Assortative Matching and Household Income Inequality: A Structural Approach *

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Abstract

We develop a model of educational investment, marriage, and household labor market search to quantify how changes in incentives to positively sort in marriage - summarized by changes in marital surplus - contributed to the rise in U.S. household income inequality. While there are always positive incentives to sort by skill and education, the former strengthened relative to the latter over time. These changes incentivized further educational attainment, especially for the high skilled. Unlike findings from previous studies, the resulting increase in like-education-skill marriages contributed to a significant rise in income equality.

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1 Introduction

Household income inequality in the United States has risen sharply since the 1980s, with the Gini coefficient rising from 0.46 to 0.52.[1] Recent work has examined whether an increase in marital sorting along education can explain this rise (Eika, Mogstad and Zafar (2019), Greenwood, Guner, Kocharkov and Santos (2014a), Chade and Eeckhout (2017), Hryshko, Juhn and McCue (2017)). An increase in educational assortative matching, it is argued, may widen the distribution of household income through the presence of more marriages of "likes" in which spouses either both earn high incomes, or both earn low incomes.

Studies that have investigated the quantitative link between sorting and inequality tend to rely on reduced-form counterfactuals, and generally conclude that this link is negligible. These exercises fix the observed marginal distributions of education in marriage, and then impose an empirically measured degree of sorting from a period of interest onto another. This yields a counterfactual distribution of household income in, say, 2020 if people were still marrying as if it were 1980, thus providing a way of gauging how changes in sorting contribute to inequality.

There are, however, some caveats to this approach. First, as argued in Chiappori et al. (2020a), many measures of realized sorting are not "monotonic", and thus fail to reflect true changes in sorting behavior. In particular, these measurements depend on the marginal distributions education in marriage, which have changed dramatically over time, especially for women. Non-monotonic measures therefore conflate changes in sorting with changes in educational attainment. Second, and separately, such an exercise does not capture that educational investments depend on the same forces that drive sorting behavior. By ignoring these coincident equilibrium responses, reduced-form approaches do not capture the full extent to which sorting incentives can affect household income inequality through educational attainment decisions. Finally, reduced-form counterfactuals cannot account for changes in sorting that may take place along unobservable dimensions.

In this paper, we confront these challenges by taking a structural approach. Rather than imposing the *realized* degree of sorting from one time period onto another, we estimate the fundamentals that shape marital surplus - the primitive behind realized sorting patterns. We then trace out how changes in these fundamentals have impacted inequality with the help of our structural model. Unlike studies which have relied on reduced-form counterfactuals, we find that changes in sorting incentives have substantially contributed to the rise in income inequality across households in the United States for the period 1980-2000: more than fifty percent.

One advantage of this approach is that realized sorting patterns are driven by the supermodularity of marital surplus (Becker (1973))). As a consequence, quantifying whether incentives to sort have risen reduces to measuring changes in marital surplus, an object which

[1] We use information from the March Current Population Survey (CPS) and calculate average household inequality for the 1980s decade and the 2000s decade, as described in Section 3.

is both well-identified in our model, and independent of the marginal distributions of education (i.e. monotonic). A second advantage is that we capture the equilibrium response of educational sorting to changes in marital surplus. As we will show, this latter force is especially important when aiming to fully account for sorting's effect on inequality. Finally, our model takes unobserved heterogeneity seriously as a potential driving force behind matching patterns. We find that sorting on unobservables has driven a large portion of the rise in household income inequality in the U.S.

Our structural framework is an equilibrium model of investment in education, marriage, and joint search in the labor market. Risk-averse individuals make forward-looking education choices, taking into account both labor and marriage market returns to education. Ex ante, individuals differ by skill, which is unobservable; ex-post they differ by education, employment, and marital status which are observed choices. Once working, skill affects offer arrival rates and separation rates, along with the average disutility observed from entering the labor force. Education affects effective labor input, and hence income earned.

Once educated, individuals form households in a frictionless, transferrable utility environment, where marital surplus is determined by the economic gains from marriage and preferences for marriage types (Chiappori et al. (2018), Choo and Siow (2006)). A household is thus characterized by its members' education, their underlying skills, and their marital status. The setup is separated into three distinct stages - schooling, marriage, and work - and gives rise to marginal distributions of education for men and women, and a joint distribution of education, skill and income across households.

Whether negative or positive sorting by skill and education emerges depends on forces that push in different directions. The ability for couples to share risk is a force towards negative sorting.[2] Those with high exposure to non-employment risk are willing to give up more in transfers to match with individuals who can offer insurance. On the other hand, there is a complementarity in the joint search process. Those paired with a spouse commanding high labor market returns can choose to be more selective, take on more non-employment risk, and hold out for better-paying jobs. Since the payoff from this is larger for individuals who themselves possess high earnings potential, this is a force towards positive sorting. As far as we know, this is the first paper to emphasize this channel.

Using data on individuals and couples from the March Current Population Survey (CPS) as well as the Basic Monthly CPS files, we estimate the model via the Expectation Maximization (EM) method to match labor market flows and income statistics for singles in both the 1980s and 2000s, separately for men and women and conditional on education and skill. Our procedure allows the shares of unobserved skill types to differ between joint and single households. We then identify the intrahousehold transfers and non-economic returns to marriage from observed marital choice probabilities. The model is able to replicate changes

[2]The idea that marriages are a form of risk sharing in the face of income risk is not new. See Kotlikoff and Spivak (1981), Hess (2004), and Chiappori and Reny (2016). Removing commitment and introducing costly consumption adjustment as in Santos and Weiss (2016) reduces the insurance provided in joint households.

in the educational distribution of marriages over time, marriage propensities, and the rise in household income inequality as measured by both the Gini coefficient and the Theil index. Moreover, we are able to replicate results from the aforementioned reduced-form literature on model-generated data.

Our estimation procedure provides direct measures of marital surplus over time. There is always a positive incentive to sort by both skill and education, but the incentives to sort by skill strengthened over time while those by education weakened. Key labor market parameters — job-finding and separation rates, as well as barriers to entering the labor force — evolved to reduce an individual's exposure to non-employment and the cost of job-loss. This weakened the incentives to match with partners for insurance motives and instead strengthened the desire to match with partners for income maximization purposes. Higher college premiums alone, however, implied that matching with a highly educated partner provided a greater buffer against non-employment as such partners now earned higher income. Consequently, incentives to positively sort by education were slightly lower in the 2000s.

These changing incentives to sort naturally have implications for household income inequality. To examine the impact of sorting, we ask what the distribution of income would have looked like in the 1980s if agents had sorted according to total marital surplus from the 2000s. This exercise isolates how changes in marital surplus affected educational attainment and marriage decisions, thereby isolating its impact on inequality through changes in household formation. We find that increased incentives to positively sort by skill - encoded in the changes in marital surplus - promoted additional educational investment, especially for the high-skilled. This generates more sorting in marriage leading to an 8 percent rise in the Gini Coefficient. We conclude that changes in marital surplus account for more than half of the increase in inequality in the data from 1980-2000. This result stands in contrast to those from the reduced form literature, which generally find a limited role for sorting's contribution towards inequality.

In a final exercise, we hold fixed the marginal distributions of education and skill among those who are married, but allow individuals to remarry based on 2000s surplus. As with the reduced-form counterfactuals, we find only small increases in inequality, but for different reasons. Intrahousehold transfers adjust so as to clear the marriage market, absorbing most of the change. We only see a substantial increase in sorting - and thus household inequality - precisely when educational investments are allowed to respond to changes in incentives to sort. Otherwise, the fixed supply of households cannot support the increase in demand for high skilled and highly educated partners brought about by the changes in sorting incentives.

Related Literature Our work is related to recent papers that study how marital surplus affects sorting behavior. In the more structural realm, Chiappori et al. (2020b) and Dupuy and Weber (2018) back out the implied marital surplus from realized marital sorting patterns; we also use realized sorting patterns for identification, and find that the supermodularity of total surplus along unobserved skill has risen over time. Our work differs in that we capture how general equilibrium effects (educational attainment and marriage propensities) are crucial for understanding the effect of increased sorting on household income inequality. Our partial equilibrium results, however, are in line with Dupuy and Weber (2018). Specifically, our general equilibrium results provide insights as to why partial equilibrium exercises tend to suggest a quantitatively small effect of sorting on inequality by highlighting the role of intrahousehold transfers.

Our work is also related to recent literature that examines how economic fundamentals affect marital patterns. Gayle and Shephard (2019) analyze how distortionary taxes can affect marital sorting and selection into marriage. While we do not consider the effect of taxes per se, our numerical exercises highlight how differential labor market outcomes can affect household composition through changes in education and skill complementarities coming through joint search. Gousse et al. (2017) consider a model of intra-household resource allocation and show how marital decisions are affected by complementarities in spouses' preferences and labor market outcomes. While Gousse et al. (2017) treat the wage process as exogenous and focus on labor supply, our paper highlights the role of joint search where both realized wage income and the decision of whether to enter the labor force are endogenous objects. Neither focus on the rise in household income inequality over time.

Closely related to our work is Greenwood et al. (2016), who study how changes in economic fundamentals, household composition, and inequality in a frictional matching environment affect inequality. Our paper differs from theirs in several important respects. First, our framework contains a single object fully summarizing sorting patterns - marital surplus. As such, it can speak directly to the reduced-form literature which seeks to measure the effect of sorting on inequality in a succinct way. Second, we allow for search in the labor market and non-employment risk as opposed to an exogenous wage processes, and also consider the role of unobservable skill. With regards to the former, entering non-employment represents a precipituous and persistent drop in income, providing a greater role for matching for insurance purposes. With regards to the latter, our model also sheds new light on how sorting by skill can reinforce decisions to acquire education, beyond what would be suggested by a higher college premium.

Finally, recent work has begun exploring the link between sorting in the marriage market and its effect on labor market outcomes of spouses. Calvo et al. (2021) examine how complementarities in spouses' hours worked can promote positive sorting in both the marriage and labor markets, and thus act as a force towards higher income inequality across households. We contribute to this burgeoning literature by combining sorting in the marriage market with joint household search in the labor market (Guler et al. (2012)), and highlight how complementarities in joint search can be a force towards positive sorting in the marriage market.

The rest of the paper proceeds as follows. In Section 2 we describe the model. In Section 3, we describe how we estimate the model using data from the Current Population Survey, and discuss our estimates as well as the fit of the model to the data. Section 4 uses the estimated model to study the effects of changes in the labor market on household income inequality through its effects on educational investment and household formation. Section 5 concludes.

2 Model

Time is continuous. The economy is populated with a measure $\mathcal{N}_f$ of risk-averse females and a measure $\mathcal{N}_m$ of risk averse males who discount the future at rate ρ and derive utility $\nu(c, Q)$ over a private consumption good c and public consumption good Q, where $\nu'(\cdot) > 0$ and $\nu''(\cdot) < 0$ in both arguments. There exists no savings technology; at any point in time, agents spend all their current income on either private consumption goods and/or public consumption goods.[3] Ex-ante, individuals can differ by their gender $s \in \{m, f\}$; we denote by i a particular male identity and by j a particular female identity. Within a gender s, workers also vary by their skill $x_s \in N_x$, which are unobservable to the econometrician, but observable to agents in our model. Individuals can either be high-skilled, $x_s = H$, or low-skilled, $x_s = L$. Ex-post, males and females will also differ in their marital status as well as their educational attainment, both of which are endogenous choices.

There are three stages of life which we will describe separately in the following sections: schooling, marriage, and work. In the first stage, every male i and female j draws a vector of schooling costs, where each element of this vector, $k^i_m(\mathcal{E}_m)$ for males and $k^j_f(\mathcal{E}_s)$ for females, represents the cost for each possible level of educational attainment $\mathcal{E}_s \in N_{\mathcal{E}}$. Schooling costs are drawn from some exogenous distribution $\mathcal{K}_s(k), s \in \{m, f\}$. The benefit of acquiring education is that it scales the worker's effective labor input by $A(\mathcal{E}_s)$ where A is an increasing function of education $\mathcal{E}_s$. At this point, both males and females are unmatched and choose their level of schooling $\mathcal{E}_s \in N_{\mathcal{E}}$ taking as given their realized costs of schooling and the expected gain from educational attainment coming from both the marriage market and the labor market, to be specified below. These schooling choices give rise to endogenous marginal distributions of education among males of skill x_m and females of skill x_f, which we denote by $\Phi(\mathcal{E}_m|x_m)$ and $\Phi(\mathcal{E}_f|x_f)$, respectively.

In the second stage, given their education and skill, males and females match in a frictionless marriage market, giving rise to an endogenous distribution of households across education and marital states. The equilibrium in the marriage market is determined as a function of economic marital surplus $\mathcal{S}(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$, which we derive explicitly as arising from the third stage, the labor market.[4] Because matching will occur in a frictionless marriage market, economic marital surplus $\mathcal{S}(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ affects who marries whom, as well as who marries at all. We begin by describing the final stage - the labor market - mimicking the numerical solution to the model.

[3] See Pilossoph and Wee (2020) for the household search model with savings.

[4] We eventually introduce an additional taste component of surplus a lá Choo and Siow (2006), but only introduce it once we estimate the model in Section 2.2.1.

2.1 Labor Market

Once individuals enter the labor market stage, the distribution of males and females of each skill across education and marriage has been realized and is assumed to be irreversible.[5] Within the labor market stage, individuals are either employed or non-employed. Among the non-employed, we further distinguish between the unemployed and those out of the labor force. A non-employed individual's decision of whether to enter the labor force in any given period is affected by their i.i.d. draw of a flow disutility, ψ, from entering the labor force. This flow disutility from entering the labor force is drawn from a gender and skill-specific distribution, $H(\psi; x_s)$, which has support over $\left[\underline{\psi}, \overline{\psi}\right]$.[6] We characterize the non-employed who choose to incur the disutility at that instant as the *unemployed*, and those who choose not to incur the disutility as *out-of-the-labor-force* (OLF). Employed workers do not incur any disutility as they are already in the labor force. To be clear, ψ is a disutility incurred if one chooses to *transition* from OLF to unemployment. Once in the labor force, this disutility is no longer incurred. One can thus think of ψ as the effort required for the individual to become "labor market ready."

Non-employed individuals of gender $s \in \{m, f\}$ receive home production b_s per unit of effective labor input. We assume that individuals inelastically supply one unit of labor towards production, where production can be either home production or market production. Education augments a worker's productivity, and hence effective labor input is given by $A(\mathcal{E}_s)$. Both unemployed and employed individuals search for job offers. Offers are sampled from an exogenous distribution which is specific to one's gender. Specifically, $F_s(w)$ is the exogenous wage offer distribution with support $w \in [\underline{w}, \overline{w}]$ for an individual of gender $s \in \{m, f\}$. Arrival and separation rates vary by both gender and skill. The unemployed receive job offers at rate $q(x_s)$, while the employed receive job offers at rate $\lambda(x_s)$. Employed singles are exogenously displaced into non-employment at rate $\delta^{sin}(x_s)$ while employed workers who are married exit employment at rate $\delta^{mar}(x_s)$.

A single can be either non-employed or employed. Following Guler et al. (2012), a joint household can be in one of four states: (i) both non-employed, (ii) the male employed at wage w_m, the female non-employed, (iii) the female employed at wage w_f, the male non-employed, or (iv) both the male and female employed at wage w_m and w_f respectively. In what follows, we describe the single and joint household problems. All value functions are expressed *net* of the disutility incurred from entering the labor force.[7]

[5]The irreversibility assumption on marriage is certainly restrictive; given the high levels of divorce and remarriage in the United States, explicitly modeling marriage and divorce would be ideal, but doing so is computationally burdensome. Our current formulation allows the model to be solved in separate blocks; allowing for divorce and remarriage would require that labor and marriage markets be computed simultaneously. One would also need to solve for the marriage market that clears in every instant, rather than a single market clearing before entry to the labor market. We discuss this issue in further detail in Section 3.

[6]For singles, the spouse's education is represented by $\emptyset$.

[7]Online Appendix H shows how we derive the value functions net of the disutility from their gross values. Working with the net values rather than joint values simplifies the state space. For joint households, we have 16 employment state combinations to deal with when working with the net values, as opposed to 33 employment state combinations when we use the gross values.

2.1.1 Singles

The net value of a non-employed single of gender s, education $\mathcal{E}_s$ and skill x_s is given by:

$$\begin{aligned} \rho U^{sin}(\mathcal{E}_s, x_s) &= \max_{c,Q} \ \nu(c,Q) + \int_{\underline{\psi}}^{\overline{\psi}} \max\left\{-\psi + \Xi^{sin}(\mathcal{E}_s, x_s), 0\right\} dH(\psi; x_s) \\ \text{s.t.} \quad c + p_Q Q &= A(\mathcal{E}_s) b_s \end{aligned} \tag{1}$$

where

$$\Xi^{sin}(\mathcal{E}_s, x_s) = q(x_s) \int_{\underline{w}}^{\overline{w}} \max\left[T^{sin}(w, \mathcal{E}_s, x_s) - U^{sin}(\mathcal{E}_s, x_s), 0\right] dF_s(w)$$

$\Xi^{sin}(\mathcal{E}_s, x_s)$ represents the expected change of value from participating in the labor market by searching for a job. $A(\mathcal{E}_s)b_s$ is the value of home production an individual with education $\mathcal{E}_s$ produces and p_Q is the price of the public consumption good relative to the price of the private good which is normalized to 1. From Equation 1, we define $\psi_s^{sin}(\mathcal{E}_s, x_s)$ as the highest disutility an individual of gender s is willing to incur and is defined by the following indifference condition:

$$\psi_s^{sin}(\mathcal{E}_s, x_s) = q(x_s) \int_{\underline{w}}^{\overline{w}} \max\left[T^{sin}(w, \mathcal{E}_s, x_s) - U^{sin}(\mathcal{E}_s, x_s), 0\right] dF_s(w) \tag{2}$$

For any $\psi > \psi_s^{sin}(\mathcal{E}_s, x_s)$, the non-employed individual strictly prefers to remain out of the labor force and chooses not to incur the disutility of transitioning into unemployment.

The net value of an employed single with wage w, gender s, education $\mathcal{E}_s$ and skill x_s is:

$$\begin{aligned} \rho T^{sin}(w, \mathcal{E}_s, x_s) &= \max_{c,Q} \ \nu(c,Q) + \delta^{sin}(x_s)\left[U^{sin}(\mathcal{E}_s, x_s) - T^{sin}(w, \mathcal{E}_s, x_s)\right] \\ &\quad + \lambda(x_s) \int_{\underline{w}}^{\overline{w}} \max\left\{T^{sin}(y, \mathcal{E}_s, x_s) - T^{sin}(w, \mathcal{E}_s, x_s), 0\right\} dF_s(y) \\ \text{s.t.} \quad c + p_Q Q &= A(\mathcal{E}_s) w \end{aligned} \tag{3}$$

where at rate $\delta^{sin}(x_s)$ the employed worker is displaced into non-employment. At rate $\lambda(x_s)$, he/she receives a job offer of wage y drawn from distribution $F_s(y)$. If that offer is accepted, he/she gets a change of value $T^{sin}(y, \mathcal{E}_s, x_s) - T^{sin}(w, \mathcal{E}_s, x_s)$.

2.1.2 Joint Households

Dual Non-employed: Consider a household of type $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$. We will denote $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) = (\mathbf{E}, \mathbf{x})$ for simplicity. Net of the flow disutility, the value of being dual non-employed for this household with Pareto weight $\alpha(\mathbf{E},\mathbf{x})$ can be expressed as:

$$\begin{aligned}
\rho U(\mathbf{E},\mathbf{x}) &= \max_{c_m,c_f,Q} \; [1-\alpha(\mathbf{E},\mathbf{x})]\,\nu(c_m,Q)+\alpha(\mathbf{E},\mathbf{x})\nu(c_f,Q) \qquad (4)\\
&+ \int_{\underline{\psi}}^{\overline{\psi}} \max\{-\psi_m+\Xi_m^{mar}(\mathbf{E},\mathbf{x}),0\}dH\left(\psi_m;x_m\right)\\
&+ \int_{\underline{\psi}}^{\overline{\psi}} \max\{-\psi_f+\Xi_f^{mar}(\mathbf{E},\mathbf{x}),0\}dH\left(\psi_f;x_f\right)\\
\text{subject to} \quad & c_m+c_f+p_Q Q = A(\mathcal{E}_m)b_m+A(\mathcal{E}_f)b_f
\end{aligned}$$

where

$$\Xi_s^{mar}(\mathbf{E},\mathbf{x}) = q(x_s)\int_{\underline{w}}^{\overline{w}} \max\left[\Omega_s(y,\mathbf{E},\mathbf{x})-U\left(\mathbf{E},\mathbf{x}\right),0\right]dF_s\left(y\right)$$

Joint households pool their income and choose their consumption of private and public goods given a Pareto bargaining weight $\alpha(\mathbf{E},\mathbf{x})$ for the female and $1-\alpha(\mathbf{E},\mathbf{x})$ for the male. We follow Chiappori et al. (2018) and assume that the Pareto weight is determined endogenously before marriage, with full commitment after marriages are formed.

From the above, each party receives income from home production, $A(\mathcal{E}_s)b_s$ for $s\in\{m,f\}$, and - given Pareto weights $\alpha\left(\mathbf{E},\mathbf{x}\right)$ - enjoys flow utility from optimally chosen current consumption. Given realized disutility ψ_s and the expected returns from searching, the couple decides whether each spouse enters the labor force. If a spouse does enter the labor force, they receive a job offer at rate $q\left(x_s\right)$. If they accept, the household receives change of value $\Omega_s(y,\mathbf{E},\mathbf{x})-U\left(\mathbf{E},\mathbf{x}\right)$. While income is split according to the Pareto weights, the disutility incurred by any household member from entering the labor force is shared by the household.[8]

Worker-Searcher Household The net value of a worker-searcher couple with education-skill combination $(\mathbf{E},\mathbf{x})$ where only the husband is employed at wage w_m is defined as:

$$\begin{aligned}
\rho\Omega_m\left(w_m,\mathbf{E},\mathbf{x}\right) &= \max_{c_m,c_f,Q} \; \left(1-\alpha(\mathbf{E},\mathbf{x})\right)\nu\left(c_m,Q\right)+\alpha(\mathbf{E},\mathbf{x})\nu\left(c_f,Q\right) \qquad (5)\\
&+\int_{\underline{\psi}}^{\overline{\psi}} \max\Big\{-\psi+\Xi(w_m,\mathbf{E},\mathbf{x}),0\Big\}dH(\psi;x_f)\\
&+\lambda(x_m)\int_{\underline{w}}^{\overline{w}} \max\left[\Omega_m\left(y,\mathbf{E},\mathbf{x}\right)-\Omega_m\left(w_m,\mathbf{E},\mathbf{x}\right),0\right]dF_m(y)\\
&+\delta^{mar}(x_m)\left[U\left(\mathbf{E},\mathbf{x}\right)-\Omega_m\left(w_m,\mathbf{E},\mathbf{x}\right)\right]\\
\text{subject to} \quad & c_m+c_f+p_Q Q = A(\mathcal{E}_m)w_m+A(\mathcal{E}_f)b_f
\end{aligned}$$

where $\Xi(w_m,\mathbf{E},\mathbf{x}) = q(x_f)\int_{\underline{w}}^{\overline{w}}\max\left[T(w_m,y,\mathbf{E},\mathbf{x})-\Omega_m(w_m,\mathbf{E},\mathbf{x}),0\right]dF_f(y)$. When the male spouse is working, only the non-employed female faces a disutility from entering the labor

[8]Here, the disutility is a public disutility, and represents the loss from having an absent member in the household when he/she transitions into the labor force. An alternative is to consider a monetary cost of entering the labor market, but, as discussed in Online Appendix I, we chose this route for technical reasons.

force. If she chooses to enter the labor force, she receives a job offer at rate $q(x_f)$ and upon acceptance, the household transitions to a dual employed household. Since the employed male is already in the labor force, he faces no disutility from entering. Instead, the employed male member receives a new job offer at rate $\lambda(x_m)$ and chooses whether to accept that offer.[9] Finally, at rate $\delta^{mar}(x_m)$, the employed male spouse exogenously loses his job, and the joint household enters into dual non-employment. An analogous expression exists for the female-headed worker-searcher household.

