Reconciling Macroeconomics and Finance for the US Corporate Sector: 1929 - Present*

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Abstract

The Integrated Macroeconomic Accounts of the United States offer a unified data set for the income statement, cash flows, and balance sheet of the U.S. Corporate Sector. We use these data together with a stochastic growth model with factorless income to revisit the question of the the extent to which fluctuations in aggregate cash flows to owners of firms drive fluctuations in the market value of U.S. Corporations. We find in these data that payout-price ratios do forecast growth of future cash flows and that the volatility of future cash flows is more than enough to account for observed volatility of corporate valuations even in the absence of fluctuations in expected excess returns on equity. Our data are consistent with the view that the failure to find cash flow predictability in data on publicly traded firms is due to dividend smoothing on the part of those firms and long run changes in payout policies by public firms. Our model measurement exercise is consistent with the view that relatively small fluctuations in investors' expectations of the share of factorless income in the long run have driven a large part of stock market fluctuations, particularly since WWII. Our model measurement exercise does uncover puzzling long-run behavior of the excess return on investment in tangible capital over the past 100 years. That return to capital was quite high from World War II until the early 1980's but has been close to the riskless interest rate since then.

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1 Introduction

As noted by Cochrane (2017), the literature in macro-finance aimed at understanding the tremendous volatility of the market value of the U.S. Corporate sector implied by data on the equity of public traded firms "stands quite apart" from the literature in macroeconomics on stocks and flows for that sector. We believe that part of this disconnect between macro-finance and macroeconomics is due to the lack of a data set common to both fields. Macro-finance tends to rely on data for publicly traded firms while macroeconomists focus on the National Income and Product Accounts. The main premise of this paper is that, in recent years, there has been significant progress towards developing such a comprehensive data set known as the Integrated Macroeconomic Accounts (henceforth IMA). This IMA data set was developed as a joint project between the Bureau of Economic Analysis and the Federal Reserve that put together data from the National Income and Product Accounts (NIPA) on macroeconomic flows and stocks with comprehensive data on financial flows and balance sheets with equity measured at market value drawn from the Financial Accounts of the United States.[1]

In this paper, we use these IMA data to ask three questions.

First, are these IMA data for the U.S. Corporate Sector a useful unified data set for studying macroeconomic flows and stocks, financial flows, and market valuations and returns for the entire U.S. Corporate Sector over the past century? That is, do these IMA data offer a picture of the returns to claims to the U.S. Corporate Sector and the volatility of the valuation of the firms in that sector in line with the data on public firms available from the CRSP database? In the first part of this paper, we argue that the answer to this question is yes.

Second, we ask whether these IMA data shed new light on the drivers of the volatility of the market valuation of U.S. Corporations over the past 100 years in comparison with previous results found with data on public firms? Again, we argue that the answer to this question is yes. In particular, echoing prior results by Larraine and Yogo (2008), we find that Campbell-Shiller regressions conducted with these data are consistent with the hypothesis that dividend-price ratios forecast growth in future cash flows to firm owners in contrast to what is perhaps the conventional wisdom based on data on public firms that dividend-price ratios do not forecast future growth in cash flows. We document that the different results obtained with the IMA versus CRSP data are likely due to the policies of firms to smooth dividends over the business cycle and the trend changes in payout policies for public firms. We also present a simple valuation model to document that there is ample volatility in cash flows to firm owners in the IMA data to resolve the *Excess Volatitlity puzzle* of Shiller (1981).

[1]The Financial Accounts of the United States produced by the Federal Reserve were formerly known as the Flow of Funds. See Cagetti et al. (2013) for an introduction to the construction of these data.

Finally, we build a parsimonious macroeconomic model that we use to provide an accounting of the relationship between the high returns and volatility of market valuations of U.S. Corporations and the realized returns on physical capital and the choice of aggregate investment by those corporations. We use our model as a measurement device in the spirit of the Chari, Kehoe, and McGrattan (2007) Business Cycle Accounting model for uncovering where a fairly standard model of macroeconomic stocks and flows and valuation matches the data and where it has difficulties indicating that new models are required. We focus on the question of whether the excess returns an unlevered claim to the corporate sector the same as the excess returns to the capital stock? Or are the returns to a claim to physical capital closer to the risk free rate than the realized return on corporate equities?

Results from our accounting model indicated that the realized excess return to capital and its relationship to the market valuation of U.S. Corporations has changed dramatically over the 1929-2022 time period. Through the lens of our model, we measure a long-term secular decline in the realized return to capital after WWII. From WWII to 1980, realized excess returns on capital were extremely high – in line on average with the realized excess return on corporate equity as a whole. The Sharpe ratio for investment in physical capital during this time period implied by our model is implausibly high. From the early 1980's to the present, however, it appears that realized excess returns on capital have been much smaller and perhaps close to zero. Since the early 1980's the realized return to physical capital has declined in line with the decline in safe interest rates over this time period. It appears that cash flows to owners of physical capital are now so low that our economy may now violate the positive cash flow to owners of physical capital condition for dynamic efficiency laid out in Abel et al. (1989). We interpret this apparent change over time in the returns to physical capital as calling for new models of and/or data on the cash flows to and valuation of physical capital in the U.S. Corporate Sector for reasons that we spell out in section 2.

The paper is organized as follows.

In section 3, we document that the measures of corporate valuations and returns to claims on the corporate sector derived from these IMA data are remarkably similar to data on market valuations and annual equity returns found in CRSP data on public firms. Thus, these IMA data offer a consistent set of income statements, cash flow statements, and balance sheets measured at market values for the U.S. corporate sector as a whole over a longer time period than is available for public firm data.[2]

We then turn to our second question of whether these IMA data shed new light on the role of fluctuations in expected cash flows in driving the volatility of the market valuations of

[2]This is perhaps the most important contribution we can make with this paper in terms of encouraging those studying macroeconomics and asset pricing to agree on a common data set for future research.

U.S. Corporations. First, we observe that one of the most striking features of these IMA data is that the cash flows to owners of firms are extremely volatile as a fraction of output for the U.S. Corporate Sector. As shown in Figure 1, cash flows to owners of U.S. Corporations as a ratio to corporate after-tax output follow a pronounced U-shape over the period 1929-2022, falling from roughly 14% to 6% of after tax Gross Value Added between 1929 and World War II and rising from 6% to roughly 14% of after-tax output again from the late 1990's to 2022. The ratio of the market value of U.S. Corporations to corporate sector output follows a similar U-shaped pattern over this time period. As shown in Figure 2, it is clearly evident that a large portion of the observed swings in the market valuation of U.S. corporations over the 1929-2022 time period can be "accounted" for, in a mechanical sense, by the fall and then rise in cash flows to owners of these corporations when valued at a constant dividend-price ratio. Based on these data, it is no surprise that the U.S. stock market has boomed relative to the size of the economy over the past 20 years — cash flows to owners of U.S. corporations have boomed proportionately.

At the same time, it is also clear in these data that the ratio of corporate valuations to cash flows to firm owners, that is dividend-price ratios, have also fluctuated a great deal over the past century. A full accounting of the drivers of fluctuations in the market valuation of U.S. Corporations must also account for what drives aggregate ratios of cash flows to value. Is it fluctuations in investors' expectations of future cash flows? Or fluctuations in the rate of return that investors demand to hold claims against the corporate sector?

Campbell-Shiller (CS) regressions are one highly cited methodology for addressing these questions.[3] It is well known that there are many reasons to be skeptical about findings based on these regressions — the standard errors are huge, estimates in different sample periods are unstable, and that these regressions fail to forecast out of sample.[4] But conventional wisdom based on CS regression results using public firm data is that fluctuations in dividend-price ratios do not forecast the growth of payouts to firm owners.[5] As a result, conventional wisdom based on these CS regression results with public firm data concludes that fluctuations in dividend-price ratios for the aggregate corporate sector are driven primarily by fluctuations in the the rate of return that investors demand to hold claims against the corporate sector.[6]

In section 4, we use CS regressions with IMA data to revisit this conventional wisdom on what drives fluctuations in dividend-price ratios for aggregate claims on the U.S. Corporate

[3] See Campbell and Shiller (1988).

[4] See, for example, the papers cited in Campbell (2018) Chapter 5.4 regarding econometric issues with these regressions, in particular, Stambaugh (1999), and failure of these regressions to predict returns out of sample, in particular Goyal and Welch (2008) and Goyal, Welch, and Zafirov (2023).

[5] See Cochrane (2008) and Campbell (2018) page 141. See also Koijen and Van Nieuwerburgh (2011) for a broader survey of this literature.

[6] See, for example, Cochrane (2011).

sector. We find very different results than those found with public firm data. In particular, as shown in Table 2, CS regression results with IMA data favor the hypothesis that fluctuations in investors' expectations of future cash flows drive a substantial part of observed fluctuations in aggregate dividend-price ratios. We find in particular that the payout to price ratio for the U.S. Corporate Sector predicts growth in the ratio of Free Cash Flow to corporate output and not growth in corporate output. In this regard, our results corroborate important earlier findings with Flow of Funds data in Larraine and Yogo (2008).

There are two likely reasons for these different CS regression results revealed in IMA data relative to those found in public firm data. One is that, as shown in Figure 3, U.S. firms in the aggregate clearly use financial policies to smooth out business-cycle fluctuations in their operating cash flows when paying out dividends. The other is that, as shown in Figure 5, changes in public firm payout policies mean that public firm data on dividends do not show the huge growth in payouts to firm owners over the past two decades evident in IMA data. In contrast, data on total payouts from publicly traded (S&P1500) firms over the 1994-2018 time period from Zeng and Luk (2020) indicate that a comprehensive measure of total payouts from these public firms show the same dramatic upward trend seen in the IMA data. Thus, it appears that the failure of CS regressions to find predictability in future cash flows to firm owners may be an artifact of the public firm data rather than a reflection of the underlying drivers of fluctuations in dividend-price ratios.[7]

To address questions with these IMA data of whether the market valuation of U.S. Corporations show excess volatility relative to cash flows and the relationship between the returns to market claims on U.S. Corporations and returns to the physical capital in those corporations, we ask whether one can build a parsimonious macrofinance model that reconciles the high average realized rate of return on corporate equity and volatility of the market valuation of the U.S. Corporate sector with the data on macroeconomic flows and stocks in the IMA? We focus in particular with modeling the relationship between equity returns and market valuations and the investment done by and physical capital stock held by the U.S. Corporate Sector over this time period. We present such a model to address these questions in this paper.

To build our model, we combine the production side of a stochastic growth model as our model of output and incomes for workers and owners of the U.S. corporate sector with

[7] Boudoukh et al. (2007), Larraine and Yogo (2008), Davydiuk et al. (2023) make similar arguments using public firm data. Conceptually, with careful measurement of returns in terms of changes in price per share and dividends per share, CS regressions should give consistent results with those based on returns in terms of total market valuation and total payouts to investors. Campbell (2018) discusses on page 141 how "a shift towards repurchases in the 1980's might have increased the growth rate of expected dividends per share in a persistent manner that is not easily captured in historical regression analysis" thus accounting for the different results using these two measures of returns in finite sample.

a flexible specification of the dynamics of macroeconomic variables and the pricing kernel developed to account for a wide range of financial data as in Lustig, Van Nieuwerburgh, and Verdelhan (2013) and Jiang et al. (2022).

The one modification we make to our model of output and incomes relative to the most basic stochastic growth model is that we assume that firms face a time-varying wedge between total revenue and total costs that leads to a pure rent for firm owners that we refer to as *factorless income* following Karabarbounis and Neiman (2019). We allow this wedge to result in a factorless income share that can be positive or negative. We interpret market power on the part of firms as a force driving towards positive factorless income.

Firms' investment decisions in the model are guided by a standard capital Euler equation taking the model's pricing kernel as given. We derive closed-form expressions for the valuation of future factorless income based on this pricing kernel that appear to be new to the literature. These valuation formulas allow, in general, rich dynamics of risk premia on the different aggregate risks investors in the corporate sector in the model face. These include risks to growth of aggregate corporate output, risks to undepreciated capital, and risks of shocks to current and long-term shares of factorless income. We present this model in Section 5. We present the valuation formulas in Section 7 and Appendix section B.

We lay out the procedure with which we use the structure of our model to account for the annual observations in the IMA data sequentially in steps in which we look to use only the minimum structure of the model to do our accounting in each step starting in Section 5. In our first step we note that our model implies that the market valuation of U.S. corporations measured in the IMA can be divided into a portion corresponding to a valuation of the physical capital within these corporations and a portion due the valuation of the future factorless income that owners of these firms expect to earn. Because we assume no adjustment costs to investment in physical capital, we can do this decomposition of U.S. Corporate Enterprise Value into these two components directly from the data. That is, in our model, deviations of Tobin's Q measured as the gap between the Enterprise Value of U.S. Corporations and the replacement value of their installed capital are accounted for by the valuation of future factorless income, positive or negative.

We present that decomposition of the valuation of U.S. corporations into components due to the replacement value of their physical capital and to the discounted present value of their future factorless income in Figure 6. What is striking here is that, in the time period from 1929 to World War II, the large fluctuations in the value of U.S. corporations in this time period appear to be accounted for primarily by fluctuations in the capital output ratio, just as one would expect in the simplest stochastic growth model. In contrast, over the long time period from World War II to the present, fluctuations the value of U.S. corporations appear to be accounted for almost entirely by fluctuations in the value of factorless income, with the measured capital to output ratio being remarkably stable.

In our second step, we use our model to measure the realized share of factorless income and the realized return on physical capital. We conduct this measurement in Section 6. Here we use our assumption that production in the model is carried out with a Cobb-Douglas production function that has been stable over the entire 1929-2022 time period. With this assumption, our model's implications for the realized factorless income share and realized returns to physical capital can be read off data on the labor share of output given an assumed value of the share α of capital in production. We show the dynamics of the factorless income share implied by this measurement procedure in Figure 7.

We then develop formulas to price future factorless income in Section 7. We use these formulas to ask whether it is possible to account for the level and volatility of observed market valuations of U.S. Corporations based on a specification of our pricing kernel in which all risk premia are constant over time? We intend this accounting exercise in the spirit of the calculation in Shiller (1981) leading to the Excess Volatility puzzle.

We find that it is easy to account for the volatility of the dividend-price ratio for U.S. corporations in the aggregate based on small fluctuations in investor's expectations of the share of factorless income in the long-run as long as one assumes that the risk premium on a claim to aggregate corporate output (both labor compensation and Gross Operating Surplus) in perpetuity is low. That is, if one assumes that the relevant $r - g$ for a claim to the aggregate output of the corporate sector is low so that price-dividend ratio for a claim to aggregate output of the corporate sector is high. In contrast, the excess volatility puzzle remains a puzzle in our model if the risk premium on this claim to aggregate corporate output is high.

The intuition for this result is simple. A claim to future factorless income is a claim to a stochastic fraction of aggregate output of the corporate sector. In section 7, we show analytically that the marginal impact of a shock to the expected value of factorless income in the long-run on the current valuation of factorless income is determined by the price-dividend ratio of a claim to aggregate output of the corporate sector and not by the price-dividend ratio for a claim to corporate equity overall.[8] Hence, if that price-dividend ratio of a claim to aggregate output of the corporate sector is high, then shocks to expectations of factorless income in the long run have a powerful impact on current valuations of corporate equity, with this effect becoming arbitrarily large as the price-dividend ratio on a claim to aggregate output gets large.

[8]It appears that this result that it is the price-dividend ratio for a claim to output that is relevant for evaluating the marginal impact of a change in expected cash flows over the long run on the market valuation of the corporate sector is new to the literature.

What is the price-dividend ratio for a claim to the aggregate output of the U.S. corporate sector? We do not know, but we argue that it is a key valuation benchmark for a number of important questions in macroeconomics. We know from the data that the average price-dividend ratio for a claim to aggregate equity is relatively low, say 25, and hence relatively large fluctuations in expectations about cash flows in the long run would be needed to account for observed equity volatility if this is the relevant valuation ratio for claims to factorless income. But it can be argued that a claim to the aggregate output of the corporate sector is a safer claim than a claim to corporate equity and hence it has a higher price-dividend ration. Lustig, Van Nieuwerburgh, and Verdelhan (2013) argue that it is quite high, perhaps 50 or 80 or even more. Since a claim to future fiscal primary surpluses is similar to a claim to factorless income in that its payoff is a stochastic share of aggregate output, those such as Blanchard (2019) who argue that high levels of government debt can be sustained with expectations of small changes in future primary surpluses are implicitly arguing that the price dividend ratio for a claim to aggregate output is quite high, even infinite.[9]

We then use our model to ask in Section 8 what is the relationship between observed realized returns on corporate Enterprise Value and equity on the one hand and investment realized returns on physical capital in the corporate sector on the other? We present our decomposition of the average realized return to Enterprise Value in a component coming from the average realized return to physical capital and a component coming from a claim to factorless income. We also present our model's implications for the dynamics of the returns to physical capital in that section.

In section 9, we examine the relationship between the price-earnings ratio implied by the IMA data and that of Shiller's CAPE smoothed measure of a price earnings ratio for publicly traded firms. Outside of the period from 1929-WWII. These two measures correspond quite closely.

Finally, in section 10, we conclude. We see the IMA data as a rich data set that we hope allows for a closer integration of macro-finance and macroeconomics in future work. We see this paper as a small step towards that goal.

2 Related Literature

There is a huge literature in macro-finance that aims to develop models of the marginal utility of the marginal investor to resolve the famous equity premium puzzle of Mehra and Prescott (1985) manifest here as the high unconditional average rate of return observed on claims on the corporate sector in both the public firm and IMA data. We do not attempt

[9]See also Abel and Panageas (2022) for an exposition of this point in a model economy with capital.

such an economic model of the marginal investor. Instead, we use the model as an accounting framework to both account for the observed fluctuations in the value of U.S. corporations year-by-year since 1929 and to decompose the observed realized returns to claims on the corporate sector into a portion due to the return on physical capital and a portion due to claims on factorless income.

Our focus on shocks to current and future factorless income are closely related to the arguments of Lustig and Van Nieuwerburgh (2008) and Greenwald, Lettau, and Ludvigson (2023) that shocks to the distribution of income between workers and owners of firms have been an important driver of fluctuations in the valuation of U.S. Corporations. Our principal contribution relative to these papers is to add consideration of physical capital and investment. We follow a large recent literature in macro-finance that builds on these ideas. See, for example, Farhi and Gourio (2018), Crouzet and Eberly (2018), Philippon (2019), Barkai (2020), Eggertsson, Robbins, and Wold (2021), Greenwald, Lettau, and Ludvigson (2023), and Crouzet and Eberly (2023). With the notable exception of Crouzet and Eberly (2023), these papers do not account year-by-year for both corporate valuations and changes in capital investment over a long time period. We see our use of the IMA data in a way that allows for consistent comparison of macroeconomic and valuation and realized return data across a range of standard valuation metrics as the primary contribution of our paper relative to this prior literature.

