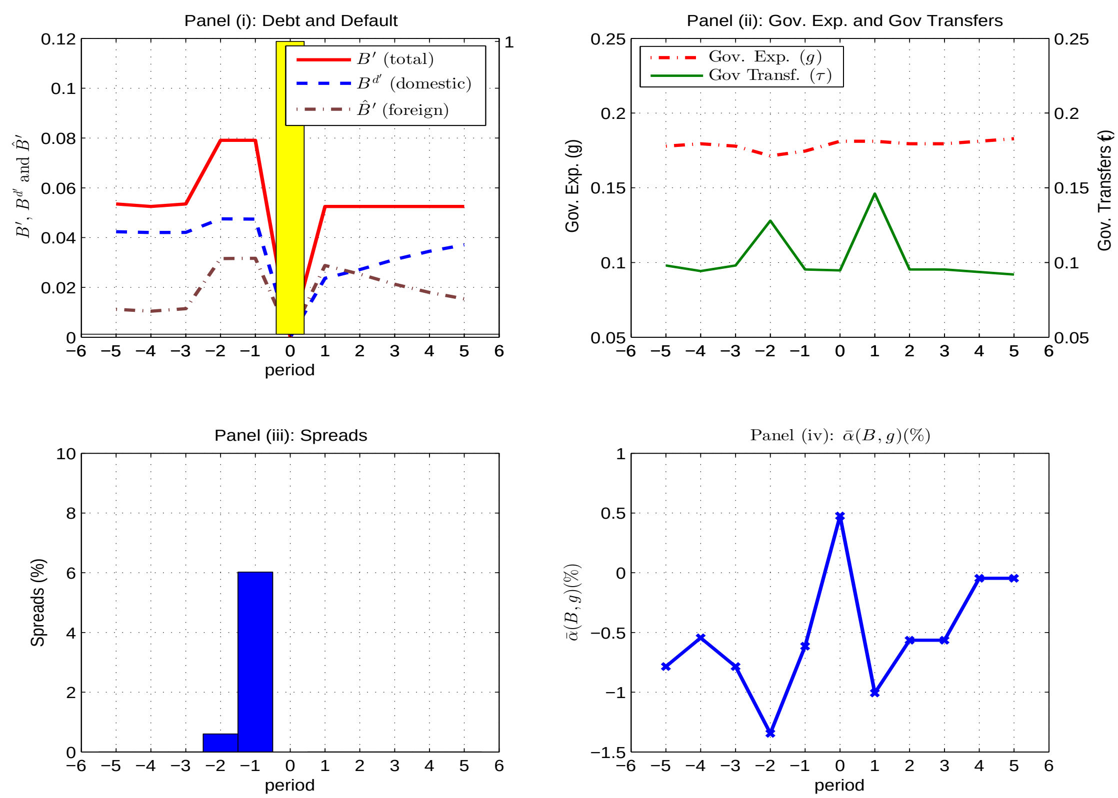
Note: $hhdi$ denotes household disposable income. In the model, $hhdi = (1-\tau^y)Y + \tau$ and $TB = Y - C - g$. $hhdi$ and C are logged and HP filtered with the smoothing parameter set to 6.25 (annual data). GDP ratios are also HP filtered with the same smoothing parameter. Standard deviations are ratios to the standard deviations of $hhdi$, which are 1.37 and 1.16 in data and model, respectively. Since the data sample for spreads is short (2002-2012) and for a period characterized by a sustained rise in spreads since 2008, we generate comparable model data by isolating events spanning 10 years before spikes in spreads, defining spikes as observations in the 95 percentile. The standard deviation of spreads is demeaned to provide a comparable variability ratio. See Appendix A-2 for details on data sources.

Given the parsimonious structure of the model, it is noteworthy that it can approximate well several key moments of the data, including most co-movements. The model does a good job at approximating the standard deviation of disposable income, as well as the relative standard deviations of consumption, the trade balance, and total debt. On the other hand, the model overestimates the variability of spreads and underestimates that of domestic debt. The correlations with government expenditures produced by the model line up very well with those found in the data. The correlations with debt, domestic debt and spreads are of particular importance for the mechanism driving the model. As we document later in this section, the model predicts that periods with relatively low g weaken default incentives and thus enhance the government's borrowing capacity. Accordingly, the model yields a negative correlation of government expenditures with spreads (-0.23 versus -0.22 in the data) and with domestic debt (-0.22 versus -0.1 in the data), and nearly uncorrelated debt and government expenditures. The model is also very close to matching the correlation between the trade balance and spreads (0.15 in the data versus 0.09 in the model, respectively), which is driven by the same mechanism, since trade deficits are financed with the share of the public debt sold abroad.

The model also approximates well the income correlations of total and domestic debt, and relatively well that of the trade balance. The correlation of consumption with disposable income is close to 1 in the model v. 0.43 in the data, and the model yields uncorrelated spreads and disposable income while in the data the correlation is -0.44.