Dual Employed: Finally, the net value of a dual-employed couple with wage pair $\mathbf{w} = (w_m, w_f)$ and of education-skill combination $(\mathbf{E}, \mathbf{x})$ is given by:

$$\begin{aligned} \rho T(\mathbf{w}, \mathbf{E}, \mathbf{x}) &= \max_{c_m, c_f, Q} \ [1 - \alpha(\mathbf{E}, \mathbf{x})] \, \nu(c_m, Q) + \alpha(\mathbf{E}, \mathbf{x}) \nu(c_f, Q) \\ &+ \ \delta^{mar}(x_f) \left[\Omega_m(w_m, \mathbf{E}, \mathbf{x}) - T(\mathbf{w}, \mathbf{E}, \mathbf{x})\right] \\ &+ \ \delta^{mar}(x_m) \left[\Omega_f(w_f, \mathbf{E}, \mathbf{x}) - T(\mathbf{w}, \mathbf{E}, \mathbf{x})\right] \\ &+ \ \lambda(x_m) \int_{\underline{w}}^{\overline{w}} \max\left\{T(y, w_f, \mathbf{E}, \mathbf{x}) - T(\mathbf{w}, \mathbf{E}, \mathbf{x}), 0\right\} dF_m(y) \\ &+ \ \lambda(x_f) \int_{\underline{w}}^{\overline{w}} \max\left\{T(w_m, y, \mathbf{E}, \mathbf{x}) - T(\mathbf{w}, \mathbf{E}, \mathbf{x}), 0\right\} dF_f(y) \\ \text{s.t.} \qquad & c_m + c_f + p_Q Q = A(\mathcal{E}_m) w_m + A(\mathcal{E}_f) w_f \end{aligned} \tag{6}$$

At rate $\lambda(x_s)$, the spouse of gender s receives a job offer while at rate $\delta^{mar}(x_s)$ they are displaced into non-employment.

2.1.3 Optimal Consumption and Search Behavior

Optimal search behavior implies a reservation wage policy for each non-employed spouse in dual-non-employed households, $\{w^*_{U,m}(\mathbf{E}, \mathbf{x}), w^*_{U,f}(\mathbf{E}, \mathbf{x})\}$; for the non-employed female in the male-headed worker-searcher household, $w^*_{\Omega_m,u}(w_m, \mathbf{E}, \mathbf{x})$, and for the non-employed male in the female-headed worker-searcher household, $w^*_{\Omega_f,u}(w_f, \mathbf{E}, \mathbf{x})$, each defined implicitly using:

$$\begin{aligned} \Omega_f\left(w^*_{U,f}(\mathbf{E}, \mathbf{x}), \mathbf{E}, \mathbf{x}\right) - U\left(\mathbf{E}, \mathbf{x}\right) &= 0 \quad \forall \mathbf{x} \in N_x \times N_x \ , \forall \mathbf{E} \in N_{\mathcal{E}} \times N_{\mathcal{E}} \\ \Omega_m\left(w^*_{U,m}(\mathbf{E}, \mathbf{x}), \mathbf{E}, \mathbf{x}\right) - U(\mathbf{E}, \mathbf{x}) &= 0 \quad \forall \mathbf{x} \in N_x \times N_x \ , \forall \mathbf{E} \in N_{\mathcal{E}} \times N_{\mathcal{E}} \\ \Omega_m\left(w_m, \mathbf{E}, \mathbf{x}\right) - T\left(w_m, w^*_{\Omega_m,u}\left(w_m, \mathbf{E}, \mathbf{x}\right), \mathbf{E}, \mathbf{x}\right) &= 0 \quad \forall \mathbf{x} \in N_x \times N_x \ , \forall \mathbf{E} \in N_{\mathcal{E}} \times N_{\mathcal{E}} \\ \Omega_f\left(w_f, \mathbf{E}, \mathbf{x}\right) - T\left(w^*_{\Omega_f,u}\left(w_f, \mathbf{E}, \mathbf{x}\right), w_f, \mathbf{E}, \mathbf{x}\right) &= 0 \quad \forall \mathbf{x} \in N_x \times N_x, \ , \forall \mathbf{E} \in N_{\mathcal{E}} \times N_{\mathcal{E}} \end{aligned}$$

All employed workers accept job offers that pay higher than their current wage.

[9] We abstract from endogenous quits here. The disutility of transitioning into the labor force in this environment makes quits less likely to occur since employed individuals would have to incur the disutility in non-employment to actively search for work.

Optimal participation We can also characterize a non-employed spouse's decision of when to enter the labor force. Denote by $\psi_s^c(\cdot)$ the highest disutility that a spouse of gender s in a particular joint household is willing to incur to enter the labor force. For the dual non-employed household, the marginal individual who is indifferent between entering and remaining non-employed incurs a disutility which satisfies:

$$\psi_s^c(\mathbf{E}, \mathbf{x}) = q(x_s) \int_{\underline{w}}^{\overline{w}} \max\left[\Omega_s(y, \mathbf{E}, \mathbf{x}) - U(\mathbf{E}, \mathbf{x}), 0\right] dF_s(y) \tag{7}$$

and for the worker-searcher household, the highest disutility the non-employed spouse is willing to incur before moving to a dual employed household is given by:

$$\psi_s^c(w, \mathbf{E}, \mathbf{x}) = q(x_s) \int_{\underline{w}}^{\overline{w}} \max\left[T(w, y, \mathbf{E}, \mathbf{x}) - \Omega_s(w, \mathbf{E}, \mathbf{x}), 0\right] dF_s(y) \tag{8}$$

Unlike the case of the dual non-employed household, Equation (8) shows that the largest disutility the joint worker-searcher household is willing to incur also depends on the employed spouse's wage and not just on the pair's education-skill combination $(\mathbf{E}, \mathbf{x})$.

Optimal Consumption For joint households, the sharing rule $\alpha(\mathbf{E}, \mathbf{x})$ will determine the split of joint income, $\mathcal{I}$, into consumption of the public good Q and consumption of the private good for each spouse following:

$$\max_{c_m, c_f, Q} \left[1 - \alpha(\mathbf{E}, \mathbf{x})\right] \nu(c_m, Q) + \alpha(\mathbf{E}, \mathbf{x}) \nu(c_f, Q)$$

$$\text{s.t.} \quad c_m + c_f + p_Q Q = \mathcal{I}$$

Optimal private consumption therefore must satisfy:

$$\frac{\nu_c(c_m, Q)}{\nu_c(c_f, Q)} = \frac{\alpha(\mathbf{E}, \mathbf{x})}{1 - \alpha(\mathbf{E}, \mathbf{x})}$$

and the trade-off between the male's private consumption and public consumption is:

$$\frac{\nu_c(c_m, Q)}{\nu_Q(c_m Q)} = \frac{1}{1 - \alpha(\mathbf{E}, \mathbf{x})} \frac{1}{p_Q}$$

Proposition 1 (Sharing Rule)**.** *Assuming preferences are logarithmic, search behavior is independent of the sharing rule* $\alpha(\mathbf{x})$*.*

Proof. See Appendix A □

This assumption on preferences allows us to separate the decision of whom to marry in the marriage market from the search decisions in the labor market. That is, if search behavior is independent of the Pareto weight, then the labor market can be studied without knowledge of the Pareto weight ex-ante, but the returns in the labor market will dictate, in equilibrium, the Pareto weights necessary to clear the marriage market.[10]

For this reason, we assume preferences over private and public consumption are logarithmic. Optimal c_m, c_f and Q satisfy:

$$c_m^*(\mathbf{E},\mathbf{x}) = [1-\alpha(\mathbf{E},\mathbf{x})]\cdot\left(\frac{\mathcal{I}}{2}\right) \quad , c_f^*(\mathbf{E},\mathbf{x}) = \alpha(\mathbf{E},\mathbf{x})\cdot\left(\frac{\mathcal{I}}{2}\right), \quad Q^*(\mathbf{E},\mathbf{x}) = \left(\frac{\mathcal{I}}{2p_Q}\right) \tag{9}$$

where $\mathcal{I}$ stands in for the income of the household in that period.

2.1.4 Steady State Distributions of Household Income

At the end of each period, for each couple type $(\mathbf{E},\mathbf{x})$, there are four possible states an individual spouse can be in. First, they can be in a dual non-employed household. Second, they can be a female-headed worker-searcher household or a male-headed worker-searcher household. Finally, they could be in a dual employed household. For singles, they can either be employed or non-employed. In Appendix B, we describe the full transition of individuals into and out of these states, and show how to arrive at the equilibrium distribution of couples across these states, which gives rise to an equilibrium distribution of household income, which we denote by $G(\mathcal{I};\mathbf{E},\mathbf{x})$. We will denote by $G(\mathcal{I};\mathcal{E}_m,\emptyset,x_m,\emptyset)$ and $G(\mathcal{I};\emptyset,\mathcal{E}_f,\emptyset,x_f)$ the steady state distribution of income for single males and females, respectively.

2.2 Marriage Market

Having described the labor market above, we now describe the individual's problem when entering the marriage market. Assuming all agents enter the labor market as non-employed and out of the labor force, if an individual of sex s, education $\mathcal{E}_s$ and skill x_s chooses to remain single, they enter the labor market expecting to earn a lifetime value of $U^{sin}(\mathcal{E}_s, x_s)$. If instead they choose marriage, they earn a certain share of the value of dual non-employment, $U(\mathbf{E},\mathbf{x})$, which itself depends on their spouse's education and skill level.

We express the value of marriage of education-skill combination $(\mathbf{E},\mathbf{x})$ to a male and female as $V_m^U(\mathbf{E},\mathbf{x};\alpha(\mathbf{E},\mathbf{x}))$ and $V_f^U(\mathbf{E},\mathbf{x};\alpha(\mathbf{E},\mathbf{x}))$ respectively, and the value of being single for males and females at every education level as $V_m^U(\mathcal{E}_m,\emptyset,x_m,\emptyset)$ and $V_f^U(\emptyset,\mathcal{E}_f,\emptyset,x_f)$. The value to the male of being in a household of education-skill combination $(\mathbf{E},\mathbf{x})$ is given by:[11]

$$\rho V_m^U(\mathbf{E},\mathbf{x};\alpha(\mathbf{E},\mathbf{x})) = \log[1-\alpha(\mathbf{E},\mathbf{x})] + \rho\widetilde{U}(\mathbf{E},\mathbf{x}) \tag{10}$$

[10]The preferences we impose here are more restrictive than the ISHARA class of Mazzocco (2007). The reason is that with ISHARA preferences, the inclusion of the disutility from entering the labor market generates decision rules which would depend on the Pareto weights. Online Appendix I shows that if the disutility cost were a monetary cost, then ISHARA preferences would suffice for the result. We opted for the non-monetary cost for two reasons. First, using the monetary cost would require working with gross values instead of net values (since one would need to keep track of realized costs in the budget constraint) which expands the number of states a household can be in and complicates the solution of the model. Second, our formulation allows for a simple probability representation of participation.

[11]Appendix A formally shows that $\widetilde{U}(\mathbf{E},\mathbf{x})$ is independent of $\alpha(\mathbf{E},\mathbf{x})$

where

$$\widetilde{U}(\mathbf{E},\mathbf{x}) = U(\mathbf{E},\mathbf{x}) - \frac{\mathcal{Z}(\mathbf{E},\mathbf{x})}{\rho}$$

and $\mathcal{Z}(\mathbf{E},\mathbf{x}) = \alpha(\mathbf{E},\mathbf{x}) \log \alpha(\mathbf{E},\mathbf{x}) + [1-\alpha(\mathbf{E},\mathbf{x})] \log[1-\alpha(\mathbf{E},\mathbf{x})]$. Similarly, for females we have:

$$\rho V_f^U(\mathbf{E},\mathbf{x};\alpha(\mathbf{E},\mathbf{x})) = \log \alpha(\mathbf{E},\mathbf{x}) + \rho\widetilde{U}(\mathbf{E},\mathbf{x}) \tag{11}$$

Since our decision rules are independent of the Pareto weights, $\alpha(\mathbf{E},\mathbf{x})$, we apply the result from Schulhofer-Wohl (2006), who shows that expected utility has the transferable utility (TU) property for some transformation of preferences. Thus, we can use an alternative cardinalization of these utilities which delivers TU. Specifically, we take an exponential transformation of these preferences and add them together, so that we arrive at:

$$\exp(\rho V_m^U(\mathbf{E},\mathbf{x};\alpha[\mathbf{E},\mathbf{x}])) + \exp(\rho V_f^U(\mathbf{E},\mathbf{x};\alpha[\mathbf{E},\mathbf{x}])) \;=\; \exp(\rho\widetilde{U}(\mathbf{E},\mathbf{x})) \tag{12}$$

Since the marriage market is frictionless and preferences feature transferable utility, the results from matching with transfers can be applied, where economic marital surplus for a household of type $(\mathbf{E},\mathbf{x})$ is defined through ex-post labor market outcomes as:

$$\begin{aligned} \mathcal{S}(\mathbf{E},\mathbf{x}) \;=\;& \exp[\rho V_m^U(\mathbf{E},\mathbf{x};\alpha(\mathbf{E},\mathbf{x}))] - \exp[\rho V_m^U(\mathcal{E}_m,\emptyset,x_m,\emptyset)] \\ &+ \exp[\rho V_f^U(\mathbf{E},\mathbf{x};\alpha(\mathbf{E},\mathbf{x}))] - \exp[\rho V_f^U(\emptyset,\mathcal{E}_f,\emptyset,x_f)] \end{aligned} \tag{13}$$

Because $\mathcal{S}(\mathbf{E},\mathbf{x})$ is a multidimensional object, sorting can occur either along education or along skill or both. In our empirical implementation, we assume two education types, $\mathcal{E} \in \{Col, HS\}$ representing college and high school; and two skill types $x \in \{H, L\}$, representing high and low-skilled individuals. Holding fixed a skill pair $\mathbf{x}$, we define $D_{\mathcal{E}}(\mathcal{S} \mid \mathbf{x})$ as the degree of supermodularity along education:

$$\begin{aligned} D_{\mathcal{E}}(\mathcal{S} \mid \mathbf{x}) \;=\;& \mathcal{S}(Col, Col \mid \mathbf{x}) - \mathcal{S}(Col, HS \mid \mathbf{x}) \\ &+ \mathcal{S}(HS, HS \mid \mathbf{x}) - \mathcal{S}(HS, Col \mid \mathbf{x}) \end{aligned} \tag{14}$$

Holding fixed an education pair $\mathbf{E}$, we define $D_x(\mathcal{S} \mid \mathbf{E})$ as the degree of supermodularity along skill:

$$D_x(\mathcal{S} \mid \mathbf{E}) = \mathcal{S}(H,H \mid \mathbf{E}) - \mathcal{S}(H,L \mid \mathbf{E}) + \mathcal{S}(L,L \mid \mathbf{E}) - \mathcal{S}(L,H \mid \mathbf{E}) \tag{15}$$

A positive value of $D_{\mathcal{E}}(\mathcal{S}|\mathbf{x})$ indicates that there is positive sorting along education for skill pair $\mathbf{x}$ while a positive value of $D_x(\mathcal{S}|\mathbf{E})$ indicates positive sorting along skill for a specific education pair $\mathbf{E}$.[12]

[12]In Appendix G, we discuss further measures used to uncover the complementarities across skill and education. Specifically, Equation 35 in Appendix G allows us to examine whether an individual of skill x_s

What determines whether there is positive or negative marital sorting in our model? While we cannot solve in closed-form for analytical expressions for $\mathcal{S}(\mathbf{E}, \mathbf{x})$, we can provide some intuition for what determines the nature of sorting in this environment (absent non-economic components to surplus, which we introduce next). First, since joint households can pool their income, risk sharing is a force towards negative sorting; individuals with low labor market returns are willing to transfer a larger share of surplus to high earning spouses in return for insurance against income risk. Changes in primitives which result in lower non-employment risk for married individuals increase the degree of supermodularity along skill as non-employment risk varies by skill in our model. That is, both the likelihood of entering non-employment and the likelihood of exiting non-employment are affected by offer arrival rates, separation rates and the disutilities incurred from entering the labor force, all of which are functions of skill x_s. On the other hand, joint search is a force towards positive sorting, as having a higher earning spouse allows a couple to move up the job ladder more efficiently; each spouse can take on more non-employment risk in order to wait for better offers if their spouse earns more. Therefore, lower participation disutility will raise the degree of supermodularity along skill, since increased participation rates by one spouse allows joint households to take further advantage of joint search. In contrast, higher returns to education in the form of higher effective labor input, $A(\mathcal{E}_s)$, raise the degree of supermodularity along education. Joint households maximize their lifetime income and climb the job ladder more efficiently when paired with a highly educated spouse with high labor market returns.

2.2.1 Non-Economic Marital Surplus

Without any further assumptions, conditional on some marginal distributions of education and skill, there will either be perfect positive or perfect negative sorting depending on the supermodularity of economic surplus, $\mathcal{S}(\mathbf{E}, \mathbf{x})$. Moreover, there may be no singles in certain education-skill categories, as marriage provides a higher value than singlehood due to risk sharing. To capture issues that affect marriage patterns not captured by economic surplus $\mathcal{S}(\mathbf{E}, \mathbf{x})$ - that are related for example to societal norms, search frictions, etc. - we introduce random utility shocks á la Choo and Siow (2006). We assume that in period zero - when agents draw their educational attainment costs - they additionally draw a vector of marital preferences, $\beta_m^i(\mathbf{E}, \mathbf{x})$ from an exogenous distribution $\mathbf{B}_m(\beta_m; \mathbf{E}, \mathbf{x})$ and $\beta_f^j(\mathbf{E}, \mathbf{x})$ from $\mathbf{B}_f(\beta_f; \mathbf{E}, \mathbf{x})$ one for each possible state (singlehood, and marriage with each possible spouse type). Stability in this environment implies that for any particular male i with education-skill $(\mathcal{E}_m, x_m)$, the optimal marriage choice must satisfy:

$$\bar{V}_m(\mathcal{E}_m, x_m) = \underset{y \in \{N_x \cup \emptyset\}, z \in \{N_{\mathcal{E}} \cup \emptyset\}}{\operatorname{argmax}} \exp\left[\rho V_m^U(\mathcal{E}_m, z, x_m, y; \alpha(\mathcal{E}_m, z, x_m, y))\right] + \beta_m^i(\mathcal{E}_m, z, x_m, y)$$

and education $\mathcal{E}_s$ garners a larger gain from matching with a highly-skilled spouse or from matching with a highly-educated spouse. Equation 36 allows us to examine if this relative gain is increasing in one's own skill type.

while the optimal marriage choice for a particular female j must satisfy:

$$\bar{V}_f(\mathcal{E}_f, x_f) = \underset{y\in\{N_x\cup\emptyset\}, z\in\{N_\mathcal{E}\cup\emptyset\}}{\operatorname{argmax}} \exp\left[\rho V_f^U(z, \mathcal{E}_f, y, x_f; \alpha(z, \mathcal{E}_f, y, x_f))\right] + \beta_f^j(z, \mathcal{E}_f, y, x_f)$$

The next proposition establishes that the *level* of supermodularity defined in Equations (14) and (15) affects the *degree* of sorting along education and skill respectively. Sorting along either dimension is measured as the share of like marriages relative to what random matching would imply.

Proposition 2 (Sorting). *Assume (i) fixed marginal distributions of education-skill in marriage for both sexes and (ii) fixed distributions of marital preferences,* $\mathbf{B}_m(\beta_m; \mathbf{E}, \mathbf{x})$ *and* $\mathbf{B}_f(\beta_f; \mathbf{E}, \mathbf{x})$*. Holding fixed a skill pair, if the level of supermodularity in economic surplus rises from some* $D_\mathcal{E}(\mathcal{S}|\mathbf{x})$ *to* $D'_\mathcal{E}(\mathcal{S}|\mathbf{x}) > D_\mathcal{E}(\mathcal{S}|\mathbf{x})$*, matching becomes more assortative in the sense that there are relatively more marriages between like education levels relative to random matching under* $D'_\mathcal{E}(\mathcal{S}|\mathbf{x})$ *than* $D_\mathcal{E}(\mathcal{S}|\mathbf{x})$*. Similarly, holding fixed an education pair if* $D'_x(\mathcal{S}|\mathbf{E}) > D_x(\mathcal{S}|\mathbf{E})$*, more positive sorting along skill arises under* $D'_x(\mathcal{S}|\mathbf{E})$ *than* $D_x(\mathcal{S}|\mathbf{E})$*.*

Proof. See Appendix C □

2.3 Schooling Choices

Having described the marriage and labor markets, we can now turn to the first choice, which is the educational attainment choice. Given their realized costs of schooling, individuals optimally choose their education level, taking into account how education $\mathcal{E}_s$ affects both their marriage and labor market returns. The optimal school choice for males of a particular skill x_s is given by:

$$\mathcal{E}_m^*|x_m = \underset{\mathcal{E}_m\in N_\mathcal{E}}{\operatorname{argmax}} \bar{V}_m(\mathcal{E}_m, x_m) - k_m^i(\mathcal{E}_m) \tag{16}$$

where $k_m^i(\cdot)$ is the cost of various levels of schooling for males and $\bar{V}_m(\mathcal{E}_m, x_m)$ represents the optimal marriage choice conditional on choosing education level $\mathcal{E}_m$ and with skill x_m. A similar problem exists for females:

$$\mathcal{E}_f^*|x_f = \underset{\mathcal{E}_f\in N_\mathcal{E}}{\operatorname{argmax}} \bar{V}_f(\mathcal{E}_f, x_f) - k_f^j(\mathcal{E}_f) \tag{17}$$

Because the payoffs to marriage and schooling depend on the endogenous choices of the opposite sex, multiple equilibria may exist. Following Chiappori et al. (2018) who exploit a result in Noldeke and Samuelson (2015), we solve the "auxiliary" game in which educational attainment and marriage decisions are chosen simultaneously. The solution to this problem delivers an outcome that is also an equilibrium for the original formulation.[13] Therefore, to simplify the problem, when we estimate the model in Section 3 we assume instead that individuals draw a composite preference shock, $\chi_m^i(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ or $\chi_f^j(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$, that

[13] This does not rule out the multiplicity problem in general, but is a way to select a particular equilibrium.

affects their decision of whether to attain schooling and form a joint household with a partner with education-skill $(\mathcal{E}_{s'}, x_{s'})$ for $s' \neq s$. In other words:

$$\chi_m^i(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) = \beta_m^i(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) - k_m^i(\mathcal{E}_m).$$

for males and

$$\chi_f^j(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) = \beta_f^j(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) - k_f^j(\mathcal{E}_f).$$

for females. For $(\mathbf{E}, \mathbf{x}) = (\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$, if $\chi_m^i(\mathbf{E}, \mathbf{x})$ is drawn from a Type 1 Extreme Value distribution with mean $\overline{\chi}_m(\mathbf{E}, \mathbf{x})$, the probability that a male of skill x_m chooses education $\mathcal{E}_m$ and marries a female of type $(\mathcal{E}_f, x_f)$ takes the familiar discrete choice probability form:

$$\pi_m(\mathcal{E}_m, \mathcal{E}_f, x_f | x_m, \alpha(\mathbf{E}, \mathbf{x})) = \frac{\exp\left[\exp\left(\rho V_m^U\left[\mathbf{E}, \mathbf{x} | \alpha(\mathbf{E}, \mathbf{x})\right]\right) + \bar{\chi}_m(\mathbf{E}, \mathbf{x})\right]}{\varphi} \tag{18}$$

where

$$\varphi = \sum_{y \in N_{\mathcal{E}}} \sum_{z \in \{N_{\mathcal{E}} \cup \emptyset\}} \sum_{j \in \{N_x \cup \emptyset\}} \exp\left[\exp\left(\rho V_m^U\left[y, z, x_m, j\right]\right) + \bar{\chi}_m(y, z, x_m, j)\right]$$

An analogous expression exists for females. Equation (18) illustrates that the choice probabilities for an individual depend on the endogenous returns to marriage for each spouse, $\exp(\rho V_s^U[\mathbf{E}, \mathbf{x}])$, as well as the composite preference shock $\bar{\chi}_s(\mathbf{E}, \mathbf{x})$.