Tallarini (2000) and Kaltenbrunner and Loechster (2010) are important papers showing how to reconcile standard business cycle fluctuations with a high equity premium in standard stochastic growth models with a representative agent with recursive preferences. These models have an advantage of being fairly tractable using standard approximation techniques. We conjecture that incorporation of shocks to factorless income in models such as these might be a fruitful avenue for developing fully equilibrium macrofinance models, albeit with constant risk premia over time. Gourio (2012), Ilut and Schneider (2014), Basu and Bundick (2017), Hall (2017), Cambell, Pflueger, and Viceira (2020), and Basu et al. (2023) develop business cycle models based on time-varying risk premia arising from a variety of different sources.

In our modeling of the returns to physical capital, we have ignored adjustment costs to investment in physical capital as in Tobin (1969), Hayashi (1982). We see consideration of such adjustment costs as an important area for future work. It is standard in macro-finance models of the equity premium to include such adjustment costs. See, for example,Cochrane (1991), Jermann (1998) and Jermann (2010). See also Philippon (2009) and Merz and Yashiv (2007). As indicated in these papers, the first-order condition for optimal investment should be related to market returns on claims to the corporation.[10]

[10]Of course, a full reconciliation of a model with adjustment costs with the IMA data would also require

Relative to these papers, we see the main puzzle that arises in the IMA data interpreted through our simple model is the dramatic change in the realized excess returns to physical capital between the period from WWII to the early 1980's and afterwards. Did the nature of investment adjustment costs change sharply in the early 1980's? Or was it some change in tax policy that impacted the marginal incentives to invest? We see further investigation of the impact of changes in tax policy as a promising area for possible explanations of this puzzling behavior of returns to capital. In that vein, see McGrattan and Prescott (2005) and Barro and Furman (2018).

In our measurement, we have abstracted from the role of unmeasured intangible capital in accounting for fluctuations in the value of the U.S. corporate sector. Many papers consider the role of unmeasured intangible capital in driving the boom in the market valuation of U.S. firms in recent decades. See, for example, McGrattan and Prescott (2010) and Crouzet et al. (2022). Eisfeldt and Papanikolaou (2014), Belo et al. (2022), Eisfeldt, Kim, and Papanikolaou (2022) and the papers cited therein argue that measured of intangible capital drawn from firms' accounting statements that is not included in the National Income and Product Accounts help account for the valuation of firms in the cross section. We see this as a fruitful avenue for future research, but we see two hurdles that should be overcome in developing this hypothesis.

First, the aggregate data cited in Corrado et al. (2022) are not favorable to the hypotheses that changes in the stock of unmeasured capital have contributed much to the fluctuations in the value of the U.S. Corporate sector as in these aggregate data on capital stocks not measured by the BEA, there is no trend in the stock of such capital relative to value added over the past decade or more. Hence, incorporating these estimates of unmeasured capital would not serve to explain much of the rise in the market valuation of U.S. corporations over the past decade. Second, we suggest that a model of the variability of the market valuation of the U.S. Corporate sector over the past century based on fluctuations in the stock of unmeasured capital held by U.S. Corporations should also account for observed flows of free cash flow to owners of these corporations, as these cash flows are invariant to failure to measure investment. See Atkeson (2020). We have seen the the fluctuations in firm value are large, and hence the corresponding fluctuations in the ratio of the stock of unmeasured capital to output would also have to be large, presumably corresponding to large fluctuations in unmeasured investment as a fraction of output.

We now turn to our discussion of the IMA data.

an accounting for how the adjustment costs are recorded in NIPA data on free cash flow.

3 Measures of Corporate Value, Cash Flows, and Returns

In this paper, we focus on valuation and cash flow measures in the data from the Integrated Macroeconomic Accounts (IMA) closest to those concepts in a standard macroeconomic stochastic growth model. In particular, we consider a model in which firms are entirely equity financed, have no financial assets, and pay out all of their after-tax gross operating surplus less investment expenditures each period to firm owners. Following standard practice in finance, we refer to this measure of value as *Enterprise Value*. We use the We use the IMA data on construct a measure of Enterprise Value for the U.S. Corporate Sector as the sum of the market value of the equity and financial liabilities less the financial assets of U.S. corporations.[11]

We use the IMA data to construct a corresponding measure of cash flows to owners of these corporations that we term *Free Cash Flow*. Our measure of Free Cash Flow in the data is equal to after-tax Gross Operating Surplus less investment expenditures of U.S. corporations. These valuation and cash flow measures are similar to those used in Hall (2001). Full details of our data construction for these and all other variables used in the paper are given in Appendix section D.

We plot our valuation and cash flow measures relative to the after-tax Gross Value Added of the U.S. Corporate Sector in Figure 1. We show Enterprise Value in the left panel in blue and Free Cash Flow in the right panel in red. We see that both Enterprise Value and Free Cash Flow are quite volatile relative to the after-tax Gross Value Added of the U.S. Corporate Sector.

[11]This measure of Enterprise Value for the Financial and Non-Financial Corporate Sectors is reported on Table B1 "The Derivation of U.S. Net Wealth" of the Financial Accounts of the United States.

Figure 1: Left Panel: The Enterprise Value of U.S. Corporations over Corporate Gross Value Added less Taxes. Right Panel: Free Cash Flow from U.S. Corporations over Corporate Gross Value Added less Taxes. 1929-2022

Fluctuations in the ratio of the Enterprise Value to after-tax Gross Value Added of U.S. Corporations can be usefully decomposed into fluctuations in the ratio of Free Cash Flow to Enterprise Value (a dividend price ratio) and fluctuations in Enterprise Value that would be predicted by observed Free Cash Flow if this dividend price ratio was constant over time. We show this decomposition in Figure 2. In the left panel of this figure, we show the ratio of Free Cash Flow to Enterprise Value. This valuation ratio shows considerable business cycle fluctuations but appears to be stable over the long term. The right panel of this figure shows the ratio of Enterprise Value to after-tax Gross Value Added in blue and a predicted value of this ratio if Enterprise Value were a constant multiple of Free Cash Flow in red.[12] We see in this panel that the low frequency fluctuations in the ratio of Enterprise Value to after-tax Gross Value Added appear to be fairly well accounted for by low frequency fluctuations in the ratio of Free Cash Flow to after-tax Gross Value Added valued at a constant price dividend ratio.[13]

[12] We use a valuation multiple of Free Cash Flow of $1/0.032 = 31.25$ in this calculation.

[13] A simple variance decomposition of fluctuations in the log of the ratio of Enterprise Value is consistent with each of these components playing an important role. The variance of the log of the ratio of Enterprise Value to after-tax output from 1929-2022 is close to 0.14. The variances of the log of the ratio of Free Cash Flow to to after-tax output and the log of the ratio of Enterprise Value to Free Cash Flow are both close to 0.18 and the covariance between these two series is close to -0.11.

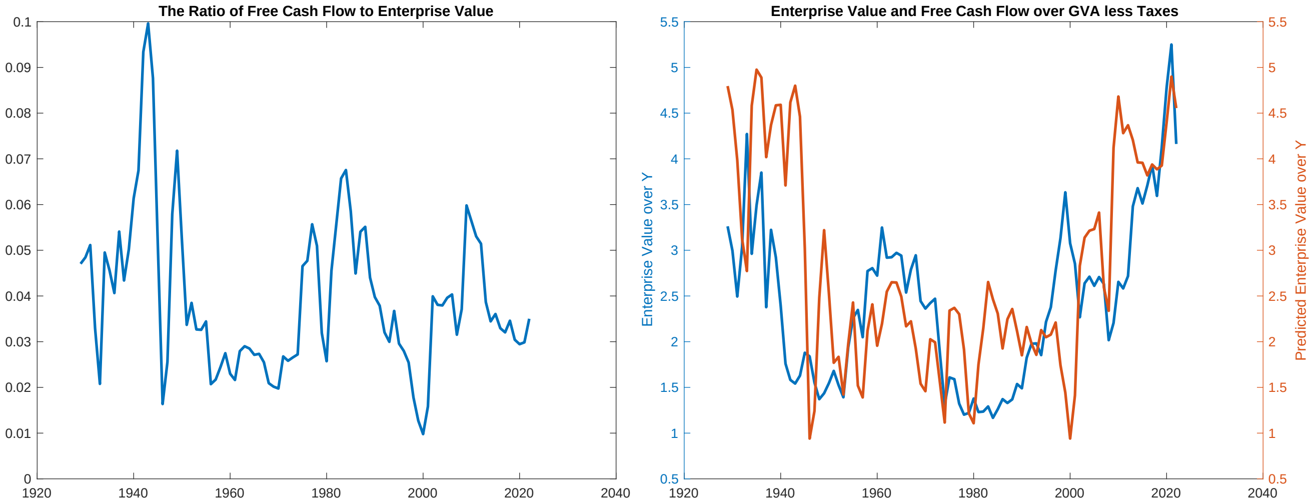

Figure 2: Left Panel: Free Cash Flow from U.S. Corporations over Enterprise Value. Right Panel: Enterprise Value (left axis) and Corporate Free Cash Flow (right axis) over Gross Value Added less Taxes. 1929-2022

We also use the IMA to construct a market valuation of the equity of U.S. Corporations (both publicly traded and closely held corporations) and a corresponding cash flow measure of monetary dividends paid to the owners of these corporations. We use these alternative measures of valuation and cash flow to further comparisons between the IMA data and work using data from CRSP and Compustat for publicly traded firms. We show these measures from the IMA of the value of Equity and dividends relative to after-tax Gross Value Added for the US Corporate Sector in red in the left and right panels of Figure 3. For comparison purposes, we show our measures of Enterprise Value and Free Cash Flow in blue.[14]

We see in the left panel of Figure 3 that the fluctuations in the market value of Equity and Enterprise Value for U.S. Corporations are tightly linked. By comparing the different scales for Enterprise Value (left axis) and Equity (right axis), we see that Enterprise Value is consistently about 50 percentage points of after-tax Gross Value Added larger than the market value of Equity. This difference between Enterprise Value and Equity Value corresponds to net debt of the U.S. Corporate Sector.

[14]We show an analogous plot for Enterprise Value and the market value of publicly traded equities both relative to after-tax Gross Value Added of the U.S. Corporate Sector in Appendix Figure A.1.

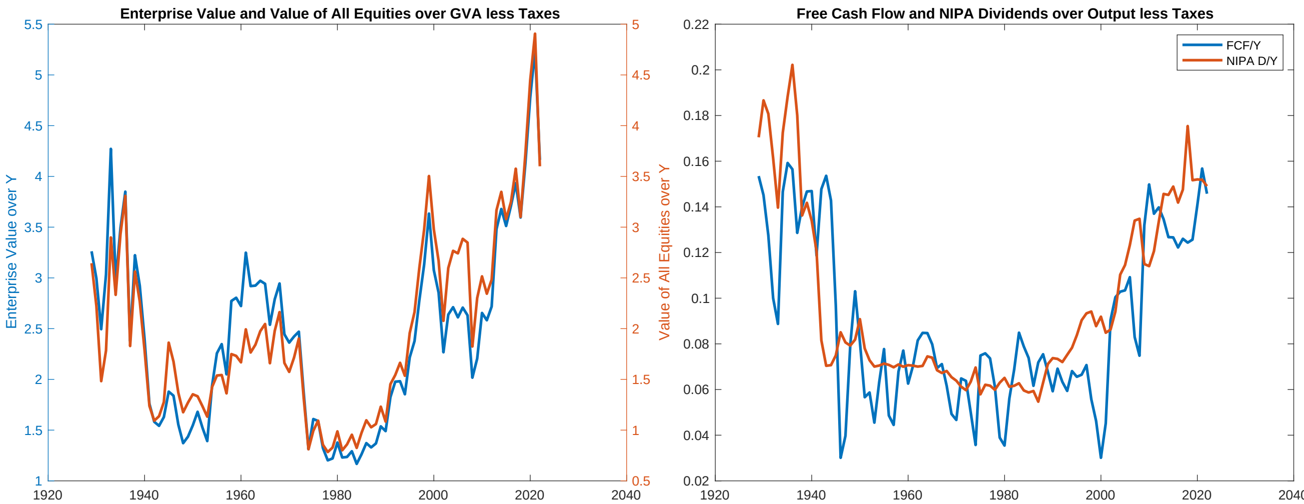

Figure 3: Left Panel: Enterprise Value (left axis) and Equity Value (right axis) of U.S. Corporations over Corporate Gross Value Added less Taxes. Right Panel: Free Cash Flow and NIPA Monetary Dividends Paid over Gross Value Added less Taxes. 1929-2022

We now consider properties of the annual returns on Enterprise Value and Equity implied by these two sets of valuation and cash flow measures from the IMA. We compute the returns on Enterprise Value from the perspective of a household in a stochastic growth model that owns the entire corporate sector and receives all cash paid out by that sector. Using that perspective, we denote Enterprise Value at the end of period t as V_t, Free Cash Flow in period $t+1$ as FCF_{t+1}, and construct realized returns on Enterprise Value each year as

$$\exp(r^V_{t+1}) = \frac{FCF_{t+1} + V_{t+1}}{V_t}$$

We deflate these and all nominal returns by the growth in the PCE deflator to compute realized real returns.

We compute realized returns on Equity from the perspective of a household that purchases equity at the end of period t at price V^E_t, collects dividend payments in year $t+1$, D^{IMA}_{t+1}, and sells that equity realizing a capital gain corresponding to the IMA reported revaluation of outstanding equity at $t+1$, $REVAL^E_{t+1}$. We compute this realized return as

$$\exp(r^E_{t+1}) = \frac{D^{IMA}_{t+1} + REVAL^E_{t+1}}{V^E_t}$$

This calculation of returns is closer to what is done in CRSP or Standard and Poors' data for public firms.

We report some basic statistics of the mean and standard deviations of log returns using these two return concepts as well as analogous return and dividend growth statistics computed using CRSP returns on the value-weighted portfolio of NYSE, AMEX, and NASDAQ stocks in Table 1. We see in this table that all three measures of returns have similar means and standard deviations. We also see that the standard deviation of growth in log Free Cash Flow is much higher than for the other two measures of dividends. This difference in volatility is readily visible in the right panel of Figure 3. We see in that figure that IMA Dividends smooth out the higher frequency fluctuations in Free Cash Flow likely due to business cycle fluctuations in investment.

Table 1: Mean and Standard Deviation of Real Log Returns and Log Dividend Growth on Enterprise Value, IMA Equity, and CRSP Value Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std D growth
Enterprise Value	1929-2022	0.072	0.146	0.28
IMA Equity	1929-2022	0.076	0.173	0.073
CRSP VW	1929-2022	0.061	0.194	0.138

We next examine the extent to which these measures of realized real returns on Enterprise Value and on IMA Equity line up with measures of realized real returns computed using the CRSP value-weighted portfolio in Figure 4. In the left panel, we show a scatter plot of realized annual returns on the CRSP portfolio on the x-axis and returns on Enterprise Value on the y-axis. The red line in the figure is a 45 degree line. We show the corresponding scatter plot for CRSP returns and realized returns on IMA Equity in the right panel. We see in the figure that both measures of returns constructed from the IMA data line up quite well with measures of equity returns from the CRSP database.[15]. The correlation of returns on Enterprise Value with those on the value-weighted CRSP portfolio is 0.943 for the period 1929-2022. The corresponding correlation for IMA Equity returns with CRSP returns is 0.981.[16]

[15] We show the same scatter plots using data from the 1946-2022 time period in Appendix Figure A.2

[16] Note that one would expect some deviation of returns on Enterprise Value from returns on equity given the presence of net debt documented in the left panel of Figure 3.

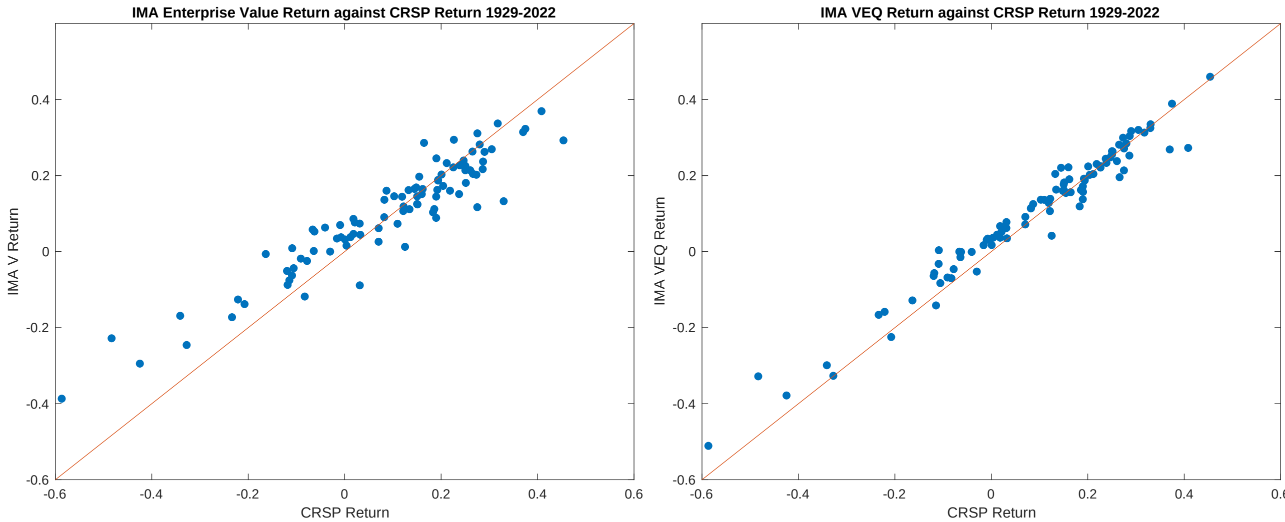

Figure 4: Left Panel: Realized Returns on Enterprise Value vs. CRSP Value Weighted Return 1929-2022. Right Panel: Realized Returns on IMA Equity Value vs. CRSP Value Weighted Return: 1929-2022

This close correspondence between measures of value and returns for claims on the U.S. Corporate Sector from the Integrated Macroeconomic Accounts with measures of value and returns constructed from CRSP data on public firms gives us some confidence that these Integrated Macroeconomic Accounts are a useful data source for further work in macrofinance aimed at offering an integrated account of aggregate corporate valuations and cash flows. We start on that agenda in the remainder of this paper.

