Inflation is Conflict *

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This paper isolates the role of conflict—defined as a disagreement on relative prices—on inflation in two ways. In the first part of the paper, we present a stylized model, kept purposefully away from traditional macro ones, where inflation arises despite the complete absence of money, credit, interest rates, production, and employment. Inflation is due to conflict; it cannot be explained by monetary policy or departures from a natural rate of output or employment. The second part of the paper, instead, develops a flexible framework that nests many traditional macroeconomic models with a general network of interconnected N "sectors", representing both goods and factors. We define conflict as the degree of incompatibility in aspirations for relative price held across sectors. Our main result shows that "conflict" drives inflation in two ways: at any point in time, generalized inflation across sectors is only possible with conflict; for a given sector, averaging inflation over a long enough period of time is conflict. Our framework sits on top of a wide set of particular models that endogenize conflict, helping generalize and clarifying their common inflation mechanism.

1 Introduction

Inflation is a messy phenomenon. Despite much experience and evidence, economists still debate its origins and precise mechanisms at play. Economic models can provide a lens to tell a more transparent story. This paper offers two lenses to explore and expand the perspective that the most proximate cause of inflation is "conflict"—defined below as a disagreement on relative prices.

Many economists confidently agree that extreme and persistent inflation episodes are understood as largely driven by the growth in money supply, often prompted by a need for seignorage. But how exactly does money transmit to inflation? The simplest idea is

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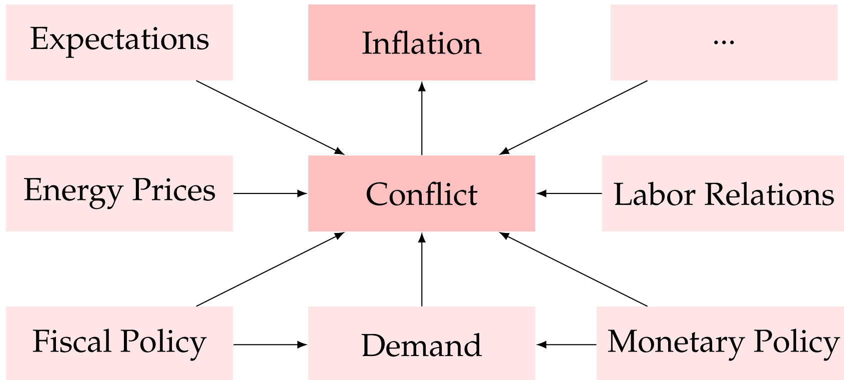

Figure 1: Inflation DAG: Conflict as Proximate with other Root Causes.

"too much money chasing too few goods." Formalizing this involves the quantity equation or more general forms of money demand. On closer inspection, one may still wonder what "money chasing goods" means and how the prices in good markets adjusts to clear the money market. The story feels incomplete, as it requires out-of-equilibrium intuitions for a general-equilibrium macroeconomic situation—simple microeconomic ideas of supply and demand may not be a proper guide. Nevertheless, it is fair to say that this idea is very well rooted in economists' thinking and that money supply is central to all these stories.

For moderate and transitory inflations, things are less clear-cut and there is much less agreement. After all, money may chase goods and increase output instead of prices. Indeed, one important notion is that the transmission mechanism works from economic activity to inflation. Higher production and employment drive up costs and the real wage, leading firms to raise prices. According to these theories, inflation must be avoided by keeping the economy at the right "natural level" of output or employment, to avoid "too much spending chasing too few goods." Monetary policy, managed through interest rates shapes economic activity and inflation. Money supply is not central to this story, economic activity and its management through interest rates is.

Theories formalizing these ideas rely, among other things, on nominal rigidities. Indeed, the explicit modeling of agents setting prices greatly clarifies the mechanism generating inflation—a positive evolution relative to "money chasing goods". However, in these models, nominal rigidities are complemented with many additional assumptions to provide a complete but rather specific model and reach conclusions about natural output or natural interest rate.

To sum up, both traditional inflationary stories contain elements of truth and are not necessarily at odds with each other. In our view, these existing theories of inflation are either incomplete about the mechanism, or overly specific to cover the broad issues surrounding inflation. As such, they may describe the root causes of inflation in some situations, but fall short of isolating the more general and proximate cause of inflation. This leads to the question we address in this paper: What is the most minimal and general framework that spells out the mechanism for inflation and describes its most proximate causes?

This paper argues that the most proximate and general cause of inflation is conflict or disagreement. In this view, inflation results from incompatible goals over relative prices, with economic agents having only partial or intermittent control over nominal prices. Due to nominal rigidities, agents occasionally change a subset of prices that are under their control. Whenever they do, they adjust them to influence relative prices in their desired direction. When coupled with staggered prices this conflict manifests itself in a finite level of inflation: the conflict over relative prices is largely frustrated. Despite a stalemate in relative prices, the changes in prices motivated by this conflict gives rise to general and sustained inflation in all prices.

We argue that the conflict perspective is both insightful and general. First, we will present a situation where inflation cannot be easily understood using the traditional stories. Second, we will argue that most traditional stories of inflation are best viewed as special cases of the conflict perspective: they simply provide a theory endogenizing conflict, which then leads to inflation. Thus, nothing needs to be lost or tossed out. Instead, there are significant gains from the conflict perspective. Figure 1 illustrates our view, with conflict as the proximate cause of inflation; however, conflict is possibly affected, in turn, by many other forces; for example, conflict may be affected by the level of demand, which affects output and employment; demand, in turn may be affected by monetary and fiscal policy; thus, this would summarize very traditional stories or models of inflation. At the same time, other stories and models can fit under the conflict framework: conflict may be directly affected by labor market institutions, such as unions, or by outside shocks to the price of energy.

Our contribution consists of two separate but complementary parts. In the first part (Sections 2 and 3), we develop a stylized model that isolates conflict and helps develop intuition for the economic concept. We purposefully stay away from standard models, such as the New Keynesian model. This has a few advantages. One advantage is that it lends itself to a self-contained presentation based on basic microeconomic concepts, without requiring familiarity or adherence to particular standard macroeconomic models.

The other advantage of our stylized model is that it isolates the conflict perspective, leaving out traditional features: agents trade endowments of goods via barter using staggered prices, there is no money, no saving nor credit, no interest rates, and no way to affect the level of output. Inflation arises from the agents' desire to exercise market power, providing a clear illustration of the role of conflict in driving inflation. Since money is absent, inflation cannot result from "too much money chasing too few goods", since output cannot be affected by monetary policy, there is no natural level of output or natural interest rate to prevent inflation. We provide some extensions of our stylized model that emphasize the conflict perspective.

The contribution of this stylized model is to isolate the role of conflict in inflation. Indeed, it is meant as a shock to the system that may sow the seeds of doubt in economists, like ourselves, raised on the notion that to speak of inflation requires first and foremost a discussion of money and interest rates, complemented perhaps with the concepts of natural levels of output, employment or interest rates. The stylized model leaves no natural interpretation for inflation except for conflict.

The second part of the paper (Sections 4 and 5) is in some ways the polar opposite of the first part. We provide a general framework that helps bridge a conflict perspective with more traditional macro models. This framework distills the staggered prices in New Keynesian models, without adopting other special assumptions of these models.

We set up a general N sector network model, where each sector represents a particular good and its price or labor and its wage. The interconnections in the network represent an input-output in production or the relevant consumption basket of workers. We then provide a decomposition of inflation into a common conflict component and another component that has to do with the adjustment of relative prices. The adjustment component only induces transitory changes in prices, no long run inflation is possible. Moreover, at all times the adjustment component must be positive for some goods and negative for others, so it cannot deliver generalized inflation. In contrast, we show that the conflict component is essential in that only it can generate persistent and generalized inflation across all goods.

Although this second part of the paper creates a bridge with standard models, it is also more general. Indeed, one benefit of the conflict perspective is that it offers a framework that sits on top of specific models, that can be seen as modeling the sources of conflict, while providing insights that are common to them.[1] For example, a conflict perspective can be made consistent with the simplest New Keynesian models of the labor market, where the marginal disutility of labor drives real wages. However, it can also easily ac-

[1] This is not unlike how economists have benefitted from thinking about growth accounting or isolating the mechanisms behind consumption smoothing choices. The economic insights gained by taking such broader perspectives become largely portable across a wide swath of models and questions.

commodate other considerations, such as labor market institutions, search frictions, labor unions, behavioral biases. While each of these dimensions could be explicitly modeled, the conflict perspective acts as an overarching layer to think about these alternatives.

In our view, the conflict perspective should be viewed in this general and broad manner. Some may associate a conflict perspective, instead, with certain specific inflation episodes, but not others—such as during times of powerful labor unions and strained labor relations. Likewise, some may associate it with an advocacy for income or price control policies, or as a critique of conventional interest rate policies. While these possibilities fit our general framework, none of them follow without adopting further specific modeling assumptions, which we do not undertake here. The perspective we offer in this paper is broader. Our goal is to provide a framework to think about conflict as the proximate cause of inflation. Specific models may then be thought of as endogenizing conflict in different ways, leading to different conclusions about the root causes of inflation, or about the effects of different policies.

The conflict view on inflation is by no means new, yet this perspective is largely unknown to most economists. It was developed and embraced by a relative minority associated with a Post-keynesian tradition. Rowthorn (1977) provided the seminal contribution, while Lavoie, 2022 contains an overview of the more recent literature. Our contribution extends the conflict perspective, providing new results and building bridges with traditional models. We hope our paper may help bring the conflict perspective to greater awareness among a broader set of economists.

Some work does not isolate a conflict perspective of inflation but gets close in spirit in discussions, in intuitions or in the nature of the exercises undertaken. We provide two examples. Blanchard (1986) models prices and wages rigidities and studies a permanent money supply shock. Although the analysis is carried out with a relatively standard neoclassical labor-supply framework, some of the discussions and intuitions transcend the boundaries of this territory: "attempts on both sides to maintain the same wage and price in the face of an adverse supply shock [...] lead to "cost push" inflation". Relatedly, Blanchard and Gali (2007) extend a standard New Keynesian model by adding an ad hoc real wage rigidity. In our view, this departure from the neoclassical labor-supply framework can be seen as an exploration of alternative real wage aspirations for workers from a conflict perspective.