3 Estimation

3.1 Data

In the following analysis we use data from the Current Population Survey (CPS) March Supplement and Basic Monthly files. The March CPS data contain information on hourly wage income, employment rates, non-employment rates and the share of the civilian population who are out of the labor force (OLF shares). The matched monthly CPS files provide us with data on monthly labor force flows.[14] As our baseline, we estimate the model twice for two separate time periods, and calculate relevant identification moments in the March Supplement and Basic Monthly CPS files covering the periods 1981-1989 and 2000-2007 separately.[15] We refer to these two samples as our 1980s and 2000s samples, respectively. Appendix D.2 constructs another two samples based instead on cohorts; we repeat all of our analysis on the cohort-based samples, and discuss the results as they relate to our baseline sample results in Appendix L.

Our ultimate goal is to run model-based counterfactuals which allow certain primitives to change across these two samples, while others are held fixed. The time-invariant parameters include the discount rate ρ, which we set to 0.004 to match an annual interest rate of 5%

[14] We do not currently use information on hours, so the assumption in the model is that all individuals work the same number of hours, which are normalized to 1.

[15] Details regarding the data construction can be found in Appendix D.1.

assuming a monthly calibration. We further restrict the price of the public good to be equal to that of the private good, i.e. $p_Q = 1$. We estimate the model in two separate blocks, the labor market block and the marriage-education market block, as the two portions of the framework can be analyzed separately following Proposition 1. We begin with the labor market block.

3.2 Labor Market Estimation

3.2.1 Singles Labor Market Returns

We first estimate the labor market parameters for singles, by gender, skill, and educational attainment separately, and then impose the identifying assumption that the parameters governing the labor market faced by couples are the same as their single counterparts, with the exception of separation rates, $\delta^{mar}(x_s)$. We allow $\delta^{mar}(x_s)$ to vary from $\delta^{sin}(x_s)$ as the non-employment rates and OLF-shares of individuals in joint households are significantly different from that of their single counterparts.

We consider two education categories for each sex s, *less than or equal to a high school diploma* (HS) and *some college or more* (Col), implying $\mathcal{E}_s \in \{HS, Col\}$. We consider two levels of skill, $x_s \in \{H, L\}$ where $x_s = H$ corresponds to high-skilled and $x_s = L$ corresponds to low-skilled. As outlined in Section 2, education only affects an individual's effective labor input, while all other labor market parameters, besides the offer distribution and home production values, vary by skill x. Specifically, we normalize $A(HS) = 1$ and allow $A(Col) \geq 1$. We set $A(\mathcal{E}_s)$ for $\mathcal{E}_s = Col$ equal to the gender-specific college premium - defined as the ratio of average wages of college individuals to the average wages of high-school individuals - observed in the time period of interest. Having extracted $A(\mathcal{E}_s)$, we then strip out this component from college wages. Next, we assume individuals draw wages from a gender-specific offer distribution $F_s(w)$, which we assume to be log-normal with mean μ_s and variance σ_s^2.[16] We set the reservation wage of the low-skilled individual in gender s to the minimum observed single wage across high school and college education following the identification arguments in Flinn and Heckman (1982).

Arrival and separation rates vary by gender and skill. Skill x_s is unobservable, but in the data, we observe outcomes for individuals across education levels. To recover the underlying skill shares in each gender-education group, we use data on singles' employment outcomes and wages, and apply the Expectations Maximization (EM) algorithm (Dempster et al. (1977)). We first estimate - given our guess of the share of individuals with education $\mathcal{E}_s$ who possess skill x_s, $p(x_s|\mathcal{E}_s)$ - the labor market parameters associated with that x_s, i.e. $\{q(x_s), \delta(x_s)\}$, as well as the offer distribution parameters that vary by gender only, $\{\mu_s, \sigma_s\}$. Given these parameters, we then update our guess of $p(x_s|\mathcal{E}_s)$. We repeat this process until we find the estimated parameters and shares that best explain the data for singles. Details

[16] Once we demean college wage earnings by the college premium, the wage distributions of college and high school are almost identical. As such, we assume that offer distribution parameters only vary by gender.

of the estimation process can be found in Appendix E.1.

Within our EM algorithm, we jointly solve for the parameters $\{b_s, \lambda(x_s), \eta(x_s)\}$ by using the following conditions. We recover the home production value for each gender b_s by using the following indifference condition for low-skilled individuals:

$$\log b_s = \log w_s^{sin*}(\mathcal{E}_s, x_s) + \frac{1}{2}\left[\lambda(x_s)\frac{\psi_s^{sin}(\mathcal{E}_s, x_s)}{q(x_s)} - \int_{\underline{\psi}}^{\psi_s^{sin}(\mathcal{E}_s,x_s)} H(\psi; x_s)d\psi\right] \tag{19}$$

Equation 19 shows that if the term in square brackets is positive, home production values can be larger than reservation wages as the presence of a disutility associated with entering the labor force can make search more costly when non-employed than when employed.[17]

We make a parametric assumption that $H(\psi; x_s)$ is exponentially distributed, and denote by $\eta(x_s)$ the single parameter which governs the distribution of participation disutilities drawn. In our model, the fraction of non-employed single individuals who are out of the labor force (OLF) for a given education, skill and gender is given by: $(1 - H[\psi_s^{sin}(\mathcal{E}_s, x_s); x_s])$. Thus, we can pin down $\eta(x_s)$ by using information on the share of non-employed singles who are out of the labor force for a given education and gender:

$$\text{OLF Share for } \mathcal{E}_s = \sum_{x_s \in \{H,L\}} p(x_s \mid \mathcal{E}_s)\left(1 - H[\psi_s^{sin}(\mathcal{E}_s, x_s); x_s]\right) \tag{20}$$

Finally, we pin down $\lambda(x_s)$ using the job-to-job (JJ) rate within each gender and education. The job-finding rate of an education-gender group can be expressed as:

$$JJ(\mathcal{E}_s) = \sum_{x_s \in \{H,L\}} \lambda(x_s)p(x_s \mid \mathcal{E}_s)\int_{w_s^{sin*}(\mathcal{E}_s,x_s)} [1 - F_s(y)]\, g_s^{sin}(y; \mathcal{E}_s, x_s)dy \tag{21}$$

Table 1 displays our estimated parameters, and the model's fit with data. We report singles' non-employment rates, OLF shares and job-to-job transition rates. We refer the reader to Figures 1 and 2 in Appendix E.1 for the model's fit with the full wage distribution over the two time periods, which is near perfect. From Table 1, within a time period, high-skilled individuals tend to have higher offer arrival rates both in employment and non-employment, as depicted by the differences in $q(x_s)$ and $\lambda(x_s)$, respectively. Across the two time periods, $\lambda(x_s)$ falls for both males and females of all skill-types, and reflects the fact that job-to-job transition rates were lower in the 2000s. Job separation rates also declined for women, consistent with findings in the literature that employed women had higher labor market attachment over time and were less likely to separate into non-employment. Job-finding rates improved for males over time while all individuals were more likely to draw lower participation disutilities in the 2000s.[18] Finally, the rise in the college premium implies that

[17]Because of log utility, $A(\mathcal{E}_s)$ drops out of all change of value terms, making $w^{sin*}(\mathcal{E}_s, x_s) = w^{sin*}(x_s)$ and $\psi^{sin}(\mathcal{E}_s, x_s) = \psi^{sin}(x_s)$.

[18]The distribution of participation disutility with a low η first order stochastically dominates (FOSD) that with a high η. Across time, $\eta(x_s)$ fell for all groups.

Panel A: Estimated parameters that vary by skill								
	Men				Women			
	1980		2000		1980		2000	
	L	H	L	H	L	H	L	H
$\delta^{sin}(x_s)$	0.041	0.031	0.042	0.046	0.039	0.021	0.028	0.017
$q(x_s)$	0.272	0.717	0.437	0.954	0.298	0.542	0.253	0.436
μ_s	2.121	2.121	2.267	2.267	1.843	1.843	1.741	1.741
σ_s	0.577	0.577	0.575	0.575	0.529	0.529	0.607	0.607
$\lambda(x_s)$	0.146	0.220	0.064	0.113	0.142	0.184	0.119	0.127
$\eta(x_s)$	0.080	0.035	0.016	0.022	0.026	0.040	0.025	0.028
b_s	1.857	1.857	1.244	1.244	4.182	4.182	4.340	4.340
Panel B: Estimated parameters and moments that vary by education								
	Men				Women			
	1980		2000		1980		2000	
	$\leq$ HS	$\geq$Col	$\leq$ HS	$\geq$ Col	$\leq$ HS	$\geq$ Col	$\leq$ HS	$\geq$ Col
$A(\mathcal{E}_s)$	1	1.287	1	1.489	1	1.426	1	1.582
$p(H \mid \mathcal{E}_s)$	0.281	0.813	0.249	0.723	0.249	0.877	0.270	0.849
Data $\mathbf{u}^{sin}$	0.222	0.107	0.248	0.142	0.365	0.128	0.338	0.156
Model $\mathbf{u}^{sin}$	0.191	0.119	0.232	0.162	0.349	0.148	0.329	0.176
Data OLF	0.521	0.567	0.713	0.670	0.792	0.702	0.808	0.760
Model OLF	0.521	0.567	0.712	0.670	0.792	0.702	0.808	0.760
Data JJ	0.041	0.036	0.031	0.027	0.035	0.034	0.026	0.026
Model JJ	0.040	0.042	0.025	0.031	0.036	0.033	0.028	0.024

Notes: Panel A shows the estimated parameters by gender and skill for individuals in joint households. Panel B shows the estimated moments as well as parameters that vary by education

Table 1: Estimated parameters and fit for singles

the effective labor input of college educated individuals increased over time, as depicted by the rise in $A(\mathcal{E}_s)$. Higher effective labor input of college graduates, increased job-finding rates for males, decreased separation rates for females and lower participation disutilities drawn on average imply an improvement in labor market returns for all individuals in the 2000s, but especially so for high-skilled college graduates.

3.2.2 Joint Household Labor Market Returns

Thus far, we have not used any data on joint households to identify the parameters single households face. We do not need to do so as we assume that joint households face exactly the same labor market parameters as singles except for the separation rates $\delta^{mar}(x_s)$. We also do not require that the skill shares to be the same across single and joint households. Instead, we jointly choose $\delta^{mar}(x_s)$ and $p(\mathbf{x}|\mathbf{E})$ to match the employment statuses and mean income within each joint household education pairs.

Because separation risks affect households' outside options and hence their reservation wages, both separation risks and skill shares affect mean income within an education pair. Thus, given data on joint households' employment status and mean income by education pair, we can back out the implied $\delta^{mar}(x_s)$ and $p(\mathbf{x}|\mathbf{E})$ that best explains these empirical

Panel A: Estimated $\delta^{mar}(x_s)$ for joint households								
	Men				Women			
	1980		2000		1980		2000	
	L	H	L	H	L	H	L	H
$\delta^{mar}(x_s)$	0.07	5e-4	0.02	2e-4	0.89	2e-4	0.20	1e-3
Panel B: Estimated skill probabilities								
	1980				2000			
	HS,HS	HS,Col	Col,HS	Col,Col	HS,HS	HS,Col	Col,HS	Col,Col
$p(L,L\|\mathbf{E})$	0.16	0.06	0.07	0.03	0.31	0.14	0.19	0.08
$p(L,H\|\mathbf{E})$	0.10	0.10	0.06	0.05	0.23	0.22	0.18	0.12
$p(H,L\|\mathbf{E})$	0.28	0.20	0.35	0.28	0.06	0.07	0.15	0.18
$p(H,H\|\mathbf{E})$	0.46	0.64	0.52	0.64	0.40	0.58	0.48	0.62
Panel C: Model Fit								
	1980				2000			
	HS,HS	HS,Col	Col,HS	Col,Col	HS,HS	HS,Col	Col,HS	Col,Col
Data $\mathbf{u}_m^{mar}$	0.14	0.10	0.07	0.04	0.14	0.11	0.09	0.06
Model $\mathbf{u}_m^{mar}$	0.14	0.10	0.07	0.04	0.14	0.11	0.09	0.06
Data $\mathbf{u}_f^{mar}$	0.45	0.28	0.44	0.33	0.39	0.21	0.36	0.27
Model $\mathbf{u}_f^{mar}$	0.45	0.27	0.43	0.32	0.38	0.21	0.35	0.26
Data avg $\mathcal{I}$	23.50	30.20	31.25	37.78	23.70	33.50	34.70	49.40
Model avg $\mathcal{I}$	23.31	30.47	31.58	37.30	23.04	34.38	36.07	47.30
Data u_u	0.07	0.03	0.03	0.01	0.06	0.03	0.04	0.02
Model u_u	0.07	0.03	0.03	0.01	0.06	0.03	0.04	0.02
Data u_Ω	0.06	0.07	0.04	0.03	0.07	0.08	0.05	0.04
Model u_Ω	0.06	0.07	0.04	0.03	0.07	0.08	0.05	0.04
Data e_Ω	0.38	0.25	0.40	0.31	0.32	0.18	0.31	0.25
Model e_Ω	0.38	0.25	0.40	0.31	0.32	0.18	0.31	0.25

Notes: Panel A shows the estimated separation rates by gender and skill for individuals in joint households. Panel B shows the estimated shares of skill types conditional on a joint household education pair. Panel C shows our model-predicted labor market moments against their data counterparts by education pair. $\mathcal{I}$ stands for household income in Panel C.

Table 2: Estimated parameters and fit for married

moments for joint households.[19]

Table 2 shows our estimated parameters and model fit. We find that separation rates declined for all individuals within joint households over the two time periods, as depicted in Panel A of Table 2. This is different from single males who experienced an increase in their separation rate in the second time period. The decline in separation rates for low-skilled married females is particularly stark and reflects their increased labor force participation. Turning to Panel B, we see that across all education groups, the majority of joint households are comprised of two high skilled individuals. This can be seen by the fact that $p(H,H|\mathbf{E})$ is the largest joint probability for all education pairs. The share of dual high-skilled individuals is greatest in households where both members are college educated. Across time however and conditional on a joint-education category, the share of dual high-skilled individuals declined

[19]Appendix E.3 shows how we can back out the underlying skill-shares within each joint household and recover $\delta^{mar}(x_s)$ from information on average household incomes.

while the share of dual low-skilled individuals rose. This rise in the share of dual low-skilled joint households reflects the fact that our estimated model predicts a decline in the share of high-skilled males across the two time periods, from 77 percent to 63 percent.

3.3 Marriage Market and Education Investment Block

Having estimated the labor market parameters, we now calculate the joint value of each marital pair net of the Pareto weights, $\rho\widetilde{U}(\mathbf{E},\mathbf{x})$, as well as the value of non-employed singles for each gender-education-skill combination. Individuals' education and marriage decisions are affected not only by their labor market returns, but also by the returns in the marriage market (their preference shocks as well as the Pareto weights which determine how much of the joint surplus they receive). We assume that preference shock for gender s is drawn from a Gumbel distribution with mean $\bar{\chi}_s(\mathbf{E},\mathbf{x})$. Therefore, what remains to be estimated are the composite preference parameters $\bar{\chi}_s(\mathbf{E},\mathbf{x})$ and the Pareto weights $\alpha(\mathbf{E},\mathbf{x})$.[20]

For a given education-skill joint household pair, we further restrict that both m and f in that pair draw their preference shocks from the same distribution, implying $\bar{\chi}_m(\mathbf{E},\mathbf{x}) = \bar{\chi}_f(\mathbf{E},\mathbf{x})$.[21] This implies we have sixteen preference parameters and sixteen Pareto weights to estimate for joint households, in addition to the preference parameters for single households. For male singles with skill type x_m and who have high school education, we normalize $\bar{\chi}_m(HS,\emptyset,x_m,\emptyset) = 0$. Given this assumption, we then use information on the implied empirical shares of each household type $(\mathbf{E},\mathbf{x})$ relative to the share of singles of gender s with high-school education and skill x_s to pin down the $\bar{\chi}_s(\mathbf{E},\mathbf{x})$. We use the fact that the measure of men in each joint household education-skill pair must be equal to the measure of females to pin down the Pareto weights, that is, the marriage market must clear. Details of our estimation procedure are in Appendix F.[22]

Because marriage tends to provide higher economic benefits than singlehood, Panels A and B of Table 3 demonstrate that to recover the observed shares of each household type, preferences for being in a single household tend to be positive while preferences for being in a joint household tend to be negative. Because the economic returns to being in a joint household also rose in the 2000s - as college premiums increased, job-finding rates improved for males and both the likelihood of drawing high participation disutilities as well as the rate at which individuals separated from their jobs declined - preferences for being in a joint household became even more negative in the second time period. Our estimation results suggests that absent changing preferences, the rising economic benefits of being in a joint household across time would have led to a decline in the share of single households.

Finally, we turn to the marriage market clearing prices, $\alpha(\mathbf{E},\mathbf{x})$. Recall that $\alpha(\mathbf{E},\mathbf{x})$ represents the female's share of marital surplus in a household with characteristics $(\mathbf{E},\mathbf{x})$.

[20] While $\alpha(\mathbf{E},\mathbf{x})$ is endogenous, we back them out from the data using the model's equilibrium conditions.

[21] We make this restriction as the parameters governing matching and educational attainment would otherwise not be identified.

[22] Table 10 in Appendix F shows how our model's estimated shares of males and females across the different education-household types line up against that observed in the data.

Panel A: Single household preferences								
	1980s				2000s			
	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
M	0.0	0.0	-14.4	13.1	0.0	0.0	-7.0	22.4
F	27.4	1.4	-0.6	5.7	23.1	-1.1	2.4	7.2
Panel B: Joint household preferences								
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	7.5	-34.7	-23.8	-86.0	-4.3	-51.4	-28.2	-110.8
HS H	-67.7	-155.0	-102.2	-238.2	-96.7	-164.2	-123.8	-259.3
Col L	-14.4	-57.0	-22.0	-89.9	-30.0	-85.0	-35.9	-134.2
Col H	-132.5	-239.1	-145.9	-311.6	-200.1	-302.7	-213.1	-403.8
Panel C: Joint household prices $\alpha(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$								
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.99	0.59	0.84	0.55	0.84	0.58	0.73	0.54
HS H	0.48	0.49	0.48	0.49	0.50	0.50	0.50	0.50
Col L	0.87	0.58	0.77	0.55	0.70	0.56	0.65	0.53
Col H	0.49	0.49	0.49	0.49	0.50	0.50	0.50	0.50

Notes: Columns 1-4 shows our estimated parameters for the 1980s while columns 5-8 shows the estimated parameter for the 2000s. Panel A shows the estimated preference parameters for single households, while Panel B shows the preference parameters for joint households. Panel C details the associated prices required to clear the marriage market for each joint household type and across time periods.

Table 3: Preference parameters and prices

Panel C of Table 3 shows that low-skilled females tend to command a higher share of the surplus when matched to low-skilled males relative to their high-skilled female counterparts. Low-skilled females have higher separation rates in joint households relative to when they are single. To encourage low-skilled females to form joint households, low-skilled males must transfer more of the marital surplus to them as compensation for the higher non-employment risk they will face. High-skilled females in joint households, on the other hand, face lower separation rates than their single counterparts, and therefore their spouses do not need to insure them as much. In fact, the surplus-split is roughly even in joint households when both members are high-skilled.

3.3.1 Marital surplus and Changes in Sorting Incentives

The key object that determines who marries whom and the nature of sorting in marriage is marital surplus, which we now have estimated for our two time periods. The estimated marital surplus is itself a function of primitives of the model, and reflects the changing economic and non-economic fundamentals underpinning the returns to marriage. Embedded in these returns are the complementarities between education and skill in marriage, which can be boiled down into a single monotonic index that captures the changes in incentives to sort (Chiappori et al. (2020a)). That is, a higher degree of supermodularity in marital surplus along some dimension (education or skill) necessarily implies a higher degree of assortative matching *in an SEV (seperable extreme value) sense.*

Panel A of Table 4 depicts marital surplus for all possible pairs. Panel B of Table 4 shows that conditional on being in a joint household and holding fixed skill pairs, there are incentives to positively sort by education. Importantly, however, those incentives weakened in the 2000s, as surplus is less supermodular along the education dimension. Conversely, as can be seen in Panel C, holding fixed education pairs, the degree of supermodularity along skill increased in the 2000s, suggesting that incentives to positively sort along skill were stronger in the 2000s. Consistent with the overall patterns in the incentives to sort over time, the SEV index in Panel D (Chiappori et al. (2020a)) of assortative matching by skill rises, while the same index by education falls.

Why do incentives to sort by skill rise over time, while incentives to sort by education decline? In our estimated model, education improves an individual's effective labor input, but does not affect their propensity to enter and exit non-employment. Skill, on the other hand, directly affects offer arrival rates, separation rates, and the distribution of participation disutilities, all of which influence an individual's exposure to non-employment risk and thus. realized income volatility. Lower exposure to non-employment risk and higher labor market returns encourage individuals to seek matches more for income maximization motives rather than for insurance purposes. As shown in Tables 1 and 2, improvements in job-finding rates, declines in separation rates and lower participation disutilities on average strengthened skill complementarities over time, giving rise to stronger incentives to sort by skill in the 2000s.[23]

4 Assortative Matching and the Rise Household Income Inequality

How do these changes in incentives to sort impact income inequality? Answering this question requires performing counterfactuals based on how people would hypothetically sort given changes in the incentives they face. Counterfactuals based solely on empirical measures of sorting other than the SEV index may mis-measure the degree of sorting to begin with, or fail to reflect full responses to changing incentives, as they do not allow for the marginal distributions (across education or marriage) to change.[24]

Unlike these reduced-form counterfactuals, our approach will tie changes in sorting incentives to changes in behavior. This is because we will simulate how people will respond to changing incentives, which have been identified and estimated using the structure of our model. Changes in behavior in response to time-varying incentives not only include who marries whom, but also (i) who gets married, and (ii) how much education people obtain.

[23]Notably, when we abstract from skill and only allow the college premium to rise over time, we still observe weaker incentives to positively sort by education in the 2000s. In this case where there are no differences in skill, all individuals want to pair with college-educated individuals when the college premium rises. Individuals with only high-school education, however, are willing to offer more in terms of intra-household transfers to college-educated individuals as their higher incomes provide a greater buffer against non-employment risk. Online Appendix M details the results from this exercise.

[24]See Chiappori et al. (2020a) for a full treatment of counterfactuals under different empirical sorting measures.

Panel A: Total Marital Surplus								
	1980s				2000s			
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	-11.6	-9.8	-43.7	-52.0	-9.8	-6.4	-28.5	-36.6
HS H	9.1	30.1	-9.0	-6.0	-31.4	15.8	-32.1	-5.9
Col L	-36.6	-27.9	-19.9	-28.9	-29.4	-21.0	-6.4	-14.1
Col H	-18.8	0.2	12.4	9.1	-44.5	-10.7	0.5	8.5
Panel B: Supermodularity, fixed (x_m, x_f)								
M/F		L	H			L	H	
L		48.7	41.2			41.7	37.1	
H		49.3	45.0			45.6	40.9	
Panel C: Supermodularity, fixed $(\mathcal{E}_m, \mathcal{E}_f)$								
M/F		HS	Col			HS	Col	
HS		19.1	11.4			43.8	34.3	
Col		10.3	5.7			25.4	15.7	
Panel D: I_{SEV}								
Education		2.36				2.17		
Skill		0.85				1.39		

Notes: Total surplus is equal to economic marital surplus plus the relative difference in mean preference shocks. Supermodularity is calculated as per equations (14) and (15) but for total surplus. Panels B and C hold fixed skill and education pairs respectively in calculating the degree of supermodularity. Positive values in B imply incentives to positively sort across education given a skill pair while positive values in C imply incentives to positively sort across skills given an education pair. Panel D shows the Separable Extreme Value Index (I_{SEV}) of assortative matching by education and by skill for the entire economy. To calculate the I_{SEV} measure for Education, we sum across all skill pairs and compute the share of married males and females with a college education. We then compute the Separable Extreme Value index following Chiappori et al. (2020a). The SEV index for skill is computed in a similar fashion.

Table 4: Model-Implied Total Marital Surplus

By construction, reduced-form counterfactuals that merely impose the degree of sorting in one period onto another cannot capture the reaction of household formation and educational investment to such shifts in incentives.

Before moving to these counterfactuals, in what follows we first show how well our model can capture the empirical movements in household income inequality over time. We also repeat the exercises done in Eika et al. (2019) and Greenwood et al. (2014b), and validate that we obtain similar results to theirs using our model-generated data. We then turn to our preferred, model-based counterfactuals to show how increased sorting incentives have indeed had a significant impact on inequality, unlike what has previously been found.