4 Revisiting Campbell-Shiller Regressions

In Figure 2 we provide evidence that the ratio of U.S. Corporate Free Cash Flow to owners of firms has fluctuated a great deal relative to after-tax output of the U.S. Corporate Sector over the period 1929-2022. The right panel of that figure provides suggestive evidence that these fluctuations in Free Cash Flow relative to after-tax output might account for a substantial portion of observed fluctuations in the Enterprise Value of U.S. Corporations relative to the after-tax output of the U.S. Corporate Sector over this time period. But it is also clear from the left panel of that figure that the ratio of Free Cash Flow to Enterprise Value (a dividend price ratio) is not constant over this time period so that fluctuations in this valuation ratio also play a significant role in accounting for fluctuations in the ratio of Enterprise Value to Corporate Sector after-tax output. Thus, to provide a full account of the drivers of fluctuations in the ratio of Enterprise Value to after-tax output of the U.S. Corporate Sector, we must also provide an account of the drivers of fluctuations in the ratio of Free Cash Flow to Enterprise Value, or, in other words, of the fluctuations over time in the dividend-price ratio for Enterprise Value.

Conceptually, a dividend price ratio can move because agents beliefs regarding the growth of future cash flows from that asset have changed or because the expected returns agents demand to hold that asset have changed. Campbell-Shiller regressions are one widely-used methodology for decomposing fluctuations in dividend-price ratios into a portion due to fluctuations in expected future rates of return and a component due to fluctuations in expected growth of future cash flows to owners of firms. Prominent papers in this literature run these regressions with data on publicly traded firms and argue that fluctuations in expected growth of future cash flows to owners of firms account for at best a small portion of the observed fluctuations in dividend-price ratios for public equity because movements in these dividend price ratios show little ability to forecast future growth of cash flows.[17]

In this section, we revisit the results of Campbell-Shiller regressions using our data on Enterprise Value and Free Cash Flow for the US Corporate Sector and IMA data for Equity Value and Dividends highlighted in Figures 3 and 4. In this regard, we draw heavily on prior work by Larraine and Yogo (2008) who argued using similar data that fluctuations in valuation ratios do have considerable ability to forecast future cash flow growth. We find similar results here. We also explore differences in the dividend series for public firms and that from NIPA that may account for these different Campbell-Shiller regression results when IMA data versus public firm data are used.

The regressions we consider are derived following the analysis in Cochrane (2011) leading to his Table II. We spell this logic out in detail for readers who may not be familiar with this standard material in Finance.

The realized return on any asset with positive dividends can be written as

$$\exp(r_{t+1}) = \frac{P_{t+1} + D_{t+1}}{P_t} = \left[\frac{\frac{P_{t+1}}{D_{t+1}} + 1}{\frac{P_t}{D_t}}\right] \frac{D_{t+1}}{D_t}$$

A loglinear approximation to this equation gives realized log returns as

$$\hat{r}_{t+1} \approx -\rho \widehat{dp}_{t+1} + \widehat{dp}_t + \hat{g}_{Dt+1}$$

with $\widehat{dp}_t$ denoting the log deviation of the dividend-price ratio from its value at the point of approximation, $\hat{g}_{Dt+1}$ denoting the log deviation of the growth rate of dividends from its

[17] See Koijen and Van Nieuwerburgh (2011) and Campbell (2018) Chapters 5.3 - 5.5 for a textbook summary of this methodology and discussion of it in the literature.

value at the point of approximation, and ρ being a constant of approximation determined by the price dividend ratio at the point of approximation given by

$$\rho \equiv \frac{\overline{P/D}}{\overline{P/D}+1}$$

We have suppressed reference to the constant term in this approximation.

Rearranging terms relates the log dividend-price ratio to future realized returns, realized divided growth, and the future realized dividend-price ratio

$$\widehat{dp}_t \approx \hat{r}_{t+1} - \hat{g}_{Dt+1} + \rho \widehat{dp}_{t+1}$$

If we iterate on this formula k times, we get a formula relating the current log dividend-price ratio to cumulative realized returns and dividend growth over horizon k and the terminal dividend-price ratio at that horizon

$$\widehat{dp}_t \approx \sum_{j=1}^{k} \rho^{j-1} \hat{r}_{t+j} - \sum_{j=1}^{k} \rho^{j-1} \hat{g}_{Dt+j} + \rho^k \widehat{dp}_{t+k}$$

This formula holds for all realizations, so it holds in expectation as well. Thus, a movement in the dividend-price ratio for this asset at time t should correspond to a linear combination of movements in expected future returns, expected future dividend growth, and the expected future dividend price ratio over a horizon of k future periods. If the dividend price ratio is stationary over time, the final term in this expression should go to zero as k gets large since $\rho < 1$ by construction. This argument leads to the standard decomposition of changes in current dividend price ratios into changes in subsequent returns and dividend growth.

We now run three regressions based on this approximation formula given by

$$\sum_{j=1}^{k} \rho^{j-1} \hat{r}_{t+j} = \alpha_r^k + \beta_r^k \widehat{dp}_t + \epsilon_{rt+k}$$

$$\sum_{j=1}^{k} \rho^{j-1} \hat{g}_{Dt+j} = \alpha_{gD}^k + \beta_{gD}^k \widehat{dp}_t + \epsilon_{gDt+k}$$

$$\rho^k \widehat{dp}_{t+k} = \alpha_{dp}^k + \beta_{dp}^k \widehat{dp}_t + \epsilon_{dpt+k}$$

Observe that if one imposes the log return approximation, the slope coefficients in these regressions satisfy the constraint

$$\beta_r^k - \beta_{gD}^k + \beta_{dp}^k = 1$$

That is, one can interpret these slope coefficients as indicative of the extent to which fluctuations in log-dividend price ratios are accounted for by fluctuations in expected returns, expected dividend growth, and expected future dividend price ratios. Conventional wisdom is that there is little evidence when using data from publicly traded firms that movements in that dividend-price ratios forecast future growth in dividends. That is, typical estimates of the slope coefficient β_{gD}^k using these data are small. Because the slope coefficients from these three regressions are linked as above, this failure to find evidence of predictability of dividend growth is interpreted as evidence that fluctuations in dividend price ratios for the stock market as a whole are driven primarily by fluctuations in future expected returns.

We revisit results from running these regressions over the 1929-2022 time period using annual data on real returns from CRSP on value-weighted returns, dividend-price ratios, and dividend growth, as well as our data on Enterprise Value Returns and Free Cash Flow and IMA Equity Value Returns and NIPA Dividends. Following the presentation of results in Cochrane (2011) Table II, in our Table 2 we report estimated slope coefficients corresponding to future returns (β_r^k), future dividend growth (β_{gD}^k), and the future price dividend ratio (β_{dp}^k) using a regression in which we construct the cumulated returns and dividend growth terms on the right side of the first two regressions directly in the data at a fifteen year horizon, and then run the three regressions.[18]

We find three main results from these regression results. First, we confirm that in CRSP data, the estimated slope coefficients on future dividend growth are relatively small. Second, we find much larger coefficients on future dividend growth when we run these regressions using Enterprise Value and Free Cash Flow, suggesting that perhaps two thirds of fluctuations in the log of the ratio of Free Cash Flow to Enterprise Value are due to fluctuations in expected growth in these cash flows over a 15-year horizon. Third, with IMA Equity and NIPA dividend data, we find coefficients on dividend growth in between those with CRSP data and with Enterprise Value and Free Cash Flow.

We note that it is possible to decompose the predictability of growth in Free Cash Flow shown in this table into a component due to predictability of growth in the log of after tax

[18]We also report regression results using an alternative VAR methodology and for the 1946-2022 sample period separately in Appendix Tables A.2 and A.3. When we run these regressions using the same sample period as in Cochrane (2011) with nominal returns and nominal dividend growth, we reproduce the results in Table II in that paper.

Table 2: Campbell-Shiller 15-year Horizon Regression Coefficients

	return β_r^k	dividend growth β_{gD}^k	future dp ratio β_{dp}^k
CRSP Data 1929-2022			
Direct $k = 15$	0.57	-0.23	0.20
IMA FCF and V Data 1929-2022			
Direct $k = 15$	0.46	-0.71	-0.17
NIPA D and VEQ Data 1929-2022			
Direct $k = 15$	0.65	-0.37	-0.02

gross value added and growth in the log of the ratio of Free Cash Flow to after-tax Gross Value added. Running these regressions separately reveals that all of the predictability in the growth of Free Cash Flow comes from the predictability in its share in output rather than in the growth of output.

There appear to be two reasons that these regressions with different valuation and cash flow measures yield different results regarding the predictability of future cash flow growth. One is that, as is evident in the right panel of Figure 3, NIPA dividends appear to be a smoothed version of Free Cash Flow. As a result, as shown in Table 1, Free Cash Flow growth is much more volatile than NIPA dividend growth or CRSP dividend growth. The business cycle volatility in Free Cash Flow shown in Figure 3 is consistent with what one would expect from a standard stochastic growth model — investment, and hence free cash flow, is volatile over the business cycle. It appears in the NIPA dividend data that firms use financial policies to smooth out these business cycle fluctuations in free cash flow.

A second potential reason for the differences in these regression results is evident in Figure 5. In the left panel of this figure, we show NIPA dividends over corporate after-tax output in blue. The red line shows a compounding of CRSP dividend annual growth rates starting from an assumed initial ratio of CRSP dividends to output of 0.16. As is evident in this figure, the long-run trends in the two dividend series are very different.

In the right panel of Figure 5, we show the dividend-price ratios in each of these three data sets. We see that the different long-run trends in cash flows for these series correspond to different long-run trends in dividend price ratios, with the dividend price ratio in the CRSP data following to what appears to be a permanently lower level after 2000.

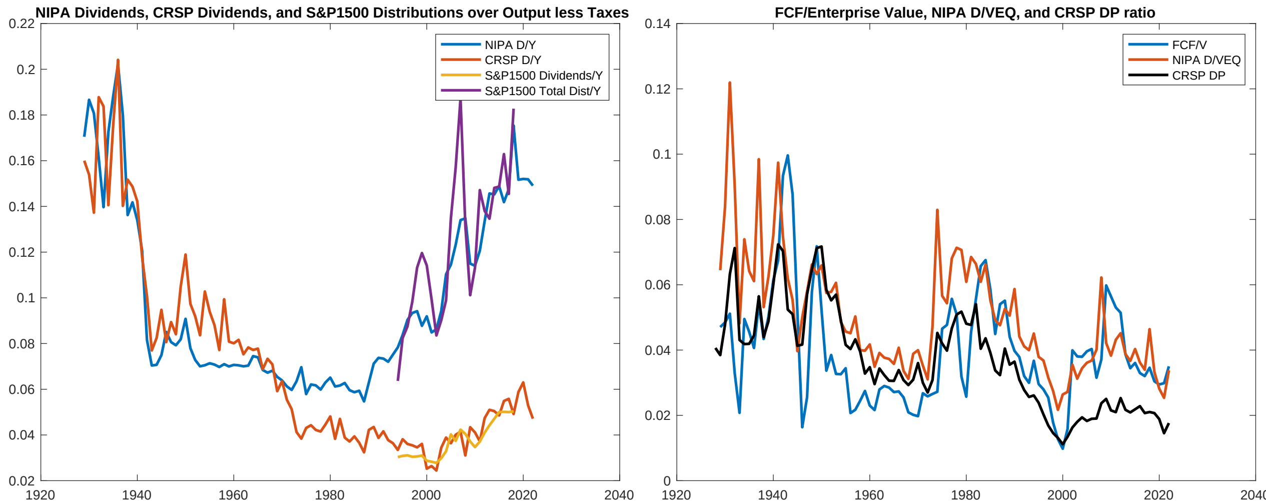

Figure 5: Left Panel: NIPA and CRSP Dividends over US Corporate Gross Value Added less Taxes 1929-2022. Right Panel: Free Cash Flow over Enterprise Value, NIPA Dividends over IMA Equity Value, and CRSP Dividend-Price Ratio 1929-2022

There are two frequently discussed explanations for why the IMA dividend (blue) and CRSP dividend (red) series shown in the left panel of Figure 5 show such divergent trends since the 1980's. One is that closely held corporations, particularly S Corporations, pay out more of their earnings as dividends.[19] A second is that public firms have changed their payout policies to favor payments to owners in the form of buybacks and expenditures on acquisitions rather than dividends.

Zeng and Luk (2020) provide evidence on the quantitative importance of these changes in payout policies for public firms by compiling estimates of the total dollar value of dividends, buybacks, and acquisitions made by S&P1500 firms over the period 1994 through 2018. In the left panel of Figure 5, we report their estimates for these public firm dividends relative to after-tax corporate value added in yellow and their estimates of the total payouts through dividends, buybacks, and acquisitions relative to after-tax corporate value added in purple.

We see in this figure that these changes in public firm payout policies have had a very large impact on the trends in public firm total payouts as compared to public firm dividends. In particular, the data shown in purple suggest that total payouts to owners of public firms show the same low frequency trends that we observe in IMA measures of Free Cash Flow and Dividends for the Corporate Sector as a whole. Thus, it appears to be the case for public firms that increased valuations of these firms in recent years may be justified by increased cash flows.

[19]See, for example, the material on S corporation dividends available here from the Bureau of Economic Analysis `https://www.bea.gov/help/faq/318`.

We take away from these results that the conventional wisdom that fluctuations in expected future cash flows are not an important driver of fluctuations in dividend price ratios and that these valuation ratios are driven primarily, or even exclusively, by fluctuations in expected returns seems worth reconsidering in light of these IMA data. We begin that reconsideration in the next section with a model that we use for accounting the the valuation of U.S. corporations, their cash flows, and their choices of investment in physical capital.

5 Model

We now introduce the model we use to account for these data on the dynamics of macroeconomic quantities and valuations of the U.S. Corporate sector over the 1929-2022 time period.

Our model has two main components. The first component is model of production and incomes in the corporate sector based on the model of production in a standard stochastic growth model. Thus, the model provides an accounting of the relationship between capital and labor and aggregate output through the production function and of the division of income (equal to output) as taxes, compensation of labor, and cash flows to owners of firms and the physical capital in those firms. The one modification we make to the standard model is that we assume that firms maximize profits subject to a time-varying wedge between total revenue and total costs that results in a portion of total income that corresponds to a pure rent paid to the owners of firms. Following Karabarbounis and Neiman (2019), we refer to this income as *factorless income*.

The second component of our model is the pricing kernel M_{t+1}. The dynamics of this pricing kernel together with the dynamics of the driving exogenous shocks in the model is specified exogenously. The pricing kernel is used in the model to value the flows of income from the production side of the model and it to rationalize the choices of investment in physical capital observed in the data through the capital Euler equation.

5.1 Output and Income Shares

Our model of production and incomes in the corporate sector is as follows. Aggregate output, corresponding to Gross Value Added of the corporate sector, is given by a Cobb-Douglas production function

$$GVA_t = K_t^{\alpha}(Z_t L)^{1-\alpha}$$

where K_t is the stock of physical capital in units of capital services, L is labor which is inelastically supplied, and Z_t is a shock to aggregate productivity. We maintain the assumption that the share of capital in production, denoted by α, is constant over time.

The evolution of the stock of capital services is given by

$$K_{t+1} = (1 - \delta_t)K_t + I_t$$

where δ_t is a time-varying physical depreciation rate for capital services and I_t is investment in new capital services. Note that we assume here that there are not investment adjustment costs.

The terms K_t and I_t are not directly measured in the data. Instead, the IMA report end of period nominal values of the stock of capital at replacement cost, nominal investment expenditures, nominal consumption of fixed capital, and nominal revaluations of the stock of capital carried into the period due to changes in the replacement cost of that capital. In our model, we use P_t to denote the nominal price level and Q_t to denote the real price of capital goods. We right the real end-of-period t replacement cost of capital as Q_tK_{t+1}, real investment expenditure in period t as Q_tI_t, real consumption of fixed capital as $\delta_tQ_tK_t$, and real revaluations of the replacement value of capital carried into period t by $(Q_t - Q_{t-1})K_t$. Thus, we write the capital transition equation in the model as

$$Q_tK_{t+1} = \left[(1 - \delta_t)\frac{Q_t}{Q_{t-1}}\right] Q_{t-1}K_t + Q_tI_t \tag{1}$$

Note that all of the terms in this equation can be constructed from the IMA data using the mapping

$$Q_tK_{t+1} = Q_{t-1}K_t + \underbrace{(Q_t - Q_{t-1})K_t}_{Reval_t + Other_t} - \underbrace{\delta_tQ_tK_t}_{CFC_t} + \underbrace{Q_tI_t}_{Investment_t}$$

Output and total income in the model is given by gross value added of the corporate sector GVA_t. This total income in the model is divided into four shares. The government takes a share of income as taxes τ_tGVA_t. We treat all taxes in the model as a tax on value added and compute this tax share from the IMA data using the sum of taxes on production and imports less subsidies, taxes on income and wealth, and business transfers, as a fraction of Gross Value Added. To conserve on notation, going forward, we denote after-tax Gross Value Added as $Y_t = (1 - \tau_t)GVA_t$.

A share of after-tax Gross Value Added Y_t is paid as compensation to labor W_tL_t/Y_t in the model and is measured as in the IMA data. The remaining portion of income, comprising after-tax Gross Operating Surplus in the IMA data, is split in the model into a share corresponding to rental cost of the physical capital stock R_tK_t (or, equivalently, $\frac{R_t}{Q_{t-1}}Q_{t-1}K_t$) and share corresponding to a wedge μ_t between total income after-tax and the costs of labor compensation and capital rentals that is the source of factorless income in the model.

We assume that firms choose to employ labor and capital within each period to minimize costs given their Cobb-Douglas production function. Given our assumptions that taxes are levied on value added and that the wedge μ_t between after-tax revenue and costs is exogenous, we have that the shares of labor compensation and capital rentals in after-tax income are given by

$$\frac{W_t L}{Y_t} = (1-\alpha)(1-\kappa_t) \tag{2}$$

and

$$\frac{R_t}{Q_{t-1}} \frac{Q_{t-1} K_t}{Y_t} = \alpha(1-\kappa_t) \tag{3}$$

where

$$\kappa_t \equiv \frac{\mu_t - 1}{\mu_t} \tag{4}$$

where κ_t denotes the share of factorless income in total after-tax income.