The second part builds on our own work on wage-price spirals Lorenzoni and Werning (2022). That paper studied price and wages in the presence of nominal rigidities as in Erceg et al. (2000) and Smets and Wouters (2007). This remains an important example in the present paper but we develop a more general network framework, with any number

N of interconnected price setting sectors (the simple price and wages case is $N = 2$).

Our network approach relates to recent work on inflation in network economies, especially Rubbo (2020) and Afrouzi and Bhattarai (2023). Our analysis differs in two important ways. First, existing models allow for general input-output relations across sectors, but include a single labor factor provided by the household sector—modeled separately from the input-output network. In contrast, our framework allows for any number of factors (e.g. labor) and integrates them into a general unified closed network (matrix). Such an integrated approach is key for our network centrality characterization. Second, existing papers work with fully specified models and study the relation between output, inflation or monetary policy. From our vantage point, these models, thus, endogenize aspirations in particular ways (as does Lorenzoni and Werning, 2022, for the simplest wage-price network). In contrast, here we purposefully abstain from taking particular stances on aspirations so as to highlight the general role of conflict in inflation.

2 Inflation from Conflict without Money

In the first part of the paper we develop a stylized model that isolates inflation and conflict. The microeconomics involved is simple and fully spelled out. To ensure inflation is not driven by money we first assume that trade takes place through barter, with money and credit completely absent. To ensure inflation is not driven by high output, we assume fixed endowments. Abstracting from labor, we focus on prices only, instead of prices and wages.

2.1 Assumptions: Barter with Staggered Prices

Technology and Preferences. Consider two agents, A and B, and two goods, also labeled A and B (e.g., Apples and Bananas). Each period, agents have an endowment of their respective good—A owns A, B owns B. We normalize the endowment to one. Goods are perishable and must be consumed within each period. There is no storage technology or capital.

Preferences within a period are symmetric and given by the utility function

$$u(c, c')$$

where c denotes consumption of the own good and c' denotes consumption of the other good (e.g., for agent A, c is good A and c' is good B). The function u is concave and twice differentiable. Utility is the discounted sum

$$\sum_{t=0}^{\infty} \beta^t u(c_t, c_t')$$

for some discount factor $\beta < 1$.

The symmetry of preferences across agents is not required for our main results, as we later show. Although in our baseline preferences are symmetric in terms of c and c', agents do not have the same preferences over goods A and B, unless we further impose symmetry across goods: $u(c, c') = u(c', c)$. For example, $u(c, c') \neq u(c', c)$ allows for "home bias" with agents preferring their own good.[2]

Regarding demographics, there are two interpretations of this baseline model. In the first, only two individuals exist in the economy. In the second, there may be many individuals of each type (possibly infinite), but each individual is permanently paired with an individual of the opposite type.[3] For concreteness, in our presentation we only refer to two individuals A and B.

Competitive Equilibria and Edgeworth Box. Our baseline model is simply an exchange economy, repeating without change each period. Allocations can be pictured in an Edgeworth-box diagram.

The market arrangement in our model will feature staggered pricing and market power. However, competitive equilibria provides a benchmark, useful for comparison purposes. By symmetry of preferences and endowments, a symmetric equilibrium must exist, with a relative price of unity. To avoid trivial outcomes, we assume preferences are such that there is non-zero trade at this competitive equilibrium.[4]

Staggered Prices. Prices are set in a staggered fashion and remain unchanged for two periods: agent A sets prices in even periods and agent B in odd periods. Let P_t^* denote a newly reset price: it denotes the price of good A in even periods and the price of B in odd

[2] One may think of "home bias" in preferences as a stand in for the costs of exchange. For example, starting from a common utility over goods, if a fraction of the fruit that is exchanged becomes harmed, then in reduced form this induces preferences with home bias.

[3] Section 3 introduces a variant of the baseline model with an infinite number of individuals of each type that meet in random matches.

[4] Quantities at this symmetric equilibrium are symmetric, with both agents consuming $c = 1 - c' \geq 0$ for some $c' \geq 0$. We assume $c' > 0$. Note that the equilibrium does not generally have $c = c'$, unless utility is symmetric across goods: $u(c, c') = u(c', c)$.

periods:

$$P_t^* = P_t^A = P_{t+1}^A \qquad t = 0, 2, \ldots$$
$$P_t^* = P_t^B = P_{t+1}^B \qquad t = 1, 3, \ldots$$

The price of B at $t = 0$ is given. Using the above conditions, the sequence of reset prices $\{P_t^*\}$ determines all prices in this economy $\{P_t^A, P_t^B\}$. Thus, our goal is to solve for the equilibrium sequence $\{P_t^*\}$.

Prices are simply numbers expressed in a common unit of account. In other words, prices are quoted in "nominal" terms, best understood as expressed in terms of some physical or digital currency, perhaps due to convention. Importantly, however, we assume that agents have no access to such currencies in our baseline model. That is, they hold no currency and come into no contact with currency. Nor do they have access to any durable good or record-keeping device. Likewise, we will not consider trading strategies that depend on past trades,

That is, there is no money, no commodity money, no storage, and no way to save or borrow.

Trade by Barter. In our baseline model all trade takes place by barter using as terms of trade the ratio of the currently posted prices. In even periods $t = 0, 2, ...$ the relative price of A is

$$\frac{P_t^A}{P_t^B} = \frac{P_t^*}{P_{t-1}^*}$$

while in odd periods $t = 1, 2, ...$ the relative price of A is

$$\frac{P_t^A}{P_t^B} = \frac{P_{t-1}^*}{P_t^*}$$

Quantities are determined by a take-it-or-leave-it offer proposed by one of the two agents. In our baseline, we assume that this is done by the agent who did not reset its price that period. We call this agent a "buyer" and call the agent setting the price a "seller", for that period.

In more detail, in even periods, after A has reset its price, agent B acts as the buyer and offers to buy c_t^A units of good A and pay a quantity of good B determined by the ratio of nominal prices,

$$\frac{P_t^A}{P_t^B} c_t^A = \frac{P_t^*}{P_{t-1}^*} c_t^A.$$

Upon receiving this offer, agent A can decide to accept or reject it. If the offer is accepted, they execute the barter exchange and consume; if the offer is rejected, both agents consume their endowment in that period.

In odd periods the trading protocol is identical, but reversing the roles of A and B.

2.2 Equilibrium Inflation

We now solve for an equilibrium, which turns out to be very simple. We solve backwards, starting with the buyer problem, then turning to the seller problem setting its price. We focus on Markov equilibria, that is, equilibria that depend on the relevant state variables.[5][6]

Buyer Problem. In a given period t, after prices are set, the buyer can buy the good of the seller at the relative price

$$p_t = \frac{P_t^*}{P_{t-1}^*}.$$

Notice that if t is even, then p_t is the price of good A in terms of B, while if t is odd it is the price of good B in terms of A. Given symmetry, this notation allows us to characterize the buyer's and seller's problem in any period.

Dropping time subscripts, the buyer solves

$$V(p) = \max_{c,c'} u(c, c')$$

subject to

$$c = 1 - pc',$$

$$u(1 - c', pc') \geq u(1, 0).$$

The first constraint is simply the buyer's budget constraint; the second is a participation constraint to ensure that the seller is willing to accept the buyer's offer.[7] The solution can

[5]The relevant state variables are the given nominal prices: P_{t-1}^* at the start of period t before the current price is set; (P_{t-1}^*, P_t^*) after the current price is set in period t.

[6]One could also study non-Markov, sub-game perfect equilibria, by allowing for strategies that depend on the history of past play. This gives a wider set of potential outcomes. Indeed, it may be possible to obtain the first-best with constant nominal prices $P_t^* = P_{-1}^*$ which gives the competitive equilibrium with a unitary relative price. This could be sustained by strategies that follow any deviation by reverting to the Markov equilibrium we study. For sufficiently high $\beta \to 1$ this punishment will dissuade deviations and sustain the first best.

However, we abstract from non-Markov equilibria in this paper because we find, in the present context, the Markov equilibrium concept more appealing. In addition, sustaining non-Markov equilibria becomes more involved in the extensions we develop with random matching (i.e., require greater monitoring).

[7]As usual, this is without loss of generality. A buyer can offer the autarky allocation with $c' = 0$ and $c = 1$, yielding the same outcome as a rejected offer. Thus, we can focus on equilibria where sellers accept

be written as a demand schedule

$$c' = D(p),$$

together with $c = 1 - pD(p)$. As the notation suggests, the buyer's problem is a classical consumer demand problem except for the presence of the participation constraint. However, in our baseline model this extra constraint never binds in equilibrium, as sellers choose prices that make themselves strictly better off. Indeed, the equilibrium in this baseline model is unchanged if we drop the participation constraint altogether.[8]

Seller Problem. Going backwards, consider now the seller who chooses P_t^* at the beginning of period t, taking as given the price P_{t-1}^* set by the other agent in the previous period. By varying P_t^* the seller is able to freely determine the relative price $p_t = P_t^*/P_{t-1}^*$ faced by the buyer.

Note that P_{t-1}^* has no direct impact on the set of relative prices available to the seller. In the Markovian equilibria we focus on, this implies that P_{t-1}^* will not affect the equilibrium relative price chosen by the seller, but simply scales the nominal price P_t^*. For the same reason, the seller in the current period anticipates that P_t^* will have no impact on p_{t+1}. This implies that the seller problem we study is static and given by

$$p^* \equiv \arg\max_p v(p), \tag{1}$$

where

$$v(p) \equiv u(1 - D(p), pD(p)).$$

We assume an optimum p^* exists and is unique.[9] The first-order condition $v'(p^*) = 0$ gives

$$p^* = \frac{1}{1 - 1/\epsilon(p^*)} \cdot \frac{u_c(c, c')}{u_{c'}(c, c')}, \tag{2}$$

where $c = 1 - D(p)$, $c' = pD(p)$, and $\epsilon(p)$ denotes the local demand elasticity $-D'(p)p/D(p)$.[10] Just as for a standard monopolist, the relative price is set at a markup $\frac{1}{1-1/\epsilon}$ over the relevant marginal cost—which in this case is the marginal rate of substitution $u_c/u_{c'}$.