4.1 Model Validation

For measures of inequality, we focus on the Gini coefficient and the Theil T index. Across the two time periods, our model-implied Gini coefficient rose from 0.46 to 0.53 while the model-implied Theil index rose from 0.37 in the 1980s to 0.49 in the 2000s. Relative to the data, our model captures roughly all of the rise in household income inequality as measured by the Gini coefficient, and 75 percent of the rise as measured by the Theil index.

Panel A	Gini Coefficient		Theil T	
	1980	2000	1980	2000
model	0.46	0.53 (15%)	0.37	0.49 (30%)
Data	0.40	0.46 (15%)	0.29	0.40 (40%)
Panel B	Sorting Counterfactuals			
	Gini Coefficient		Theil T	
2000s sorting, 1980 imposed (EMZ)	0.47	-	0.38	-
2000s sorting, 1980 imposed (GGKS)	0.47	-	0.39	-

Notes: Panel A reports the Gini coefficient and Theil Index generated by the model for both time periods. Terms in parentheses show the percent increase relative to the 1980s. Panel B shows the empirical shift-share analysis using the counterfactual sorting implemented as in Eika et al. (2019) or as in Greenwood et al. (2014a).

Table 5: Shift-Share Analysis of Inequality with Model-Generated Data

As a validation test of our model, we perform empirical shift share analyses as in Eika et al. (2019) and Greenwood et al. (2014a) on model-generated data, without appealing to the structure of our model.[25] Panel B of Table 5 shows that data generated from our model can replicate the "reduced form" counterfactual results as in Eika et al. (2019) and Greenwood et al. (2014a). Imposing realized sorting using these methods suggests that sorting matters little towards increasing household income inequality: the Gini rises by at most 1 percentage point, while the Theil by at most 2.

4.2 Counterfactuals Using A Structural Approach

To implement our counterfactual exercise, we take the following steps. First, we assume that the economy begins with a population dictated by the 1980s environment, where the shares of high- and low- skilled males and females are equal to their 1980's counterpart. We then impose that, at time zero and prior to making any educational attainment or household formation decisions, individuals *perceive* that they face marital surplus as estimated from the 2000s, thus making their decisions of whether to marry, whom to marry, and how much education to obtain based on the 2000s returns. Once these choices have been made, within each household type, we assign income that would have been realized under 1980s primitives.

This exercise effectively performs a shift-share type of analysis in which household composition changes, but realized income within each household type does not. However, unlike a standard shift-share analysis, the counterfactual distribution of households is determined through the model as an equilibrium response to the perceived returns, rather than by imposing the actual realized distribution of households in the 2000s.[26] This allows us to quantify the full impact of changing incentives to sort on household income inequality through changes in household formation decisions.

Table 6 shows the results. Column I reports our model predictions for the 1980s. Column

[25]Online Appendix K provides more detail how we implement the sorting counterfactuals following Eika et al. (2019) and Greenwood et al. (2014a).

[26]Specifically, these two objects differ since the underlying skill share is different in the 1980s and 2000s.

II shows our counterfactual results, when individuals sort according to 2000s marital surplus. Column III shows our the model predictions for the 2000s. Strikingly, we find that the Gini coefficient rises from .46 to .50 when individuals sort according to 2000s surplus, accounting for more than half of the overall increase in inequality between the 1980s and 2000s. Similarly, the rise in household income inequality as measured by the Theil T index accounts for slightly over half the estimated increase in inequality over the two time periods. This stands in stark contrast to results found using the reduced-form approaches, as depicted in Table 5.

There are several coincident changes which contribute towards the rise in inequality under our counterfactual. First, as discussed in Section 3.3, preferences for being in a joint household fell in the 2000s. Therefore, when individuals make decisions based on 2000s perceived returns they are more likely to form single households, as depicted in Panel B by the higher single share under our GE counterfactual. More single households are a force towards raising inequality, because relative to joint households, singles have more volatile household income.

Second, conditional on marriage there are more households "on the diagonals" (the case where individuals match with their exact education-skill counterpart), and in particular at the upper tail of the diagonals, as depicted in Table 7. This movement can be thought of in two related parts: changes in realized sorting in response to changing incentives (which may occur even without changes in the marginal distribution of types in marriage), and changes in the marginal distribution of types in marriage, which can amplify changes in realized sorting in response to any given change in incentives. It is the combination of these two channels which results in the rise household inequality *within* joint households of 8% as measured by the Gini coefficient (Gini_{joint}) and by 20% as measured by the Theil Index (Theil_{joint}) (see Panel A of Table 6).

Realized sorting in marriage will necessarily rise under our counterfactual when marginals are fixed (See Proposition 2).[27] Because economic fundamentals that vary by skill are perceived to have improved as in the 2000s, this leads individuals to expect to spend less time in non-employment and for incomes to be less volatile. Consequently, the desire to seek partners out for insurance purposes weakens, and income maximization motives matter relatively more. This - combined together with a rising college premium - increases demand by high-skilled individuals for high-skilled counterparts who are also college-educated. This is because household incomes, and thus individual payoffs, are largest in pairs where high-skilled individuals are matched with their exact skill-education counterpart.

But the second effect - the change in marginals within marriage - works to amplify the effect of increased incentives to sort. The counterfactual change in surplus leads high-skilled individuals - more so than the low-skilled - to increasingly demand education, such that higher incentives to sort lead to a greater mass of joint households in the upper tail of

[27] Under higher incentives to positively sort, decisions in whom to marry can never translate into less realized positive sorting. There are knife edge cases, such as when individuals are already perfectly sorted, where an increase in incentives to positively sort do not translate into any further increase in realized sorting.

Panel A: Inequality measures			
	1980s	GE	2000s
	(I)	(II)	(III)
Gini	0.46	0.50 (8%)	0.53 (15%)
Theil T	0.37	0.43 (16%)	0.49 (30%)
Gini_{joint}	0.28	0.30 (8%)	0.32 (15%)
Theil_{joint}	0.15	0.18 (20%)	0.19 (25%)
Gini_{single}	0.47	0.46 (-3%)	0.49 (5%)
Theil_{single}	0.42	0.40 (-5%)	0.46 (8%)
Panel B: College and education shares			
Single HH share	0.40	0.48	0.47
College share	0.43	0.62	0.61
Panel C: Premia			
Education (M,L)	-1.17	-0.41	-0.42
Δ		(0.76)	(0.75)
Marital education (M,L)	0.16	0.36	0.40
Δ		(0.20)	(0.24)
Single education (M,L)	-1.33	-0.77	-0.82
Δ		(0.56)	(0.51)
Education (M,H)	0.20	1.00	0.94
Δ		(0.80)	(0.74)
Marital education (M,H)	-1.10	-0.51	-0.55
Δ		(0.59)	(-0.55)
Single education (M,H)	1.30	1.51	1.49
Δ		(0.21)	(0.19)
Education (F,L)	-1.18	-0.35	-0.38
Δ		(0.83)	(0.80)
Marital education (F,L)	1.40	1.53	1.30
Δ		(0.13)	(-0.10)
Single education (F,L)	-2.58	-1.88	-1.68
Δ		(0.70)	(0.90)
Education (F,H)	0.02	1.00	0.90
Δ		(0.98)	(0.88)
Marital education (F,H)	-1.09	-0.76	-0.84
Δ		(0.33)	(0.25)
Single education (F,H)	1.11	1.76	1.74
Δ		(0.65)	(0.63)

Notes: Column 1 shows the results under the 1980s baseline model. Column 2 shows the outcomes under GE where skill shares are fixed to 1980s values. Column 3 shows the results under the 2000s estimated model. For GE, 1980s realized income returns are imposed. Panel A reports aggregate inequality, Panel B reports college and single shares, and Panel C reports the education premia by gender and skill. Δ refers to the change relative to its 1980 level.

Table 6: GE counterfactual exercises

the diagonal. To see this, we calculate both the education premium (EP) and the marital education premium (MEP) for each skill level (Chiappori et al. (2015), Chiappori et al. (2017)), which are (here, outlined for males) given by:

$$EP_m(x_m) = \left[\mathbf{E} \left\{ \max_{\{\mathcal{E}_f, x_f\}} \exp\left(\rho V_m(Col, \mathcal{E}_f, x_m, x_f)\right) + \chi^i_m(Col, \mathcal{E}_f, x_m, x_f) \right\} - \mathbf{E} \left\{ \max_{\{\mathcal{E}_f, x_f\}} \exp\left(\rho V_m(HS, \mathcal{E}_f, x_m, x_f)\right) + \chi^i_m(HS, \mathcal{E}_f, x_m, x_f) \right\} \right] \quad (22)$$

$$\begin{aligned} MEP_m(x_m) = & \Bigg(\mathbf{E} \Big\{ \max_{\mathcal{E}_f, x_f} \big[\exp\left(\rho V_m(Col, \mathcal{E}_f, x_m, x_f)\right) + \chi^i_m(Col, \mathcal{E}_f, x_m, x_f) \\ & - \exp\left(\rho V_m(Col, \emptyset, x_m, \emptyset)\right) - \chi^i_m(Col, \emptyset, x_m, \emptyset) \big] \Big\} \\ & - \mathbf{E} \Big\{ \max_{\mathcal{E}_f, x_f} \big[\exp\left(\rho V_m(HS, \mathcal{E}_f, x_m, x_f)\right) + \chi^i_m(HS, \mathcal{E}_f, x_m, x_f) \\ & - \exp\left(\rho V_m(HS, \emptyset, x_m, \emptyset)\right) - \chi^i_m(HS, \emptyset, x_m, \emptyset) \big] \Big\} \Bigg) \end{aligned} \quad (23)$$

In words, the former is the gain from receiving a college education relative to a high school education for skill x_m, while the latter is the additional gain on the marriage market from having a college education. Panel C of Table 6 shows that under perceived 2000s surplus, both education and marital education premia rise for males and females across all skill types. Single education premia is also reported in Panel C of Table 6 and is computed as the difference between the education premia and the marital education premia.

The increase in the education premia is especially stark for the high-skilled, with the rise in marital educational premia only serving to reinforce these strong incentives to acquire education. Focusing on females first, the increase in marital education premia constitutes a greater proportion of the total increase in education premia for high-skilled females than for low-skilled females (one-third versus 16 percent, respectively). Similarly, when we focus on males, we find that while the majority of the rise in the education premia is explained by the change in marital education premia for high-skilled males (74 percent), the rise in marital education premia only accounts for a small share for low-skilled males (26 percent). For both genders, increased incentives to sort raised the returns to education more for high-skilled individuals than for low-skilled individuals, with the proportion of the increase in education premia stemming from the rise in marital education premia generally being larger for the high-skilled.

Why is the change in marital education premia a larger driver behind the increase in total education premia for high-skilled individuals than for low-skilled individuals? In part, this occurs because investments in education for the highly skilled have an additional pay off: a higher likelihood of matching with a like education-skill partner. Joint households where both members are highly-skilled and highly-educated receive the highest payoffs and offer the highest income maximization opportunities. Because the highly-skilled become more educated, the composition of married individuals changes, tilting towards high skill-high education individuals. The higher supply of college-educated high-skilled individuals provides support for increased incentives to sort to actually translate into significant changes in realized sorting behavior. This is exactly how our counterfactual plays out - with a large increase in the share of joint households on the extreme end of the diagonal, an ensuing rise in joint household income inequality, and thus a significant increase in aggregate household income inequality.

Overall, our results suggests that its crucial to take into account how the decisions of how much education to attain and whether to marry respond to changing incentives to sort. The endogenous change in the supply of married individuals (in response to increased incentives to sort) reinforces and supports the extent to which sorting incentives can translate into actual realized sorting patterns. In other words, individuals can only take advantage of increased incentives to positively sort when there is a ready supply of like education-skill counterparts that they can match with.

Panel A: Probability shares within marriage only								
	1980s				GE			
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.07	0.05	0.00	0.01	0.07	0.03	0.02	0.02
HS H	0.13	0.21	0.02	0.05	0.02	0.11	0.01	0.08
Col L	0.01	0.01	0.01	0.01	0.02	0.01	0.03	0.04
Col H	0.06	0.08	0.09	0.20	0.03	0.06	0.13	0.32

Notes: This table reports probability shares conditional on marriage for the 1980s (columns 1-4) and for GE (columns 5-8). Shaded cells highlight the shares of joint households where both members share the same education and skill under the GE counterfactual.

Table 7: Significant increase in college-college-high-skill-high-skill pairs

4.3 Partial Equilibrium Counterfactuals

Thus far, we have shown through the GE counterfactual that sorting has a non-trivial impact on the rise in household income inequality, unlike the results from reduced form counterfactuals. Beyond the contribution from the rise in single households, we showed that there were two additional channels which contributed to the rise in inequality: (i) the change in realized sorting conditional on some fixed marginal distributions of education and skill in marriage and (ii) the endogenous change in the marginal distributions themselves in response to increased incentives to sort.

In this section, we demonstrate through a "partial equilibrium" exercise that the latter force is a key component of the overall rise in inequality that we document. By "partial equilibrium" (PE), we mean a counterfactual in which we do not allow the marginal distributions of education and skill to change in marriage so that the only outcome that can change is whom marries whom, conditional on marriage. Given this restriction, we ask how agents would sort differently in marriages if their decisions of whom to marry were based on

2000s total surplus. This gives us a sense of the quantitative importance of the first channel above relative to the second.

	Inequality measures					
	Aggregate		Within			
	Gini	Theil	Gini_{joint}	Theil_{joint}	Gini_{single}	Theil_{single}
1980s	0.46	0.37	0.28	0.15	0.47	0.42
PE	0.47	0.39	0.29	0.16	0.47	0.42
$\%\Delta$	1%	3%	2%	9%	0%	0%

Notes: Row 1 shows the outcomes under the 1980s benchmark model. Row 2 refers to the PE exercise where the supply of married individuals by skill and education is fixed to its 1980s levels. Row 3 shows the percent change in the inequality measure in PE relative to its level in 1980.

Table 8: Impact on inequality from decisions of whom to marry and whether to marry

Table 8 shows our results. Similar to results from the more "reduced-form" counterfactual exercises, the PE exercise shows household income inequality rising by 1 - 3 percent depending on the measure of inequality used. In this case, none of the increase comes from a changing share of single households as we hold the supply of married individuals by sex, education and skill fixed. All of the increase comes from the rise in *within* joint household income inequality, which rises 2 % as measured by the Gini_{joint} and by 9% as measured by the Theil_{joint}. Notably, even within joint households, this rise in inequality is less than that observed under GE. Table 6 shows that under the GE counterfactual, the Gini_{joint} and Theil_{joint} rose by 8 and 20 percent, respectively.

While our PE exercise features the same small increase in aggregate household income inequality as the reduced form exercises, it arises for different reasons. When the supply of married individuals is fixed, intrahousehold transfers, $\alpha(\mathbf{E}, \mathbf{x})$, instead adjust to clear the marriage market. This adjustment in transfers partially absorbs the increased incentives to positively sort by skill and education, resulting in a smaller increase in realized positive sorting relative to our GE counterfactual.

In this fixed-marginal world, among those who marry there is a shortage of high-skilled females relative to high-skilled males: 83 percent of married males are high-skilled while only 62 percent of married females are high-skilled (Table 7). Similarly, there is an excess of low-skilled females relative to low-skilled married males. To clear the marriage market, the share of surplus for high-skilled females is generally higher in PE, i.e., $\alpha(\mathbf{E}, x_m, x_f = H)$ increased, reducing some of the incentives for high-skilled males to form households with high-skilled females (Panel A of Table 9). Consequently, the share of households where both members are high-skilled and college educated declines under PE, as depicted in Panel B of Table 9.[28]. Further, the shares of surplus that low-skilled high-school educated females can obtain becomes more uniform across different household types in order to induce more marriages with this group of females who are in excess supply. This in turn gives rise to a more uniform distribution of low-skilled high-school educated females across household types.

[28]However, following Proposition 2, the sum of the diagonal elements rises from .49 to .52.

Panel A: $\alpha(\mathbf{E}, \mathbf{x})$								
	1980's				PE			
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.99	0.59	0.84	0.55	0.67	0.61	0.86	0.60
HS H	0.48	0.49	0.48	0.49	0.55	0.55	0.59	0.55
Col L	0.87	0.58	0.77	0.55	0.56	0.56	0.70	0.57
Col H	0.49	0.49	0.49	0.49	0.52	0.52	0.54	0.53
Panel B: Probability shares within marriage								
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.07	0.05	0.00	0.01	0.09	0.03	0.01	0.01
HS H	0.13	0.21	0.02	0.05	0.09	0.23	0.01	0.07
Col L	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01
Col H	0.06	0.08	0.09	0.20	0.06	0.08	0.09	0.19

Notes: Columns 1-4 show the outcomes under the 1980s baseline model. Column 5-8 shows the outcomes under counterfactual PE where the supply of married individuals by skill and education is fixed to its 1980s levels. Panel A reports marriage market clearing prices. Panel B reports the distribution of joint households.

Table 9: Prices and joint household distribution under PE

Overall, the adjustment in prices dampens the incentives to positively sort, resulting in a small increase in realized sorting as depicted in Panel B of Table 9, in particular at the top end.[29] Since there is no quantitatively substantial increase in realized positive sorting, there is also no quantitatively large rise in aggregate income inequality. Notably, our results highlight the importance of allowing marginals to react to incentives to sort. Without a changing supply of married individuals which can meet the increased demand for like-education-skill partners, increased incentives to sort largely get absorbed by prices, resulting in small changes in actual sorting patterns, and hence less of a rise in inequality. Taking stock, our results suggest that counterfactual exercises where marginals are held fixed do not allow for sorting's full effect to be analyzed as prices are forced to instead adjust to clear the marriage market.

5 Conclusion

In this paper, we develop a model of educational attainment, marriage and the labor market to examine how changes in incentives to sort can affect household income inequality over time. Using our estimated model, we measure increased incentives to sort by the degree of supermodularity in total marital surplus. Across the two time periods, there exists complementarities in education and skill within marital surplus, giving rise to positive incentives

[29]In Appendix N, we show how if instead prices were fixed at their 1980s levels, and the marriage market was instead cleared using a stable matching algorithm, the increased incentives to sort do then translate into a sizable change in realized sorting as depicted in Table 25. The significant increase in realized positive sorting then leads to a non-trivial change in household income inequality as shown in Table 26 in Appendix N. In fact, when prices are instead fixed and we assume stable matching, household income inequality, rises by a non-trivial 4 and 9 percent as measured by the Gini coefficient and Theil index, respectively. These increases represent 20 to 30 percent of the total estimated increase in household income inequality across the two time periods.

to sort. Incentives to positively sort by skill especially rose across the two time periods as primitives that are a function of skill – namely that of job-finding rates, job separation rates, and disutilities from participation – all moved in a direction to lower an individual's exposure to non-employment risk. Increased incentives to sort raised the returns to education by more for high-skilled individuals, causing the composition of joint households to tilts toward having more highly skilled, highly-educated pairs, raising household income inequality among those who are married. In summary, we find that increased incentives to sort can explain more than half of the empirical rise in household income inequality over time.

By focusing on the fundamentals such as the supermodularity in marital surplus, our paper sheds light on the impact of increased incentives to sort on inequality. In particular, our paper provides a tractable framework for understanding how the incentives to sort affect household formation and educational attainment decisions. We highlight how modeling the response of these decisions to increased incentives to sort is crucial for correctly accounting for the increase in household income inequality. Holding fixed marginals, the quantitative impact of sorting on inequality is small as intrahousehold transfers absorb the changes in incentives to sort. Our results underscore the fact that for incentives to sort to have a significant impact on inequality, the supply of individuals must be able to respond in such a way that increased incentives can translate into increased realized sorting.

Because this is the first paper which links labor market search to marital sorting, we see a number of interesting fruitful extensions using our framework. For example, changes in the supply and generosity of unemployment insurance might impact the nature of sorting. We leave this and other interesting questions for future research.

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Appendix

A Independence from α

Proof of Proposition 1. We suppress the dependence on $(\mathbf{E}, \mathbf{x})$ for ease of exposition. Under log utility, optimal consumption is given by:

$$c_m = (1-\alpha)\frac{\mathcal{I}}{2}, \quad c_f = \alpha\frac{\mathcal{I}}{2}, \quad Q = \frac{\mathcal{I}}{2p_Q}$$

And thus, we can write the net value of the dual non-employed joint household as:

$$\begin{aligned}
\rho U &= \mathcal{Z} + \log\left(\frac{\mathcal{I}_U}{2}\right) + \log\left(\frac{\mathcal{I}_U}{2p_Q}\right) \\
&+ \int_{\underline{\psi}}^{\psi_m^c} \left\{ -\psi + q_m \int_{w_{Um}^*}^{\overline{w}} \left[\Omega_m(y) - U\right] dF_m(y) \right\} dH_m(\psi) \\
&+ \int_{\underline{\psi}}^{\psi_f^c} \left\{ -\psi + q_f \int_{w_{Uf}^*}^{\overline{w}} \left[\Omega_f(y) - U\right] dF_f(y) \right\} dH_f(\psi)
\end{aligned}$$

where $\mathcal{Z} = \alpha \log \alpha + (1-\alpha)\log(1-\alpha)$ and $\mathcal{I}_U$ is the joint income when both members are dual non-employed. We guess and verify that the transformed problem is independent of α. In particular, we guess:

$$\widetilde{U} = U - \frac{\mathcal{Z}}{\rho}, \qquad \widetilde{\Omega}_i(w) = \Omega_i(w) - \frac{\mathcal{Z}}{\rho}$$

Working with the transformed problem $\widetilde{U}$, we have:

$$\begin{aligned}
\rho \widetilde{U} &= \log\left(\frac{\mathcal{I}_U}{2}\right) + \log\left(\frac{\mathcal{I}_U}{2p_Q}\right) \\
&+ \int_{\underline{\psi}}^{\psi_m^c} \left\{ -\psi + q_m \int_{w_{Um}^*}^{\overline{w}} \left[\widetilde{\Omega}_m(y) - \widetilde{U}\right] dF_m(y) \right\} dH_m(\psi) \\
&+ \int_{\underline{\psi}}^{\psi_f^c} \left\{ -\psi + q_f \int_{w_{Uf}^*}^{\overline{w}} \left[\widetilde{\Omega}_f(y) - \widetilde{U}\right] dF_f(y) \right\} dH_f(\psi)
\end{aligned}$$

The above verifies that $\widetilde{U}$ is independent of α. Since all the expected change in values in the transformed problem are exactly the same as the original problem, this implies that all decision rules in the transformed problem are the solution to the original problem. To see this, observe that the indifference condition governing the reservation wage rule out of dual non-employment takes the following form:

$$\widetilde{\Omega}_m(w^*_{Um}) - \widetilde{U} = \Omega_m(w^*_{Um}) - \frac{\mathcal{Z}}{\rho} - \left[U - \frac{\mathcal{Z}}{\rho}\right] = \Omega_m(w^*_{Um}) - U = 0$$

Thus, the solution to the reservation wage rule from the transformed problem extends to the original problem and does not depend on α. Further, the participation cut-offs as given by Equation (7) can be represented by:

$$\psi^c_m = q_m \int_{w^*_{Um}}^{\overline{w}} \left[\widetilde{\Omega}_m(y) - \widetilde{U}\right] dF_m(y) = q_m \int_{w^*_{Um}}^{\overline{w}} \left[\Omega_m(y) - U\right] dF_m(y)$$

where the above shows that the participation cut-off is also independent of α. Because all α terms enter into the joint household's problem additively, they do not affect any labor market decision rules and only affect how the household splits income.

$\square$

A.1 Separating out individual gains

Since all labor market decision rules for the joint household are independent from α and the pareto weights only affect how the household splits income, we can write the value for a male and female member of the dual non-employed joint household as:

$$\rho V^U_m(\mathbf{E}, \mathbf{x}) = \log(1 - \alpha[\mathbf{E}, \mathbf{x}]) + \rho\widetilde{U}(\mathbf{E}, \mathbf{x}) \text{ and } \rho V^U_f = \log(\alpha[\mathbf{E}, \mathbf{x}]) + \rho\widetilde{U}(\mathbf{E}, \mathbf{x})$$

The value of a dual non-employed joint household is the weighted sum of its member where the weights are given by α:

$$\rho U(\mathbf{E}, \mathbf{x}) = (1 - \alpha[\mathbf{E}, \mathbf{x}])\rho V^U_m(\mathbf{E}, \mathbf{x}) + \alpha[\mathbf{E}, \mathbf{x}]\rho V^U_f(\mathbf{E}, \mathbf{x}) = \mathcal{Z}(\mathbf{E}, \mathbf{x}) + \rho\widetilde{U}(\mathbf{E}, \mathbf{x})$$

B Steady State Laws of Motion

In what follows, we suppress the dependence of arrival rates, separation rates and distributions on gender, education and skill. In the benchmark model, all rates and distributions depend on the individual's gender $s \in \{m, f\}$, education $\mathcal{E}_s \in N_{\mathcal{E}}$ and skill, $x_s \in N_x$.