These equations imply that the ratio of after-tax gross operating surplus to after-tax Gross Value Added in the model is given by $1-(1-\alpha)(1-\kappa_t)$ and that the ratio of Free Cash Flow to after-tax Gross Value Added is given by

$$\frac{FCF_t}{Y_t} = 1-(1-\alpha)(1-\kappa_t) - \frac{Q_t I_t}{Y_t} \tag{5}$$

where investment expenditures $Q_t I_t$ in the model correspond to expenditures on Gross Fixed Capital Formation in the IMA data.

5.2 Investment and Returns to Physical Capital

We assume that firms in the model choose investment expenditures and hence end of period capital $Q_t K_{t+1}$ to satisfy the following capital Euler equation taking the pricing kernel M_{t+1} as given

$$1 = \mathbb{E}_t M_{t+1} \left[\alpha(1-\kappa_{t+1}) \frac{Y_{t+1}}{Y_t} \frac{Y_t}{Q_t K_{t+1}} + (1-\delta_{t+1}) \frac{Q_{t+1}}{Q_t} \right] \tag{6}$$

where we have used equation 3 to substitute out for the rental rate on capital. We refer to the term in square braces in this equation as the realized return on physical capital and denote this gross return as $\exp(r^K_{t+1})$.

Note that this equation implies that the expected excess return to physical capital is given by

$$\mathbb{E}_t \exp(r^K_{t+1}) - \exp(r^f_t) = -(1+r^f_t)\mathbb{C}ov_t\left(M_{t+1}, (1+r^K_{t+1})\right) \tag{7}$$

where $\exp(r^f_t)$ is the risk free rate and the term on the right side of this equation is the risk premium on physical capital. In a standard macro model used for business cycle analysis, this risk premium is typically assumed to be quite small.

The model implies that Free Cash Flow to owners of firms defined in equation 5 can be decomposed into a component due to factorless income $\kappa_t Y_t$ and a component due to capital rentals less investment expenditure. We refer to this second component as *Free Cash Flow to Capital* defined in the model as

$$FCF_{Kt} = \alpha(1-\kappa_t)Y_t - Q_t I_t \tag{8}$$

Using equation 1, we can also compute realized returns to physical capital between period t and $t+1$ in the capital Euler equation 6 as

$$\exp(r^K_{t+1}) = \frac{FCF_{Kt+1} + Q_{t+1}K_{t+2}}{Q_t K_{t+1}} \tag{9}$$

5.3 Firm Valuation

Enterprise Value in our model is the expected discounted present value of Free Cash Flow to owners of firms with those present values computed using the model's pricing kernel M_{t+1}. Given our division of Free Cash Flow FCF_t into a component that is factorless income $FCF_{\Pi t} = \kappa_t Y_t$ and a component that is Free Cash Flow to Capital FCF_{Kt}, it is natural to compute Enterprise Value, denoted by V_t, as the sum of the values of these two cash flows

$$V_t = V_{Kt} + V_{\Pi t} \tag{10}$$

where V_{Kt} denotes the value of future Free Cash Flow to Capital and $V_{\Pi t}$ denotes the value of future factorless income. It is a standard result that, regardless of the dynamics of the pricing kernel and regardless of the parameters of the production function and capital accumulation, as long as firms choose investment according to the capital Euler equation 6, then the value of future Free Cash Flow to Capital is given by the end of period replacement value of the capital stock $V_{Kt} = Q_t K_{t+1}$.[20] Given this result, we measure the value of factorless income using the IMA data using the difference between Enterprise Value and the replacement value of the capital stock:

$$V_{\Pi t} = V_t - Q_t K_{t+1} \tag{11}$$

We show the breakdown of Enterprise Value relative to after-tax output into these two components in Figure 6. In the left panel of this figure, we show Enterprise Value (in blue) and the replacement value of the capital stock (in red) both relative to after-tax output of the Corporate Sector. In the right panel of this figure, we show Enterprise Value (in blue)

[20] We provide a proof of this result in Appendix section B.

and the Value of Factorless Income (in red) both relative to after-tax output of the Corporate Sector. We see in the left panel of this figure that between 1929 and World War II (WWII), fluctuations in the capital output ratio account for much of the fluctuations in Enterprise Value relative to output, but that after WWII, the ratio of capital to output has remained remarkably stable. In the left panel of this Figure, we see that it is fluctuations in the value of factorless income that account for the majority of fluctuations in Enterprise Value to Output. These data in this figure are the key valuation facts that we wish to account for — the dynamics of corporate valuations due to movements (or lack thereof) in the capital to output ratio and movements in the valuation of factorless income.

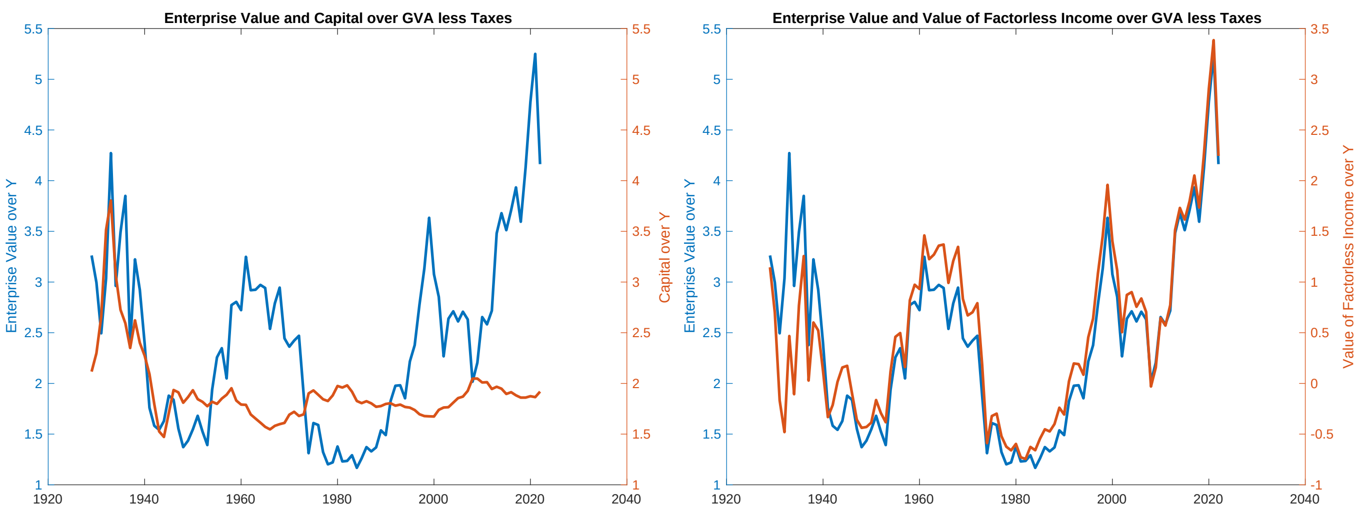

Figure 6: Left Panel: Enterprise Value (left axis) and Replacement Value of Capital Stock (right axis) over US Corporate Gross Value Added less Taxes. Right Panel: Enterprise Value (left axis) and Value of Factorless Income (right axis) over US Corporate Gross Value Added less Taxes

To summarize, the model that we use to account for the macroeconomic dynamics and valuation of the U.S. Corporate Sector over the period 1929-2022 is comprised of three parts. We develop each part sequentially in the next three sections of our paper.

First, we model production in this sector with a Cobb-Douglas production function that has a coefficient on the aggregate capital stock α that has been constant over this time period. We examine the implications of this component of our model for the measurement of the realized share of factorless income κ_t in after-tax Gross Value Added of the U.S. Corporate Sector in Section 6. We also develop the implications of this model of production for the share of Free Cash Flow accounted for by returns to physical capital in this section.

Second, in Section 7, we develop a model of the dynamics of the stochastic discount factor M_{t+1} and agents' expectations for the future dynamics of the share of factorless income in output to account for the observed dynamics of the value of future factorless income shown in the right panel of Figure 6.

Third, given the results in these two prior sections, in section 8, we examine the dynamics of the expected returns to investment in physical capital needed to rationalize the observed behavior of the capital to output ratio shown in the left panel of Figure 6.

Finally, in section 9, we use the results in these three sections to review how our model accounts for observed fluctuations in price-dividend and price-earnings ratios for the U.S. Corporate sector over the 1929-2022 time period.

6 Measuring Income Shares

In our model, we assume that output is given by a Cobb-Douglas production function in which the coefficient on physical capital α has been constant over the 1929-2022 time period. With this assumption, given a choice of α, we can identify the realized share of factorless income in after tax output κ_t from data on the share of labor compensation in after tax output. In the left panel of Figure 7, we shown the data on the share of labor compensation in after-tax Gross Value Added of the U.S. Corporate Sector from 1929-2022. In the right panel of this figure, we show the implied value of the share of factorless income in after tax output κ_t given a choice of $\alpha = 0.2646$. We see in this figure considerable volatility in this factorless income share, particularly in recent decades.

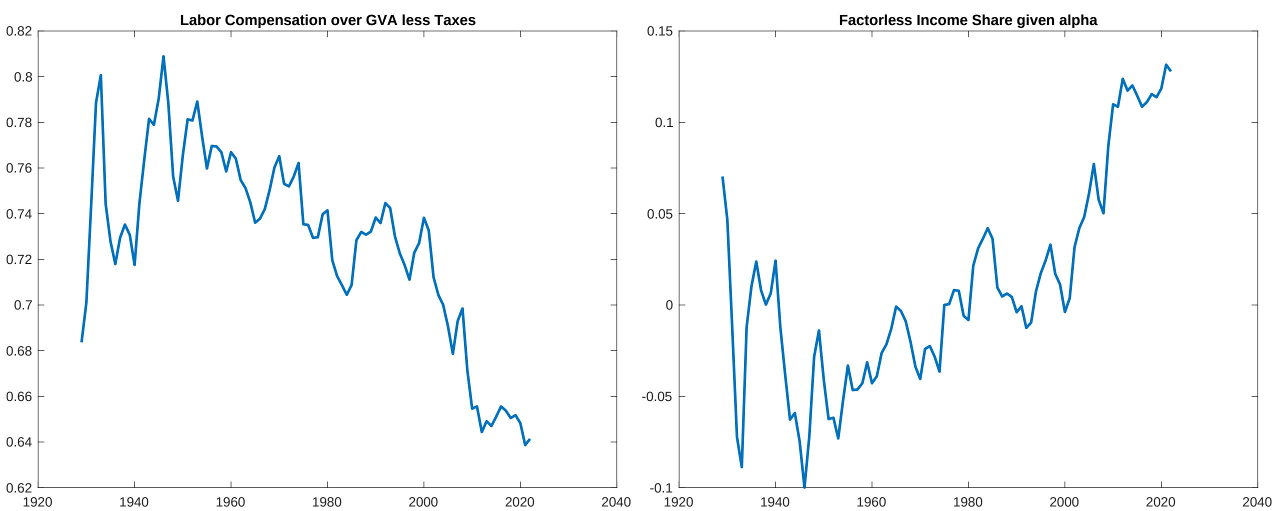

Figure 7: Left Panel: Share of Labor Compensation in US Corporate Gross Value Added less Taxes. Right Panel: Implied share of Factorless Income in US Corporate Gross Value Added less Taxes.

We take the capital share parameter $\alpha = 0.2646$ as a baseline value of this parameter. With this value of α, the sample mean share of factorless income in after tax output in Figure 7 is one percent. Note here that alternative choices of the capital share parameter α simply shift the implied share of factorless income up or down, but the alternative series for factorless income show similar volatility. With $\alpha = 0.25$, the sample mean for the factorless income share is just under 3 percent. If we choose $\alpha = 0.2725$, the implied sample mean for the factorless income share is equal to zero.

Of course, it is not immediately evident how the patterns of the labor share and implied share of factorless income shown in Figure 7 map into the data on the share of Free Cash Flow in after tax output shown in the right panel of Figure 1. In our model, the ratio of Free Cash Flow to after-tax output fluctuates for three reasons. The first two are fluctuations in the tax share in output and in the factorless income share. These combine to result in fluctuations in the ratio of after-tax Gross Operating Surplus to after-tax output.[21] The third are fluctuations in the ratio of investment to after-tax output. We show these two components of Free Cash Flow relative to after-tax output in the left panel of Figure 8. We see in this figure that both components contribute to the business cycle fluctuations in Free Cash Flow and that low frequency movements in the difference between these two series drive the low frequency movements in the ratio of Free Cash Flow to after-tax output shown in the left panel of Figure 2.

We next examine our model's implications for Free Cash Flow to Capital. In the right panel of Figure 8, we show the capital rental share (in blue) and the overall share of Free Cash Flow to Capital in after-tax output in red implied by our choice of $\alpha = 0.2646$ and the corresponding measure of the factorless income share κ_t shown in the right panel of Figure 7. We see in the right panel of Figure 8 that our model implies a modest decline in the share of capital rentals in after-tax output in blue (due to the rise in factorless income) but a dramatic secular decline the ratio of Free Cash Flow to Capital relative to after-tax output in red.

Note that this finding of a secular decline in Free Cash Flow to Capital results from a minimum of model structure: our assumption of a Cobb-Douglas production function with a constant capital share in costs α. Alternative values of α simply shift this measure of Free Cash Flow to Capital up or down without changing the clear secular trend. We discuss how this decline in Free Cash Flow to Capital impacts our measurement of the returns to investment in physical capital in Section 8.

[21] Note that in a standard stochastic growth model with Cobb-Douglas production and a constant tax share, the ratio of after-tax Gross Operating Surplus to after-tax output is constant over time.

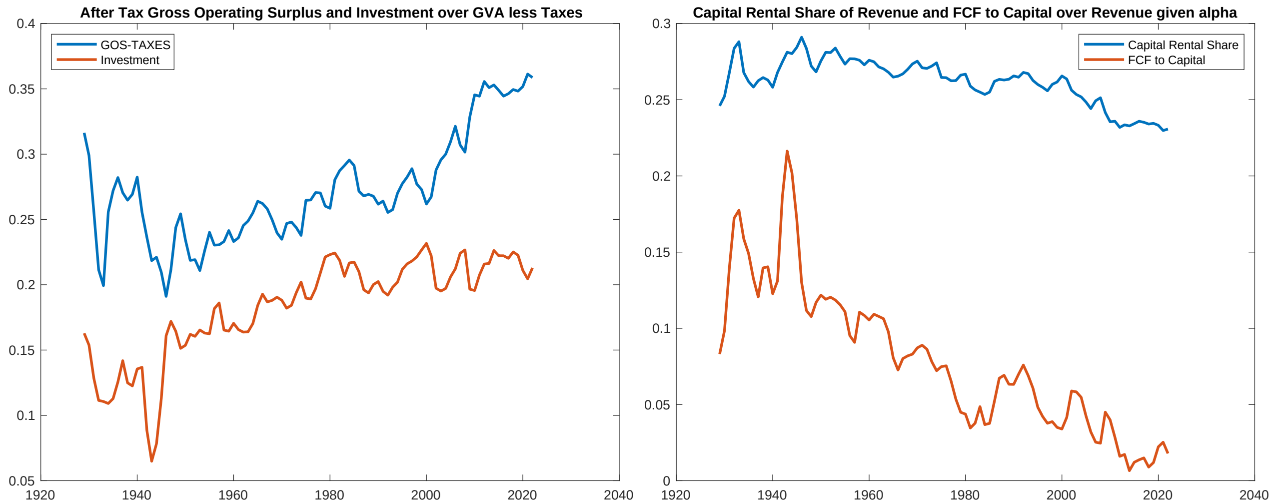

Figure 8: Left Panel: Shares of Gross Operating Surplus and Investment in US Corporate Gross Value Added less Taxes. Free Cash Flow over US Corporate Gross Value Added less Taxes is the difference between the blue and red lines in this panel. Right Panel: Implied Shares of Capital Rental payments (blue) and Free Cash Flow to Capital (red) in US Corporate Gross Value Added less Taxes

We next turn to our model of the dynamics of the pricing kernel M_{t+1} and agents' expectations of the dynamics of the future share of factorless income κ_t that we use to account for our measurement of the value of factorless income $V_{\Pi t}/Y_t$ shown in the right panel of Figure 6.

7 Valuing Factorless Income

In this section we develop our formula for valuing a claim to future factorless income as a function of the current and long-run expected factorless income share and apply that formula to measure the movements in the long-run expected share of factorless income required to justify the model-implied valuation of factorless income shown in Figure 6. We evaluate whether this model resolves the excess volatility puzzle of Shiller (1981) based on the volatility of this long-run expected factorless income share required to account for the observed volatility in the valuation of factorless income.

We value a claim to factorless income as follows. The price at t relative to output at t for a claim to factorless income at $t+k$ is given as

$$\frac{P_{\Pi t}^{(k)}}{Y_t} = \mathbb{E}_t M_{t,t+k} \frac{Y_{t+k}}{Y_t} \kappa_{t+k}$$

where $M_{t,t+k}$ is the pricing kernel between periods t and $t+k$ and the value of a claim to all factorless income from $t+1$ on is given by

$$\frac{V_{\Pi t}}{Y_t} = \sum_{k=1}^{\infty} \frac{P_{\Pi t}^{(k)}}{Y_t}$$

Note that we can write the prices of claims to factorless income at different horizons as

$$\frac{P_{\Pi t}^{(k)}}{Y_t} = \frac{P_{Yt}^{(k)}}{Y_t} \mathbb{E}_t \kappa_{t+k} + \mathbb{C}ov_t \left(M_{t,t+k} \frac{Y_{t+k}}{Y_t}, \kappa_{t+k} - \mathbb{E}_t \kappa_{t+k} \right) \tag{12}$$

where we define the price at t of a claim to output at $t+k$ relative to output at t by

$$\frac{P_{Yt}^{(k)}}{Y_t} = \mathbb{E}_t M_{t,t+k} \frac{Y_{t+k}}{Y_t}$$

Thus, from equation 12 we have that the price of a claim to factorless income at horizon k relative to output can move for three reasons. First, the price of a claim to output at horizon k relative to output at t given by $\frac{P_{Yt}^{(k+1)}}{Y_t}$ might move. Second, the expected factorless income share might move. And third, the risk premium in the covariance term might move.

Note that the pricing equation 12 differs from the standard pricing equation for an asset whose cash flows are always positive in that the covariance term representing risk impacts the level of the price of a claim to factorless income. Thus, even if expected factorless income $\mathbb{E}_t \kappa_{t+k}$ is positive, the price of a claim to that income can be negative. This property of our pricing model helps us account for observations of the value of factorless income that are below zero corresponding to measurement of values of Tobin's Q that are below one.