The equilibrium is illustrated in Figure , using an Edgeworth box diagram.

all offers.

[8]This is no longer the case in some of our extensions. The participation constraint plays a more vital role in these extensions, even when it does not bind in equilibrium, by helping ensure that the price-setting problem is well defined.

[9]This is generically true, i.e., only in knife-edge specifications of u we have multiple global optima. Thus, it is relevant to focus on the uniqueness cases.

[10]As usual, a necessary condition for p^* to be optimal is $\epsilon(p^*) > 1$.

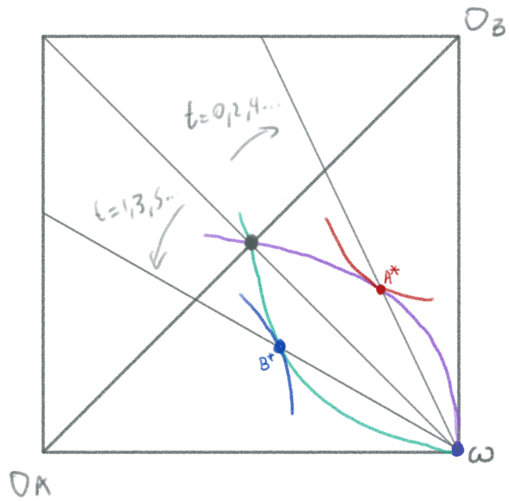

Figure 2: Edgeworth box diagram. The offer curve for A (green) and B (purple) intersect at the competitive equilibrium (assumed here with $c = c' = 1/2$). The optimum for seller A (red) and seller B (blue) feature a tangency between the offer curves and indifference curve. At the equilibrium, the relative price P_{At}/P_{Bt} cycles back and forth between these optima.

Equilibrium Inflation. Taking stock, we have seen that the rate of change in nominal prices is constant and entirely determined by preferences and endowments. Indeed, in this baseline model the solution is entirely driven by static considerations—it does not depend on the discount factor β nor on expected inflation. The next proposition collects these observation and shows that inflation is strictly positive.

Proposition 1. *Inflation is constant and positive, unaffected by β and given by*

$$\frac{P_t^*}{P_{t-1}^*} > 1$$

where $\epsilon^* = \epsilon(P_t^*/P_{t-1}^*)$ is the local elasticity of demand.

The proof of strictly positive inflation is quite intuitive. It is natural to expect $p^* > 1$ but it cannot be read off directly from (2): the markup satisfies $\frac{1}{1-1/\epsilon} > 1$ but typically $u_c/u_{c'} < 1$. We first show that that seller always optimally chooses a relative price that is strictly above that of all competitive equilibria. Then, since there exists a symmetric competitive equilibrium with unity relative price, this implies that $P_t^*/P_{t-1}^* > 1$.[11]

[11] This does not assume that other, non-symmetric, equilibria do not exist.

Quasilinear Iso-elastic Example. To develop further intuition, we work out a simple example in closed form using the quasilinear iso-elastic utility function

$$u(c,c') = c + \bar{d}^{\frac{1}{\epsilon}} \frac{(c')^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\epsilon}}$$

with $\epsilon > 1$ and $\bar{d} \in (0,1)$. This yields a familiar demand curve $D(p) = \bar{d}p^{-\epsilon}$ with constant elasticity over $p \geq 1$ (the participation constraint is not binding). [12] Condition (2) becomes

$$p^* = \frac{1}{1-1/\epsilon} \bar{d}^{-\frac{1}{\epsilon}} (p^* D(p^*))^{\frac{1}{\epsilon}}$$

which can be solved for[13]

$$p^* = \left(\frac{1}{1-1/\epsilon}\right)^{\frac{1}{2-1/\epsilon}} > 1.$$

As before, inflation is strictly positive. It follows that p^* is strictly decreasing in ϵ with $p^* \to \infty$ as $\epsilon \downarrow 1$ and $p^* \downarrow 1$ as $\epsilon \to \infty$. Thus, inflation can take any positive value as the elasticity varies.

Discussion: the Role of Conflict and Staggering. Inflation—a persistent and generalized rise in nominal prices—obtains in this model despite having abstracted from money, credit or savings, interest rates, production or employment. Standard culprits for inflation, or interpretations of the process, will just not do: inflation cannot be explained by an increase in money, nor the improper management of nominal interest rates (too low or too high), nor the level of demand, production or employment, or their stimulus by policy. By stripping almost everything away, the stylized model helps us see more clearly what remains.

Inflation results from the disagreement or conflict regarding relative prices in conjunction with the staggered setting of nominal prices. Both agents would like to enjoy a relative price that favors them, a higher price for the good they sell. Agents alternate attaining such conflicting aspirations, but these efforts lead to a constant rate of nominal price increases. Also, agents end up with an unfavorable relative price every other period. Indeed, on average over time relative prices are at a stalemate. The persist pursuit of conflicting aspirations over relative prices has no real winners and its energy gets canalized into a persistent increase in nominal prices.

As hinted above, disagreement or conflict over relative prices creates a force for infla-

[12] We restrict $\bar{d} < 1$ so that $c > 0$ for all $p \geq 1$, which is the relevant range of prices.

[13] This illustrates that $1 < p^* < \frac{1}{1-1/\epsilon}$ since x^θ for any $x > 1$ and $\theta < 1$ and $x = \frac{1}{1-1/\epsilon} > 1$ and $\theta = \frac{1}{2-1/\epsilon} \in (1/2, 1)$ for $\epsilon > 1$.

tion, but staggering is also crucial. Formally, it ensures an equilibrium with a finite level for inflation. To see this, consider a trading game similar to that of our baseline model, but where nominal prices are set simultaneously at the start of each period by both agents. No equilibrium exists for this game: the best response of each agent is to set a price higher than the other. Intuitively, staggering spreads the inflationary force of conflict over time, ensuring an equilibrium with finite inflation.

Particular models help see certain aspects of reality, but not others. Our model without money is purposefully stripped down, to isolate the conflict perspective, while preventing other interpretation. As such, it can be helpful to explore some questions, but not to answer others. One may ask: Is inflation necessarily always and everywhere a monetary phenomena? Is inflation necessarily due to excess demand or improper management of interest rates? The answers are 'no'. The stylized model forces one to look for other proximate causes of inflation.

However, this stylized model is certainly special and was not built to answer other questions. One should not conclude from this analysis that money or interest rates do not affect inflation, nor to deny an association between inflation and output or employment. In our view, many useful models and much empirical work supports such notions. However, we will argue that these models, which are also quite particular, are consistent with the conflict perspective.

Our view is that the more proximate cause of inflation is always conflict. However, conflict is typically endogenous. Many models may endogenize conflict in different ways. For example, monetary policy, via money supply or interest rates may have an influence on the economy and, thus, on conflict and inflation. The second part of our paper aims to provide a bridge between standard models and the conflict perspective in this light.

2.3 Two Simple Extensions

Here we discuss two very simple extensions. The next section provides a more significant variant of the model.

Random Roles for Buyer vs. Seller. We continue to consider two individuals A and B that are permanently matched. Agent A resets its price in even periods, while agent B does so in odd periods. Previously, we assumed that agent B acted as buyer in even periods and A did so in odd periods. That is, when agents reset their price they were seller. We now relax this assumption.

We now suppose that after an agent resets its price they will act as sellers with probability $\alpha \in (0,1]$ and as buyers with probability $1-\alpha \in [0,1)$ (across periods draws are independent). Thus, in even periods agent B acts as buyer with probability α; in odd periods A acts as buyer with probability α. Our baseline model amounts to the case with $\alpha = 1$; the case with $\alpha = 1/2$ is a simple case of interest since each agent has equal chances of acting as a buyer or seller in each period.

An agent resetting its price at any t now maximizes

$$\bar{v}(p) \equiv \alpha v(p) + (1-\alpha)V(1/p),$$

where $v(p)$ and $V(p)$ are the seller and buyer indirect utilities defined earlier. We assume that this problem is well posed with a finite and unique solution $p^* < \infty$. We note that this is more delicate than it was previously for the baseline model. Here the participation constraint of the seller imposed on the buyer plays a role in making the problem well posed.[14]

The thrust of Proposition 1 extends to this more general case. Once again there is positive inflation

$$\frac{P^*_{t+1}}{P^*_t} = p^* > 1.$$

We can now consider comparative statics on α. The solution must be in the range where $V(1/p)$ is increasing, it follows that p^* is decreasing in α. This implies that inflation is higher whenever $\alpha < 1$ than in the baseline model. Indeed, in the limit as $\alpha \to 0$ we obtain $p^* \to \infty$.

Non-Symmetric Preferences. For simplicity, we consider each extension separately, so let us return to the case with $\alpha = 1$. We now explore non-symmetric utility functions and show that the thrust of Proposition 1 still goes through.

Agent A has utility $U_A(c_A, c_B)$ and agent B has $U_B(c_A, c_B)$. We no longer impose that $U_A(c, c') = U_B(c', c)$ as in the original model.

Take any competitive equilibrium relative price for the static endowment economy

$$\hat{p} = \frac{P_A}{P_B}.$$

Applying the same logic as in Proposition 1, when A resets its price, it will act as a mo-

[14]To see this, note that if α is near 0 then $\bar{v}(p)$ approaches $V(1/p)$. Without the participation constraint this function is everywhere increasing and, thus, has no maxima; the participation constraint prevents this.

nopolist and ensure a relative price that is higher than this competitive equilibrium

$$\frac{P_t^*}{P_{t-1}^*} = p_A^* > \hat{p}.$$

Symmetrically, when B resets its price

$$\frac{P_t^*}{P_{t-1}^*} = p_B^* > \frac{1}{\hat{p}}$$

Note that the comparison is to $1/\hat{p}$ because it represents the relevant relative price P_B/P_A whenever B is reseting its price as a seller. Combining the two inequalities gives

$$\frac{P_{t+1}^*}{P_t^*}\frac{P_t^*}{P_{t-1}^*} = \frac{P_{t+1}^*}{P_{t-1}^*} = p_A^* p_B^* > 1$$

for any $t = 0, 1, \ldots$ Thus, for each good, prices are always reset (every two periods) at a strictly higher price than they were previously in proportion $p_A^* p_B^* > 1$ for both goods.