B.1 Singles

Inflows into non-employment for singles stem from exogenous separations from employment while outflows from non-employment stem from drawing a flow disutility below the threshold

ψ^c and drawing $w \geq w_u^{sin^*}$. The steady state measure of non-employed singles is:

$$\mathbf{u}^{sin} = \frac{\delta}{\delta + H\left(\psi^{sin}\right) q \left[1 - F(w_u^{sin^*})\right]} \tag{24}$$

where $\mathbf{u}^{sin}$ represents the total non-employed singles. $H\left(\psi^{sin}\right)$ represents the probability that a non-employed single enters the labor force in the current period while $q\left[1 - F(w_u^{sin^*})\right]$ is the probability they find and accept a job offer.

The measure of non-employed individuals who remain out of the labor force is equal to the fraction of non-employed who draw a disutility above ψ^{sin}:

$$\text{Measure out of labor force} = \left[1 - H(\psi^{sin})\right] \mathbf{u}^{sin} \tag{25}$$

Denote $G^{sin}(w)$ as the cdf of all employed single individuals earning a wage less than w. Inflows into this group stem from the non-employed successfully entering the labor force, and receiving a job offer with wage less than or equal to w. Outflows stem from exogenous separations and job-to-job movements to jobs paying above w. The distribution of employed individuals with wages less than or equal to w is:

$$G^{sin}\left(w\right) = \frac{\delta}{\left[1 - F\left(w_u^{sin^*}\right)\right]} \frac{\left[F\left(w\right) - F\left(w_u^{sin^*}\right)\right]}{\delta + \lambda\left[1 - F\left(w\right)\right]} \tag{26}$$

B.2 Joint Households

Dual non-employed Inflows into the dual non-employed stem from exogenous separations of the employed member in worker-searcher households while outflows stem from each individual of gender $s \in \{m, f\}$ drawing a disutility below the threshold $\psi_s^c(\mathbf{x})$ and receiving a job offer above the reservation wage, $w_{U,s}^*(\mathbf{x})$:

$$\delta_m \mathbf{e}_\Omega + \delta_f \mathbf{u}_\Omega = \left\{H\left(\psi_f^c\right) q_f \left[1 - F_f\left(w_{U,f}^*\right)\right] + H\left(\psi_m^c\right) q_m \left[1 - F_m\left(w_{U,m}^*\right)\right]\right\} \mathbf{u}_u$$

where $\mathbf{u}_u$ is the measure of dual non-employed, $\mathbf{e}_\Omega$ is the measure of male-headed worker-searcher households, and $\mathbf{u}_\Omega$ is the measure of female-headed worker-searcher households.

Worker-Searcher Households Inflows into the male-headed worker searcher household earning a wage $\leq w$ stem from exogenous separations of the female employed in a dual employed household with her husband earning a wage less than or equal to w, and from the male in the dual non-employed entering and finding a job that pays above the reservation wage, but below w. Outflows stem from exogenous separations, transitions to wages above

w, and from the non-employed female partner finding a job.

$$\left\{(\delta_m + \lambda_m [1 - F_m(w)]) G^m(w) + q_f \int_{\underline{w}}^{w} H\left[\psi_f^c(y)\right]\left[1 - F_f\left(w^*_{\Omega_m,u}[y]\right)\right] g^m(y)\, dy\right\} \mathbf{e}_{\boldsymbol{\Omega}}$$
$$= H\left(\psi_m^{u,c}\right) q_m \left[F_m(w) - F_m\left(w^*_{U,m}\right)\right] \mathbf{u}_u + \delta_f \int_{\underline{w}}^{w} \left\{\int_{\underline{w}}^{\overline{w}} g^T(z,\epsilon)\, d\epsilon\right\} dz \mathbf{e}_T$$

where $G^m(w)$ is the cumulative distribution of the employed in the male-headed worker-searcher household earning less than or equal to w, $g^m(w)$ is the associated probability density. $\mathbf{e}_T$ is the measure of dual employed households and $g^T(z,\epsilon)$ is the density of dual employed households where m earns z and f earns ϵ. In the limit, when $w \to \overline{w}$, we implicitly get the measure of male-headed worker-searcher households from the equation below:

$$\left\{\delta_m + q_f \int_{\underline{w}}^{\overline{w}} H[\psi_f^c(y)]\left[1 - F_f\left(w^*_{\Omega_m,u}[y]\right)\right] g^m(y)\, dy\right\}\mathbf{e}_\Omega = H\left(\psi_m^{u,c}\right) q_m \left[1 - F_m\left(w^*_{U,m}\right)\right] \mathbf{u}_u + \delta_f \mathbf{e}_T$$

An analogous expression exists for the female-headed worker-searcher household.

Dual employed Inflows into the dual employed where m earns a wage $w \leq w_m$ and f earns a wage $w \leq w_f$ occur whenever the non-employed spouse from a worker-searcher household where the employed partner is currently earning $w \leq w_m$ ($w \leq w_f$), enters the labor force draws a wage below or equal to w_f (w_m) but above their reservation wage. Outflows stem from exogenous separations, and from transitions to a wage above w_m or w_f.

$$\sum_{i \in \{m,f\}} \{\delta_i + \lambda_i [1 - F_i(w_i)]\} G^T(w_m, w_f)\, \mathbf{e}_T$$
$$= \left\{q_f \int_{\underline{w}}^{w_m} H\left[\psi_f^c(y)\right]\left[1 - F_f\left(w^*_{\Omega_m,u}[y]\right)\right] \mathbb{I}\left(w^*_{\Omega_m,u}[y] \leq w_f\right) g^m(y)\, dy \mathbf{e}_\Omega \right.$$
$$\left. + q_m \int_{\underline{w}}^{w_f} H\left[\psi_m^c(y)\right]\left[1 - F_m\left(w^*_{\Omega_f,u}[y]\right)\right] \mathbb{I}\left(w^*_{\Omega_f,u}[y] \leq w_m\right) g^f(y)\, dy \mathbf{u}_\Omega\right\}$$

where $G^T(w_m, w_f)$ is the cumulative joint distribution of the dual employed earning where m earns $w \leq w_m$ and f earns $w \leq w_f$. $\mathbb{I}\left(w^*_{\Omega_s,u}[y] \leq w_{\neq s}\right)$ equals 1 when the reservation wage of the non-employed spouse in an s-headed worker-searcher household where the employed spouse is currently earning y is less than equal to $w_{\neq s}$ for $s \in \{m, f\}$.

As both w_m and $w_f \to \overline{w}$, we can implicitly recover the measure of dual employed from:

$$\sum_{i \in \{m,f\}} \delta_i \mathbf{e}_T = \left\{q_f \int_{\underline{w}}^{\overline{w}} H\left[\psi_f^c(y)\right]\left[1 - F_f\left(w^*_{\Omega_m,u}[y]\right)\right] \mathbb{I}\left(w^*_{\Omega_m,u}[y] \leq \overline{w}\right) g^m(y)\, dy \mathbf{e}_\Omega \right.$$
$$\left. + q_m \int_{\underline{w}}^{\overline{w}} H\left[\psi_m^c(y)\right]\left[1 - F_m\left(w^*_{\Omega_f,u}[y]\right)\right] \mathbb{I}\left(w^*_{\Omega_f,u}[y] \leq \overline{w}\right) g^f(y)\, dy \mathbf{u}_\Omega\right\}$$

Finally, the following accounting identity must hold in every period:

$$\mathbf{u}_u + \mathbf{e}_\Omega + \mathbf{u}_\Omega + e_T = 1$$

To derive the overall joint distribution of income, $G(i_m, i_f)$ and assuming measure 1 of type $\mathbf{x}$ households, for a given $\mathbf{x}$ joint household we use the following accounting identity:

$$g(i_m, i_f) = \begin{cases} \mathbf{u}_u & \text{if } i_m = Ab_m, \quad i_f = Ab_f \\ g^f(w)\mathbf{u}_\Omega & \text{if } i_m = Ab_m, \quad i_f = Aw \\ g^m(w)\mathbf{e}_\Omega & \text{if } i_m = Aw, \quad i_f = Ab_f \\ g^T(w_m, w_f)\mathbf{e}_T & \text{if } i_m = Aw_m, \quad i_f = Aw_f \end{cases} \Bigg\}$$

We then integrate over the joint density $g(i_m, i_f)$ to arrive at the cdf $G(i_m, i_f)$.

C Supermodularity and Sorting

Proof of Proposition 2. Optimal risk sharing implies:

$$\exp\left[\rho V_m^U(\mathbf{E}, \mathbf{x}; \alpha[\mathbf{E}, \mathbf{x}])\right] = (1 - \alpha[\mathbf{E}, \mathbf{x}]) \exp\left(\rho \widetilde{U}(\mathbf{E}, \mathbf{x})\right) \tag{27}$$

$$\exp\left[\rho V_f^U(\mathbf{E}, \mathbf{x}, \alpha[\mathbf{E}, \mathbf{x}])\right] = \alpha[\mathbf{E}, \mathbf{x}] \exp\left(\rho \widetilde{U}(\mathbf{E}, \mathbf{x})\right) \tag{28}$$

Denote the measure of marriages of type $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ as:

$$\mathcal{M}^m_{\mathcal{E}_m, \mathcal{E}_f, x_m, x_f} = m(\mathcal{E}_m, x_m)\, \pi(\mathcal{E}_f, x_f | \mathcal{E}_m, x_m, \alpha[\mathbf{E}, \mathbf{x}])$$

where $m(\mathcal{E}_m, x_m)$ is the measure of males with education $\mathcal{E}_m$ and skill x_m who marry and $\pi(\mathcal{E}_f, x_f | \mathcal{E}_m, x_m, \alpha[\mathbf{E}, \mathbf{x}])$ is the conditional probability a male chooses to marry a female of education $\mathcal{E}_f$ and skill x_f given that he has education-skill $(\mathcal{E}_m, x_m)$. This conditional probability takes the form of:

$$\pi_m(\mathcal{E}_f, x_f | \mathcal{E}_m, x_m, \alpha(\mathbf{E}, \mathbf{x})) = \frac{\exp\left[\exp\left(\rho V_m^U\left(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f; \alpha\left(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f\right)\right) + \bar{\chi}_m\left(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f\right)\right)\right]}{\sum_{\mathcal{E}_f^k} \sum_{x_f^k} \exp\left[\exp\left(\rho V_m^U\left(\mathcal{E}_m, \mathcal{E}_f^k, x_m, x_f^k; \alpha\left(\mathcal{E}_m, \mathcal{E}_f^k, x_m, x_f^k\right)\right)\right) + \bar{\chi}_m\left(\mathcal{E}_m, \mathcal{E}_f^k, x_m, x_f^k\right)\right]}$$

Consider a skill pair (x_m, x_f). Take the natural log of $\mathcal{M}^m_{\mathcal{E}_m, HS, x_m, x_f}$ and $\mathcal{M}^m_{\mathcal{E}_m, Col, x_m, x_f}$ and subtract them to get:

$$\begin{aligned} \log\left(\frac{\mathcal{M}^m_{\mathcal{E}_m, Col, x_m, x_f}}{\mathcal{M}^m_{\mathcal{E}_m, HS, x_m, x_f}}\right) &= \frac{\pi_m(Col, x_f \mid \mathcal{E}_m, x_m, \alpha[\mathcal{E}_m, Col, x_m, x_f])}{\pi_m(HS, x_f \mid \mathcal{E}_m, x_m, \alpha[\mathcal{E}_m, HS, x_m, x_f])} \\ &= (1 - \alpha[\mathcal{E}_m, Col, x_m, x_f]) \exp\left[\rho \widetilde{U}(\mathcal{E}_m, Col, x_m, x_f)\right] \\ &\quad - (1 - \alpha[\mathcal{E}_m, HS, x_m, x_f]) \exp\left[\rho \widetilde{U}(\mathcal{E}_m, HS, x_m, x_f]\right) \\ &\quad + \bar{\chi}_m(\mathcal{E}_m, Col, x_m, x_f) - \bar{\chi}_m(\mathcal{E}_m, HS, x_m, x_f) \end{aligned}$$

The above is the relative probability that a married male of education $\mathcal{E}_m$ and skill x_m chooses to marry a college educated female over a high school educated female of skill x_f.

We apply the same calculations for women. Let $\mathcal{M}^f_{\mathcal{E}_m,\mathcal{E}_f,x_m,x_f}$ be the measure of marriages between a female of education-skill type $(\mathcal{E}_f, x_f)$ to males of education-skill type $(\mathcal{E}_m, x_m)$.

$$\begin{aligned}
\log\left(\frac{\mathcal{M}^f_{Col,\mathcal{E}_f,x_m,x_f}}{\mathcal{M}^f_{HS,\mathcal{E}_f,x_m,x_f}}\right) &= \frac{\pi_f(Col, x_m \mid \mathcal{E}_f, x_f, \alpha[Col, \mathcal{E}_f, x_m, x_f])}{\pi_f\left(HS, x_m \mid \mathcal{E}_f, x_f, \alpha[HS, \mathcal{E}_f, x_m, x_f]\right)} \\
&= \alpha\left(Col, \mathcal{E}_f, x_m, x_f\right)\exp[\rho\widetilde{U}\left(Col, \mathcal{E}_m, x_m, x_f\right)] \\
&\quad -\alpha\left(HS, \mathcal{E}_f, x_m, x_f\right)\exp[\rho\widetilde{U}\left(HS, \mathcal{E}_f, x_m, x_f\right)] \\
&\quad +\bar{\chi}_f\left(Col, \mathcal{E}_f, x_m, x_f\right) - \bar{\chi}_f\left(HS, \mathcal{E}_f, x_m, x_f\right)
\end{aligned}$$

The above is the corresponding relative probability that a married female of education-skill $(\mathcal{E}_f, x_f)$ chooses to marry a college male over a high school educated male of skill x_m.

Holding fixed a skill pair $(x_m, x_f) = \mathbf{x}$, and summing across the relative probabilities of marrying a spouse of the same education level, we arrive at:

$$\begin{aligned}
\log\left(\frac{\mathcal{M}^m_{Col,Col,\mathbf{x}}}{\mathcal{M}^m_{Col,HS,\mathbf{x}}}\cdot\frac{\mathcal{M}^m_{HS,HS,\mathbf{x}}}{\mathcal{M}^m_{HS,Col,\mathbf{x}}}\cdot\frac{\mathcal{M}^f_{Col,Col,\mathbf{x}}}{\mathcal{M}^f_{HS,Col,\mathbf{x}}}\cdot\frac{\mathcal{M}^f_{HS,HS,\mathbf{x}}}{\mathcal{M}^f_{Col,HS,\mathbf{x}}}\right) &= \rho\widetilde{U}\left(Col, Col, \mathbf{x}\right) + \rho\widetilde{U}\left(HS, HS, \mathbf{x}\right) \\
&- \rho\widetilde{U}\left(HS, Col, \mathbf{x}\right) - \rho\widetilde{U}\left(Col, HS, \mathbf{x}\right) \\
&+ \mathbb{X} \\
&= D_{\mathcal{E}}(\mathcal{S}|\mathbf{x}) + \mathbb{X}
\end{aligned}$$

where $D_{\mathcal{E}}(\mathcal{S}|\mathbf{x})$ is the measure of supermodularity across education for a fixed skill pair $\mathbf{x}$ of the *net* economic marital gains to marriage and $\mathbb{X}$ denotes the sum of the marital preference gain terms. In equilibrium, the $\alpha\left(\mathbf{E}, \mathbf{x}\right)$ adjust to clear markets, such that $\mathcal{M}^f_{\mathbf{E},\mathbf{x}} = \mathcal{M}^m_{\mathbf{E},\mathbf{x}} = \mathcal{M}_{\mathbf{E},\mathbf{x}}$. Thus,

$$\begin{aligned}
D_{\mathcal{E}}(\mathcal{S}|\mathbf{x}) + \mathbb{X} &= \log\left(\left(\frac{\mathcal{M}_{Col,Col,\mathbf{x}}}{\mathcal{M}_{Col,HS,\mathbf{x}}}\right)^2\cdot\left(\frac{\mathcal{M}_{HS,HS,\mathbf{x}}}{\mathcal{M}_{HS,Col,\mathbf{x}}}\right)^2\right) \qquad (29) \\
&= 2\left[\sum_{\mathcal{E}_m\in\{HS,Col\}}\log\left(\mathcal{M}_{\mathcal{E}_m,\mathcal{E}_f=\mathcal{E}_m,\mathbf{x}}\right) - \log\left(\mathcal{M}_{\mathcal{E}_m,\mathcal{E}_f\neq\mathcal{E}_m,\mathbf{x}}\right)\right]
\end{aligned}$$

For given skill pair $(x_m, x_f) = \mathbf{x}$, suppose now that $\mathcal{M}_{\mathcal{E}_m,\mathcal{E}_f,\mathbf{x}} = \frac{m(\mathcal{E}_m,x_m)}{\sum_{\mathcal{E}_m} m(\mathcal{E}_m,x_m)}\frac{m\left(\mathcal{E}_f,x_f\right)}{\sum_{\mathcal{E}_f} m\left(\mathcal{E}_f,x_f\right)}$, $\forall\mathcal{E}_m, \mathcal{E}_f \in \{HS, Col\}$, which is the allocation under random matching. Plugging this into the right hand side of Equation (29) gives a zero which implies that $D_{\mathcal{E}}(\mathcal{S}|\mathbf{x}) + \mathbb{X}$ is zero, we arrive at the random matching allocation. Finally, suppose we move from some $D_{\mathcal{E}}(\mathcal{S}|\mathbf{x})$ to some $D'_{\mathcal{E}}(\mathcal{S}|\mathbf{x}) > D_{\mathcal{E}}(\mathcal{S}|\mathbf{x})$. Holding all else constant, such an increase implies that the (log) measure of symmetric joint households where individuals are married to their counterparts in lke education, $\sum_{\mathcal{E}_m}\log\mathcal{M}_{\mathcal{E}_m,\mathcal{E}_f=\mathcal{E}_m,\mathbf{x}}$, must rise relative to the (log) measure of joint households where individuals are married to a partner with a different education level, $\sum_{\mathcal{E}_m} \log \mathcal{M}_{\mathcal{E}_m, \mathcal{E}_f \neq \mathcal{E}_m, \mathbf{x}}$. In other words, when the measure $D_{\mathcal{E}}(\mathcal{S}|\mathbf{x})$ rises, there is more positive sorting ceteris paribus. The increase in $D_{\mathcal{E}}(\mathcal{S}|\mathbf{x})$ supports more positive sorting along education amongst joint households of skill $\mathbf{x}$. A similar argument can be made for $D_x(\mathcal{S}|\mathbf{E})$. $\square$

D Data

D.1 Baseline Samples

We use data from the March CPS and Basic monthly files for the years 1981-1989 for the 1980's estimation and 2000-2007 for the 2000's estimation. Our sample consists of singles between the ages of 25-54 and married couples in which both spouses are within the same age range. We drop single individuals who are missing information on their hourly wages when employed, and married couples in which either spouse satisfies the same criterion. We also drop singles who are missing labor force status information and couples in which either spouse is missing this information. To construct household income, we take hourly wage income for singles if they are employed and impose an income of zero if they are not employed. For couples, we take the geometric mean of hourly wages, where we replace a spouse's hourly wage with a zero if they are not employed. To construct hourly wage income, we divide individual income by reported hours or by 40 hours (if full time employed and missing hours) and 25 hours (if missing hours but reported part-time). Finally, we winsorize the top and bottom 1% of individual hourly wages within each year, education, sex, and marital status bin.

D.2 Cohort-Based Samples

The cohort-based samples follow the same cleaning, except we do not pool years 1981-1989 and 2001-2007. Instead, our 1980's sample consists of single individuals who are ages 40-49 in 1985, 41-50 in 1986, and so on, up until those who are 44-53 in 1989. For married individuals, we keep couples for which both spouses are either ages 40-49 in 1985, 41-50 in 1986, and so on, up until couples in which both spouses are 44-53 in 1989. We do the same thing for the 2000s sample (individuals ages 40-49 in 2000, etc). Otherwise, the sample construction is the same in both datasets.

E Labor Market Estimation

E.1 EM algorithm for Labor Market Block

Consider a particular education and gender. Given data on $\mathcal{N}_m$ single males with education $\mathcal{E}$, of which $\mathcal{N}_m^E$ are employed, $\mathcal{N}_m^U$ are non-employed in the March Current Population Survey. Within $\mathcal{N}_m^U$ non-employed, suppose a subset of these of mass $\mathcal{N}_m^{OLF}$ are out of the labor force (OLF). Further, for those employed we observe their hourly wage w_i. Since there exists both high and low-skilled individuals in across the two education groups, we make the identifying assumption that the reservation wage $w^{sin_s^\star}(\mathcal{E}_s, L)$ equals the minimum observed wage in high school and college education groups for that gender s. We use the inflow-outflow equations established in Section B to solve for the probability that a randomly picked single male of education x earns a wage $\leq w$, $G^{sin}(w, \mathcal{E}_s, x_s)$, the probability that he is non-employed $\mathbf{u}^{sin}(\mathcal{E}_s, x_s)$ and the probability that a non-employed individual is out of the labor force, $H(\psi_m^{sin}(\mathcal{E}_m, x_m); x_m)$. We back out $A(\mathcal{E}_s = Col)$ by setting it equal to the college premium, i.e., ratio of mean college wage to mean high school wage:

$$A(\mathcal{E}_s) \mid_{\mathcal{E}_s = Col} = \frac{\text{mean college wage}}{\text{mean high school wage}}$$

We assume that $A(\mathcal{E}_s = HS) = 1$. Finally, we assume that all individuals draw wages from the same distribution $F_s(w)$ within gender. From the perspective of our model, the earnings of college individuals in the data include the college premium $A(\mathcal{E}_s = Col)$, we extract this component and divide the wages of college individuals by $A(\mathcal{E}_s = Col)$. This leaves us with only the accepted wage offers of college individuals, rather than the earnings of college individuals that is augmented by their higher effective labor input.

We implement the following procedure for each individual of gender $s \in \{m, f\}$, education $\mathcal{E}_s \in \{HS, Col\}$ and skill $x_s \in \{H, L\}$:

1 **Expectation step**: for each education-gender combination, guess the shares of underlying skill types: $p(x_s \mid \mathcal{E}_s)$, with the requirement that $p(L \mid \mathcal{E}_s) + p(H \mid \mathcal{E}_s) = 1$

2 **Maximization step**: given this initial guess, solve for $\delta^{sin}(x_s)$ such that the model-implied employment-to-non-employment (EN) rate is equal to its empirical counterpart, that is, we solve:

$$\delta^{sin}(\mathcal{E}_s) = \sum_{x_s} \delta^{sin}(x_s) p\left(x_s \mid \mathcal{E}_s\right)$$

3 Given $p(x_s \mid \mathcal{E}_s)$, and $\delta^{sin}(x_s)$, we then guess $\{q_s(x), \mu_s, \sigma_s\}$. Consider the following log likelihood:

$$\begin{aligned} \log L_s \;=\; & \sum_{x_s} \sum_{\mathcal{E}_s} p\left(x_s \mid \mathcal{E}_s\right) \Big[\mathcal{N}_s^U\left(\mathcal{E}_s\right) \log \mathbf{u}^{sin}(\mathcal{E}_s, x_s) \\ & + \mathcal{N}^E\left(\mathcal{E}_s\right) \log\left(1 - \mathbf{u}^{sin}(\mathcal{E}_s, x_s)\right) + \sum_{i \in \mathcal{N}^E(\mathcal{E}_s)} \log g^{sin}\left(w_i; \mathcal{E}_s, x_s\right) \Big] \end{aligned}$$

where $\mathbf{u}^{sin}(\mathcal{E}_s, x_s)$ is as defined in equation 24, $g^{sin}(w_i; \mathcal{E}_s, x_s)$ is the associated density of individual i earning wage and is as defined in equation 26.