Note as well that the covariance term representing risk in equation 12 represents the risk attached to innovations to the share of factorless income. Since, to a first order, changes in the share of factorless income do not impact aggregate output, this risk is not the standard risk to aggregate output or consumption considered in many asset pricing models. Instead, it requires that innovations to the marginal utility of the marginal investor be correlated with innovations to the share of factorless income. Greenwald, Lettau, and Ludvigson (2023) present a model with such a risk premium due to the assumption that the marginal investor derives all of his or her wealth from a claim to cash flows to owners of firms. In our baseline parameterization of this model, our valuation of factorless income implies that this risk premium is zero.

Now we calculate what fluctuations in the expected factorless income share in the long-run are needed to account for observed fluctuations in $V_{\Pi t}/Y_t$ under the assumption that the discount rate for claims to factorless income are constant over time. We do not intend to make the claim here that the discount rate for claims to factorless income are constant over time in the data. Instead, we intend this calculation in the spirit of the calculation in Shiller (1981). We expand on the connection between our calculation here and that in Shiller (1981) in greater detail in Appendix section C.

We find in this calculation that relatively small fluctuations in the expected factorless income share in the long run are sufficient to account for observed volatility in the value of factorless income. We do not intend this calculation as an argument that this is the only factor driving fluctuations in the valuation of factorless income. Instead, we intend it only as a demonstration that small fluctuations in the expected share of factorless income in the long run have a powerful impact on our model's implications for the volatility of the valuation of U.S. Corporations.

Given equation 12, pricing factorless income with no time variation in risk premia amounts to assuming that the terms $P_{Yt}^{(k)}/Y_t$ are constant over time and that the conditional covariance term in that equation at each horizon k is also constant over time. We now discuss conditions under which these terms are indeed constant over time.

We denote the price of a claim at t to aggregate after-tax output at $t+k$ by $P_t^{(k)}$. Note that these prices satisfy the recursion

$$\frac{P_{Yt}^{(k+1)}}{Y_t} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \frac{P_{Yt+1}^{(k)}}{Y_{t+1}}$$

with

$$\frac{P_{Yt}^{(0)}}{Y_t} = 1$$

The value of a claim to aggregate output from $t+1$ on is given by

$$\frac{V_{Yt}}{Y_t} = \sum_{k=1}^{\infty} \frac{P_{Yt}^{(k)}}{Y_t}$$

Lemma 1: If the ratio of the price to a claim to output one period ahead to the current value of output, $P_{Yt}^{(1)}/Y_t$, is constant over time at $P_Y^{(1)}/Y$, then the price to a claim to output at all future dates relative to current output is constant over time as well V_{Yt}/Y_t and is given by

$$\frac{V_Y}{Y} = \frac{\frac{P_Y^{(1)}}{Y}}{1 - \frac{P_Y^{(1)}}{Y}} \tag{13}$$

For the proof of this lemma, see Appendix Section B.

Remark on Lemma 1: By definition

$$\frac{P_{Yt}^{(1)}}{Y_t} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} = \mathbb{E}_t M_{t+1} \mathbb{E}_t \frac{Y_{t+1}}{Y_t} + \mathbb{C}ov_t \left(M_{t+1}, \frac{Y_{t+1}}{Y_t} \right) \tag{14}$$

Note that

$$\mathbb{E}_t M_{t+1} \mathbb{E}_t \frac{Y_{t+1}}{Y_t} = \frac{1}{1 + r_t^f} \mathbb{E}_t \frac{Y_{t+1}}{Y_t}$$

is a comparison of the expected growth rate of output one period ahead and the one period risk free interest rate. The conditional covariance term in the equation above is a risk premium on a claim to aggregate output one period ahead. Thus, the assumption that $P_{Yt}^{(1)}/Y_t$ is constant over time is equivalent to assuming that movements in the gap between the risk free interest rate and the expected growth rate of output correspond to offsetting movements in the risk premium on a claim to output one period ahead. In Appendix section B.2.3, we present a parameter restriction on an essentially affine model with time-varying risk premia that could be used to assess via estimation the extent to which this assumption of a constant value of $\frac{P_{Yt}^{(1)}}{Y_t}$ is contradicted by the data.

Now we turn to developing conditions under which the conditional covariance term in equation 12 at each horizon k is also constant over time.

In developing these conditions, we assume that the factorless income share follows an AR1 with a shifting endpoint

$$\kappa_{t+1} - x_{\kappa_{t+1}} = \rho_\kappa (\kappa_t - x_{\kappa_t}) + \epsilon_{\kappa t+1} \tag{15}$$

$$x_{\kappa_{t+1}} = x_{\kappa_t} + \epsilon_{x_\kappa t+1} \tag{16}$$

These equations imply that

$$\mathbb{E}_t \kappa_{t+k} = \rho_\kappa^k (\kappa_t - x_{\kappa_t}) + x_{\kappa_t}$$

and

$$\mathbb{E}_t \kappa_{t+\infty} = x_{\kappa_t}$$

This model of the dynamics of the share of factorless income is similar to that used in Kozicki and Tinsley (2001) to model the dynamics of the short-term interest rate. The motivation for this assumed structure is the same in the two applications. Theory suggests that both the short rate and the share of factorless income should be stationary time series. Yet, in the data, both series look like they might be non-stationary and simple valuation models based on either the expectations hypothesis of the term structure or the valuation of factorless income with constant risk adjustments call for very high persistence in these series to account for either the volatility of long-term interest rates or observed volatility of valuations of factorless income. Here we have treated the endpoint x_{κ_t} as a non-stationary variable, but we would get very similar results if we assumed it was also an AR1 with very high persistence.

With these dynamics of the factorless income share, we have for $k \geq 0$, innovations to expectations of the factorless income share at horizon k are given by

$$\mathbb{E}_{t+1}\kappa_{t+k+1} - \mathbb{E}_t\kappa_{t+k+1} = \rho_\kappa^k(\kappa_{t+1} - x_{\kappa_{t+1}}) - \rho_\kappa^{k+1}(\kappa_t - x_{\kappa_t}) + x_{\kappa_{t+1}} - x_{\kappa_t} = \rho_\kappa^k \epsilon_{\kappa t+1} + \epsilon_{x_\kappa t+1}$$

We interpret these dynamics of the share of factorless income as follows. Standard theory argues that competition in product and factor markets should push the wedge between total revenue and total costs towards a relatively small level in the long term. We model the speed of these dynamics with ρ_κ and this long-term level of factorless income by x_{κ_t}.

Lemma 2: Assume that $P_{Yt}^{(1)}/Y_t$ is constant over time and, for all k and all t

$$\mathbb{C}ov_t\left(M_{t+1}\frac{Y_{t+1}}{Y_t}, \epsilon_{\kappa t+1}\right) = C$$

and

$$\mathbb{C}ov_t\left(M_{t+1}\frac{Y_{t+1}}{Y_t}, \epsilon_{x_\kappa t+1}\right) = D$$

That is, assume that the covariance of innovations to the expected factorless income share and the product of the pricing kernel and output growth are constant over time. Then, for $k \geq 1$,

$$\frac{P_{\Pi t}^{(k)}}{Y_t} = \left(\frac{P_Y^{(1)}}{Y}\right)^k \mathbb{E}_t\kappa_{t+k} + \left(\frac{P_Y^{(1)}}{Y}\right)^{k-1} (\sum_{s=0}^{k-1} \rho_\kappa^s C + kD)$$

For the proof of this lemma, see Appendix section B.

When the conditions of Lemma 2 are satisfied, we can write the value of a claim to factorless income as

$$\frac{V_{\Pi t}}{Y_t} = \left[\frac{\rho_\kappa \frac{V_Y}{Y}}{1 + (1-\rho_\kappa)\frac{V_Y}{Y}}\right](\kappa_t - x_{\kappa_t}) + \frac{V_Y}{Y}x_{\kappa_t} + F \tag{17}$$

where

$$F = \sum_{k=1}^{\infty}\left(\frac{P_Y^{(1)}}{Y}\right)^{k-1}\left(\left(\frac{1-\rho_\kappa^k}{1-\rho_\kappa}\right)C + kD\right)$$

and the price dividend ratio for a claim to aggregate output is given by equation 13.

We explore this quantitative implication of our equation 17 for valuing factorless income in the next subsection.

Remark on Lemma 2: Note that the assumptions that we need for our two lemmas do not require that all expected excess returns be constant. In Appendix section B, we present a flexible specification of an essentially affine pricing kernel with time-varying volatility of the stochastic discount factor as in Campbell (2018) chapter 8.3.3 that would allow for a rich model of the term structure of interest rates with time-varying expected excess returns to holding long term bonds and a full set of time-varying risk premia on other claims as described in Lustig, Van Nieuwerburgh, and Verdelhan (2013) and Jiang et al. (2022). We show how to derive the implications of this model for the term structure of interest rates and for the prices for claims to output at different horizons using standard calculations. We also develop a new formula for pricing claims to factorless income when the pricing kernel and output growth are conditionally lognormal but innovations to the factorless income share are normal. The assumptions that we have made in our previous two lemmas amount to parameter restrictions in this model that could be evaluated in an estimation exercise. We do not intend to claim that these conditions hold in the data. We leave evaluation of that question to future research.

7.1 Accounting for Observed Valuations of Factorless Income

We use equation 17 together with our measurement of κ_t from section 6 to back out estimates of the time series for $x_{\kappa t}$ needed to reconcile equation 17 with the data on the value of factorless income relative to output shown in the right panel of Figure 6.

We evaluate the outcome of this measurement based on the implied volatility of the time series for x_{κ_t} needed to reconcile equation 17 with the data on the value of factorless income relative to output. If our model's measurement of x_{κ_t} has no variability, then the data on the valuation of factorless income can be reconciled simply as a result of observed fluctuations in the current share of factorless income projected to decay to a constant unconditional mean at rate ρ_κ. If the estimated series for x_{κ_t} is highly variable, we regard it as improbable that expected fluctuations in the share of factorless income alone can reasonably account for observed valuations of factorless income as such expectations would require highly variable movements in the share of factorless income expected over the long-term. We discuss the relationship between our accounting exercise and those based on Shiller (1981) in Appendix section C.

Equation 17 has three parameters. The first is the value of $P_Y^{(1)}/Y$ or, equivalently, the price dividend ratio for a claim to aggregate output V_Y/Y in equation 13. The second is the persistence of the factorless income share ρ_κ. The third is the constant risk premium F on a claim to factorless income due to the covariances of innovations to κ_t and $x_{\kappa t}$ with the product of the pricing kernel and aggregate growth.

We are not aware of any direct measurement of the price-dividend ratio V_Y/Y for a claim to the future aggregate output of the U.S. Corporate Sector. Lustig, Van Nieuwerburgh, and Verdelhan (2013) argue that the price dividend ratio for a claim to aggregate consumption is quite high. Based on that analysis, we consider a baseline value of $V_Y/Y = 50$ and consider alternative values of $V_Y/Y = 25$ and $V_Y/Y = 100$.

We do note that in our data, the difference between the realized growth of nominal corporate after-tax gross value added and the one-year risk free nominal rate at the end of the prior year has a mean of 2.9% and a standard deviation of 8.5% in our data from 1929-2022. Thus, if the price of a claim to output one period ahead relative to current output is one, corresponding to a value of $V_Y/Y = \infty$, then the Sharpe Ratio on that claim would be 0.34, since realized growth in output in excess of the short rate would correspond to the realized excess returns on that claim to output one period ahead. In contrast, if the price of a claim to output one period ahead relative to current output were 0.98, corresponding to a value of $V_Y/Y = 50$ as in our baseline case, then the Shape Ratio on that claim to output one period ahead would be 0.58, which seems quite high relative to standard estimates of the Sharpe Ratio for the stock market. Regardless, we take this as a baseline value. In sum, output growth in our data is not nearly as volatile as returns on Enterprise Value or equity and thus a claim to output one period ahead should not have such a high risk premium, leading to a high value of V_Y/Y.

We explore a wide range of values of the persistence parameter ρ_κ. We consider $\rho_\kappa = 0.90$ as a baseline and consider alternative values of $\rho_\kappa = 0.10$ and $\rho_\kappa = 0.99$ as alternatives.

Consider first the results of our baseline measurement exercise with $\alpha = 0.2646$, $V_Y/Y = 50$, and $\rho_\kappa = 0.9$. Note that with this baseline value of α, the unconditional sample mean value of $\kappa_t = 0.01$. Thus, from equation 17, since we have set $V_Y/Y = 50$, we match the unconditional sample mean value of $V_{\Pi t}/Y_t = 0.50$ with a series for the expected long-run factorless income share x_{κ_t} with an unconditional sample mean equal to that for the realized factorless income share κ_t and a risk premium parameter in our valuation equation 17 of $F = 0$. Thus, with these parameters for the capital share α and the price dividend ratio for a claim to aggregate output V_Y/Y, our model accounts for the average valuation of factorless income with no bias in long run expectations and no need for a further risk adjustment due to shocks to the factorless income share.

We now consider the dynamics of the long-run factorless income share expected in the long run in this baseline case shown in Figure 9. We see in this figure that the implied time series for x_{κ_t} is relatively smooth and shows no trend over time. That result obtains because, with these parameters, the coefficient on x_{κ_t} in equation 17 is large. That is, small movements in the expected share of factorless income in the long run, x_{κ_t}, have a powerful impact on the implied value of factorless income. In particular, the coefficient on κ_t in valuation equation 17 given by $\rho_\kappa P_Y^{(1)}/Y/(1-\rho_\kappa P_Y^{(1)}/Y)$ is equal to 7.5 while the combined coefficient on x_{κ_t} in this equation is 42.5. Thus, a movement in the the expected share of factorless income of one percentage point in the long-run accounts for a movement in the value of factorless income relative to output of 42.5 percentage points.

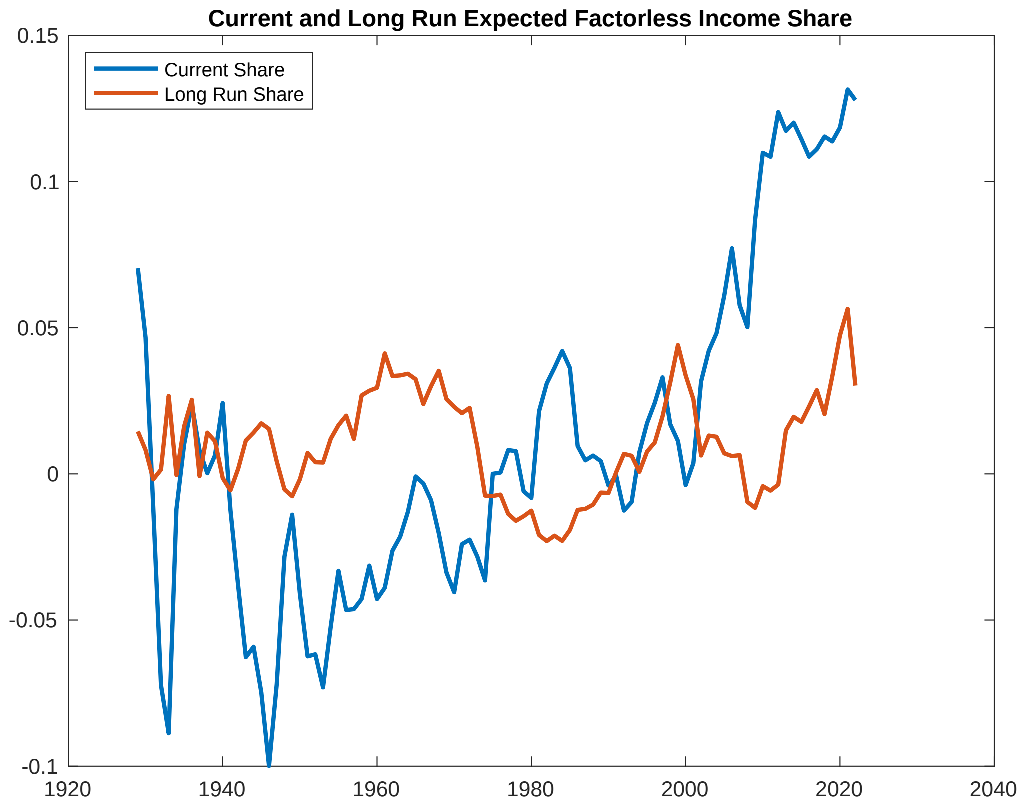

Figure 9: Implied share of Factorless Income in US Corporate Gross Value Added less Taxes (blue) and Expected Factorless Income Share in the Long Run (red). Expected Long Run Share Calculated at Baseline Values $\frac{V_Y}{Y} = 50$ and $\rho_\kappa = 0.9$

The ratio of the price of a claim to aggregate output relative to current output plays an important role in our analysis as it impacts the size of the coefficients on κ_t and x_{κ_t} in equation 17. Here we conduct a sensitivity analysis of our measurement of agents' expectations of the long-run factorless income share x_{κ_t} with respect to this parameter. We present the results of this exercise in Figure 10. When we conduct our sensitivity analysis to alternative choices of V_Y/Y below, we adjust the parameter α and hence the sample realized value of κ_t to ensure that our model continues to account for the sample mean valuation of factorless income realative to output $V_{\Pi t}/Y_t$ with no bias in long run expectations and no need for a risk adjustment due to shocks to the factorless income share.

In the left panel of this figure, we show in red the estimate of x_{κ_t} when $\frac{V_Y}{Y} = 25$, rather than our baseline value of $\frac{V_Y}{Y} = 50$, and $\rho_\kappa = 0.9$. The choice of α in this case is 0.2571 corresponding to a sample mean of κ_t equal to 0.02 rather than 0.01 in our baseline case. This lower price-dividend ratio on a claim to output of $\frac{V_Y}{Y} = 25$ corresponds to a substantially higher risk premium on such a claim. This price dividend ratio would be closer to that seen for a claim to equity.

We see in this figure that the implied fluctuations in the expected long-run share of factorless income needed to account for the data on the valuation of factorless income are roughly twice as large as in our baseline case. This implication of our model arises from the observation that, with these parameters, the coefficient on κ_t in equation 17 is 6.4, which is close to its value in our baseline case, but the coefficient on x_{κ_t} in that equation is only 18.6, less than half its value with our baseline parameters. Thus, in this case, we find substantially more variability in our estimate of x_{κ_t} in this case than we did in our baseline case shown in Figure 9.