3 Random Matching: Inflation Expectations and Money

We now explore a more significant extension, allowing for random matching, a common assumption in the search literature. The price-setting problem now becomes forward looking and creates a role for inflation expectations. Overall, our main results are unchanged: inflation emerges for the same reasons as it did earlier. Random matching leads to a higher equilibrium rate of inflation than the baseline model with fixed matches. However, inflation may rise or fall with inflation expectations.

We then extend the analysis by adding money. The nominal unit of account now represents an object that is explicitly included in the model. Will money affect inflation? Can a steadfast commitment to keep money supply fixed stop inflation, at least eventually? No: We show that conflict inflation prevails even when the nominal money supply is held constant. Indeed, nominal prices rise without bound and real money balances asymptotically vanish. Thus, along the equilibrium path money is always present and used, but over time we approach the cashless economy studied earlier.

3.1 Random Matching and Inflation Expectations

Previously we assumed just two agents, A and B, that were perpetually matched. This implied that the price setting problem was static and that expectations about the future did not play any role.

Next, we consider a variant with infinitely many individuals of each type, A and B. Agents meet in pairs to trade, but are matched at random each period against someone in the opposite type. Random matching is a more standard assumption in the macro-search literature. This extension makes the price reset problem dynamics and opens the door for expectations of future inflation to matter, impacting actual inflation today.

The timing is as follows. First, sellers reset prices. Then each seller is matched with a random buyer from the opposite type. Importantly, prices are reset without knowing the price of their trading partner. As before, type A agents are sellers in even periods and buyers in odd periods, and vice versa for B.

To simplify we focus on stationary equilibria, where prices are expected to increase at a constant rate: $P_t^* = \Pi P_{t-1}^*$ for some $\Pi > 0$. An agent resetting its price takes as given the nominal price set in the previous period P_{t-1}^* by the agent of the opposite type. Looking forward to their next period, they expect a nominal price $P_{t+1}^* = \Pi^2 P_{t-1}^*$. The seller chooses its price P_t^* in proportion to P_{t-1}^*

$$P_t^* = p^*(\Pi) P_{t-1}^*$$

where the optimal price increment $p^*(\Pi)$ is given by

$$p^*(\Pi) \equiv \arg\max_p \left\{ v(p) + \beta V(\Pi^2/p) \right\}$$

Intuitively, by resetting their price agents determine the relative price faced by their current buyers p and, thus, obtain $v(p)$ as sellers in the current period; further, they anticipate that next period the relative price they will face when acting as buyers is Π^2/p, obtaining utility $V(\Pi^2/p)$. That is, due to random matching price setters influence the terms of trade they face in the next period. Since V is strictly decreasing in the relevant range it follows that p^* is higher with random matching, compared to the baseline with permanent matches. Additionally, p^* is increasing in β and approaches the case with permanent matches as $\beta \to 0$.

In equilibrium, inflation is given as a solution to the fixed point

$$\Pi = p^*(\Pi).$$

In general, the function $p^*(\Pi)$ may be increasing or decreasing. In the quasilinear case (with $\epsilon > 1$) $p^*(\Pi)$ is decreasing.[15] Intuitively, for the current price setter, higher expected

[15] The first order condition is

$$v'(p) - \beta V'\left(\frac{\Pi^2}{p}\right)\frac{\Pi^2}{p^2} = 0$$

inflation Π, all things the same, implies that agents anticipate a higher price in the next period, when it is their turn to be buyers. This higher price may discourage purchases. This, in turn, may make them care less about influencing this future price using their current price setting choice.[16]

Now consider the model extension with $\alpha = 1/2$ so that being a buyer or seller is purely random. In this case we concluded that the optimal reset price maximizes $\bar{v}(p) = \frac{1}{2}v(p) + \frac{1}{2}V(1/p)$. However, with random matching the problem becomes

$$\max_p \{\bar{v}(p) + \beta\bar{v}(p/\Pi^2)\}.$$

Note that when there is no expected inflation so that $\Pi^e = 1$ then the solution is as before and maximizes $\bar{v}(p)$. It is easy to see that locally around $\Pi = 1$ then $p^*(\Pi)$ is increasing but less than one for one with Π.

For any α, a stationary equilibrium again solves $\Pi = p^*(\Pi)$. We have verified the existence and uniqueness of a stationary equilibrium in the quasilinear case; we suspect this conclusion holds for other utility functions of interest, although exploring the conditions is not our focus. One can also explore non-stationary rational expectations equilibria. For the quasilinear case we find that there are a continuum of equilibrium paths, indexed by the starting rate of change in prices. All these paths converge to the stationary one we have focused on.

3.2 Adding Money

We now extend the random-matching model further to add money. Currency is held in the form of cash and used to facilitate some transactions. It also plays the role of unit of account: nominal prices are set in units of cash. Trade takes place using barter or cash, but cash has a distinct advantage.

Basics: Motivating Money. To motivate the use of money, we assume some transactions cannot be performed via barter. In particular, a fraction of the time agents suffer a

To see this note that $-V'(x)x = u_c D(x)x$ where $x = \Pi^2/p$ is the expected terms of trade. With quasilinear utility $u_c = 1$ and $D(x)x = x^{1-\epsilon}$ so that $V'(x)x$ falls with Π for fixed p. Assuming $v'' < 0$ the result then follows.

[16]On the other hand, there is a countervailing effect due to the concavity of the utility function: low purchases may imply high marginal utility, making them care more about improving their future terms of trade. This can be seen following the previous footnote and noting that $u_c(1 - D(x), xD(x))$ where $x = \Pi^2/p$ is not generally constant.

shock and have no endowment to barter with.[17] This shock creates a need for liquidity that money can fill, as a medium of exchange.

There are many ways of modeling the details, and we chose them with an eye towards tractability. In particular, a well-known complication is keeping track of the evolution in the distribution of money across agents. To sidestep this issue we set up the model and focus on equilibria where money fully swaps hands each period: buyers fully spend all their cash balances, so that money travels back and forth each period between agents A and B.

Specifics: Barter and Money. To keep everything else as simple as possible, suppose $\alpha = 1$ so that agents resetting their prices play the role of sellers. Just as before, the price P_t^* is reset by the seller at the very beginning of the period and stays fixed in place for two periods.

The new aspects of the model are as follows. Each period is composed of a continuum of "instants" indexed by $s \in [0,1)$. Agents are matched for an entire period, but meet each instant to trade and consume. Utility within a period is the simple integral over instants. Lifetime utility is thus

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 u(c_t(s), c_t'(s))\, ds$$

where $(c_t(s), c_t'(s))$ denotes consumption at instant s within period t.

A fraction $1-\delta$ of instants $s \in [0, 1-\delta)$ are just as before and we call them "regular": both agents have their endowments. Buyers can trade by making take it or leave it offers and paying using the ratio of nominal prices P_t^*/P_{t-1}^* as the terms of trade.

The remaining fraction δ of instants $s \in [1-\delta, 1)$ are "disasters": the buyer has no endowment so barter is impossible. However, the buyer can pay using cash. Only the seller price P_t^* is relevant in this case; the buyer price P_{t-1}^* is irrelevant.

We construct an equilibrium where buyers spend cash during disasters only and they spend all their cash balances. We proceed by conjecturing this is the case and then characterizing the seller pricing problem. We then return to verify the conjecture.

For simplicity, we assume additively separable utility

$$u(c, c') = F(c) + H(c')$$

[17] An equivalent interpretation is that a fraction of the time their endowment is intact, but cannot be used in exchange for the other good. One can tell many specific stories: that a fraction of the time the endowment cannot be transported, or the buyer "left the house without it", or the quality of the endowment (or its delivery) cannot be assured or verified by the counterparty, or the endowment is of a particular variety that the counterparty finds distasteful, etcetera, etcetera.

with F and H concave and $-H''(c')c'/H'(c') < 1$ or equivalently that $H'(c')c'$ is increasing. The quasilinear iso-elastic case with $\epsilon > 1$ satisfies these conditions.

Seller Problem. The seller price is set for two periods. A fraction $1-\delta$ of transactions are regular barter ones; but a fraction δ involve cash and require further consideration. The seller takes the entire sequence of prices $\{P_t^*\}$ as given and choose $\tilde{P}_t^*$ to solve

$$(1-\delta)v(\tilde{p}_t) + \delta F(1 - m_t/\tilde{p}_t) + \beta\left((1-\delta)V\left(\frac{p_t p_{t+1}}{\tilde{p}_t}\right) + \delta H\left(\frac{m_t}{p_t p_{t+1}}\right)\right)$$

where $\tilde{p}_t = \tilde{P}_t^*/P_{t-1}^*$, $p_t = P_t^*/P_{t-1}^*$ and $m_t = M/P_{t-1}^*$. The first order condition for $\tilde{p}_t^*$ evaluated at the equilibrium $\tilde{p}_t = p_t$ gives

$$v'(p_t)p_t + m_{t+1}\frac{\delta}{1-\delta}u(1-m_{t+1},0) = \beta V'(p_{t+1})p_{t+1}. \tag{3}$$

Together with the condition that $m_{t+1} = m_t/p_t$, this defines a dynamical system for $\{p_t, m_t\}$. The steady state of this system is $(p^*, 0)$, the steady-state of the moneyless model.

Buyer Cash-Management Problem. We now come back and check the conjecture that buyers spend their cash only during disasters and spend all of it then. This requires to possible deviations, providing two first-order conditions.

First, buyers can deviate from spending all their cash and hold on to a small amount, then spend it all two periods later during disasters. The first-order condition associated with this deviation is

$$H'(m_t) \geq \beta^2 \frac{1}{p_t p_{t+1}} H'(m_{t+2})$$

a standard intertemporal Euler that compares the marginal utility today to the discounted marginal utility times the real rate of return of money. Equivalently (multiplying both sides by m_t)

$$H'(m_t)m_t \geq \beta^2 m_{t+2} H'(m_{t+2}). \tag{4}$$

This condition will be automatically satisfied in the presence of positive inflation: we assume $H'(m)m$ is increasing and $m_{t+2} < m_t$.