4 Within each iteration and given guess of $\{q(x_s), \mu_s, \sigma_s\}$, we jointly solve the following conditions to recover $\{b_s, \lambda(x_s), \eta(x_s)\}$. Since we set $w_s^{sin*}(\mathcal{E}_s, L)$ to equal the minimum

reservation wage for gender s, we can use Equation 19 to back out b_s. Similarly, once we have b_s, we can use the same equation 19 to back out $w_s^{sin*}(\mathcal{E}_s, H)$. We choose $\lambda(x_s)$ such that the model-implied job-to-job transition rates by gender and education replicate their empirical counterparts. The model implied job-to-job rate (JJ) is given by equation 21. Using integration by parts and a change of variable where we define $k = F_s(y)$:

$$JJ(\mathcal{E}_s) = \sum_{x_s} \lambda(x_s) p(x_s \mid \mathcal{E}_s) \frac{\delta^{sin}(x_s)}{1 - F_s^\star} \int_{F_s^\star}^{1} \frac{k - F_s^\star}{\delta^{sin}(x_s) + \lambda(x_s)[1-k]} dk$$

where for ease of notation, we denote $F_s^* = F_s(w^{sin*}[\mathcal{E}_s, x_s])$. Finally, we choose $\eta(x_s)$ such that the model-implied OLF share is equal to its empirical counterpart. Assuming measure 1 of individuals, this implies making sure that Equation (25) is equal to the OLF share in the data. To compute this, we note that the threshold participation disutility is given by:

$$\psi_s^{sin*}(\mathcal{E}_s, x_s) = q(x_s) \int_{w^{sin}(\mathcal{E}_s, x_s)}^{\overline{w}} \frac{2}{y} \frac{1 - F_s(y)}{\rho + \delta(x_s) + \lambda(x_s)[1 - F_s(y)]} dy$$

Given $\psi_s^*(\mathcal{E}_s, x_s)$, we can recover the OLF share.

4 Having found the implied $\{b_s, \lambda(x_s), \eta(x_s)\}$, we choose $\{q(x_s), \mu_s, \sigma_s\}$ to maximize the log-likelihood above.

5 Having recovered parameters $\{\delta(x_s), q(x_s), \mu_s, \sigma_s, \lambda(x_s), \eta(x_s), b_s\}$ and endogenous variables $\{w_s^{sin}(\mathcal{E}_s, H), \psi_s^{sin}(\mathcal{E}_s, x_s)\}$ for all education and skill types, we then update the probability of having skill x. Specifically, in the *expectation* step, we update the probability with $p'(x_s \mid \mathcal{E}_s)$:

$$\begin{aligned} p'(x_s \mid \mathcal{E}_s) &= \frac{1}{N} \Bigg\{ \sum_{\mathbf{i}} \mathbb{I}_{\mathbf{E}}(\mathbf{w_i}) p(x_s \mid \text{employed at } w_i, \mathcal{E}_s) \\ &\quad + \sum_i (\mathbf{1} - \mathbb{I}_{\mathbf{E}}) p(x_s \mid \text{non-employed}, \mathcal{E}_s) \Bigg\} \end{aligned}$$

where $\mathbb{I}_E(w_i)$ is an indicator function that equals to one if individual i of gender s and education $\mathcal{E}_s$ is employed at wage w_i and $\mathbb{I}_E$ is an indicator function that is equal to one if the individual is employed. $p(x_s \mid \text{non-employed}, \mathcal{E}_s)$ is given by our model-implied non-employment rate $\mathbf{u}^{sin}(\mathcal{E}_s, x_s)$ while $p(x_s \mid \text{employed at } w_i, \mathcal{E}_s)$ is given by $g^{sin}(w_i, \mathcal{E}_s, x_s)$

6 We repeat this procedure until we arrive at the estimated parameters and shares that best explain the data for singles.

E.2 Model Fit: Wage distribution

Figures 1 and 2 show the fit of our model-implied wage distribution against that observed in the data for the respective time periods of 1980s and 2000s.

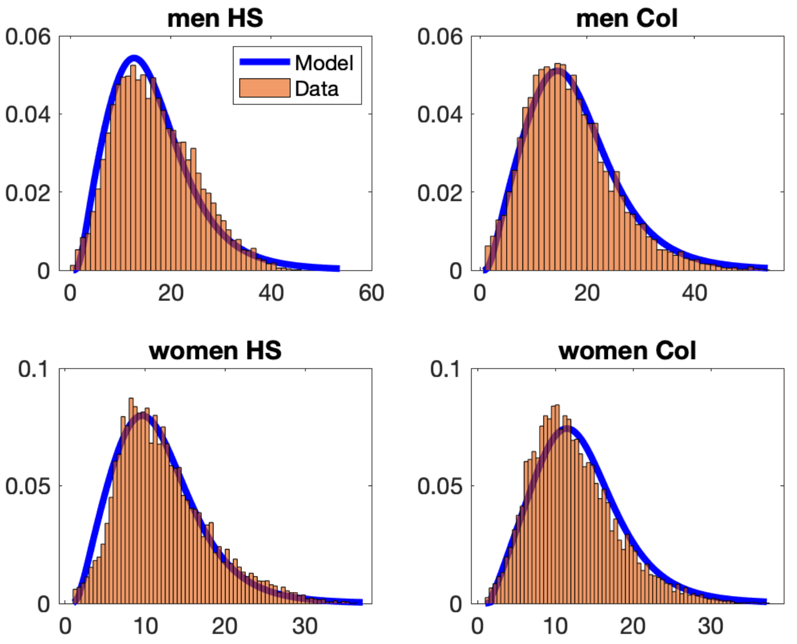

Figure 1: Model fit: 1980s wage distribution

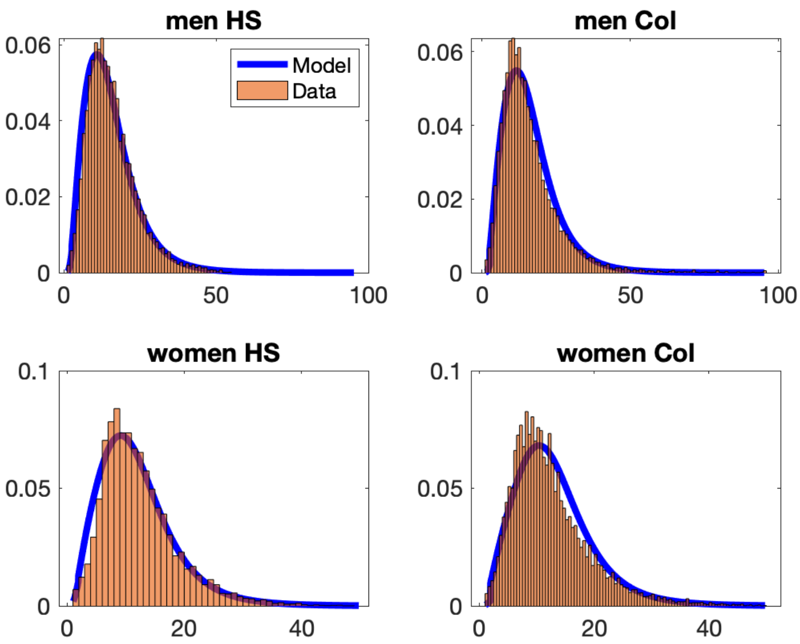

Figure 2: Model fit: 2000s wage distribution

E.3 Joint household estimation of labor market parameters

Following the same notation as in the model where $\mathcal{N}_s$ is the measure of individuals of gender s, we denote $\mathcal{N}_m(\mathcal{E}_m, \mathcal{E}_s) = \mathcal{N}_f(\mathcal{E}_m, \mathcal{E}_f) = \mathcal{N}(\mathcal{E}_m, \mathcal{E}_f)$ as the measure of individuals of gender s in joint households of education pair $(\mathcal{E}_m, \mathcal{E}_f)$. Further denote $\mathcal{N}^{ab}(\mathcal{E}_m, \mathcal{E}_f)$ for $a, b \in \{u, e\}$ be the measure of individuals in joint households of education pair $(\mathcal{E}_m, \mathcal{E}_f)$ with joint employment status ab.

We jointly choose $\delta^{mar}(x_s)$ and $p(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f)$ to match the employment statuses and mean incomes of joint households within an education pair. In the data we observe the share of households in each education pair that are in dual non-employment, in a male worker-searcher household, and in a female worker-searcher household. This give us three moments for each education pair to help pin down $p(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f)$ for each $x_m, x_f \in \{H, L\}$. Specifically, we observe:

$$\frac{\mathcal{N}^{uu}(\mathcal{E}_m, \mathcal{E}_f)}{\mathcal{N}(\mathcal{E}_m, \mathcal{E}_f)} = \sum_{x_m \in \{H,L\}} \sum_{x_f \in \{H,L\}} u_u\left(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f\right) p\left(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f\right) \tag{30}$$

$$\underbrace{\frac{\mathcal{N}^{eu}(\mathcal{E}_m, \mathcal{E}_f)}{\mathcal{N}(\mathcal{E}_m, \mathcal{E}_f)}}_{\text{male worker-searcher}} = \sum_{x_m \in \{H,L\}} \sum_{x_f \in \{H,L\}} e_\Omega\left(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f\right) p\left(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f\right) \tag{31}$$

$$\underbrace{\frac{\mathcal{N}^{ue}(\mathcal{E}_m, \mathcal{E}_f)}{\mathcal{N}(\mathcal{E}_m, \mathcal{E}_f)}}_{\text{female worker-searcher}} = \sum_{x_m \in \{H,L\}} \sum_{x_f \in \{H,L\}} u_\Omega\left(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f\right) p\left(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f\right) \tag{32}$$

We further use the fact that $\sum_{x_m \in \{H,L\}} \sum_{x_f \in \{H,L\}} p(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f) = 1$ to reduce one of the parameters we have to solve for in each $(\mathcal{E}_m, \mathcal{E}_f)$ education pair. Notably, separation risks also affect joint households' employment statuses via the model-implied rates u_u, u_Ω, e_Ω. Thus, we also target the mean income within a an education pair and use it to pin down $\delta^{mar}(x_s)$. The mean income within an education pair is given by:

$$\mathcal{I}(\mathcal{E}_m, \mathcal{E}_f) = \sum_{x_m \in \{H,L\}} \sum_{x_f \in \{H,L\}} \sum_i \sum_j (w_i + w_j) g^{mar}(w_i, w_j | \mathcal{E}_m, \mathcal{E}_f, x_m, x_f) p(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f)$$

We choose $\delta^{mar}(x_s)$ and $p(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f)$ to minimize the distance between our model-implied and empirical moments.

F Choice Probabilities and Identification for Marriage Market Block

Consider an individual of gender m with skill x_m. Note that an individual's skill is not a choice but rather a fixed effect. There are ten choices available for m with skill x_m: first, he can choose to remain single and his education level. This constitutes two choices. Alternatively, he can choose to obtain some education $\mathcal{E}_m$ and form a joint household with a female of skill x_f and education $\mathcal{E}_f$. Conditional on his education $\mathcal{E}_m$, this constitutes another 4 choices. Since $\mathcal{E}_m \in \{HS, COL\}$, this makes it a total of 8 choices of joint household pairs. Given his skill x_m, the the choice probabilities for the male for each $(\mathcal{E}_m, \mathcal{E}_f, x_f)$ combination (inclusive of singlehood where $\mathcal{E}_f, x_f = \emptyset$) can be characterized as:

$$\pi_m(\mathcal{E}_m, \mathcal{E}_f, x_f|x_m) = \frac{\exp\left(\frac{\widehat{V}_m^U(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)+\bar{\chi}_m(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)}{\gamma}\right)}{\sum_{\mathcal{E}_m}\sum_{\mathcal{E}_f}\sum_{x_f}\exp\left(\frac{\widehat{V}_m^U(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)+\bar{\chi}_m(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)}{\gamma}\right)}$$

where γ is the standard Euler constant, and $\widehat{V}_m^U(\mathbf{E}, \mathbf{x}) = \exp\left(\rho V_m^U\left[\mathbf{E}, \mathbf{x}|\alpha(\mathbf{E}, \mathbf{x})\right]\right)$ is the value the male gets from being in that joint household given the associated Pareto weight. Thus for a male with skill x_m, we can now write down the probability of being in a household of type $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ relative to the probability of being single with a high school level of education. For m, this takes the form of:

$$\frac{\pi_m(\mathcal{E}_m, \mathcal{E}_f, x_f|x_m)}{\pi_m(HS, \emptyset, \emptyset|x_m)} = \frac{\exp\left(\frac{\widehat{V}_m^U(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)+\bar{\chi}_m(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)}{\gamma}\right)}{\exp\left(\frac{\widehat{V}_m^U(\mathcal{E}_m,\emptyset,x_m,\emptyset)+\bar{\chi}_m(\mathcal{E}_m,\emptyset,x_m,\emptyset)}{\gamma}\right)}$$

with a similar equation for females.

In the data, we do not observe the choice probabilities conditional on skill. However, using our estimated model, we are able to recover the joint probability of observing a household of $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ type. From our estimation for joint households, we recovered $p(x_m, x_f|\mathcal{E}_m, \mathcal{E}_f)$. The joint probability of an individual of sex s being in household $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ is given by

$$p_s\left(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f\right) = p\left(x_m, x_f \mid \mathcal{E}_m, \mathcal{E}_f\right)\frac{\mathcal{N}_s\left(\mathcal{E}_m, \mathcal{E}_f\right)}{\mathcal{N}_s} \tag{33}$$

where $\frac{\mathcal{N}_s\left(\mathcal{E}_m,\mathcal{E}_f\right)}{\mathcal{N}_s}$ is the empirical share of sex s that is in a joint household of education pair $(\mathcal{E}_m, \mathcal{E}_f)$. Because we observe all the empirical shares of sex s in a particular education-pair and across marital status, and since we have the underlying skill shares conditional on an education pair, as well as conditional on being single with a particular education, we can compute the left-hand-side of equation 33. Summing $p_s\left(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f\right)$ across $(\mathcal{E}_m, \mathcal{E}_f, x_f)$, we recover the population share of males that have skill x_m, $p_s(x_m)$. This then allows us to write the conditional probabilities $p_s(\mathcal{E}_m, \mathcal{E}_f, x_f \mid x_m) = p_s(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)/p_s(x_m)$.

Given these choice probabilities, we choose $\bar{\chi}_m(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ such that the distance between our model-implied ratio $\frac{\pi_m(\mathcal{E}_m,\mathcal{E}_f,x_f|x_m)}{\pi_m(HS,\emptyset,\emptyset|x_m)}$ and the ratio implied in the data $\frac{p_m(\mathcal{E}_m,\mathcal{E}_f,x_f|x_m)}{p_m(HS,\emptyset,\emptyset|x_m)}$ is minimized. In doing so, we normalize $\bar{\chi}_m(HS, \emptyset, L, \emptyset) = 0$ and $\bar{\chi}_m(HS, \emptyset, H, \emptyset) = 0$, and restrict $\bar{\chi}_m(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) = \bar{\chi}_f(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ for joint households. For singles, we allow $\bar{\chi}_m(\mathcal{E}_m, \emptyset, x_m, \emptyset) \neq \bar{\chi}_f(\emptyset, \mathcal{E}_f, \emptyset, x_f)$. This leaves us with 22 preference parameters to estimate for both males and females across all skill types.

Finally, to recover $\alpha(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$, we require that the measure of males in a household with $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ characteristics must equal the measure of females in that household. Denote the measure of males in a particular education-skill pair as: $\mathcal{N}_m(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$. Then, dividing both sides by the total number of females, the following accounting identity must be true:

$$\begin{aligned}
\mathcal{N}_m(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) &= \mathcal{N}_f(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) \\
\frac{\mathcal{N}_m(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)}{\mathcal{N}_m}\frac{\mathcal{N}_m}{\mathcal{N}_f} &= \frac{\mathcal{N}_f(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)}{\mathcal{N}_f} \\
p_m(\mathcal{E}_m, \mathcal{E}_f, x_f|x_m)\, p_m(x_m)\frac{\mathcal{N}_m}{\mathcal{N}_f} &= p_f(\mathcal{E}_m, \mathcal{E}_f, x_m|x_f)\, p_f(x_f)
\end{aligned} \tag{34}$$

The ratio $\mathcal{N}_m/\mathcal{N}_f$ reflects the fact that the population of males and females need not be the same in the data. Since the joint probabilities are a product of the conditional choice probability and the population share of x_s, we can solve for the underlying Pareto weights by solving the following equation for each education-skill pair:

$$\pi_m(\mathcal{E}_m, \mathcal{E}_f, x_f|x_m)\, p_m(x_m)\frac{\mathcal{N}_m}{\mathcal{N}_f} = \pi_f(\mathcal{E}_m, \mathcal{E}_f, x_m|x_f)\, p_f(x_f)$$

where $\pi_m(\mathcal{E}_m, \mathcal{E}_f, x_f|x_m)$ is the model-implied choice probability conditional on skill.

Table 10 shows how our model's estimated shares of males and females across different education and household types compares against their empirical counterparts across the two time periods. Because we only observe education in the data, we aggregate across skill to derive the final share in each household and education category.

Probability shares across education and marital status								
	1980s				2000s			
	M		F		M		F	
	data	model	data	model	data	model	data	model
(HS,HS)	0.36	0.36	0.32	0.32	0.19	0.19	0.17	0.17
(HS,Col)	0.07	0.07	0.06	0.06	0.09	0.09	0.09	0.09
(Col,HS)	0.13	0.13	0.11	0.11	0.08	0.08	0.08	0.08
(Col,Col)	0.24	0.24	0.22	0.22	0.36	0.36	0.33	0.33
(HS,∅)	0.10	0.10	0.17	0.17	0.12	0.12	0.14	0.14
(Col,∅)	0.11	0.11	0.11	0.11	0.16	0.16	0.20	0.20

Notes: Columns 1-4 shows how the model-implied and empirical distributions of household across education and marital status for the 1980s. Column 5-8 shows the same distributions for the 2000s.

Table 10: Estimated probabilities shares across education and household type

G Complementarities in skill and education

To calculate the relative gain from matching with a high skilled individual than from matching with a highly educated individual, we proceed with the following exercise, fix the skill and education of the male individual. We then calculate whether the economic surplus for this male is increasing more if he is married to a high-skilled high school female, or if it increases by more if he marries a highly-educated low-skilled female. To do this, we define the following:

$$\Delta\mathcal{S}_{x_f}(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) = \mathcal{S}(\mathcal{E}_m, HS, x_m, H) - \mathcal{S}(\mathcal{E}_m, HS, x_m, L)$$

$$\Delta\mathcal{S}_{\mathcal{E}_f}(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) = \mathcal{S}(\mathcal{E}_m, Col, x_m, L) - \mathcal{S}(\mathcal{E}_m, HS, x_m, L)$$

The relative gain for a male with education $\mathcal{E}_m$ and skill x_m from matching with a high skilled female than from matching with a highly educated female is given by the ratio

$$(\text{Relative Gain} \mid \mathcal{E}_m, x_m) = \Delta\mathcal{S}_{x_f}(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)/\Delta\mathcal{S}_{\mathcal{E}_f}(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f) \tag{35}$$

Note that a ratio greater than 1 implies that the male with education $\mathcal{E}_m$ and skill x_m has a larger gain from matching with a high-skilled female, than with a highly-educated female. We further define the change in this relative gain with respect to the male's skill as:

$$\Delta_x(\text{Relative Gain} \mid \mathcal{E}_m, x_m) = (\text{Relative Gain} \mid \mathcal{E}_m, H) - (\text{Relative Gain} \mid \mathcal{E}_m, L) \tag{36}$$

A positive value for $\Delta_x(\text{Relative Gain} \mid \mathcal{E}_m, x_m)$ implies that this relative gain is larger for high-skilled males than for low-skilled males of the same education level $\mathcal{E}_m$. Note that this measure is similar to the cross-partial derivative in Postel-Vinay and Lindenlaub (2017). Finally, note that $(\text{Relative Gain} \mid \mathcal{E}_f, x_f)$ and $\Delta_x(\text{Relative Gain} \mid \mathcal{E}_f, x_f)$ are the analogous measures for females.

Columns 1,2,4 and 5 of Table 11 shows that for all gender-education-skill combinations, the gain from matching with a high-skilled partner outweighs the gain from matching with a highly-educated partner. Further, this gain is increasing in one's skill, suggesting there exist strong complementarities in skill.

Panel A: Relative gain from matching with high skill vs. high education						
	1980s			2000s		
	L	H	Δ_x Relative Gain	L	H	Δ_x Relative Gain
M HS	1.95	2.11	0.16	1.64	1.69	0.05
M Col	1.98	2.13	0.15	1.67	1.70	0.03
F HS	2.62	2.90	0.28	1.72	1.81	0.09
F Col	2.68	3.02	0.34	1.75	1.87	0.12

Notes: Columns 1,2,4 and 5 represent the relative gain from matching with a high-skilled partner vs. matching with a highly-educated partner, i.e., $\Delta\mathcal{S}_{x_s}(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)/\Delta\mathcal{S}_{\mathcal{E}_s}(\mathcal{E}_m,\mathcal{E}_f,x_m,x_f)$, for the respective gender s and education-skill combination. Columns 3 and 6 represent the change in the relative gain across the individual's skill x.

Table 11: Relative gains from matching with high-skill vs. with high education

Appendix For Online Publication

H Gross vs. Net values

We show how we derived our continuous time value functions net of current disutility costs from end-of-period value functions in a discrete time setting. We then take limits to arrive at our continuous time set-up.

Consider the following timing: A) at the start of the period, the non-employed draw ψ, B) separations occur, and the newly separated cannot search since they did not receive a chance to draw ψ which affects their decision of whether to enter the labor force, C) search and matching occurs, and finally D) there is production and consumption. We assume that no two events can happen at the same time in the discrete time version, as in the continuous time case. We focus on the problem for single households because the derivation is similar for joint households. For ease of exposition, we suppress all dependence on gender s, education $\mathcal{E}_s$ and skill x_s. Below shows the gross and net values of singles in discrete time.