In the right panel of this figure, we show in red the estimate of x_{κ_t} when $\frac{V_Y}{Y} = 100$ and $\rho_\kappa = 0.9$. This higher price-dividend ratio on a claim to output corresponds to a substantially lower risk premium on such a claim than in our baseline case. The choice of α in this case is 0.2683 corresponding to a sample mean of κ_t equal to 0.005 rather than 0.01 in our baseline case. We see in this figure that the fluctuations in the long run expected value of factorless income needed to account for the data on the fluctuations in the valuation of that income are quite small. This is because, with these parameters, the coefficient on κ_t in equation 17 is 8.2, which is close to its value in our baseline case, but the coefficient on x_{κ_t} in that equation now 91.8. As a result, in this case, we find substantially less variability in our estimate of x_{κ_t} than we did in Figure 9.

Some argue that the risk premium on a claim to aggregate output might be quite low and, as a result, the price dividend ratio for such a claim might be quite high, even infinite. If we set this parameter quite high (for example to $\frac{V_Y}{Y} = 500$) we find that the estimated time series for x_{κ_t} nearly converges to a constant and the corresponding capital share parameter required to justify the average valuation of factorless income converges to $\alpha = 0.2725$ at which point the sample mean of the implied factorless income share κ_t converges to zero.

The main implication of these results is that if the price-dividend ratio for a claim to aggregate output is large, then relatively small shifts in agents' expectations for the share of factorless income in the long run are sufficient to account for the observed variability of the value of a claim to factorless income relative to output. Thus, the answer to the question of whether there is "excess volatility" of the market valuation of U.S. Corporations relative to the volatility of their cash flows to owners of these firms is contingent on what assumptions one makes about this price-dividend ratio of a claim to aggregate output.

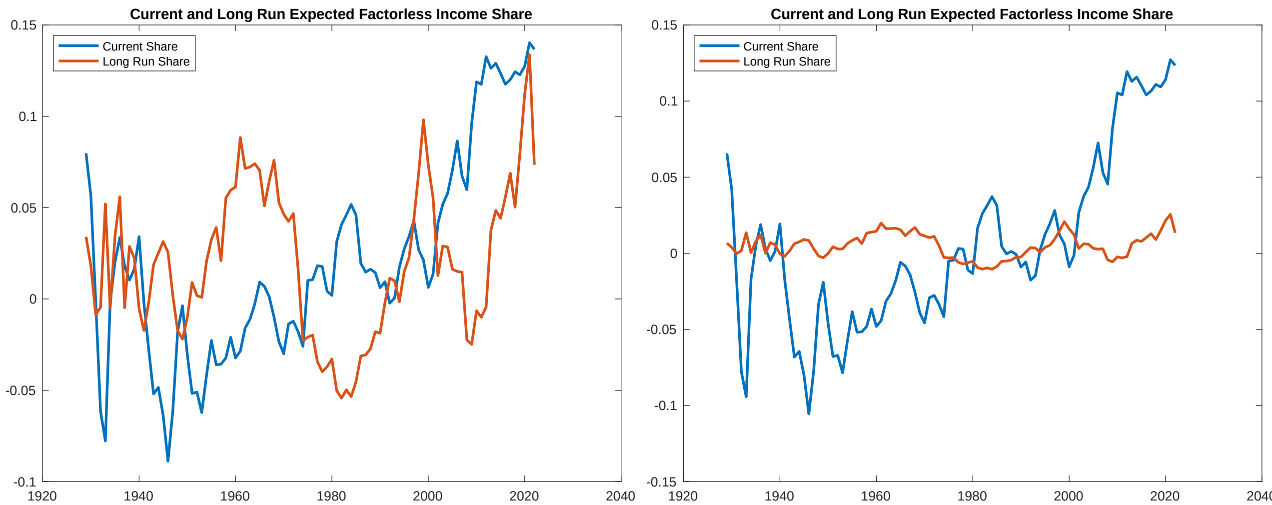

Figure 10: Left Panel: Implied share of Factorless Income in US Corporate Gross Value Added less Taxes (blue) and Expected Factorless Income Share in the Long Run (red). Expected Long Run Share Calculated at $\frac{V_Y}{Y} = 25$ and $\rho_\kappa = 0.9$. Right Panel: Implied share of Factorless Income in US Corporate Gross Value Added less Taxes (blue) and Expected Factorless Income Share in the Long Run (red). Expected Long Run Share Calculated at $\frac{V_Y}{Y} = 100$ and $\rho_\kappa = 0.9$

We now consider implications of alternative assumptions about the persistence parameter ρ_κ in equation 17 on our estimate of agents' expectations of the share of factorless income in the long run x_{κ_t} derived from that equation. Results are presented in Figure 11.

In the left panel of this figure, we show in red the estimate of x_{κ_t} when $\frac{V_Y}{Y} = 50$ and $\rho_\kappa = 0.1$ rather than our baseline value of $\rho_\kappa = 0.9$. With these parameters, the coefficient on κ_t in equation 17 falls to 0.11 while the coefficient on x_{κ_t} in that equation becomes 49.9. Thus, in this case, we find similar variability in our estimate of x_{κ_t} to what we found in Figure 9. This results indicates that our baseline finding is not sensitive to the choice of the persistence parameter ρ_κ for a wide range of choices of this parameter.

In the right panel of this figure, we show in red the estimate of x_{κ_t} when $\frac{V_Y}{Y} = 50$ and $\rho_\kappa = 0.99$ rather than our baseline value of $\rho_\kappa = 0.9$. With these parameters, the coefficient on κ_t in equation 17 rises to 33.0 while the coefficient on x_{κ_t} in that equation falls to 17.0. Thus, in this case, we find very large variability in our estimate of x_{κ_t} compared to what we found in Figure 9. In this case, the observed fluctuations in κ_t imply very large fluctuations in the value of factorless income. As a result, implausibly large countervailing fluctuations in the estimate of x_{κ_t} are needed to offset the impact of κ_t on the value of a claim to factorless income.

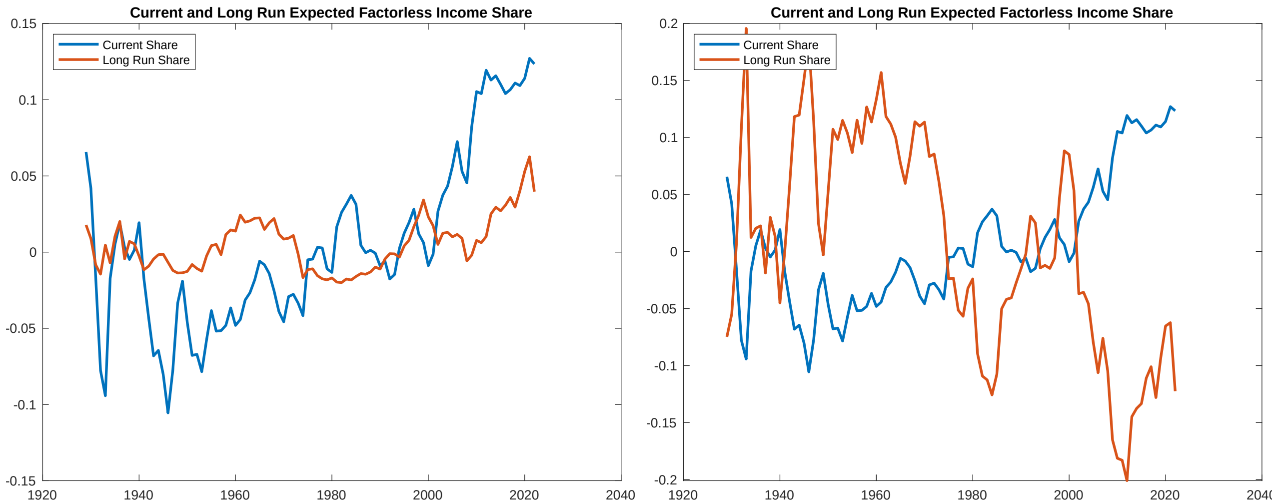

Figure 11: Left Panel: Implied share of Factorless Income in US Corporate Gross Value Added less Taxes (blue) and Expected Factorless Income Share in the Long Run (red). Expected Long Run Share Calculated at $\frac{V_Y}{Y} = 50$ and $\rho_\kappa = 0.1$. Right Panel: Implied share of Factorless Income in US Corporate Gross Value Added less Taxes (blue) and Expected Factorless Income Share in the Long Run (red). Expected Long Run Share Calculated at $\frac{V_Y}{Y} = 50$ and $\rho_\kappa = 0.98$

Again, the main implication of these accounting exercises is that, if the price-dividend ratio for a claim to aggregate output is large, then relatively small fluctuations in agents' expectations of the share of factorless income in the long run can justify large swings in the valuation of factorless income relative to output even if there are no movements in risk premia. Thus, the observed volatility of the valuation of U.S. Corporations relative to corporate output does not seem to be much of a puzzle if a claim to aggregate output has a small risk premium.

8 Returns to Physical Capital

As documented in Table 1, the data on Enterprise Value and Free Cash Flow from the IMA reveal a high average realized real return to owners of the U.S. Corporate Sector in line with that found using CRSP data on public firms. In our model, Enterprise Value is comprised of the sum of the value of a claim to physical capital and a claim to factorless income. In this section we ask, what are our model's implications for how much of this high average return on Enterprise Value is due to the average realized return to a claim to physical capital versus risk in a claim to factorless income? And what are our model's implications for the dynamics of the annual realized return to capital over the 1929-2022 time period?

As a matter of accounting, we can write the realized arithmetic return on Enterprise Value as the sum of the realized arithmetic return to capital times the weight of physical capital in Enterprise Value and the product of the return to a claim to factorless income and the share of factorless income in Enterprise Value as follows.[22]

$$\frac{FCF_{t+1} + V_{t+1}}{V_t} = \left(\frac{FCF^K_{t+1} + Q_{t+1}K_{t+2}}{Q_t K_{t+1}}\right)\frac{Q_t K_{t+1}}{V_t} + \left(\frac{\kappa_{t+1}Y_{t+1} + V_{\Pi t+1}}{V_t}\right) \tag{18}$$

All of the entries in this equation are given as data in the IMA except for the division of overall Free Cash Flow FCF_{t+1} into the component going to physical capital FCF^K_{t+1} and the component that is factorless income $\kappa_{t+1}Y_{t+1}$ as in equation 8. This decomposition in our model is determined by the choice of the parameter α with a higher value of α corresponding to a lower average level of factorless income κ_t and thus an attribution of a greater portion of observed Free Cash Flow to Free Cash Flow to Capital.

With this observation, we start with a discussion of the relationship between the average return to Enterprise Value and the average return to physical capital implied by our model. From the IMA data, we have an average arithmetic real return to Enterprise Value from 1929-2022 of 8.6%. At our baseline choice of the capital share parameter $\alpha = 0.2646$, the average arithmetic real return to physical capital over this time period implied by our model is also quite high at 7.4%. This corresponds to an average realized excess return on capital relative to a one year riskless interest rate of 6.9% and an average excess return relative to a ten-year riskless interest rate of 5.7%. Thus, the baseline specification of our model, the average real return and realized excess return to capital in our model is quite high.

From an asset pricing perspective applied to the Euler equation for physical capital in equation 7, this observation is something of a puzzle because the volatility of realized returns and excess returns to physical capital are quite low (the standard deviation of realized excess returns to capital over the short rate is only 6.5%) relative to those for claims to Enterprise Value of corporate equities shown in Table 1. Thus, in our model, physical capital has an unconditional Sharpe Ratio at an annual frequency over one, which is well in excess of estimates for equity.

This observation that our model implies a high average realized real return to capital is robust across the alternative values of α that we considered in our sensitivity exercise regarding the valuation of factorless income shown in Figure 10. In particular, with a lower

[22] Note that we do not expand the second term on the right side to its traditional form

$$\left(\frac{\kappa_{t+1}Y_{t+1} + V_{\Pi t+1}}{V_t}\right) = \left(\frac{\kappa_{t+1}Y_{t+1} + V_{\Pi t+1}}{V_{\Pi t}}\right)\frac{V_{\Pi t}}{V_t}$$

because $V_{\Pi t}$ can be zero and negative.

value of $\alpha = 0.2571$ corresponding to the valuation of factorless income when a claim to aggregate output is risky, the implied arithmetic average real return to capital is 6.9% as opposed to 7.4% in our baseline case. With a higher value of $\alpha = 0.2683$ corresponding to the valuation of factorless income when a claim to aggregate output has a very low risk premium, the implied arithmetic average real return to capital is 7.7%. Thus, in all three of these cases, the model implied average realized real return to capital is quite high.[23]

In the literature, this discrepancy between the observed volatility of realized returns to capital and estimates of the level of these returns has been addressed with adjustment costs as in Cochrane (1991) and Jermann (2010) or with unrealized disaster risk as in Gourio (2012). We leave consideration of such factors for future work. Instead, we turn to our model's implications for the dynamics of these returns. We ask when were these realized returns high? And when were they not?

To begin to address this question, observe that equations 6 and 9 offer two equivalent perspectives on the realized return to capital. Following equation 6, figure 12 decomposes the real return to capital into a portion due to the rental rate (in the left panel) and a portion due to the return to undepreciated capital (in the right panel). As is evident in the figure, at an annual horizon, most of the return to capital comes from the return to undepreciated capital. in the left panel, we see a large increase in the model-implied rental rate for capital from 1929 into WWII and then a small secular decline after that.

In the right panel, we see also a secular decline in the return to undepreciated capital due to an increase in the observed depreciation rate for physical capital. Following equation 9, the right panel of Figure 8 presents the evolution of model-implied Free Cash Flow to Capital with our baseline choice of α. This figure shows a large secular decline in Free Cash Flow to Capital since WWII.

[23] We must reduce α to a very low share, such as 0.18, for our model to imply average realized excess returns on capital close to zero, as would be the case if capital were close to a risk free asset. With this low value of α, the implied average share of factorless income in the economy is over 11%, which seems implausibly high.

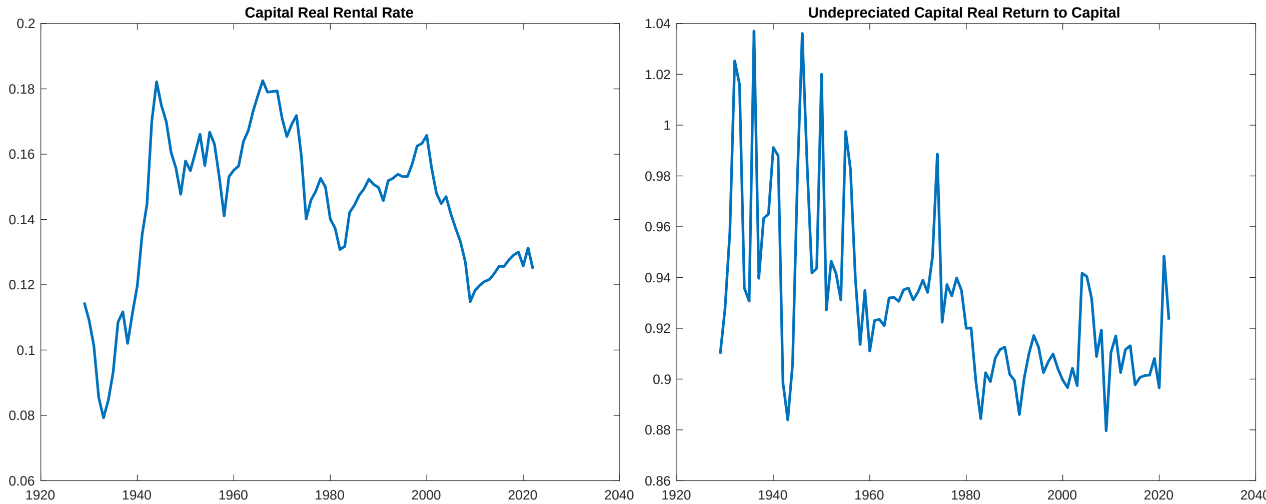

Figure 12: Left Panel: Implied realized real rental rate on physical capital. Right Panel: Implied realized real return on undepreciated capital. These two returns sum to the realized real return on capital.

We put these components of the realized return to capital together using equation 9 and plot these model-implied realized returns to capital in Figure 13. In the left panel of this figure, we plot the realized real return on capital. We see in this figure a large increase in the realized return to capital over the period 1929 into WWII (with large fluctuations) and then a long decline in the realized real return to physical capital after WWII.

In the right panel of this figure, we show the realized excess return to physical capital computed as the difference between the nominal return to physical capital between years t and $t+1$ less the nominal one-year rate at the end of period t. We are motivated to do so by equation 7 to evaluate the evolution of the implied risk premium on physical capital over the decades. This figure also shows a dramatic rise in the excess return to physical capital from 1929 into WWII, then a period of relatively high realized excess return from WWII until the early 1980's, and then a much smaller average excess return to physical capital after that time period.

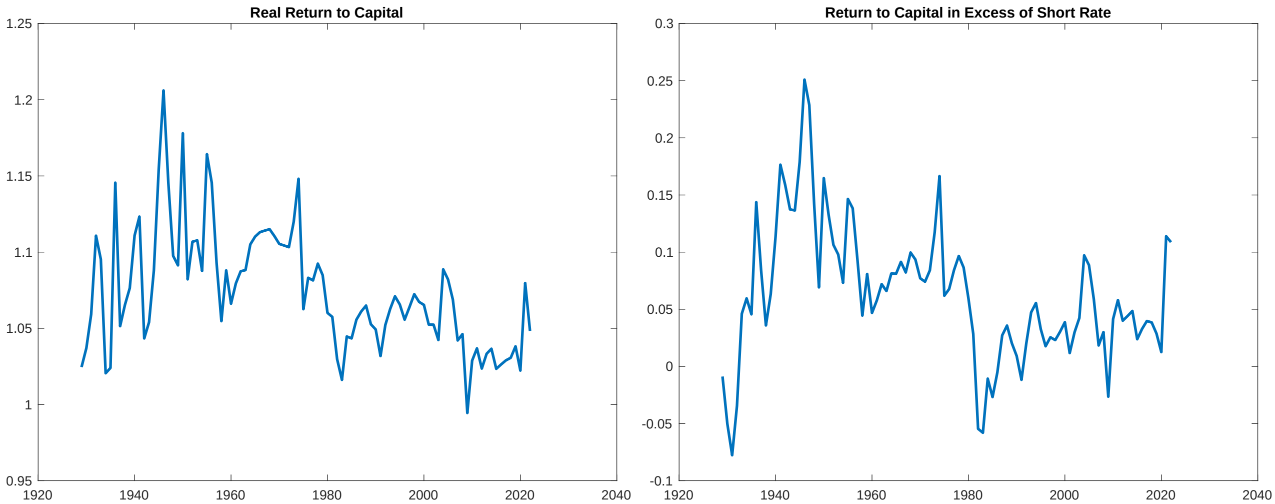

Figure 13: Left Panel: Implied realized real return on physical capital. Right Panel: Implied return on capital in excess of the one year interest rate computed as the nominal return on capital at $t+1$ minus the end of year nominal rate in t.