Second, buyers can also deviate by also spending during regular instants $s \in [0, 1-\delta)$, not just during disasters. The first-order condition associated with this deviation is

$$G'(D(p_t)) \leq G'(m_t/p_t), \tag{5}$$

the marginal utility must be higher during a disaster. This condition simplifies to

$$D(p_t)p_t > m_t$$

which is satisfied whenever m_t is low enough. Intuitively, if buyer were sufficiently "cash rich" they would also want to spend this cash during regular instants.

Equilibrium with Money Swapping. A sequence $\{p_t, m_t\}_{t=0}^{\infty}$ is an equilibrium where money swaps hands each period if and only if (3) and (4) and (5) hold. We then have the following result.

Proposition 2 (Vanishing Money)**.** *For sufficiently small initial money balances $m_0 = M/P_{-1}$ there exist equilibria with strictly positive inflation. All these equilibria converge over time to the steady-state of the moneyless model.*

The point of this extension was to show that our results were not sensitive to a complete absence of money. They survive in an equilibrium where money exists and forever changing hands and used in transactions. In this model, money is valuable and, thus, demanded by agents. Even with a fixed supply of money, inflation from conflict prevails: the price level rises without bound, sending real money balances asymptotically towards zero.

How is this possible? Money is demanded for its real balances so pitting this demand against a constant money supply and rising prices should eventually create money scarcity that stops inflation in its tracks, or not?

Technically, in our model, note that money "demand" is not properly captured by any static condition such as "$M/P_t = L(\cdots)$". Thus, with fixed M, we cannot argue supposing L is constant (or bounded below) to conclude that P_t is pinned down (or bounded above). Instead, in our model, money holdings satisfy appropriate Euler equations, developed and checked above; these are dynamic relations, not static ones, and are fully consistent with shrinking real money balances.

To be sure, shrinking real money balances is unfortunate, it reduces liquidity and hurts trade. Intuitively, however, it does not create any obvious force for sellers to halt the rise in prices. Sellers still aspire to a favorable relative price, one above unity, and this generates inflation, as before. In our model, inflation continues to be generated by a disagreement or conflict in relative prices of goods; real money balances are simply a casualty that does not get in the way of these conflicting aspirations to stop inflation.

4 Inflation and Conflict in a Network Economy

The second part of our paper is in some ways the complete opposite of the first part. Earlier, we introduced a stylized and fully-specified model. That model was not built to be realistic, but to help isolate conflict as the driver of inflation. The model was purposefully kept distant from familiar benchmark models in macroeconomics. It featured barter in a setting with no money, no saving, no credit, no labor nor production. Straying away from familiar territory can liberate the mind from habits or engrained conclusions and intuitions, e.g., inflation generated by monetary expansion or output gaps. We solved for the rational expectations equilibrium of this stylized model.

In contrast, in this second part we develop a general framework with staggered price setting and any number of interdependent sectors. By a framework we mean a subset of the equations of a specific and full model. This framework is general enough to fit many familiar models. Indeed, it nests and distills the essence of the New Keynesian model, staggered pricing, while avoiding its other special aspects.

The relative generality of our analysis is possible by virtue of our approach taking as given any path for *aspirations* affecting price setting. Of course, aspirations are endogenous in a full-fledged model, but our goal is to elucidate the mapping from aspirations to inflation in a way that is useful across a wide set of models or situations. In this way, our approach is consistent with the diagram in Figure 1 at the outset: we are studying the proximate causal channel, from conflict in aspirations to inflation. As we argue, inflation emerges from conflict precisely through these interdependencies across sectors—a generalized "wage-price spiral" of sorts.

Of course, studying the root causes of inflation or performing policy counterfactuals may require endogenizing aspirations. Doing so is a worthy enterprise and we have pursued it ourselves in prior work (Lorenzoni and Werning, 2022).

On first pass, our analysis may be confused as entirely mechanical, since we take as given the crucial aspirations. However, this is wrong. At the heart of our framework are interdependencies across sectors mediated through endogenous prices. Indeed, inflation emerges from this interplay across sectors.

Finally, our analysis is fully general and applies to *any* path for aspirations, including those endogenously generated by a fully-specified models. We pursue endogenizing aspirations in a limited way in the next section, to adjust for inflation expectations and solve for the rational expectations benchmark.

4.1 Conflict as Infeasibility

We begin by introducing minimal ingredients needed to formalize the idea of conflict. Our basic building block is an economy with multiple, interdependent sectors. The key ingredient is a notion of net gains for marginal transactions in each sector. These net gains depend on relative prices throughout the economy. There is a set of feasible net gain. This the set of gains that agents can, in principle, aspire to obtain, without raising a fundamental problem of incompatibility. Our definition of "conflict" is then simply the complement of the feasibility set, the set of net gains that are not feasible or incompatible.

The present subsection is completely silent on the determination of prices. It will make no reference to to price setting nor dynamics. The next two subsection develops our price setting framework and then studies the dynamics of prices.

Wage-Price Example. Before developing our general framework, we introduce our leading wage-price example, which we repeatedly come back to in the rest of the paper. The dynamics in this wage-price case are studied in greater detail in Lorenzoni and Werning (2022).

In this example economy, firms produce consumption goods with nominal price P. Households, in turn, provide labor to firms, at nominal wage W. Both P and W are expressed in logarithms.

Firms gain at the margin in accordance to their profit margin $\mathcal{A}_P(P, W) = P - W$. Similarly, workers gain at the margin in accordance to their real wage $\mathcal{A}_W(P, W) = W - P$. Naturally and trivially, these gains are mirror images of each other: $\mathcal{A}_P(P, W) = -\mathcal{A}_w(P, W)$. In this economy the circular flow of goods is extremely simple: firms take labor as an input (hence the wage is subtracted), households consume goods produced by firms (hence the price is subtracted).

Suppose now that firms and workers have some *aspirations* for these gains: firms seek a_P while workers seek a_W. Any full model endogenizes these aspirations. For example, in the simplest models a_P represents the desired firm markup minus productivity, while a_W equals the marginal rate of substitution between labor and consumption plus a potential markup from labor.

In the present analysis, we do not model nor take a specific stand on aspirations. Instead, we study the implications of any general aspirations. We first ask, when are aspirations mutually compatible and when are they in conflict? The answer in this simple case is trivial. The condition

$$a_P + a_W = 0$$

is both necessary and sufficient for the existence of a pair (P, W) yielding $a_P = \mathcal{A}_P(P, W)$ and $a_w = \mathcal{A}_W(P, W)$. When $a_P + a_W \neq 0$ aspirations are in conflict in the sense that there is no price and wage that attains both aspirations. The sum $a_P + a_W$ is, thus, a measure of conflict.

Note that we have merely made observations about mutual feasibility of some aspirations $a = (a_P, a_W)$, without discussing the determination of prices and wages. Later, after we develop a framework for price and wage adjustment, we will see that the degree of conflict is an important determinant in inflation.

The General Network Economy. The previous case was very special and only had two sectors. We now develop a more general framework with N sectors $n = 1, 2, \dots, N$. The log price in sector n is labeled P_n and we define the price vector as

$$P = (P_1, P_2, \dots, P_N)'.$$

In keeping with our previous example, some goods may represent labor services, in which case their prices should be interpreted as wages.

The marginal gain for a seller in sector n is

$$\mathcal{A}_n(P) = P_n - \sum_{n'} m_{nn'} P_{n'}$$

where $\bar{P}_n = \sum_{n'} m_{nn'} P_{n'}$ is a relevant price index for sector n with $m_{nn'} \geq 0$ and $\sum_{n'} m_{nn'} = 1$. This price index, which in general varies across sectors, is an important element of our framework. First, economically, it ensures that the gains $\mathcal{A}$ are in real terms, that is, only a function of relative prices and invariant to the overall price level (in logarithms, an additive constant). Second, it captures various considerations. For firms selling goods, the index captures the cost of production through the price of its inputs, including labor;[18] for workers selling labor, the price index represents the cost of the basket of goods they consume, so that $P_n - \bar{P}_n$ represents the relevant real wage.

Defining $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N)'$ then

$$\mathcal{A}(P) \equiv (I - M)P = AP$$

with $A \equiv I - M$ and where $M = (m_{nn'})$ stacks the price index coefficients of each sector in its rows and thus has non-negative entries and rows that add up to one.

[18] We assume that $m_{nn'} \geq 0$. This captures most cases of macroeconomic interest. However, this does rule out strategic substitution across producers, whereby good n may want to lower the price if good n' raises its price.

Network Centrality. The network matrix M encapsulates the interdependencies in our economy. It is analogous to an input-output matrix for a production economy. As mentioned earlier, this is one important part of the interpretation for M in our framework, but some sectors may represent labor in which case M also summarizes the consumption expenditure shares of workers.

We make the following assumptions on the matrix M. First, it is natural to assume that each sector n acts as input to some other sector, so that for each n we assume $m_{n'n} > 0$ for some $n' \neq n$—otherwise, sector n would lack any economic relevance. We further assume that non-zero entries are pervasive enough that the matrix M is irreducible.[19] Economically, this represents a situation where sectors are sufficiently interconnected so the economy cannot be split off into two independent economies.

Each row of $A = I - M$ adds up to zero, so A cannot have full rank N. However, from the irreducibility of M it follows that A has rank $N - 1$ (see Berman and Plemmons, 1979) which also ensures the existence of a unique vector satisfying the following definition.

Definition (Network Centrality)**.** The unique strictly positive vector $\gamma > 0$ satisfying $\sum_n \gamma_n = 1$ and

$$\gamma' = \gamma' M, \tag{6}$$

or, equivalently, $\gamma' A = 0$, is the *network centrality* vector*, with* $\gamma_n > 0$ representing the network centrality of sector n.

In what follows, network centrality γ is used as a weight to define relevant averages or indices.

Mathematically, M could represent a Markov transition matrix in which case γ satisfying (6) represents an invariant distribution. However, economically M does not represent a probability matrix, it summarizes economic connections across sectors. Network theory develops network centrality as a measure of the overall economic importance of sectors, taking into account their direct and indirect effects on other sectors. Formally, condition (6) says that γ is a left eigenvalue for M associated with unit eigenvalue. For this reason, our notion of network centrality may be referred more specifically as "left-hand eigenvector centrality" (Stachurski and Sargent, 2022).