We study next dynamics around default events. Figure A.9 shows a set of event analysis charts based on the simulated data set with its 73 defaults. The plots show 11-year event windows centered on the year of default at $t = 0$ starting from the median debt level of all default events at $t = -5$. Panel (i) shows total public debt (B) and domestic and foreign debt holdings (B^d and $\hat{B}$, respectively). Panel (ii) shows g and τ. Panel (iii) shows bond spreads. Panel (iv) shows the social welfare gain of default denoted $\overline{\alpha}$.

Figure A.9: Default Event Analysis



The event analysis plots show that a debt crisis in the model appears to emerge suddenly, after seemingly uneventful times. Up to three years before the default, debt is barely moving, spreads are zero, and government expenditures, transfers, and the social welfare gain of default are also relatively stable. In the two years before the default everything changes dramatically. Debt rises sharply by nearly 300 basis points, with both foreign and domestic holdings rising but the former rising faster. Spreads rise very sharply to 100 and 600 basis points in the second and first year before the default, respectively. This follows from a slight drop in g coupled with a larger rise in τ and a sharp drop in $\overline{\alpha}$ at $t = -2$, and then a modest increase in g, and reversals in τ and $\overline{\alpha}$ at $t = -1$.

The reason for the rapid, large changes at $t = -2$ is that the decline in g weakens the government's incentives to default, because the exogenous default cost rises as g falls. The resulting higher borrowing capacity enables the government to redistribute more resources and provide more liquidity to credit-constrained agents by issuing more debt and paying higher transfers. The sharp drop in $\overline{\alpha}$ shows that using the newly gained borrowing capacity in this way is indeed socially optimal. Foreign debt holdings rise more than domestic holdings because domestic agents already have sizable debt holdings for self-insurance, although higher spreads still attract agents with sufficiently high (b, y) to buy more debt.

At $t = -1$, g rises only slightly while debt, and hence transfers, remain unchanged. The higher debt, together with the positive autocorrelation of the g process, strengthen default incentives ($\overline{\alpha}$ rises) and cause an increase in the probability that a default may occur in the following period, causing the sharp increase in spreads to 600 basis points. Then at $t = 0$, g rises slightly again but, at the higher debt, this is enough to cause a large change in $\overline{\alpha}$ by about 100 basis points from -0.5 to 0.5 percent, causing a "sudden" default on a debt ratio practically unchanged from two years prior. In addition, default occurs with relatively low external debt, which is roughly 46 percent of total debt. The surge in spreads at $t = -1$ and the default that followed, both occurring with an unchanged debt, could be viewed as suggesting that equilibrium multiplicity or self-fulfilling expectations were the culprit, but in this simulation this is not the case.

In the early years after a default, g hardly changes but, since the agents' precautionary savings were wiped out, domestic debt holdings rise steadily from 0 to 4 percent of GDP by $t = 5$. This reflects the optimal (gradual) buildup of precautionary savings by agents that draw relatively high income realizations. Total debt and transfers rise sharply in the first year, as the social value of debt starting from zero debt is very high and debt that is not sold at home is sold abroad at zero spread, because repayment incentives are strong ($\overline{\alpha}$ is around -1 percent). By $t = 5$, debt and its foreign and domestic component are approaching the levels they had at $t = -5$. Repayment incentives are weak but still enough to issue debt at zero spread.

A-9 Algorithm Endogenous Partial Default

This Appendix describes the algorithm we constructed to solve for the model's CRME and RME when there is endogenous partial default. The algorithm performs a global solution using value function iteration. We approximate the solution of the infinite horizon economy by solving for the equilibrium of a finite-horizon version of the model for which the finite number of periods (T) is set to a number large enough such that the distance between value functions, government policies and bond prices in the first and second periods are the same up to a convergence criterion. The corresponding first-period functions are then treated as representative of the solution of the infinite-horizon economy.

1. Problem in iteration T, for each $\{b, y\}$ and $\{B', B, g\}$:

   - Government Debt choice: $B'_T(B, g) = 0$.
   - Price Debt: $q_T(B', g) = 0$.
   - Tax no default:
   $$\tau_T^{d=0}(B', B, g) = B + g - \tau^y Y$$
     - Define negative consumption flag:
   $$\text{flag}_t^{c<0}(B', B, g) = I_{\{(1-\tau^y)y_1 + b_1 - \tau_T(B', B, g) \leq 0\}}$$
   - Define household value (note that at T it does not depend on $\tilde{B}$ since $q_T(B', g) = 0$)
   $$\tilde{V}_T^{d=0}(\tilde{B}, y, b, B, g) = u((1-\tau^y)y + b - g - B + \tau^y Y)$$
     - If $\text{flag}_t^{c<0}(\tilde{B}, B, g) = 1$ set $\tilde{V}_T^{d=0}(\tilde{B}, :, :, B, g) = -\infty$
   - In period T, government debt choice $B' = 0$.
   - Household value in $d = 0$: $V_T^{d=0}(y, b, B, g) = \tilde{V}_T^{d=0}(0, y, b, B, g)$
   - Tax under default:
   $$\tau_T^{d=1}(B, g, \varphi) = (1-\varphi)B + g - \tau^y Y$$
   - Value in default
   $$\tilde{V}_T^{d=1}(y, b, B, g, \varphi) = u((1-\tau^y)y(1-\phi(g)) - g + [1-\varphi](b - B) + \tau^y Y)$$
   - In period T, government $\varphi = 1$, $V_T^{d=1}(y, b, B, g) = \tilde{V}_T^{d=1}(y, b, B, g, 1)$