The results in these figures suggest that, in terms of evaluating the empirical success of our simple model for accounting for both the valuation and investment behavior of U.S. Corporations, the glass is half-full. That is, if we consider the time period since the early 1980's, our model tells a story for the boom in the market valuation of U.S. corporations resulting from a boom in the current cash flows to owners of these firms both in terms of Free Cash Flow, IMA dividends, and payouts from public firms (Figures 2 and 5) that is expected to gradually return to more normal historical levels (Figure 9). The right hand panel of Figure 13 suggests that realized excess returns on physical capital have been small, so that, according to equation 7, the investment behavior of U.S. Corporations during this time period appears to be consistent with the view that the cost of capital guiding investment has fallen in line with the secular decline in interest rates over this time period. The combination of a stable value of a claim to future output V_{Yt}/Y_t and a falling return to capital over this time period are implicitly reconciled by a decline in the expected growth rate of output and perhaps some movement in the risk premium on a claim to future output as described in equation 14.

In contrast, the realized excess returns on physical capital prior to 1980 shown in the right panel of Figure 13 present two puzzles to be resolved. First, we see the dramatic movements in those realized excess return from 1929 into WWII. Certainly, the Great Depression must have been a difficult time for capital investment. But what change in the economic environment (or measurement) led to the dramatic decline in the capital-output ratio from 1929 to WWII and beyond?

Second, we see a very large realized excess return to physical capital in the right panel of Figure 6 that persisted from WWII to the early 1980's. These observations raise the question of what drove the large movements in the returns to capital and the capital output ratio during this time period before WWII and the high average realized return to capital between WWII and the early 1980's? And why has the realized excess return on capital been comparatively low since the early 1980's?

In light of these puzzles, one must ask whether these findings are a matter of economics or a problem of measurement? Is it a matter of the impact of taxes on capital investment prior to 1980? Is it a matter of changes in the production function corresponding to changes in the share parameter α? Or is it some other factor? We leave the answers to these questions to future research.

9 Accounting for Valuation Ratios

Finally, we turn to consideration of the implications of our model for other stock market valuation ratios. One popular valuation ratio is the ratio of the market value of U.S. Corporations relative to GDP, sometimes called the *Buffett Ratio*. A second is the ratio of the market value of U.S. Corporations to the replacement value of the capital stock in those corporations, otherwise known as *Tobin's Q*. Our valuation reproduces these ratios for the U.S. Corporate sector over the 1929-2022 time period. A third popular ratio is the ratio of the market value of U.S. Corporations to a smoothed measure of their earnings, otherwise known as *Shiller's CAPE*. We compare this valuation ratio to an analogous measure in our model constructed a the ratio of the Enterprise Value of U.S. Corporations to the after-tax Net Operating Surplus of the U.S. Corporate Sector.

We show the fluctuations in these valuation ratios in Figure 14. In the left panel of that figure, we show the log of three ratios each relative to their respective mean log value. These ratios are our model version of the Buffett Ratio (the ratio of Enterprise Value to after-tax Output), our model version of Tobin's Q (the ratio of Enterprise value to the replacement value of the capital stock), and Shiller's CAPE computed from S&P public firm data.

We see in that figure that, particularly after WWII, the log deviations of each of these series relative to their respective means are quite similar. It is as if the valuation of U.S. Corporations is volatile while, at least relative to each other, after-tax corporate output, the replacement value of the capital stock, and corporate earnings are all stable. Our model matches these three valuation ratios over this time period based on an assumption of relatively small fluctuations in the expected value of factorless income in the long run.

In the right panel of Figure 14, we confirm that, at least in terms of the fluctuations in its log value relative to its mean log value, our measure of the ratio of Enterprise Value to after-tax Net Operating Surplus shows similar fluctuations after WWII as Shiller's CAPE.

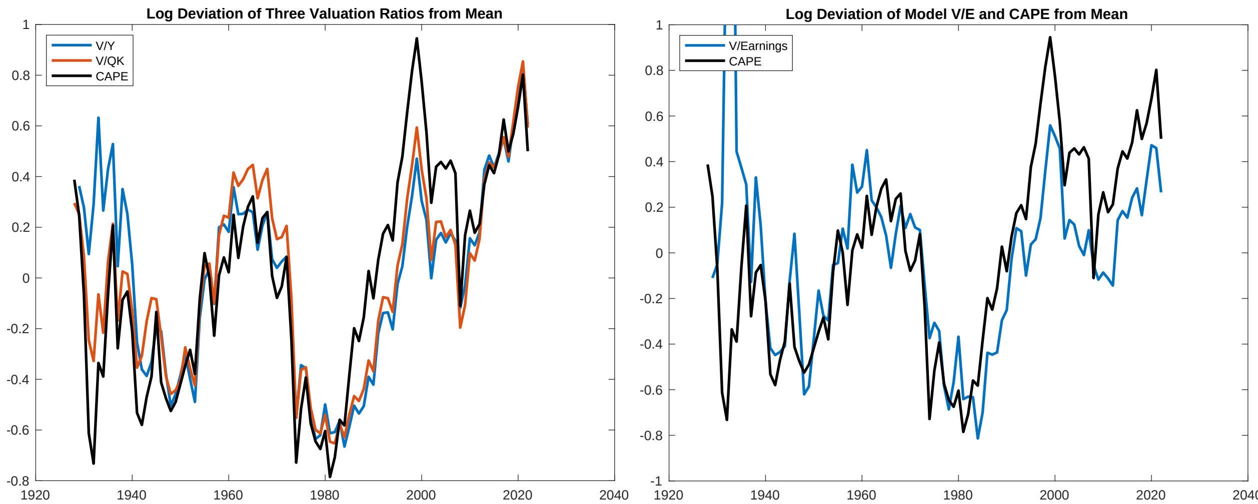

Figure 14: Left Panel: The log of the ratio of Enterprise Value to after-tax Output, of the ratio of Enterprise Value to the Replacement Value of the Capital Stock, and of Shiller's CAPE, each minus their respective mean log value. Right Panel: The log of the ratio of Enterprise Value to after-tax Net Operating Surplus and Shiller's CAPE each minus their respective mean log value.

We conclude from this Figure that our model does a good job of capturing a wide range of popular stock market valuation indicators.

10 Conclusion

In this paper we have explored the hypothesis that the data in the Integrated Macroeconomic Accounts form a useful unified data set for work in macroeconomics and finance. To illustrate the potential utility of this data set, we first explored the correspondence between measures of returns and valuation in these IMA data with corresponding measured obtained from public firm data. We then used these data to revisit some important questions in macrofinance regarding the drivers of the volatility of the market valuation of U.S. Corporations using both Campbell-Shiller regression analysis and a simple valuation model comparable to that in Shiller (1981). Finally, we developed a simple variant of a standard stochastic growth model to provide an accounting of the relationship between the realized returns to physical capital and financial claims on firms and used that to raise a new puzzle regarding the observed trends in returns on physical capital in the United States.

This paper is structured as an exploratory tour of the rich information in the Integrated Macroeconomic Accounts. Clearly more research will be needed to resolve any one of the issues we have raised using these data. It is our aim to do that more focused study in subsequent papers. We hope readers of this paper will be motivated to use the Integrated Macroeconomic Accounts to address these and other macro-finance questions in their research so that we might finally have a full reconciliation of macroeconomics and finance based on a common set of data.

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Appendices

A Data Statistics from 1945-2022

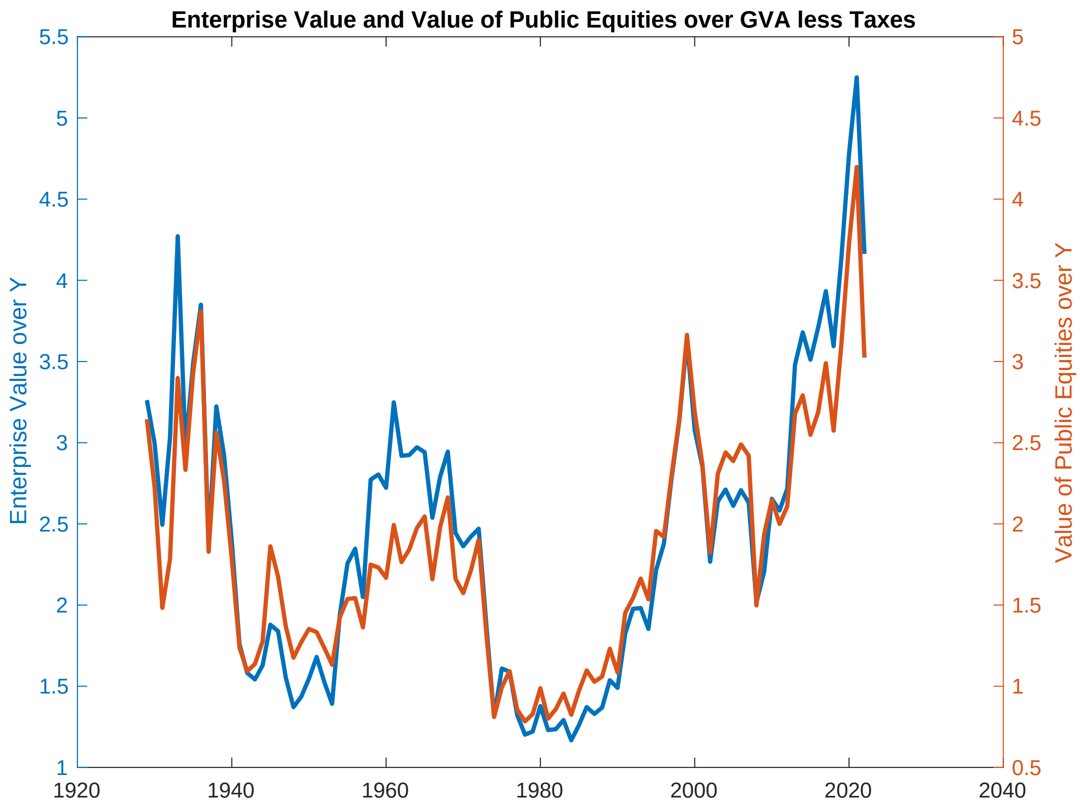

Figure A.1: Enterprise Value (left axis) and the Market Value of Corporate Public Equities (right axis) over Gross Value Added less Taxes. 1929-2022

The Integrated Macroeconomic Accounts start with measures of end of year balance sheet items in 1945. In this section, we report statistics computed using only the data from these accounts.

Table A.1: Mean and Standard Deviation of Real Log Returns and Log Dividend Growth on Enterprise Value, IMA Equity, and CRSP Value Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std D growth
Enterprise Value	1946-2022	0.08	0.132	0.279
IMA Equity	1946-2022	0.082	0.15	0.15
CRSP VW	1946-2022	0.069	0.172	0.132

Figure A.2 For V returns correlation with CRSP 1946-2022 0.936. For EQ returns correlation with CRSP 1946-2022 0.968

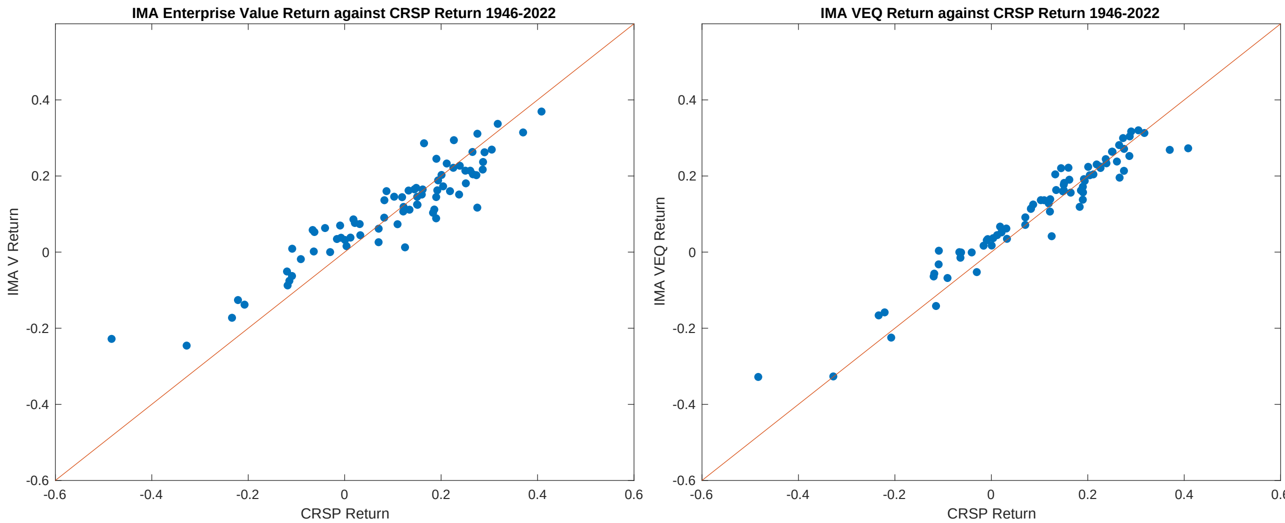

Figure A.2: Left Panel: Realized Returns on Enterprise Value vs. CRSP Value Weighted Return 1946-2022. Right Panel: Realized Returns on IMA Equity Value vs. CRSP Value Weighted Return: 1946-2022

Table A.2: Campbell-Shiller 15-year Horizon Regression Coefficients

	return β_r^k	dividend growth β_{gD}^k	future dp ratio β_{dp}^k
CRSP Data 1929-2022			
Direct $k = 15$	0.57	-0.23	0.20
VAR $k = 15$	0.64	-0.10	0.26
VAR $k = \infty$	0.87	-0.13	0.00
IMA FCF and V Data 1929-2022			
Direct $k = 15$	0.46	-0.71	-0.17
VAR $k = 15$	0.28	-0.71	0.01
VAR $k = \infty$	0.28	-0.72	0.00
NIPA D and VEQ Data 1929-2022			
Direct $k = 15$	0.65	-0.37	-0.02
VAR $k = 15$	0.35	-0.63	0.02
VAR $k = \infty$	0.36	-0.64	0.00

Table A.3: Campbell-Shiller 15-year Horizon Regression Coefficients

	return	dividend growth	future dp ratio
CRSP Data 1946-2022			
Direct $k = 15$	0.64	-0.26	0.10
VAR $k = 15$	0.87	0.15	0.28
VAR $k = \infty$	1.22	0.22	0.00
IMA FCF and V Data 1946-2022			
Direct $k = 15$	0.64	-0.56	-0.20
VAR $k = 15$	0.41	-0.58	0.01
VAR $k = \infty$	0.42	-0.58	0.00
NIPA D and VEQ Data 1946-2022			
Direct $k = 15$	1.18	0.01	-0.16
VAR $k = 15$	0.65	-0.32	0.02
VAR $k = \infty$	0.67	-0.33	0.00

B Appendix on Pricing Kernel and Pricing Formulas

We start this section with proofs of the result that $V_{Kt} = Q_t K_{t+1}$ and Lemmas 1 and 2 and then present a general essentially affine model of the pricing kernel and a solution of that model.

B.1 Proofs of Lemmas 1 and 2:

Proof that $V_{Kt} = Q_t K_{t+1}$.

To be filled in. This result is the standard result that Tobin's Q is equal to one under constant returns to scale in production and no adjustment costs for output.

Proof of Lemma 1: By definition, the ratio of the price of a claim to output one period ahead to current output is given by

$$\frac{P_{Yt}^{(1)}}{Y_t} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t}$$

Assume that this ratio is constant over time as a value $P_Y^{(1)}/Y$. We then have

$$\frac{P_{Yt}^{(2)}}{Y_t} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \frac{P_{Yt+1}^{(1)}}{Y_{t+1}} = \frac{P_Y^{(1)}}{Y} \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} = \left(\frac{P_Y^{(1)}}{Y} \right)^2$$

where the first equality is by definition, and the second and third by the assumption that a price to a claim to output one period ahead relative to current output is constant over time. Iteration on this argument proves that

$$\frac{P_{Yt}^{(k)}}{Y_t} = \left(\frac{P_Y^{(1)}}{Y} \right)^k$$

which then proves the result.

Proof of Lemma 2: We have that a price to a claim to factorless income at horizon k satisfies the recursion

$$\frac{P_{\Pi t}^{(k+1)}}{Y_t} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \frac{P_{\Pi t+1}^{(k)}}{Y_{t+1}}$$

with

$$\frac{P_{\Pi t}^{(0)}}{Y_t} = \kappa_t$$

Applying this formula for $k=1$ gives

$$\frac{P_{\Pi t}^{(1)}}{Y_t} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \kappa_{t+1} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \mathbb{E}_t \kappa_{t+1} + \mathbb{C}ov_t \left(M_{t+1} \frac{Y_{t+1}}{Y_t}, \kappa_{t+1} - \mathbb{E}_t \kappa_{t+1} \right) =$$

$$\mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \mathbb{E}_t \kappa_{t+1} + \mathbb{C}ov_t \left(M_{t+1} \frac{Y_{t+1}}{Y_t}, \epsilon_{\kappa t+1} + \epsilon_{x_\kappa t+1} \right) = \frac{P_Y^{(1)}}{Y} \mathbb{E}_t \kappa_{t+1} + C + D$$

where we have imposed our assumptions that both $P_Y^{(1)}/Y$ and the conditional covariance in this expressions are constant over time. We then prove the lemma by induction. Assume the conditions of the lemma and

$$\frac{P_{\Pi t}^{(k)}}{Y_t} = \left(\frac{P_Y^{(1)}}{Y} \right)^k \mathbb{E}_t \kappa_{t+k} + \left(\frac{P_Y^{(1)}}{Y} \right)^{k-1} (\sum_{s=0}^{k-1} \rho_\kappa^s C + kD)$$

Use the recursion to get

$$\frac{P_{\Pi t}^{(k+1)}}{Y_t} = \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \left[\left(\frac{P_Y^{(1)}}{Y} \right)^k \mathbb{E}_{t+1} \kappa_{t+k+1} + \left(\frac{P_Y^{(1)}}{Y} \right)^{k-1} (\sum_{s=0}^{k-1} \rho_\kappa^s C + kD) \right] =$$

$$\left(\frac{P_Y^{(1)}}{Y} \right)^k \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} \mathbb{E}_t \kappa_{t+k+1} + \left(\frac{P_Y^{(1)}}{Y} \right)^k \mathbb{C}ov_t \left(M_{t+1} \frac{Y_{t+1}}{Y_t}, \mathbb{E}_{t+1} \kappa_{t+k+1} - \mathbb{E}_t \kappa_{t+k+1} \right) +$$

$$\left(\frac{P_Y^{(1)}}{Y} \right)^{k-1} (\sum_{s=0}^{k-1} \rho_\kappa^s C + kD) \mathbb{E}_t M_{t+1} \frac{Y_{t+1}}{Y_t} =$$

$$\left(\frac{P_Y^{(1)}}{Y} \right)^{k+1} \mathbb{E}_t \kappa_{t+k+1} + \left(\frac{P_Y^{(1)}}{Y} \right)^k \mathbb{C}ov_t \left(M_{t+1} \frac{Y_{t+1}}{Y_t}, \rho_\kappa^k \epsilon_{\kappa t+1} + \epsilon_{x_\kappa t+1} \right) +$$

$$\left(\frac{P_Y^{(1)}}{Y} \right)^k (\sum_{s=0}^{k-1} \rho_\kappa^s C + kD) =$$

$$\left(\frac{P_Y^{(1)}}{Y} \right)^{k+1} \mathbb{E}_t \kappa_{t+k+1} + \left(\frac{P_Y^{(1)}}{Y} \right)^k (\sum_{s=0}^{k} \rho_\kappa^s C + (k+1)D)$$

which proves the result.