Conflict. Suppose we have some value $a_n \in \mathbb{R}$ for each sector n which we associate with a potential value or *aspiration* for the marginal gain $\mathcal{A}_n(P)$. We are interested in

[19] A sufficient condition is that all off diagonal entries are strictly positive, but this condition is not necessary. For example, another sufficient condition is that sector $m_{nn+1} > 0$ for $n = 1, \ldots, N-1$ and $m_{Nn'} > 0$ for some $n' \neq N$.

distinguishing when aspirations are collectively feasible or infeasible—whether they are compatible or in conflict.

Definition (Conflict)**.** Fix a vector of aspirations $a = (a_1, a_2, \ldots, a_N)' \in \mathbb{R}^N$. We say there is conflict if there exists no price vector $P \in \mathbb{R}^N$ such that

$$a = \mathcal{A}(P) = AP. \tag{7}$$

Conversely, if condition (7) holds for some P then we say there is no conflict.

Suppose there is no conflict so that $a = AP$ for some vector P. But then, $\gamma' a = \gamma' AP = 0$ using the definition of network centrality (6). Since A has rank $N - 1$ the converse is also true, as proved in the appendix. We then obtain the following.

Lemma (No Conflict)**.** *There is no conflict if and only if*

$$\gamma' a = \sum_n \gamma_n a_n = 0, \tag{8}$$

that is, when average aspirations, weighted by network centrality, are zero.

When condition (8) does not hold, we say that there is positive conflict if $\gamma' a > 0$ and negative conflict if $\gamma' a < 0$. Moreover, we adopt $\gamma' a$ as a quantitative measure of conflict.

This simple result illustrates network centrality as a relevant weight to average sectors: when average aspirations are zero they are compatible in the sense that there are prices that deliver these aspirations. Otherwise, they are in conflict and the aspirations are infeasible, regardless of prices. We next explore what happens in these situations with prices.

4.2 A Framework for Price Dynamics

Up to now we have only remarked on the mutual feasibility of aspirations or potential gains for sectors at a given point in time. However, we have had nothing to say about how prices are set or how they evolve—thus, nothing to say about inflation. We now develop develop a relatively standard price setting framework leveraging our previous concepts. Studying the dynamics of prices we then show that conflict, as defined above, plays a key role in inflation.

Preliminaries. Time is continuous $t \geq 0$. Within each sector $n = 1, 2, \ldots, N$ there are a continuum of symmetric varieties $i \in [0, 1]$. Nominal prices P_{nit} are expressed in logarithms. We summarize the price of sector n as[20]

$$P_{nt} = \int P_{nit}\, di,$$

the average price over varieties. Let P_t denote the column vector of sectoral prices $P_t = (P_{1t}, P_{2t}, \ldots, P_{Nt})'$. We next turn to the dynamical system for P_t.

Pricing Frictions. Prices of varieties are reset infrequently à la Calvo: price resetting opportunities arrive at a constant Poisson rate $\lambda_n \in (0, \infty)$ in each sector n with independent realizations across time and varieties (the flow of price resetters is constant at λ_n). The average price in sector n then evolves according to

$$\dot{P}_{nt} = \lambda_n (P^*_{nt} - P_{nt}), \tag{9}$$

where P^*_{nt} is the price of varieties in sector n that are reset at t. Intuitively, the rate of inflation equals the fraction of firms changing prices λ_n times the size of the price change $P^*_{nt} - P_{nt}$. We postulate that the reset price is

$$P^*_{nt} = a_{nt} + \sum_{n'} m_{nn'} P_{n't} \tag{10}$$

for a given path $\{a_{nt}\}$ of aspirations. This condition stipulates that, for given aspirations, the reset price is endogenous to prices through the index $\sum_{n'} m_{nn'} P_{n't}$, reflecting the notion that aspirations are real.[21]

Due to price staggering, condition (10) says that $a_{nt} = P^*_{nt} - \sum_{n'} m_{nn'} P_{n't}$ so that aspirations are always met at the moment of a price reset. However, some time $s > 0$ after a price reset we may have $a_{nt+s} \neq P^*_{nt} - \sum_{n'} m_{nn'} P_{n't+s}$; this also implies that aspirations are not necessarily met for sectoral prices $a_{nt} \neq P_{nt} - \sum_{n'} m_{nn'} P_{n't}$.

Aspirations are simply our driving force for reset prices. This does not imply that agents act naively, resetting prices to match some target aspiration, only to realize their aspirations are not met later. We later derive reset aspirations $\{a_{nt}\}$ in a forward-looking manner from some given targets $\{\hat{a}_{nt}\}$ agents have for their current and future gains. However, any model that generates aspirations endogenously implies some path $\{a_{nt}\}$ satisfying (10). We take such a path of aspirations entering (10) as given.

[20]Focusing on the average price can be justified in the usual manner as the first order approximation to any non-linear constant-returns price index.

[21]Strategic complementarity in price setting among firms in the same sector is possible and it is captured by a positive coefficient m_{nn}.

Substituting we obtain

$$\dot{P}_{nt} = \lambda_n \left(a_{nt} + \sum_{n'} m_{nn'} P_{n't} - P_{nt} \right) \qquad n = 1, 2, \ldots, N, \tag{11}$$

or

$$\pi_t \equiv \dot{P}_t = \Lambda \left(a_t - A P_t \right). \tag{12}$$

where π_t is a vector of sectoral inflation rates and Λ is a diagonal square matrix with λ_n on the diagonal.

Dynamics in the Wage-Price Example. In our leading example, following Erceg, Henderson and Levin (2000) firms and worker unions reset individual prices and wages in a staggered Calvo manner. Price setting is $P_t^* = a_{pt} + W_t$ and wage setting $W_t^* = a_{wt} + P_t$ so that

$$\pi_t^p = \dot{P}_t = \lambda_p (P_t^* - P_t) = \lambda_p (a_{pt} + \omega_t)$$
$$\pi_t^w = \dot{W}_t = \lambda_w (W_t^* - W_t) = \lambda_w (a_{wt} - \omega_t)$$

where $\omega_t = W_t - P_t$ is the real wage; the real wage evolves according to

$$\dot{\omega}_t = \lambda_p a_t^p - \lambda_w a_t^w + (\lambda_p + \lambda_w) \omega_t.$$

Thus, the dynamics for (P_t, W_t) can be expressed in terms of given aspirations $\{a_t\}$ and a single relative price $\{\omega_t\}$. We note that the real wage, a relative price is driven by differences in aspirations, unlike conflict which relates to an average of aspirations.

4.3 Inflation is Conflict

Define the aggregate price index

$$\bar{P}_t = \sum_n \psi_n P_{nt},$$

with weights satisfying $\psi_n \geq 0$ and $\sum_n \psi_n = 1$ given by

$$\psi_n = \frac{d_n}{\bar{d}} \gamma_n,$$

where $d_n = 1/\lambda_n$ is the price duration of sector n and and $\bar{d} = \sum_n \gamma_n d_n$ is average price duration, weighted by centrality. If $\lambda_n = \lambda$ then $\psi = \gamma$, otherwise, ψ shifts weights towards less flexible sectors.

It is useful to work with sectoral relative prices defined as

$$p_{nt} = P_{nt} - \bar{P}_t \tag{13}$$

These relative prices satisfy, by construction, the normalization $\sum_n \psi_n p_{nt} = 0$.

Proposition 3 (Conflict Inflation)**.** *Given network centrality γ and average duration $\bar{d}$, at any point in time t define conflict inflation by*

$$\Pi_t^C = \frac{1}{\bar{d}} \gamma' a_t. \tag{14}$$

Then inflation in the aggregate index $\bar{P}_t = \sum_n \psi_n P_{nt}$ equals conflict inflation

$$\bar{\pi}_t = \sum_n \psi_n \pi_{nt} = \Pi_t^C$$

Moreover, there exists a unique relative price vector $\mathcal{P}(a)$ such that if $p_t = \mathcal{P}(a_t)$ all sectors have inflation equal to conflict inflation

$$\pi_{nt} = \Pi_t^C \qquad n = 1, 2, \ldots, N.$$

To bring out the economic implications, let us discuss some special cases. At any point in time, zero average inflation

$$\sum_n \psi_n \pi_{nt} = 0$$

holds if and only if conflict is absent

$$\sum_n \gamma_n a_{nt} = 0.$$

Indeed, absent conflict, inflation can be zero in all sectors $\pi_{nt} = 0$ for some relative price. In particular, there is a steady state with zero inflation in all sectors. The absence of conflict is crucial to zero inflation.

In contrast, in the presence of positive conflict $\sum_n \gamma_n a_n > 0$ average inflation is positive $\sum_n \psi_n \pi_n > 0$, so inflation is positive in some sectors. In particular, there is a relative price for which inflation equalized in all sectors $\pi_{nt} = \bar{\pi}_t > 0$ for all n. Indeed, by continuity, in a neighborhood of this relative price inflation is strictly positive in all sectors. Thus, positive conflict is crucial for positive inflation.

A straightforward implication of Proposition 3 is that positive inflation in all sectors is only possible with positive conflict. Conflict inflation depends only on current aspirations a_t, and not on relative prices p_t which depend on the entire history. Thus, positive inflation in all sectors cannot be spurred by some particular relative prices or past history.

Positive inflation require current conflict in aspirations.

4.4 Adjustment Inflation and Decomposition

We now decompose the dynamics of all sectoral prices into two components: conflict inflation and the dynamic adjustment of relative prices.

Combining (13) and Proposition (3) gives

$$\dot{p}_t = \dot{P}_t - \mathbf{1}\dot{\bar{P}}_t = \Lambda\left(a_t - AP_t\right) - \mathbf{1}\frac{1}{\bar{d}}\gamma' a_t,$$

where $\mathbf{1}$ is a column vector of ones. Note that $AP_t = Ap_t$ (since the rows of A add up to 0) so we obtain

$$\dot{p}_t = \Lambda a_t - \mathbf{1}\frac{1}{\bar{d}}\gamma' a_t - \Lambda A p_t. \tag{15}$$

By construction, relative prices p_t satisfy $\psi' p = 0$; that is, they lie in the $N-1$ dimensional subspace $S = \{p : \psi' p = 0\}$.