H.1 Gross Values

H.1.1 Gross Value of Non-employed Singles

Let a period be of Δ length. Consider the gross end-of-period value of a non-employed single individual who participated in period t, but failed to find a job within that period.

$$\mathcal{U}_t^{sin}\left(\psi_t,1\right)=-\psi_t\Delta+\nu\left(c_t^*\left[b\right],Q_t^*\left[b\right]\right)\Delta+(1-\rho\Delta)\int_{\underline{\psi}}^{\overline{\psi}}\max\left\{\mathcal{A}_{t+\Delta}^{sin}\left(\psi_{t+\Delta}\right),\mathcal{B}_{t+\Delta}^{sin}\left(\psi_{t+\Delta}\right)\right\}dH\left(\psi_{t+\Delta}\right)$$

where the first argument in $\mathcal{U}_t^{sin}$ is the realized value of disutility, and the second argument takes a value of 1 when the individual chooses to incur the disutility and 0 otherwise.

$$\mathcal{A}_{t+\Delta}^{sin}\left(\psi_{t+\Delta}\right)=\mathcal{U}_{t+\Delta}^{sin}\left(\psi_{t+\Delta},1\right)+q\Delta\int_{w_u^{sin,*}}^{\overline{w}}\left[\mathcal{T}_{t+\Delta}^{sin}\left(y,\psi_{t+\Delta}\right)-\mathcal{U}_{t+\Delta}^{sin}\left(\psi_{t+\Delta},1\right)\right]dF\left(y\right)$$

and

$$\mathcal{B}^{sin}_{t+\Delta} = \mathcal{U}^{sin}_{t+\Delta}\left(\psi_{t+\Delta}, 0\right)$$

Now consider the gross end-of-period value of a non-employed who chose not to enter the labor market.

$$\mathcal{U}^{sin}_{t}\left(\psi_{t}, 0\right) = \nu\left(c^{*}_{t}\left[b\right], Q^{*}_{t}\left[b\right]\right)\Delta + \left(1-\rho\Delta\right)\int_{\underline{\psi}}^{\overline{\psi}} \max\left\{\mathcal{A}^{sin}_{t+\Delta}\left(\psi_{t+\Delta}\right), \mathcal{B}^{sin}_{t+\Delta}\left(\psi_{t+\Delta}\right)\right\} dH\left(\psi_{t+\Delta}\right)$$

H.1.2 Gross Value of Employed Singles

Consider the value of a newly employed single with wage w. The newly employed individual is one who incurred the disutility ψ_t as she was initially non-employed at the start of the period, but was successful in her job-search and hence employed by the end of period.

$$\begin{aligned}
\mathcal{T}^{sin}_{t}\left(w, \psi_{t}\right) &= -\psi_{t}\Delta + \nu\left(c^{*}_{t}\left[w\right], Q^{*}_{t}\left[w\right]\right)\Delta + \left(1-\rho\Delta\right)\mathcal{T}^{sin}_{t+\Delta}\left(w, 0\right) \\
&\quad + \left(1-\rho\Delta\right)\delta^{sin}\Delta\int_{\underline{\psi}}^{\overline{\psi}}\left[\mathcal{U}^{sin}_{t+\Delta}\left(\psi_{t+\Delta}, 0\right) - \mathcal{T}^{sin}_{t+\Delta}\left(w, 0\right)\right] dH\left(\psi_{t+\Delta}\right) \\
&\quad + \left(1-\rho\Delta\right)q\Delta\int_{w}^{\overline{w}}\left[\mathcal{T}^{sin}_{t+\Delta}\left(y, 0\right) - \mathcal{T}^{sin}_{t+\Delta}\left(w, 0\right)\right] dF\left(y\right)
\end{aligned}$$

Consider the value of a continuously employed single. Unlike newly employed singles, continuously employed individuals are already in the labor force and hence incur 0 disutility.

$$\begin{aligned}
\mathcal{T}^{sin}_{t}\left(w, 0\right) &= \nu\left(c^{*}_{t}\left[w\right], Q^{*}_{t}\left[w\right]\right)\Delta + \left(1-\rho\Delta\right)\mathcal{T}^{sin}_{t+\Delta}\left(w, 0\right) \\
&\quad + \left(1-\rho\Delta\right)\delta^{sin}\Delta\int_{\underline{\psi}}^{\overline{\psi}}\left[\mathcal{U}^{sin}_{t+\Delta}\left(\psi_{t+\Delta}, 0\right) - \mathcal{T}^{sin}_{t+\Delta}\left(w, 0\right)\right] dH\left(\psi_{t+\Delta}\right) \\
&\quad + \left(1-\rho\Delta\right)q\Delta\int_{w}^{\overline{w}}\left[\mathcal{T}^{sin}_{t+\Delta}\left(y, 0\right) - \mathcal{T}^{sin}_{t+\Delta}\left(w, 0\right)\right] dF\left(y\right)
\end{aligned}$$

H.2 Net Values

Define the value of non-employment net of disutility as:

$$U^{sin}_{t} = \mathcal{U}^{sin}_{t}\left(\psi_{t}, \mathbb{I}_{t}\left[\psi_{t}\right]\right) + \psi_{t}\Delta\mathbb{I}_{t}\left[\psi_{t}\right]$$

where $\forall\psi_t, \mathbb{I}_t\left[\psi_t\right] = 1$ if $\mathcal{U}^{sin}_{t}\left(\psi_t, 1\right) \geq \mathcal{U}^{sin}_{t}\left(\psi_t, 0\right)$. Because ψ_t is a disutility cost, we add it back in the above to net it out from the gross value, $\mathcal{U}^{sin}_{t}(\psi_t, \mathbb{I}_t[\psi_t])$

H.2.1 Net value of non-employed

Then net value (net of disutility) of non-employed at end of period who participated becomes:

$$U^{sin}_{t} = \mathcal{U}^{sin}_{t}\left(\psi_{t}, 1\right) + \psi_{t}\Delta = \nu\left(c^{*}_{t}, Q^{*}_{t}\right)\Delta + \left(1-\rho\Delta\right)\int_{\underline{\psi}}^{\overline{\psi}} \max\left\{\mathcal{A}^{sin}_{t+\Delta}\left(\psi_{t+\Delta}\right), \mathcal{B}^{sin}_{t+\Delta}\left(\psi_{t+\Delta}\right)\right\} dH\left(\psi_{t+\Delta}\right)$$

and this can further be expressed as:

$$U_t^{sin} = \nu\left(c_t^*, Q_t^*\right)\Delta + (1-\rho\Delta)\int_{\underline{\psi}}^{\overline{\psi}} \max\left\{A_{t+\Delta}^{sin} - \psi_{t+\Delta}\Delta, B_{t+\Delta}^{sin}\right\} dH\left(\psi_{t+\Delta}\right)$$

where

$$\begin{aligned} A_{t+\Delta}^{sin} &= U_{t+\Delta}^{sin} + q\Delta\int_{w_u^{sin,*}}^{\overline{w}} \left[T_{t+\Delta}^{sin}(y) - U_{t+\Delta}^{sin}\right] dF(y) \\ B_{t+\Delta}^{sin} &= U_{t+\Delta}^{sin} \\ T_t^{sin}(y) &= \mathcal{T}_t^{sin}(y,\psi_t) + \psi_t\Delta\mathbb{I}\left[\psi_t \neq 0\right] \end{aligned}$$

The net value of the non-employed is the same, whether the individual incurred the disutility and searched as an unemployed or chose not to incur the disutility and remained OLF. This is precisely because the only difference in the gross value of the non-employed who participated and the non-employed who remained OLF was whether the individual chose to incur the disutility. Further, the net value, U_t^{sin}, has to be independent of ψ since we have netted out the disutility.

H.2.2 Net value of employed single

Similarly, the net value of the employed at the end of period can be expressed as:

$$\begin{aligned} T_t^{sin}(w) = \mathcal{T}_t^{sin}(w,\psi_t) + \psi_t\Delta &= \nu\left(c_t^*[w], Q_t^*[w]\right)\Delta + (1-\rho\Delta)\left[T_{t+\Delta}^{sin}(w) - 0\Delta\mathbb{I}\left[\psi_t \neq 0\right]\right] \\ &\quad + (1-\rho\Delta)\,\delta^{sin}\Delta\int_{\underline{\psi}}^{\overline{\psi}} \left[U_{t+\Delta}^{sin} - T_{t+\Delta}^{sin}(w)\right] dH\left(\psi_{t+\Delta}\right) \\ &\quad + (1-\rho\Delta)\,q\Delta\int_{w}^{\overline{w}} \left[T_{t+\Delta}^{sin}(y) - T_{t+\Delta}^{sin}(w)\right] dF(y) \end{aligned}$$

where for the continuously employed, $\psi_t = 0$. As per the net value of the non-employed single, the net value of the employed single is the same for both the continuously employed and newly employed singles. Again this arises because the difference in their gross values only stemmed from the disutility incurred by the newly employed individual.

H.3 The Continuous Time Limit

Taking the limit $\Delta \to 0$, one can show that, in continuous time, the net value of the non-employed collapses to:

$$\rho U^{sin} = \nu\left(c^*[b], Q^*[b]\right) + \int_{\underline{\psi}}^{\overline{\psi}} \max\left\{q\int_{w_u^{sin,*}}^{\overline{w}} \left[T^{sin}(y) - U^{sin}\right] dF(y) - \psi, 0\right\} dH(\psi)$$

and the gross value of non-employment is given by:

$$\rho\mathcal{U}^{sin}(\psi,\mathbb{I}[\psi]) = \rho\left(U^{sin} - \psi\mathbb{I}[\psi]\right)$$

Similarly, the net value of the employed in continuous time is given by:

$$\rho T^{sin}(w) = \nu\left(c^*[w], Q^*[w]\right) + \delta^{sin}\left[U^{sin} - T^{sin}(w)\right] + q\int_w^{\overline{w}}\left[T^{sin}(y) - T^{sin}(w)\right]dF(y)$$

It is straightforward to show that the decision rules from solving the problem with net-values are the same as solving the problem with gross-values. As an example, the rule determining reservation wages for singles:

$$T^{sin}\left(w_u^{sin,*}\right) - U^{sin} = 0 \qquad \Longrightarrow \left[\mathcal{T}^{sin}\left(w_u^{sin,*},\psi\right) + \psi\right] - \left[\mathcal{U}^{sin}(\psi,1) + \psi\right] = 0 \quad \forall\psi\in\left[\underline{\psi},\overline{\psi}\right]$$

Hence, the reservation wage resulting from the net value functions are the same as reservation wage resulting from the gross value functions.

Analogously, one can derive the joint household net value functions from the gross values by using similar definitions. In that case, we have two indicator functions for the gross value of the non-employed $\mathcal{U}(\psi_m,\psi_f,\mathbb{I}[\psi_m],\mathbb{I}[\psi_f])$ and

$$U = \mathcal{U}(\psi_m,\psi_f,\mathbb{I}[\psi_m],\mathbb{I}[\psi_f]) + \psi_m\mathbb{I}(\psi_m) + \psi_f\mathbb{I}(\psi_f)$$

I Disutility costs do not scale proportionately

Consider the problem of the dual non-employed under CRRA preferences. We omit the utility individuals derive from the consumption of public goods and suppress all dependence on $(\mathbf{E},\mathbf{x})$ for ease of exposition.

$$\begin{aligned}
\rho U &= \max_{c_m,c_f}\alpha\frac{c_f^{1-\gamma}}{1-\gamma} + (1-\alpha)\frac{c_m^{1-\gamma}}{1-\gamma} \\
&\quad + \int_{\underline{\psi}}^{\bar{\psi}}\max\left\{-\psi + q_m\int_{w_m^R}^{\overline{w}}\left[\Omega_m(y) - U\right]dF_m(y), 0\right\}dH_m(\psi) \\
&\quad + \int_{\underline{\psi}}^{\bar{\psi}}\max\left\{-\psi + q_f\int_{w_f^R}^{\overline{w}}\left[\Omega_f(y) - U\right]dF_f(y), 0\right\}dH_f(\psi)
\end{aligned}$$

s.t.

$$c_m + c_f = \mathcal{I}_u$$

Optimal current consumption is given by:

$$c_m = \left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma} \frac{1}{1+\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma}} \mathcal{I}_u$$

$$c_f = \frac{1}{1+\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma}} \mathcal{I}_u$$

current utility ends up being:

$$\underbrace{\left[\alpha\left[\frac{1}{1+\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma}}\right]^{1-\gamma} + (1-\alpha)\left(\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma} \frac{1}{1+\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma}}\right)^{1-\gamma}\right]}_{g(\alpha)} \underbrace{\frac{\mathcal{I}_u^{1-\gamma}}{1-\gamma}}_{u(\mathcal{I}_u)}$$

Thus, we can re-write the problem of the dual employed as:

$$\begin{aligned} \rho U &= g(\alpha)\, u(\mathcal{I}^u) \\ &+ \int_{\underline{\psi}}^{\bar{\psi}} \max\left\{-\psi + q_m \int_{w_m^R}^{\overline{w}} [\Omega_m(y) - U]\, dF_m(y), 0\right\} dH_m(\psi) \\ &+ \int_{\underline{\psi}}^{\bar{\psi}} \max\left\{-\psi + q_f \int_{w_f^R}^{\overline{w}} [\Omega_f(y) - U]\, dF_f(y), 0\right\} dH_f(\psi) \end{aligned}$$

Denote $\frac{U}{g(\alpha)} = \widehat{U}$, and let us guess and verify if $\widehat{U}$ is independent of $g(\alpha)$:

$$\begin{aligned} \rho\widehat{U} &= u(\mathcal{I}^u) \\ &+ \int_{\underline{\psi}}^{\bar{\psi}} \left\{\max\left\{-\frac{\psi}{g(\alpha)} + q \int_{w_m^R}^{\overline{w}} \left[\widehat{\Omega}_m(y) - \widehat{U}\right] dF_m(y), 0\right\}\right\} dH_m(\psi) \\ &+ \int_{\underline{\psi}}^{\bar{\psi}} \left\{\max\left\{-\frac{\psi}{g(\alpha)} + q \int_{w_f^R}^{\overline{w}} \left[\widehat{\Omega}_f(y) - \widehat{U}\right] dF_f(y), 0\right\}\right\} dH_f(\psi) \end{aligned}$$

where $\widehat{\Omega}_i(y) = \Omega_i(y)/g(\alpha)$. Clearly, one can observe that from the RHS of the above equation that $\widehat{U}$ is not independent of $g(\alpha)$ which is a contradiction. More generally, the above shows that while expected benefits from the change of value from finding a job scale proportionally in $g(\alpha)$, disutility costs do not scale proportionally with $g(\alpha)$, making decision rules dependent on α.

J Modeling participation costs as a monetary cost

In this section, we show that if we had instead modeled the disutility from entering the labor force as a monetary cost instead of a disutility, one gets back ISHARA preferences and decision rules even for CRRA utilities over consumption goods would be independent of $\alpha(\mathbf{x})$. The trade-off is that one would have to work with the gross value functions which expands the state space. For ease of exposition, we again omit the utility from public goods and suppress all dependence on $\mathbf{x}$.

Denote ψ as a monetary cost the individual incurs when she chooses to enter the labor force. For the joint household with income $\mathcal{I}$, the realized budget constraints can take on four outcomes:

$$
\begin{aligned}
c_m + c_f &= \mathcal{I} && \text{if } \psi_m > \psi_m^c \quad \text{and} \quad \psi_f > \psi_f^c \\
c_m + c_f &= \mathcal{I} - \psi_m && \text{if } \psi_m \leq \psi_m^c \quad \text{and} \quad \psi_f > \psi_f^c \\
c_m + c_f &= \mathcal{I} - \psi_f && \text{if } \psi_m > \psi_m^c \quad \text{and} \quad \psi_f \leq \psi_f^c \\
c_m + c_f &= \mathcal{I} - (\psi_f + \psi_m) && \text{if } \psi_m \leq \psi_m^c \quad \text{and} \quad \psi_f \leq \psi_f^c
\end{aligned}
$$

where ψ_i^c for $i \in \{m, f\}$ here denotes the thereshold above which the household member of sex i chooses not to enter the labor force. The first line corresponds to the household where both m and f choose to be OLF. The second (third) line refers to the realized budget constraint when only m (f) enters the labor force and the fourth line refers to the case where both m and f enter the labor force.

Once we model the cost of entering the labor force as a monetary cost, we have to work with four separate value functions - one for each realized budget constraint - for the joint household where both individuals initially start off the period as non-employed. Thus the gross value of dual non-employment is given by:

$$
\mathcal{U} = \int_{\underline{\psi}}^{\overline{\psi}} \int_{\underline{\psi}}^{\overline{\psi}} \max \left\{ V^{np} \left(\psi_m, \psi_f \right), V^{mp} \left(\psi_m, \psi_f \right), V^{fp} \left(\psi_m, \psi_f \right), V^{p} \left(\psi_m, \psi_f \right) \right\} dH_m \left(\psi_m \right) dH_f \left(\psi_f \right)
$$

where

$$
\begin{aligned}
\rho V^{np}\left(\psi_m,\psi_f\right) &= (1-\alpha)\,u\left(c_m\right)+\alpha u\left(c_f\right) \\
\rho V^{mp}\left(\psi_m,\psi_f\right) &= (1-\alpha)\,u\left(c_m\right)+\alpha u\left(c_f\right)+q_m\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_m\left(y\right)-\mathcal{U},0\right]dF_m\left(y\right) \\
\rho V^{fp}\left(\psi_m,\psi_f\right) &= (1-\alpha)\,u\left(c_m\right)+\alpha u\left(c_f\right)+q_f\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_f\left(y\right)-\mathcal{U},0\right]dF_y\left(y\right) \\
\rho V^{p}\left(\psi_m,\psi_f\right) &= (1-\alpha)\,u\left(c_m\right)+\alpha u\left(c_f\right)+q_f\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_f\left(y\right)-\mathcal{U},0\right]dF_y\left(y\right) \\
&\quad +q_m\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_m\left(y\right)-\mathcal{U},0\right]dF_m\left(y\right)
\end{aligned}
$$

where $\mathcal{O}_i$ is the gross value of a worker-searcher household where individual of sex $i\in\{m,f\}$ is the employed member. Since ψ shows up only in the budget constraints and hence in the consumption values, we once again have ISHARA preferences. To see this, observe that under CRRA utility over consumption goods, optimal consumption requires:

$$
c_f=\frac{1}{1+\left[\frac{1-\alpha}{\alpha}\right]^{1/\gamma}}\mathcal{I}^{state}
$$

and

$$
c_m=\left[\frac{1-\alpha}{\alpha}\right]^{1/\gamma}\frac{1}{1+\left[\frac{1-\alpha}{\alpha}\right]^{1/\gamma}}\mathcal{I}^{state}
$$

where $state=\{np,mp,fp,p\}$ and net income $\mathcal{I}^{state}$ is different in each state. This implies that current utility can be expressed as:

$$
\underbrace{\left[\alpha\left[\frac{1}{1+\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma}}\right]^{1-\gamma}+(1-\alpha)\left(\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma}\frac{1}{1+\left[\frac{(1-\alpha)}{\alpha}\right]^{1/\gamma}}\right)^{1-\gamma}\right]}_{g(\alpha)}\frac{\left(\mathcal{I}^{state}\right)^{1-\gamma}}{1-\gamma}
$$

and the gross value functions under different states can be expressed as:

$$
\begin{aligned}
\rho V^{np}\left(\psi_m,\psi_f\right) &= g\left(\alpha\right)u\left(\mathcal{I}^u\right)\\
\rho V^{mp}\left(\psi_m,\psi_f\right) &= g\left(\alpha\right)u\left(\mathcal{I}^u-\psi_m\right)+q_m\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_m\left(y\right)-\mathcal{U},0\right]dF_m\left(y\right)\\
\rho V^{fp}\left(\psi_m,\psi_f\right) &= g\left(\alpha\right)u\left(\mathcal{I}^u-\psi_f\right)+q_f\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_f\left(y\right)-\mathcal{U},0\right]dF_y\left(y\right)\\
\rho V^{p}\left(\psi_m,\psi_f\right) &= g\left(\alpha\right)u\left(\mathcal{I}^u-\psi_m-\psi_f\right)+q_f\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_f\left(y\right)-\mathcal{U},0\right]dF_y\left(y\right)\\
&\quad +q_m\int_{\underline{w}}^{\overline{w}}\max\left[\mathcal{O}_m\left(y\right)-\mathcal{U},0\right]dF_m\left(y\right)
\end{aligned}
$$

We will guess and verify that $\widetilde{\mathcal{U}}$ is independent of α. In particular, we will guess that $\widetilde{\mathcal{U}}=\mathcal{U}/g(\alpha)$. Dividing all value functions by $g\left(\alpha\right)$, we arrive at:

$$
\begin{aligned}
\rho\widetilde{V}^{np}\left(\psi_m,\psi_f\right) &= u\left(\mathcal{I}^u\right)\\
\rho\widetilde{V}^{mp}\left(\psi_m,\psi_f\right) &= u\left(\mathcal{I}^u-\psi_m\right)+q_m\int_{\underline{w}}^{\overline{w}}\max\left[\widetilde{\mathcal{O}}_m\left(y\right)-\widetilde{\mathcal{U}},0\right]dF_m\left(y\right)\\
\rho\widetilde{V}^{fp}\left(\psi_m,\psi_f\right) &= u\left(\mathcal{I}^u-\psi_f\right)+q_f\int_{\underline{w}}^{\overline{w}}\max\left[\widetilde{\mathcal{O}}_f\left(y\right)-\widetilde{\mathcal{U}},0\right]dF_y\left(y\right)\\
\rho\widetilde{V}^{p}\left(\psi_m,\psi_f\right) &= u\left(\mathcal{I}^u-\psi_m-\psi_f\right)+q_f\int_{\underline{w}}^{\overline{w}}\max\left[\widetilde{\mathcal{O}}_f\left(y\right)-\widetilde{\mathcal{U}},0\right]dF_y\left(y\right)\\
&\quad +q_m\int_{\underline{w}}^{\overline{w}}\max\left[\widetilde{\mathcal{O}}_m\left(y\right)-\widetilde{\mathcal{U}},0\right]dF_m\left(y\right)
\end{aligned}
$$

and

$$
\widetilde{\mathcal{U}}=\int_{\underline{\psi}}^{\overline{\psi}}\int_{\underline{\psi}}^{\overline{\psi}}\max\left\{\widetilde{V}^{np}\left(\psi_m,\psi_f\right),\widetilde{V}^{mp}\left(\psi_m,\psi_f\right),\widetilde{V}^{fp}\left(\psi_m,\psi_f\right),\widetilde{V}^{p}\left(\psi_m,\psi_f\right)\right\}dH_m\left(\psi_m\right)dH_f\left(\psi_f\right)
$$

where terms with a tilde just represent being divided by $g\left(\alpha\right)$, e.g. $\widetilde{\mathcal{O}}_m(w)=\frac{\mathcal{O}_m(w)}{g(\alpha)}$. The above verifies that $\widetilde{\mathcal{U}}$ is independent of α. One can easily show that in the solution to the transformed problem is also a solution to the original problem, and hence all search behavior is independent of α.

The key difference is that since ψ enters the budget constraint, one must work with the realized gross value functions since current utility over consumption goods depends on the income net of monetary costs of entering the labor force. This enlarges the state space and requires us to work with four value functions for the joint household where both members are initially non-employed. Including the different combinations for worker-searcher households, this requires us to consider 9 different employment status combinations for a joint household of a particular education pair $\mathbf{x}$ compared to 4 employment status combinations when we worked with the net values associated with ψ as a disutility.

K Model-Implied Household Income Inequality

To conduct the sorting counterfactuals, one must first take a stance on how to construct different hypothetical distributions of marriage under different degrees of sorting. We therefore follow Eika et al. (2019) and Greenwood et al. (2014a) and first measure what the distribution of married households would look like under random matching in both years, which we refer to as $r^t(\mathbf{E}, \mathbf{x})$ for $t \in \{1980, 2000\}$. This is simply the product of the marginal distributions of education for married men and women in each time period $t \in \{1980, 2000\}$:

$$r^t(\mathbf{E}, \mathbf{x}) = \Phi_m^t(\mathcal{E}_m, x_m) \times \Phi_f^t(\mathcal{E}_f, x_f)$$

We take the realized distribution of marriages $\mathcal{M}(\mathbf{E}, \mathbf{x})$ relative to the hypothetical distribution under random matching in each time period to construct a sorting parameter matrix, $s^t(\mathbf{E}, \mathbf{x})$:

$$s^t(\mathbf{E}, \mathbf{x}) = \frac{\mathcal{M}^t(\mathbf{E}, \mathbf{x})}{r^t(\mathbf{E}, \mathbf{x})}$$

As a final step, we implement the matching algorithm outlined in Eika et al. (2019) and Greenwood et al. (2014a) to construct hypothetical distributions of marriages either (i) assuming the marginal distributions of education from the 2000's, but the sorting parameter matrix $s^t(\mathbf{E}, \mathbf{x})$ from the 1980's or (ii) the reverse. We then weight the distributions of income obtained under the 2000's estimation by the newly constructed distribution of marriages, and we generate a hypothetical distribution of household income. We impose that the share of singles is unchanged and only change marital sorting patterns. This has the flavor of asking how much did changes in sorting patterns amongst married individuals affect household income inequality. It abstracts entirely from changes in selection into marriage, and is directly comparable with the results in Eika et al. (2019).

L Cohort Analysis Results

L.1 Labor market estimation: singles

Table 12 shows our estimated labor market parameters for singles when we limit our sample to specific cohorts, while Figures 3 and 4 show how well our estimated model fits the wage data.