- Welfare values for default decision

$$W_T^{d=0}(B,g) = \int_{Y\times B} V_T^{d=0}(y,b,B,g)d\omega(b,y)$$

$$W_T^{d=1}(B,g) = \int_{Y\times B} V_T^{d=1}(y,b,B,g)d\omega(b,y)$$

- Default decision (note that $d=1$ implies a given φ)

$$d_T(B,g) = \arg\max_{d=\{0,1\}} \{W_T^{d=0}(B,g), W_T^{d=1}(g)\}$$

- Let

$$V_T(y,b,B,g) = (1-d_T)V_T^{d=0}(y,b,B,g) + d_T V_T^{d=1}(y,b,B,g)$$

2. Problem in iteration $t = T-1,\ldots,1$

   - Solve for price function $q_t(B',g)$.
     - Define fraction defaulted (default probability together with fraction defaulted)

$$p_t(B',g) = \sum_{g'} d_{t+1}(B',g')\varphi_{t+1}(B',g')F(g',g)$$

     - price is

$$q_t(B',g) = \frac{1-p_t(B',g)}{1+r}$$

   - Define Tax Function in $d=0$ state

$$\tau_t^{d=0}(B',B,g) = B + g - q_t(B',g)B' - \tau^y Y$$

     - Create flag for negative consumption: Combinations of B', B and g that imply negative consumption for y_1, b_1 when choosing b_1'

$$\text{flag}_t^{c<0}(B',B,g) = I_{\{(1-\tau^y)y_1+b_1-q_t(B',g)B'-\tau_t(B',B,g)\leq 0\}}$$

   - Solve problem household for b, y and $\{\tilde{B}, B, g\}$

$$\tilde{V}_t^{d=0}(\tilde{B},y,b,B,g) = \max_{b'} u(c) + \beta E_{g'}[V_{t+1}(b',y',\tilde{B},g')]$$

   s.t.

$$c = (1-\tau^y)y + b - q_t(\tilde{B},g)b' - \tau_t(\tilde{B},B,g)$$

- Optimal Debt choice

$$B_t'(B,g) = \arg\max_{\tilde{B}} \int \tilde{V}_t^{d=0}(\tilde{B}, y, b, B, g) d\omega(b,y)$$

- Define continuation value for the households under no default

$$V_t^{d=0}(y,b,B,g) = \tilde{V}_t^{d=0}(B_t'(B,g), y, b, B, g)$$

- Tax under default:

$$\tau_t^{d=1}(B,g,\varphi) = (1-\varphi)B + g - \tau^y Y$$

- Value in default

$$\tilde{V}_t^{d=1}(y,b,B,g,\varphi) = u((1-\tau^y)y(1-\phi(g)) - \tau_t^{d=1}(B,g,\varphi)) + \beta E_{g'}[V_{t+1}(0,y',0,g')]$$

- Optimal Fraction of Default Choice

$$\varphi_t(B,g) = \arg\max_{\varphi} \int \tilde{V}_t^{d=1}(y,b,B,g,\varphi) d\omega(b,y)$$

- Define value in default

$$V_t^{d=1}(y,b,B,g) = \tilde{V}_t^{d=1}(y,b,B,g,\varphi_t(B,g))$$

- Auxiliary functions:

$$\hat{p}_t(B,g) = \sum_{g'} d_{t+1}(B_{t+1}'(B,g),g')\varphi_{t+1}(B_{t+1}'(B,g),g'))F(g',g)$$

$$\hat{q}_t(B,g) = \frac{1-\hat{p}_t(B,g)}{1+r}$$

$$\hat{\tau}_t(B,g) = B + g - \hat{q}_t(B,g)B_{t+1}'(B,g) - \tau^y Y$$

$$\hat{\text{flag}}_t^{c<0}(B',B,g) = I_{\{(1-\tau^y)y_1 + b_1 - \hat{q}_t(B,g)B_{t+1}'(B,g) - \hat{\tau}_t(B,g) \leq 0\}}$$

- Government values

$$W_t^{d=0}(B,g) = \int_{Y\times B} V_t^{d=0}(y,b,B,g) d\omega(b,y)$$

$$W_t^{d=1}(B,g) = \int_{Y\times B} V_t^{d=1}(y,b,B,g)d\omega(b,y)$$

- Government default decision

$$d_t(B,g) = \arg\max_{d=\{0,1\}} \{W_t^{d=0}(B,g), W_t^{d=1}(B,g)\}$$

3. After done with solution to periods $t = T-1,\ldots,1$ check whether value functions, government policies and bond prices in periods 1 and 2 are sufficiently close. If they are, you are done. If not, increase T and restart.