When $a_t = a$ is constant the system (15) has a unique steady state, the relative price vector $\mathcal{P}\left(a\right)$ defined in Proposition 3.[22] The next proposition addresses stability.

Proposition 4 (Stability of Relative Prices)**.** *The differential system for relative prices* (15) *is stable. If $a_t = a$ is constant then $p_t \to \mathcal{P}(a)$. Moreover, given any bounded path $\{a_t\}$ the solution for relative prices $\{p_t\}$ is also bounded.*

When aspirations are constant, relative prices settle down. Relative prices may fluctuate when aspirations are not constant, but they do not intrinsically lead to explosive dynamics, unless aspirations themselves are explosive.

Define the relative price adjustment component of inflation as

$$\Pi_t^A \equiv \dot{p}_t.$$

Then inflation satisfies

$$\pi_t = \Pi_t^A + \Pi_t^C.$$

From $\psi' p_t = 0$ it follows that $\psi' \dot{p}_t = \psi' \Pi_t^A = 0$. We see once again that $\bar{\pi}_t = \psi' \pi_t = \Pi_t^C$.

At steady states with conflict sectors do not attain their aspirations, if

$$p = \mathcal{P}(a) \implies a_n - \mathcal{A}_n(p) = \frac{d_n}{\bar{d}}\gamma' a. \tag{16}$$

[22]The proof of that proposition shows that $\mathcal{P}\left(a\right)$ satisfies the steady state condition $\dot{p}_t = 0$.

Thus, with positive conflict $\gamma' a > 0$ aspiration are frustrated by an amount that is proportional to the degree of conflict and proportional to the relative duration of sector n. Positive conflict leads to inflation and sectors with higher durations suffer a greater deterioration with inflation. A sector with fully flexible prices attains its aspiration at all times.

Inflation over the Long Run. The observation that long-run inflation is dominated by conflict can be extended beyond the constant aspirations case. Define long-run averages for sectoral inflation and for conflict inflation

$$\bar{\pi}_n = \lim_{T\to\infty} \frac{1}{T}\int_0^T \pi_{nt}\, dt \qquad \bar{\Pi}^C = \lim_{T\to\infty} \frac{1}{T}\int_0^T \Pi_t^C\, dt.$$

Our next result links these two averages.

Proposition 5 (Long Run Inflation is Conflict)**.** *Suppose the path for aspirations $\{a_t\}$ is bounded. Then long-run average inflation equals long-run average conflict inflation*

$$\bar{\pi}_n = \bar{\Pi}^C$$

for all sectors n.

This result says that any prolonged episodes of inflation must be driven by conflict.[23] Inflation is often defined as a sustained and generalized increase in prices. Thus, in this sense, our results establish that inflation is conflict.

The intuition can be gleamed from our previous example with a_t constant: the adjustment force may move nominal prices to move to shift relative prices towards $p^*(a)$, this can produce inflation in some sector, but it is only a temporary force that does not produce sustained inflation. Proposition 5 extends this observation to the more general cases in which $\{a_t\}$ keeps fluctuating, producing never-ending fluctuations in relative prices, without reaching a steady state. Intuitively, in these cases the adjustments component creates forces for both inflation *and* deflation that average out to zero over long periods of time.

The formula for conflict inflation (14) is very simple and revealing, conflict $\gamma' a$ is divided by average duration $\bar{d}$. This confirms that average duration is the right measure of stickiness for aggregation (see Werning, 2022b). Expressed in terms of price frequencies $\bar{d} = \sum \gamma_n \lambda_n^{-1}$ which is maximized for given $\sum_n \lambda_n$ by a constant frequency $\lambda_n = \lambda$.

[23] Here we have taken time averages, but if we consider a situation with uncertainty and suppose $\{a_t\}$ are stationary stochastic processes, then the same can be said about the probabilistic average. That is, the unconditional expectation for inflation $\mathbb{E}[\pi_{nt}]$ must be proportional to the unconditional expectation of conflict $\mathbb{E}[\gamma' a_t]$.

Indeed, at the opposite extreme if there exists one sector with rigid prices $\lambda_n \to 0$ then $d_n = \infty$ and $\bar{d} = \infty$, so $\Pi_t^C = 0$. Intuitively, inflation from conflict is amplified by the feedback across sectors. When price stickiness is evenly distributed across sectors this feedback is maximized; if, instead, any one sector is rigid, the feedback loop is stopped dead in its tracks. In the wage-price example, this feedback captures one aspect of what (Lorenzoni and Werning, 2022) define as wage-price spirals (the other works through expectations).

Perhaps more surprisingly, the formula for conflict inflation reveals that there no interaction between aspirations and stickiness. If conflict is positive, it does not matter if this is due to positive aspirations in relatively flexible or sticky sectors. Long run inflation will be the same.

The intuition for this result is that conflict provides the impulse for inflation, but it plays out in the long run through the feedback across sectors and this only depends on a measure of overall stickiness. Consider an extreme example, suppose for $t < 0$ we were at a steady state with zero inflation and $a = 0$ and at $t = 0$ we find that $a_n > 0$ for some n and $a_{n'} = 0$ for $n' \neq n$. If sector n has extremely sticky prices $d_n > \bar{d}$ this slows down inflation initially. However, over time inflation in sector n leads to inflation in sectors $n' \neq n$ even though their aspirations have not changed. Indeed, since these sectors have relatively flexible prices there price adjusts more and sector n finds its price index has risen by more. This leads sector n to make larger prices changes. At the new steady state with inflation $\Pi^c > 0$ sectors with higher durations make larger price changes $P^*_{n't} - P_{nt}$.

5 Forward-Looking Aspirations

Aspirations affect reset prices. One simple interpretation, then, is that aspirations represent naive goals that are frustrated by positive inflation. This not our preferred interpretation and all our previous results hold along any equilibrium path when aspirations are endogenous. We now explore aspirations that incorporate inflation expectations, that is, we endogenizing aspirations a_t to adjust for expected inflation. We first provide formulas for aspirations for any arbitrary expectations. We then solve the (fixed-point) equilibrium outcome for the benchmark case of rational expectations.[24]

[24] Since we abstract from aggregate uncertainty, this amounts to perfect foresight. However, the solution with uncertainty and rational expectations would be analogous.

5.1 Aspirations Adjusted by Expected Inflation

We call a path $\{\hat{a}_{nt}\}$ the target aspiration to distinguish it from the reset aspiration path $\{a_{nt}\}$.We now postulate that the reset price satisfies

$$P^*_{nt} = (\rho + \lambda_n)\hat{\mathbb{E}}_{nt} \int_t^\infty e^{-(\rho+\lambda_n)(s-t)}(\hat{a}_{ns} + \sum_{n'} m_{n'n} P_{n's})ds, \tag{17}$$

for given path $\{\hat{a}_{nt}\}$, where $\rho > 0$ is a discount rate, $\hat{\mathbb{E}}_{nt}$ the subjective expectation held at time t in sector n. According to this condition, the reset price is a weighted average of future values for both the target aspirations and the price index $\sum_{n'} m_{n'n} P_{n's}$ for sector n.[25]

The reset equation (10) is consistent with (10) with an endogenous reset aspiration given by[26]

$$a_{nt} = \hat{\mathbb{E}}_{nt} \int_t^\infty e^{-(\rho+\lambda_n)(s-t)}((\rho + \lambda_n)\hat{a}_{ns} + \sum_{n'} m_{nn'} \pi_{n's})ds. \tag{18}$$

This shows that expected inflation raises aspirations $\{a_{nt}\}$ for a given path of target aspirations $\{\hat{a}_{nt}\}$, creating an additional feedback channel across sectors. Sectors are concerned with inflation in other sectors according to their own price index.

To fix ideas, consider a simple case with constant target aspirations $\hat{a}_{nt}$ and constant inflation expectations $\hat{\mathbb{E}}_{nt}[\pi_{n't+s}] = \pi^e_{nn'}$. Then

$$a_n = \hat{a}_n + \frac{\sum_{n'} m_{nn'} \pi^e_{nn'}}{\rho + \lambda_n}$$

Intuitively, prices must be reset more aggressively to attain the same targets if expected inflation is positive. It follows immediately, computing our measure of conflict

$$\gamma' a = \gamma' \hat{a} + \gamma' \frac{\sum_{n'} m_{nn'} \pi^e_{nn'}}{\rho + \lambda_n} \tag{19}$$

that conflict $\gamma' a$ increases in both $\gamma' \hat{a}$ (i.e. target conflict) and expected inflation for sector n' held by each sectors n $\{\pi^e_{nn'}\}_{n,n'}$.[27] Thus, higher expected inflation raises inflation through conflict (as in Section 4).

[25] As usual, this condition can be derived as the valid linear approximation around a steady state.

[26] This can be derived by first noting that

$$a_{nt} = (\rho + \lambda_n)\hat{\mathbb{E}}_{nt} \int_t^\infty e^{-(\rho+\lambda_n)(s-t)}(\hat{a}_{ns} + \sum_{n'} m_{n'n}(P_{n's} - P_{n't}))ds.$$

and $P_{n's} - P_{n't} = \int_0^t \pi_{n'z} dz$. Integrating by parts gives the desired result.

[27] The effect of inflation expectations is dependent on the pricing model. Werning (2022a) shows that Calvo pricing implies a relatively high effect from expected inflation.

Consider a simple example. Suppose initially $\hat{a} = 0$ and $a = 0$ with initial price vector consistent with zero inflation. However, suppose all sectors have the same (irrational) positive inflation expectations for all sectors so that $\pi^e_{nn'} = \pi^e > 0$. This generates conflict $\gamma' a > 0$ despite no intrinsic conflict in target aspirations, $\gamma' \hat{a} = 0$. As this example emphasizes, disagreement in expectations is not what generates conflict, it is the level of expectations that matters not their difference. This is because positive inflation expectations is a force for each sector to desire high prices, as protection against inflation. In the wage-price example expected inflation creates conflict if workers seek a more favorable wage *and* if firms seek a more favorable price.