Estimated Parameters from Singles Estimation								
	Men				Women			
	1980		2000		1980		2000	
	L	H	L	H	L	H	L	H
$\delta^{sin}(x_s)$	0.096	0.025	0.104	0.025	0.122	0.027	0.152	0.051
$q(x_s)$	0.518	1.223	0.585	1.216	0.554	1.167	0.698	1.416
μ_s	2.240	2.240	2.323	2.323	2.095	2.095	2.216	2.216
σ_s	0.581	0.581	0.568	0.568	0.525	0.525	0.579	0.579
$\lambda(x_s)$	0.277	0.145	0.138	0.069	0.205	0.084	0.053	0.062
$\eta(x_s)$	0.068	0.003	0.039	0.002	0.063	0.003	0.033	0.003
b_s	3.489	3.489	1.568	1.568	1.971	1.971	0.356	0.356
Estimated Moments for Singles								
	Men				Women			
	1980		2000		1980		2000	
	$\leq$ HS	$\geq$Col	$\leq$ HS	$\geq$ Col	$\leq$ HS	$\geq$ Col	$\leq$ HS	$\geq$ Col
$A(\mathcal{E})$	1	1.459	1	1.512	1	1.490	1	1.666
$p(H \mid \mathcal{E})$	0.439	0.985	0.256	0.854	0.265	0.980	0.251	0.978
Data $\mathbf{u}^{sin}$	0.267	0.119	0.295	0.157	0.321	0.127	0.338	0.167
Model $\mathbf{u}^{sin}$	0.248	0.151	0.285	0.172	0.307	0.159	0.311	0.187
Data OLF	2.746	7.423	2.387	5.383	2.119	6.875	1.956	4.978
Model OLF	0.735	0.880	0.705	0.845	0.682	0.871	0.662	0.833
Data JJ	0.031	0.030	0.023	0.023	0.031	0.032	0.023	0.024
Model JJ	0.058	0.032	0.042	0.024	0.058	0.024	0.024	0.023

Table 12: Labor market estimation for singles under cohort data

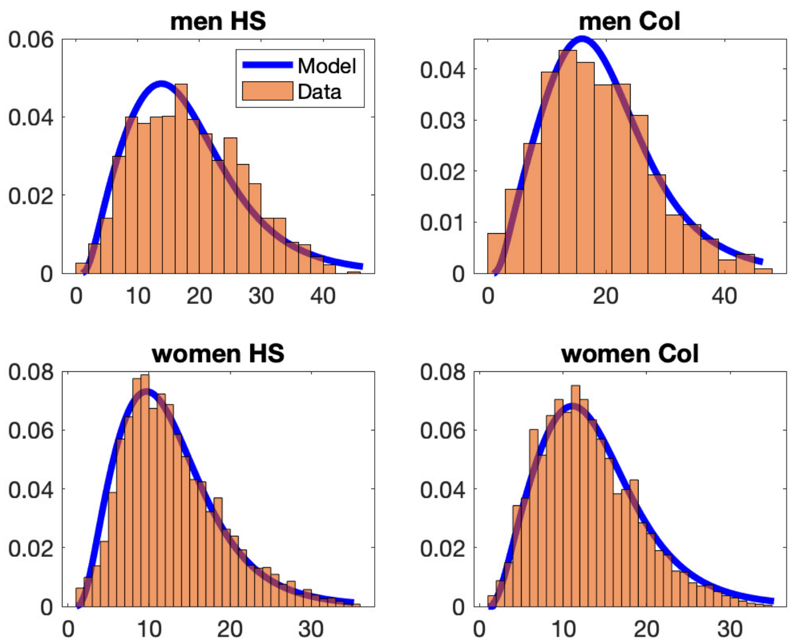

Figure 3: Model fit: 1980s wage distribution under cohort data

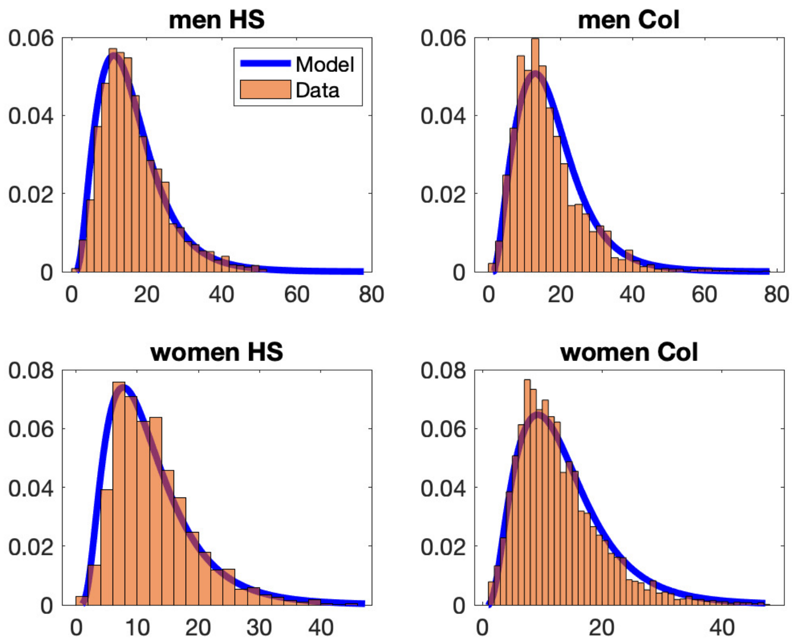

Figure 4: Model fit: 2000s wage distribution under cohort data

L.2 Labor market estimation: joint

Table 13 shows how well our model fits the targeted moments for joint households when we limit our sample to specific cohorts.

L.3 Marriage and education market estimation

Table 14 shows the implied preference parameters and prices when we limit the sample to specific cohorts.

L.4 Implied total marital surplus

Under the estimated parameters, Table 15 shows the degree of supermodularity if we fix either skill or education pairs. Note that our results for specific cohorts share the same findings as our results for the full sample, that is, incentives to sort by skill strengthened in the 2000s.

L.5 Implied reduced forms

Under the estimated parameters for cohort data, Table 16 shows that an even larger share of the rise in household income inequality is due to changes in household composition, the between component.

Panel A: Estimated δ for joint households								
	Men				Women			
	1980		2000		1980		2000	
	L	H	L	H	L	H	L	H
$\delta^{mar}(x_s)$	0.38	8e-4	0.06	2e-4	0.28	1e-3	0.32	2e-3
Panel B: Estimated skill probabilities								
	1980				2000			
	HS,HS	HS,Col	Col,HS	Col,Col	HS,HS	HS,Col	Col,HS	Col,Col
$p(L, L\|\mathbf{E})$	0.10	0.04	0.04	0.02	0.34	0.17	0.15	0.08
$p(L, H\|\mathbf{E})$	0.04	0.07	0.04	0.02	0.15	0.13	0.13	0.08
$p(H, L\|\mathbf{E})$	0.35	0.20	0.35	0.28	0.01	0.04	0.18	0.16
$p(H, H\|\mathbf{E})$	0.51	0.69	0.56	0.68	0.49	0.66	0.54	0.68
Panel C: Model Fit								
Data $\mathbf{u}_m^{mar}$	0.13	0.11	0.08	0.04	0.14	0.11	0.08	0.05
Model $\mathbf{u}_m^{mar}$	0.13	0.10	0.08	0.04	0.14	0.10	0.08	0.05
Data $\mathbf{u}_f^{mar}$	0.41	0.22	0.38	0.29	0.31	0.19	0.32	0.24
Model $\mathbf{u}_f^{mar}$	0.41	0.22	0.38	0.28	0.31	0.19	0.32	0.24
Data mean hh w	26.40	34.10	35.80	44.80	25.50	35.90	37.40	54.00
Model mean hh w	25.92	34.45	36.61	43.88	24.77	37.36	39.24	50.43
Data u_u	0.07	0.03	0.03	0.01	0.06	0.03	0.03	0.02
Model u_u	0.07	0.03	0.03	0.01	0.06	0.03	0.03	0.02
Data u_Ω	0.06	0.08	0.05	0.03	0.08	0.07	0.05	0.04
Model u_Ω	0.06	0.08	0.05	0.03	0.08	0.07	0.05	0.04
Data e_Ω	0.34	0.19	0.34	0.27	0.25	0.16	0.29	0.23
Model e_Ω	0.34	0.19	0.34	0.27	0.25	0.16	0.29	0.23

Table 13: Labor market estimation for joint households under cohort data

Panel A: Single household preferences								
	1980s				2000s			
	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
M	0.0	0.0	-43.7	0.9	0.0	0.0	-16.3	17.1
F	25.9	-0.9	-17.8	7.6	21.7	1.5	-9.8	20.6
Panel B: Joint household preferences								
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	4.4	-53.8	-26.3	-112.7	-1.7	-76.0	-16.6	-161.2
HS H	-54.6	-163.7	-88.5	-263.5	-75.7	-162.8	-76.6	-286.5
Col L	-18.3	-75.1	-23.5	-126.0	-26.6	-111.4	-19.6	-190.0
Col H	-149.9	-293.3	-159.9	-391.9	-131.9	-287.2	-124.3	-422.3
Panel C: Joint household prices $\alpha(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$								
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.83	0.49	0.73	0.49	0.98	0.53	0.90	0.51
HS H	0.47	0.47	0.47	0.48	0.47	0.48	0.48	0.49
Col L	0.71	0.49	0.65	0.50	0.73	0.52	0.70	0.51
Col H	0.48	0.48	0.49	0.49	0.49	0.49	0.49	0.49

Table 14: Marriage market estimation under cohort data

Total Surplus								
	1980's				2000's			
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	-24.2	-32.9	-38.8	-66.2	-15.7	-20.7	-10.9	-54.5
HS H	3.6	20.9	-2.9	-17.1	-68.5	13.3	-29.5	-11.1
Col L	-25.4	-18.2	12.3	-27.7	-33.8	-25.6	16.4	-22.8
Col H	-25.2	-5.7	23.6	2.5	-47.5	-14.5	12.2	2.8
Panel B: Supermodularity, fixed (x_m, x_f)								
M/F		L	H			L	H	
L		52.3	23.7			45.4	36.6	
H		55.3	46.2			20.7	41.7	
Panel C: Supermodularity, fixed $(\mathcal{E}_m, \mathcal{E}_f)$								
M/F		HS	Col			HS	Col	
HS		26.0	13.2			86.8	62.0	
Col		12.2	18.9			24.7	29.7	

Table 15: Marital surplus under cohort data

Panel A	Gini Coefficient		Theil T	
	1980	2000	1980	2000
model	0.50	0.56 (12%)	0.43	0.54 (25%)
Panel B	Sorting Counterfactuals			
	Gini Coefficient		Theil T	
1980 sorting (EMZ)	0.50	-	0.43	-
1980 sorting (GGKS)	0.50	-	0.44	-

Table 16: Reduced form findings under cohort data

L.6 Counterfactuals

Similar to our baseline model's results, we find that accounting for how the marginals change in respond to increased incentives to sort, gives us a non-trivial increase in inequality. Similar to the baseline model, income inequality within joint households rose while inequality declined within single households.

Panel A: Inequality measures				
	1980s	PE	GE	2000s
	(I)	(II)	(III)	(IV)
Gini	0.50	0.49	0.54	0.56
Theil T	0.43	0.43	0.51	0.54
Gini$_{joint}$	0.28	0.27	0.30	0.32
Theil$_{joint}$	0.14	0.14	0.18	0.18
Gini$_{single}$	0.50	0.50	0.49	0.51
Theil$_{single}$	0.46	0.46	0.45	0.49
Panel B: College and education shares				
Single HH share	0.47	0.47	0.55	0.54
College share	0.40	0.40	0.61	0.60

Table 17: Counterfactuals under cohort data

L.7 PE distribution of married

As per our baseline model, increased incentives to sort were absorbed by changes in prices, leaving little change in realized sorting.

Panel A: $\alpha(\mathbf{E}, \mathbf{x})$								
	1980s				PE			
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	1.00	0.48	0.84	0.49	0.10	0.55	0.93	0.55
HS H	0.48	0.47	0.48	0.48	0.59	0.57	0.73	0.55
Col L	0.81	0.49	0.73	0.49	0.24	0.53	0.79	0.53
Col H	0.49	0.48	0.49	0.49	0.51	0.52	0.57	0.52
Panel B: Probability shares within marriage								
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.05	0.02	0.00	0.01	0.06	0.01	0.00	0.00
HS H	0.17	0.24	0.02	0.05	0.07	0.30	0.01	0.10
Col L	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00
Col H	0.06	0.09	0.08	0.19	0.14	0.05	0.08	0.15

Table 18: PE marriage shares

L.8 GE distribution of married

When marginals respond to increased incentives to sort, we see, similar to our baseline model, an increase in inequality.

Panel A: Probability shares within marriage								
	1980's				GE			
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.05	0.02	0.00	0.01	0.04	0.02	0.01	0.01
HS H	0.17	0.24	0.02	0.05	0.01	0.13	0.01	0.09
Col L	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.02
Col H	0.06	0.09	0.08	0.19	0.04	0.06	0.14	0.37

Table 19: GE marriage shares

M Marital Surplus Over Time by Skill, Education

Here, we investigate how marital surplus changed over time by shutting down either skill or education. Because our baseline model feature both skill parameters and the college premium changing over time, this made it difficult to assess the changes stemming from each component. Thus, we conduct two exercises to show how incentives to sort changed by skill and education.

In our first exercise termed "Education Only", we assume that the economy is populated with only low-skilled individuals, $x_s = L$, and fix all labor market skill parameters,

$\{q(x_s), \lambda(x_s), \delta(x_s), \eta(x_s)\}$, to their 1980s levels. Given our 1980s estimated preferences, college premium and all other parameters that do not vary by skill, we re-estimate what the implied total marital surplus is in this environment and the $\alpha(\mathbf{E})$ required to clear the marriage market market. We then repeat this same exercise for the 2000s, but continue to fix $\{q(x_s), \lambda(x_s), \delta(x_s), \eta(x_s)\}$ to their 1980s levels, while updating all other parameters to their 2000s values. This exercise allows us to hold constant the labor market components that reduced individual's exposure to non-employment risk over time, while allowing the college premium to increase. Further, because we fix all individuals to be low-skilled, there is no ability to sort by skill in this exercise.

Table 20 shows our results. Similar to our baseline model results in Table 4, we find that while there were still positive incentives to sort by education, these incentives weakened over time. While all individuals want to be paired with a college educated partner as they now offer higher incomes than before, we find that high-school individuals are willing to pay more for such partners in the form of intrahousehold transfers, as depicted by the rise and fall in $\alpha(\mathbf{E})$ for (HS, Col) and (Col, HS) households, respectively. Intuitively, the change in $\alpha(\mathbf{E})$ in this example reflect how individuals are willing to pay more for insurance when the earnings potentials of college educated partners provides more of a buffer against non-employment risk.

Supermodularity				
	Education only		Skill only	
1980s	48.7		19.1	
2000s	39.5		43.8	
Panel B: Education only, $\alpha(\mathbf{E})$				
	1980s		2000s	
M/F	HS	Col	HS	Col
HS	1.00	0.73	0.96	0.78
Col	0.84	0.73	0.80	0.70
Panel C: Skill only, $\alpha(\mathbf{x})$				
	1980s		2000s	
M/F	L	H	L	H
L	1.00	0.59	0.81	0.57
H	0.46	0.47	0.52	0.50

Table 20: Time-varying incentives to sort by education, skill only

In our second exercise termed "Skill Only", we assume that the economy is populated with only high-school educated individuals, $\mathcal{E}_s = HS$, and $A(HS) = 1$ across all time periods. We allow all other parameters to be set to their 1980s values and estimate the marital surplus and $\alpha(\mathbf{x})$ required to clear the marriage market. As before, we repeat the same exercise for the 2000s and update all parameters to their 2000s values while holding $A(HS) = 1$. Because we abstract from education in this exercise, individuals can only sort by skill.

Table 20 shows our results. As in our baseline model, there are positive incentives to sort by skill and these incentives strengthen over time. Unlike the results in Panel B, Panel C shows that high-skilled individuals do not receive larger intrahousehold transfers from low-skilled individuals, which would have supported more negative sorting. Rather high-skilled females extract more of the surplus when paired with high-skilled males. Overall, our results suggest that incentives to sort by skill strengthened over time as labor market parameters that differed by skill evolved such that individuals faced lower non-employment risk and hence could choose to match with partners more for income maximization motives.

N Matching via deferred acceptance

Given that PE does not feature a substantial increase in realized positive sorting when prices are flexible, a natural question arises as to what might have been the realized sorting patterns if prices had not adjusted to clear markets? To examine this, we now hold fixed $\alpha(\mathbf{E}, \mathbf{x})$ to their 1980s levels. Because we no longer allow prices to adjust, we require a different mechanism to clear the marriage market. To do this, we appeal to the Deferred Acceptance Algorithm first introduced in Gale and Shapley (1962). That is, we compute what each member of the household would receive if individuals split 2000s economic marital surplus according to 1980s prices and if preference gains from being in a joint household are that derived from the 2000s. We then ask what matches would have formed if say males proposed, and females could accept or reject. We repeat this exercise until there is no male or female left unmatched and all matches are stable.

Our algorithm in detail is described as follows. To examine how much sorting would have realized if prices had not adjusted, we hold fixed prices, $\alpha(\mathbf{E}, \mathbf{x})$ to their 1980s levels. We also continue to hold fixed the marginal distributions of married individuals by skill, education and sex. Under this set-up, the payoff to a male of type $(\mathcal{E}_m, x_m)$ to being in a joint household of type $(\mathcal{E}_m, \mathcal{E}_f, x_m, x_f)$ is given by:

$$\begin{aligned} \text{payoff to male in } (\mathbf{E}, \mathbf{x}) \text{ HH} \quad &= \quad [1 - \alpha_{1980}(\mathbf{E}, \mathbf{x})] \exp(\rho \widetilde{U}[\mathbf{E}, \mathbf{x}]) - \exp(\rho V_m^U[\mathcal{E}_m, \emptyset, x_m, \emptyset]) \\ &\quad + \chi_m(\mathbf{E}, \mathbf{x}) - \chi_m(\mathcal{E}_m, \emptyset, x_m, \emptyset) \end{aligned}$$

and the payoff to the female of type $(\mathcal{E}_f, x_f)$ is:

$$\begin{aligned} \text{payoff to female in } (\mathbf{E}, \mathbf{x}) \text{ HH} \quad &= \quad \alpha_{1980}(\mathbf{E}, \mathbf{x}) \exp(\rho \widetilde{U}[\mathbf{E}, \mathbf{x}]) - \exp(\rho V_f^U[\emptyset, \mathcal{E}_f, \emptyset, x_f]) \\ &\quad + \chi_f(\mathbf{E}, \mathbf{x}) - \chi_f(\emptyset, \mathcal{E}_m, \emptyset, x_f) \end{aligned}$$

To clear the marriage market, we instead assume that males and females engage in a deferred acceptance game. Males simultaneously propose to females. Females can examine all offers and choose which one to accept or reject. If a female accepts a male, they leave the marriage market. Leftover males and females repeat the deferred acceptance game again and continue to do so until no more matches can be formed.

Note that the payoffs based on 1980s prices, $\alpha(\mathbf{E}, \mathbf{x})$, and 2000s economic marital sur-

Panel A: Perceived payoffs under fixed prices				
M/F	HS L	HS H	Col L	Col H
HS L	(-9.1, **-0.7**)	(**-8.3**, 2.0)	(-24.9, -3.6)	(-16.7, -19.9)
HS H	(-3.7, -27.7)	(**12.9**, **2.9**)	(-9.9, -22.2)	(10.1, -16.0)
Col L	(-20.5, -8.9)	(-14.5, -6.5)	(-14.8, **8.4**)	(**-3.8**, -10.2)
Col H	(-15.9, -28.6)	(-6.1, -4.5)	(0.7, -0.3)	(**11.4**, **-2.9**)
Panel B: Marginal distributions of married by skill,education and sex				
	HS L	HS H	Col L	Col H
M	0.13	0.40	0.04	0.42
F	0.26	0.35	0.12	0.28

Notes: Panel A shows the perceived payoffs each member of a household receives under fixed prices. The left cell in the bracket corresponds to the male's payoff while the right cell corresponds to the female's payoff. Bolded cells show the highest payoff for that skill-education-gender combination. Highlighted cells show the pairing that is the highest for both members. Panel B shows the marginal distributions of married by skill, education and sex.

Table 21: Round 1: Payoffs under fixed $\alpha(\mathbf{E}, \mathbf{x})$

Perceived payoffs under fixed prices, round 2		
M/F	HS L	Col L
HS L	(**-9.1**, **-0.7**)	(-24.9, -3.6)
HS H	(**-3.7**, -27.7)	(-9.9, -22.2)
Col L	(-20.5, -8.9)	(**-14.8**, **8.4**)
Col H	(-15.9, -28.6)	(**0.7**, -0.3)

Notes: This table shows the perceived payoffs each member of a household receives under fixed prices from Round 2. The left cell in the bracket corresponds to the male's payoff while the right cell corresponds to the female's payoff. Bolded cells show the highest payoff for that skill-education-gender combination. Highlighted cells show the pairing that is the highest for both members.

Table 22: Round 2: Payoffs under fixed $\alpha(\mathbf{E}, \mathbf{x})$

Perceived payoffs under fixed prices, round 3		
M/F	HS L	Col L
HS H	(**-3.7**, **-27.7**)	(-9.9, -22.2)
Col H	(-15.9, -28.6)	(**0.7**, **-0.3**)

Notes: This table shows the perceived payoffs each member receives under fixed prices from Round 3. The left cell in the bracket corresponds to the male's payoff while the right cell in the bracket corresponds to the female's payoff. Bolded cells show the highest payoff for that skill-education-gender combination. Highlighted cells show that the pairing is the highest for both members.

Table 23: Round 3: Payoffs under fixed $\alpha(\mathbf{E}, \mathbf{x})$

plus and preference gains give us the following payoffs as in Table 21 for convenience. In the first round, high-skilled college males propose to high-skilled college females, and high-skilled high-school educated males propose to high-skilled high-school educated females. Both these proposals are accepted and matches between high-skilled and their exact education counterpart are formed. All other proposals by low-skilled males are rejected at this time. Because high-skilled females are in short supply, all high-skilled females form joint households and leave the marriage market after the first round.

From round 2 of the deferred acceptance game, we observe that high-school educated low-skilled males propose to high-school educated low-skilled females, and college educated low-skilled males also propose to their exact counterpart - college-educated low-skilled females. These matches also bring the highest payoffs to low-skilled females and are thus accepted as highlighted in Table 22. As per Panel B of Table 21, low-skilled males are in relative short supply. As such, all low-skilled males exit the marriage market after the second round.

In the third round, only low-skilled females and high-skilled males are left. In this case, sorting only occurs along education as college high-skilled males propose to college low-skilled females, and high-school educated high-skilled males propose to high-school educated low-skilled females. Again these matches also bring low-skilled females the highest payoffs, and so matches are formed between like-education pairs. Because the remaining share of college males exceeds that of college females, and because high-school females exceed the supply of high-school males, we have the remainder of college high-skilled college males married to low-skilled high-school females in the fourth and final round.

Perceived payoffs under fixed prices, round 4	
M/F	HS L
Col H	(-15.9, -28.6)

Notes: This table shows the perceived payoffs each member receives under fixed prices from Round 4. The left cell in the bracket corresponds to the male's payoff while the right cell in the bracket corresponds to the female's payoff.

Table 24: Round 4: Payoffs under fixed $\alpha(\mathbf{E}, \mathbf{x})$

This then leaves us with the following final distribution of joint households, as depicted in Table 25. Notably, the significant rise in realized positive sorting leads to a substantial increase in inequality as documented in Table 26.

Probability shares within marriage								
	1980s				PE (Fixed $\alpha(\mathbf{E}, \mathbf{x})$)			
M/F	HS L	HS H	Col L	Col H	HS L	HS H	Col L	Col H
HS L	0.07	0.05	0.00	0.01	0.13	0.00	0.00	0.00
HS H	0.13	0.21	0.02	0.05	0.06	0.35	0.00	0.00
Col L	0.01	0.01	0.01	0.01	0.00	0.00	0.04	0.00
Col H	0.06	0.08	0.09	0.20	0.08	0.00	0.07	0.27

Notes: This table highlights the distribution of joint households. Columns 1-4 show the outcomes under the 1980s baseline model. Column 5-8 shows the outcomes under counterfactual where the supply of married individuals by skill-education is fixed to its 1980s levels and prices are fixed to 1980s levels. Shaded cells highlight the amount of sorting among like education-skill pairs.

Table 25: Counterfactual surplus and joint household distribution under fixed α

Inequality measures						
	Aggregate		Within			
	Gini	Theil	Gini_{joint}	Theil_{joint}	Gini_{single}	Theil_{single}
1980s	0.46	0.37	0.28	0.15	0.47	0.42
PE fixed α	0.48	0.41	0.30	0.19	0.47	0.42
$\%\Delta$	4%	9%	7%	26%	0%	0%

Notes: Row 1 shows the outcomes under the 1980s benchmark model. Row 2 refers to the PE exercise where the supply of married individuals by skill and education is fixed to its 1980s levels. Row 3 shows the percent change in the inequality measure in PE relative to its level in 1980.

Table 26: Significant rise in household income inequality under fixed prices