Adjustments for inflation disappears in two extreme cases. First, if $\pi^e_{nn'} = 0$ so agents do not anticipate inflation. One possibility is that expectations of inflation are not rational and are "well anchored" around zero inflation.[28] A variant of this idea is that agents have non-zero expectations but may not act on them at low inflation rates (Rowthorn, 1977; Werning, 2022a). Finally, even with full rationality, as $\rho \to \infty$ then aspirations in (18) become myopic and $a_t \to \hat{a}_t$.

5.2 Inflation and Conflict Under Rational Expectations

The previous subsection showed how to relate reset aspirations $\{a_t\}$ to target aspirations $\{\hat{a}_t\}$ and inflation expectations. The implications for inflation were immediate thanks to the fact that the analysis in Section 4 applied to any sequence of reset aspirations $\{a_t\}$. In this way, one can leverage the previous analysis to incorporate expected inflation.

Now we study the price dynamics under the rational expectations benchmark. That is, we endogenize expectations by taking as given target aspirations $\{\hat{a}_t\}$ and solving for the path of prices $\{P_t\}$ assuming agents have perfect foresight about this path. This involves solving a fixed point.

Combining the price setting equations above with the law of motions for sectoral prices (9), yield the second order differential equation

$$\rho \dot{P}_{nt} = \lambda_n (\rho + \lambda_n) (\hat{a}_{nt} + \sum_{n'} m_{n'n} P_{n't} - P_{nt}) + \ddot{P}_{nt},$$

or

$$\rho \dot{P} = \hat{\Lambda} (\hat{a} - AP) + \ddot{P},$$

where $\hat{\Lambda}$ is a diagonal matrix with diagonal elements equal to $\lambda_n(\rho + \lambda_n)$. The definition of network centrality γ is unchanged.

[28] The issue is somewhat symmetric: in countries experiencing high inflation and undergoing a stabilization program to lower inflation, there is concern that aspirations a_t will not fall.

Our definition of conflict inflation is now

$$\hat{\Pi}_t^C = \frac{1}{\bar{D}} \int_0^\infty e^{-\rho s} \gamma' \hat{a}_{t+s}\, ds$$

where $D_n = d_n^2/(\rho d_n + 1)$ and $\bar{D} = \sum_n \gamma_n D_n$. Conflict inflation is proportional to the present value of conflict in target aspirations, discounting at rate $\rho > 0$—a property familiar in the standard New Keynesian model. Intuitively, with rational expectations future conflict has a direct effect on current inflation, but since it also affects future inflation this has an indirect effect on current inflation through inflation expectations.

With this definition of conflict inflation we obtain results analogous. Define the average price index $\hat{P}_t = \sum_n \hat{\psi}_n P_{nt}$ with weights $\hat{\psi}_n = (D_n/\bar{D})\gamma_n$. Then at any time t average inflation in this price index equals conflict inflation,

$$\sum_n \hat{\psi}_n \pi_{nt} = \hat{\Pi}_t^C$$

and the time averaged inflation in each sector equals that of conflict inflation

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \pi_n = \lim_{T\to\infty} \frac{1}{T} \int_0^T \hat{\Pi}_t^C.$$

In addition, price dynamics are (backward) stable. That is, we can write

$$\dot{P}_t = C(P_t - \hat{P}_t - \hat{p}_t) + \Pi_t^C$$

where $\hat{p}_t$ is a linear function of future target aspirations $\{\hat{a}_{nt+s}\}$. Here $p_t = P_t - \hat{P}_t$ represents relative prices (satisfying $\hat{\psi}' p_t = 0$). We show that C implies stable dynamics for the homogenous system $\dot{P}_t = CP_t$, with all eigenvalues of C having negative real part. Since the dynamics of relative prices are driven by C relative prices are stable, as before. In particular if aspirations are constant then $\hat{p}_t = \hat{p}$ is constant and relative prices converge to it, $p_t = P_t - \hat{P}_t \to \hat{p}$.

6 Conclusion

The paper provided a perspective on the role of conflict—defined here as a disagreement on relative prices—on inflation. Individuals make efforts to adjust the absolute prices under their control to meet these relative price goals. If their goals are collectively incompatible these efforts are destined to failure, but set in motion a trend in absolute prices. Conflict over relative prices translate into inflation in absolute prices.

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Appendix

A Proof of Proposition 1

First, we introduce an upper bound on the utility of the seller $v(p)$. Define the hypothetical utility of a seller who could choose quantities (c, c'), taking the price p as given:

$$\tilde{v}(p) \equiv \max_{c,c'} u(c, c') \text{ s.t. } c' = p(1 - c).$$

The vector $(c, c') = (1 - D(p), pD(p))$ is feasible in the problem above and given the definition of $v(p)$ we have

$$v(p) = u(1 - D(p), pD(p)) \leq \tilde{v}(p) \text{ for all } p > 0$$

with strict inequality as long as $(c, c') = (1 - D(p), pD(p))$ is not optimal at p. Notice that $p = 1$ is a competitive equilibrium price and at a competitive equilibrium price, when both agents are price takers they choose quantities consistent with market clearing. This implies that

$$v(1) = \tilde{v}(1).$$

The function $\tilde{v}(p)$ is strictly increasing in p since $c' > 0$ and thus $1 - c > 0$. We conclude that for any $p < 1$, we have

$$v(p) \leq \tilde{v}(p) < \tilde{v}(1) = v(1),$$

so $p < 1$ is never optimal in problem (1). Moreover, by the envelope theorem $v'(1) = \tilde{v}'(1) > 0$ and $p = 1$ cannot be optimal. Therefore, the optimum must satisfy $p^* > 1$.

B Proof of Proposition 2

For m_0 small enough and p_0 close enough to the moneyless steady state $\bar{p} > 1$ the sequence satisfying 3 is close to the moneyless one with $m_t = 0$, which implies that $p_t > 1$, so that $m_t \downarrow 0$ and $p_t \to \bar{p}$. For small enough m_0, the condition (5) is satisfied since $m_t \leq m_0$. The condition 4 becomes

$$H'(m_t)m_t \geq \beta^2 H'(m_{t+2})m_{t+2}$$

which is satisfied since $\beta < 1$, $m_{t+2} < m_t$ and $H'(m)m$ is increasing. Thus, all the conditions for an equilibrium are satisfied.

C Proof of Lemma 4.1

Since A has rank $n-1$ the following

$$\{y = AP \text{ for some } P \in R^n\}$$

is an $n-1$ dimensional subspace of R^n. Moreover, the following

$$\{y : \gamma' y = 0\}$$

is also an $n-1$ dimensional subspace of R^n. We proved in the text that the first subspace is a subset of the second. But since they are both subspaces of R^n of equal dimension, one cannot be a strict subset of the other, and we must have

$$\{y = AP \text{ for some } P \in R^n\} = \left\{y : \gamma' y = 0\right\}.$$

D Proof of Proposition 3

Recall the ODE for nominal prices (12),

$$\pi_t = \Lambda\left(a_t - AP_t\right).$$

Subtracting conflict inflation on both sides gives

$$\pi_t - \mathbf{1}\Pi_t^C = \Lambda\left(a_t - Ap_t\right) - \mathbf{1}\frac{1}{\bar{d}}\gamma' a_t, \tag{20}$$

where $\mathbf{1}$ is a vector of ones, and where we use the fact that the rows of M sum to 1, which implies $AP_t = Ap_t$.

Given any a, define the relative price vector $\mathcal{P}(a)$ as the vector of relative prices p that satisfies

$$\Lambda\left(a - Ap\right) - \frac{1}{\bar{d}}\gamma' a = 0,$$

or, equivalently,

$$Ap = a - d\frac{1}{\bar{d}}\gamma' a, \tag{21}$$

where $d = (d_1, d_2, ..., d_N)'$. Let us prove that this vector exists and is unique. Recall from the proof of Lemma 4.1 that

$$\{y = AP \text{ for some } P \in R^n\} = \left\{y : \gamma' y = 0\right\}.$$

Then, given that

$$\gamma'\left(a - d\frac{1}{\bar{d}}\gamma' a\right) = \gamma' a - \frac{\gamma' d}{\bar{d}}\gamma' a = 0,$$

there exists a P such that

$$AP = a - d\frac{1}{\bar{d}}\gamma' a.$$

Letting $p = P - \mathbf{1}\sum \psi_n P_n$ and using again the fact that $AP = Ap$, we have a vector of relative prices that satisfies (21). Uniqueness follows from A having rank $n-1$.

Given the definition of $\mathcal{P}(a)$, it follows immediately from (20) that if $p = \mathcal{P}(a)$ then $\pi_{nt} = \Pi_t^C$ for all n.

E Proof of Proposition 5

Solving the ODE for ω_t gives

$$\omega_t = \omega_0 e^{-\left(\lambda_p+\lambda_w\right)t} + \left(\lambda_p+\lambda_w\right)\int_0^t e^{-\left(\lambda_p+\lambda_w\right)(t-s)}\tilde{\omega}_s ds,$$

so the boundedness of f_t and g_t implies that ω_t is bounded. The fact that $\dot{\omega}_t = \Pi_t^A$ implies

$$\frac{1}{T}\int_0^T \Pi_t^A dt = \frac{\omega_T - \omega_0}{T}.$$

Since the numerator is bounded this implies $\lim_{T\to\infty}\frac{1}{T}\int_0^T \Pi_t^A dt = 0$. The result follows.

F Derivation of Aspirations with Expected Inflation

Starts with

$$\begin{aligned} f_t &= (\rho+\lambda_p)\hat{\mathbb{E}}_t^p \int_0^\infty e^{-(\rho+\lambda_p)s}(\phi_{t+s} - (w_{t+s} - w_t))\, ds \\ &= (\rho+\lambda_p)\hat{\mathbb{E}}_t^p \int_0^\infty e^{-(\rho+\lambda_p)s}(\phi_{t+s} - \int_0^s \pi_{t+z}^w dz)\, ds \end{aligned}$$

Integrating the second term by parts yields the desired result. Analogous calculations apply for g_t.