Note that we can derive equation 17 from

$$\frac{V_{\Pi t}}{Y_t} = \sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y} \right)^k \left[\rho_\kappa^k (\kappa_t - x_{\kappa t}) + x_{\kappa t} \right] + \left(\frac{P_Y^{(1)}}{Y} \right)^{k-1} \left(\left(\frac{1-\rho_\kappa^k}{1-\rho_\kappa} \right) C + kD \right)$$

B.2 Full Model of Pricing Kernel

We now turn to developing our full model of the pricing kernel that allows for time-varying risk premia. We use the notation in Section 8.3 page 247 of Campbell's textbook.

We start with a specification of the dynamics of the state variables and the pricing kernel. We then develop formulas for the prices of claims to zero coupon bond, zero coupon claims to aggregate output, and zero coupon claims to factorless income.

B.2.1 State Dynamics and Pricing Kernel

There is a column vector of state variables x_t, of length N. This vector includes observable outcomes and, potentially, unobserved latent states. All macroeconomic dynamics that we take as exogenous need to be in this list of state variables as well as any states we need for asset pricing.

This vector of state variables has dynamics given by

$$x_{t+1} = \Phi x_t + \Sigma \epsilon_{t+1} \tag{19}$$

Here Φ and Σ are $N \times N$ matrices, and ϵ_{t+1} is an $N \times 1$ vector of independent standard normal random variables.

For asset pricing, we are interested in the dynamics of two random variables. The first is the log SDF m_{t+1}, whose dynamics are given by

$$m_{t+1} = -(\delta_0 + \delta_1' x_t) - \frac{1}{2} \Lambda_t' \Lambda_t - \Lambda_t' \epsilon_{t+1} \tag{20}$$

Here δ_0 is a scalar, and δ_1 and Λ_t are $N \times 1$ vectors.

The vector Λ_t controls the conditional variance of the pricing kernel. This vector is given by

$$\Lambda_t = \Sigma^{-1} (\lambda_0 + \lambda_1 x_t) \tag{21}$$

where λ_0 is an $N \times 1$ vector and λ_1 is an $N \times N$ matrix.

We are interested in using this framework to price the following in our model. We wish to develop formulas for real interest rates (both short and long), for the value of a claim to real after-tax output of the corporate sector, for a claim to factorless income, and for the capital Euler equation governing the choice of end of period capital over output.

B.2.2 Pricing Zero Coupon Bonds

Our general framework offers a model for the prices of real zero coupon bonds as described in Campbell as follows. We price a claim at time t to one unit of consumption at time $t + k$.

We denote this price as $P_t^{(k)}$ and solve for it from the recursion

$$P_t^{(k+1)} = \mathbb{E}_t \exp(m_{t+1}) P_{t+1}^{(k)}$$

starting from $P_t^{(0)} = 1$. The log of these bond prices, denoted by $p_t^{(k)} = \log(P_t^{(k)})$ has the form

$$p_t^{(k)} = A_k + B_k' x_t \tag{22}$$

with A_k a scalar and B_k and $N \times 1$ vector with $A_1 = -\delta_0$, $B_1 = -\delta_1$,

$$B_k' = B_{k-1}' (\Phi - \lambda_1) - \delta_1'$$

and

$$A_n = A_{n-1} - B_{n-1}' \lambda_0 - \delta_0 + \frac{1}{2} B_{n-1}' \Sigma \Sigma' B_{n-1}$$

These expressions are presented in Campbell and can be derived along the lines of how we derive the price of a claim to output below.

B.2.3 Pricing a claim to ouput

We now turn to pricing a claim to output. We denote the price of a claim at t to aggregate after-tax output at $t+k$ by $P_t^{(k)}$. Note that these prices satisfy the recursion

$$\frac{P_{Yt}^{(k+1)}}{Y_t} = \mathbb{E}_t \exp(m_{t+1}) \frac{Y_{t+1}}{Y_t} \frac{P_{Yt+1}^{(k)}}{Y_{t+1}} = \mathbb{E}_t \exp(m_{t+1} + g_{Yt+1}) \frac{P_{Yt+1}^{(k)}}{Y_{t+1}}$$

This recursion looks the same as that for zero coupon bonds, but now we have the price to output ratio $\frac{P_t^{(k+1)}}{Y_t}$ instead of the zero coupon bond price $P_t^{(k+1)}$ and we have the sum of the log SDF and output growth $m_{t+1} + g_{Yt+1}$ instead of simply m_{t+1}. Thus, we look for a solution of the form

$$pd_{Yt}^{(k)} \equiv \log \left(\frac{P_{Yt}^{(k)}}{Y_t} \right) = C_k + D_k' x_t$$

To derive this solution, we specifically assume that the dynamics of output growth are given by

$$g_{Yt+1} = \gamma_0 + \gamma_1' x_t + \eta_g' \epsilon_{t+1} - \frac{1}{2} \eta_g' \eta_g \tag{23}$$

where γ_0 is mean log output growth, γ_1 is a vector indicating how expected output growth varies with the state, and η_g' corresponds to $\gamma_1' \Sigma$ relevant for impulses to output growth.

Lemma: The solution for these prices is given by

$$\frac{P_{Yt}^{(k)}}{Y_t} = \exp\left(C_k + D_k' x_t\right) \tag{24}$$

with $C_0 = 0$, $D_0' = 0$,

$$D_k' = -(\delta_1' - \gamma_1') - \eta_g' \Sigma^{-1} \lambda_1 + D_{k-1}' (\Phi - \lambda_1)$$

and

$$C_k = C_{k-1} - (\delta_0 - \gamma_0) - \eta_g'\Sigma^{-1}\lambda_0 - D_{k-1}'\lambda_0 + \frac{1}{2}D_{k-1}'\Sigma\Sigma' D_{k-1}$$

We derive this formula in section B.4.

Note that the expected return on a claim to output one period ahead is given by

$$\frac{Y_t}{P_{Yt}^{(1)}}\mathbb{E}_t \exp(g_{Yt+1}) = \exp(-C_1 - D_1'x_t + \gamma_0 + \gamma_1'x_t) =$$

$$\exp\left(\delta_0 + \delta_1'x_t + \eta_g'\Sigma^{-1}(\lambda_0 + \lambda_1 x_t)\right)$$

Important Lemma: If

$$0 = (\delta_1' - \gamma_1') + \eta_g'\Sigma^{-1}\lambda_1 \tag{25}$$

then the price dividend ratio for a claim to output is constant no matter what else is going on with asset prices.

Proof: From the recursion for D_k, since we start with $D_0 = 0$, these conditions imply that $D_k = 0$ for all k. This then implies that

$$\frac{P_{Yt}^{(k)}}{Y_t} = \exp\left(C_k\right) \tag{26}$$

which is constant. Note that in this case

$$C_k = k\left[-(\delta_0 - \gamma_0) - \eta_g'\Sigma^{-1}\lambda_0\right]$$

and the ratio of the value of a claim to output relative to output is given by a constant

$$\frac{V_{Yt}}{Y_t} = \frac{1}{1 - \exp\left(C_1\right)}$$

Interpretation What does the condition

$$0 = (\delta_1' - \gamma_1') + \eta_g'\Sigma^{-1}\lambda_1$$

mean? The vector δ_1' governs how the risk free interest rate moves with the state x_t and the vector γ_1' governs how the conditional expectation of the growth of log after tax output moves with the state x_t. Thus, $\delta_1' - \gamma_1'$ governs how $r - g$ relevant for pricing a claim to output one period ahead moves with the state x_t. The vector $\eta_g'\Sigma^{-1}\lambda_1 = 0$ governs how the risk correction to a claim to output one period ahead moves with the state x_t. Thus, the condition of the lemma is a condition that all observed movements in the gap between the risk free interest rate and expected growth of after tax output are movements in the risk adjustment on a claim to after tax output one period ahead. This is a general version of the assumptions we made in the baseline case in the prior set of notes. If one were to estimate this model, we believe that this is a restriction on parameters that one could impose on the estimation and then check whether it is rejected or not.

B.2.4 Pricing claims to factorless income

Now we turn to pricing a claim to factorless income. The price at t to a claim to factorless income at $t+k+1$ satisfies

$$\frac{P_{\Pi t}^{(k+1)}}{Y_t} = \mathbb{E}_t \exp(\sum_{s=0}^{k} m_{t+s+1} + g_{Yt+s+1})\kappa_{t+k+1} =$$

$$\mathbb{E}_t \exp(\sum_{s=0}^{k} m_{t+s+1} + g_{Yt+s+1})\mathbb{E}_t\kappa_{t+k+1} + \mathbb{C}ov_t\left(\exp(\sum_{s=0}^{k} m_{t+s+1} + g_{Yt+s+1}), \kappa_{t+k+1}\right)$$

This argument gives us a key pricing relationship for a claim to factorless income at horizon $k+1$

$$\frac{P_{\Pi t}^{(k+1)}}{Y_t} = \frac{P_{Yt}^{(k+1)}}{Y_t}\mathbb{E}_t\kappa_{t+k+1} + \mathbb{C}ov_t\left(\exp(\sum_{s=0}^{k} m_{t+s+1} + g_{Yt+s+1}), \kappa_{t+k+1}\right) \tag{27}$$

The value of a claim to factorless income at t is then given by the sum of these terms across horizons k

$$\frac{V_{\Pi t}}{Y_t} = \sum_{k=1}^{\infty} \frac{P_{Yt}^{(k+1)}}{Y_t}\mathbb{E}_t\kappa_{t+k+1} + \mathbb{C}ov_t\left(\exp(\sum_{s=0}^{k} m_{t+s+1} + g_{Yt+s+1}), \kappa_{t+k+1}\right)$$

Thus, the price of a claim to factorless income at horizon $k+1$ relative to output can move for three reasons. First, the price of a claim to output at horizon $k+1$ relative to output at t given by $\frac{P_{Yt}^{(k+1)}}{Y_t}$ might move. Second, the expected factorless income share might move. And third, the risk premium in the covariance term might move.

We look to develop an analytical solution for the price to a claim to factorless income by making use of the following formula. If x and y and z are independent standard normal random variables and a, b, c, d are scalar constants, then

$$\mathbb{E}\exp(ax+by)(cx+dz) = ca\exp((a^2+b^2)/2) \tag{28}$$

We derive this formula in section B.5. It is a special case of Stein's Lemma. Note that since x, y, and z all have mean zero, $\mathbb{E}\exp(ax+by)\mathbb{E}(cx+dz) = 0$ and hence

$$\mathbb{C}ov\left(\exp(ax+by), cx+dz\right) = ca\exp((a^2+b^2)/2)$$

Also note that if ϵ is an $N\times 1$ vector of independent standard normal random variables and A and B are $N\times 1$ vectors, then

$$\mathbb{E}\exp(A'\epsilon)B'\epsilon = \exp\left(\frac{1}{2}A'A\right)A'B$$

We use this version of this formula below.

We proceed as follows. We assume that the share of factorless income in after tax output

κ_t is one of the elements of the state vector x_t and $x_{\kappa t}$ is another element of this vector. Assume that μ and the matrices Φ and Σ are consistent with

$$\kappa_{t+1} = \kappa_t + x_{\kappa_t} + \eta_\kappa' \epsilon_{t+1} \tag{29}$$

$$x_{\kappa_{t+1}} = \rho_\kappa x_{\kappa_t} + \theta_{x_\kappa}' \epsilon_{t+1} \tag{30}$$

These equations imply that

$$\mathbb{E}_t \kappa_{t+k+1} = \kappa_t + \left(\frac{1-\rho_\kappa^k}{1-\rho_\kappa} \right) x_{\kappa_t}$$

and

$$\mathbb{E}_t \kappa_{t+\infty} - \mathbb{E}_t \kappa_{t+k+1} = \rho_\kappa^k \frac{x_{\kappa_t}}{1-\rho_\kappa}$$

Thus, we can consider the gap between the expected share of factorless income from next period on to converge to its long run value at a rate ρ_κ with the initial gap being $\frac{x_{\kappa_t}}{1-\rho_\kappa}$

To solve for the value of factorless income, we have the recursion

$$\frac{P_{\Pi t}^{(k+1)}}{Y_t} = \mathbb{E}_t \exp(m_{t+1} + g_{Y t+1}) \frac{P_{\Pi t+1}^{(k)}}{Y_{t+1}}$$

with

$$\frac{P_{\Pi t}^{(0)}}{Y_t} = \kappa_t$$

Lemma: The price for a claim to factorless income at horizon k is given by

$$\frac{P_{\Pi t}^{(k)}}{Y_t} = \frac{P_{Y t}^{(k)}}{Y_t} \left(\mathbb{E}_t \kappa_{t+k} + F_k + G_k' x_t \right) \tag{31}$$

with $F_0 = 0$ and $G_0' = 0$ and

$$G_k' = G_{k-1}' \left(\Phi - \lambda_1 \right) - (\eta_\kappa' + \theta_{x\kappa}') \Sigma^{-1} \lambda_1$$

and

$$F_k = F_{k-1} + G_{k-1}' \mu + \left(\eta_\kappa' + \theta_{x\kappa}' + G_{k-1}' \Sigma \right) \left(\Sigma' D_{k-1} - \Sigma^{-1} \lambda_0 + \eta_g \right)$$

Proof: In subsection B.6

Important Special Case: Assume condition 25 holds so that $D_k = 0$ for all k the prices of claims to output at different horizons are all constant at those given by equation 26. Assume as well that $(\eta_\kappa' + \theta_{x\kappa}') \Sigma^{-1} \lambda_1 = 0$ so that the risk on factorless income share shocks is independent of the state vector x_t. Then $G_k' = 0$ for all k and thus

$$F_k = k \left(\eta_\kappa' + \theta_{x\kappa}' \right) \left(-\Sigma^{-1} \lambda_0 + \eta_g \right) = k F_1$$

In this case, we have

$$\frac{V_{\Pi t}}{Y_t} = \frac{1}{1-\exp\left(C_1\right)}\left[\kappa_t + \frac{x_{\kappa t}}{1-\rho_\kappa}\right] - \frac{1}{1-\rho_\kappa \exp\left(C_1\right)}\frac{x_{\kappa t}}{1-\rho_\kappa} + F_1\frac{\exp(C_1)}{(1-\exp(C_1))^2}$$

B.3 The Euler Equation for Capital

The Euler equation for capital plays a key role in our model. To solve the Euler equation, we assume the dynamics of the capital price Q_t are given by

$$\log(Q_{t+1}) - \log(Q_t) = \zeta_0 + \zeta_1' x_t + \eta_Q' \epsilon_{t+1} - \frac{1}{2}\eta_Q' \eta_Q$$

where ζ_0 is mean growth of Q_t, ζ_1 is an $N \times 1$ vector indicating how expected growth of Q_t varies with the state, and η_Q' corresponds to $\zeta_1' \Sigma$ relevant for impulses to the growth of the capital price.

This is given by

Lemma: Capital Output Ratio

$$\frac{Q_t K_{t+1}}{Y_t} = \tag{32}$$

$$\alpha \frac{\left(\frac{P_{Yt}^{(1)}}{Y_t} - \frac{P_{\Pi t}^{(1)}}{Y_t}\right)}{1-(1-\delta_{t+1})\exp(H_1 + J_1' x_t)}$$

with

$$J_1' = -(\delta_1' - \zeta_1') - \eta_Q' \Sigma^{-1}\lambda_1$$

and

$$H_1 = -(\delta_0 - \zeta_0) - \eta_Q' \Sigma^{-1}\lambda_0$$

where we have used the formulas for the price of a claim to output and to factorless income one period ahead in our Lemmas above.

Proof: See subsection B.7

B.4 Proof of formula (24)

We have the dynamics of output growth given by

$$g_{Yt+1} = \gamma_0 + \gamma_1' x_t + \eta_g' \epsilon_{t+1} - \frac{1}{2}\eta_g' \eta_g$$

where γ_0 is mean output growth, γ_1 is a vector indicating how expected output growth varies with the state, and η_g' corresponds to $\gamma_1' \Sigma$ relevant for impulses to output growth.

From equation 20, we can write

$$m_{t+1} + g_{Yt+1} = -(\delta_0 - \gamma_0 + (\delta_1' - \gamma_1')x_t) - \frac{1}{2}\Lambda_t' \Lambda_t - (\Lambda_t' - \eta_g')\epsilon_{t+1} - \frac{1}{2}\eta_g' \eta_g \tag{33}$$

The recursion that we need to solve for C_k and D_k is

$$\exp(C_k + D_k' x_t) =$$

$$\mathbb{E}_t \exp\left(-(\delta_0 - \gamma_0 + (\delta_1' - \gamma_1')x_t) - \frac{1}{2}\Lambda_t'\Lambda_t - (\Lambda_t' - \eta_g')\epsilon_{t+1} - \frac{1}{2}\eta_g'\eta_g + C_{k-1} + D_{k-1}'(\mu + \Phi x_t + \Sigma\epsilon_{t+1})\right)$$

Note that the conditional variance of this term in parentheses is given by

$$(\Lambda_t' - \eta_g' - D_{k-1}'\Sigma)(\Lambda_t - \eta_g - \Sigma' D_{k-1}) = \Lambda_t'\Lambda_t - 2\eta_g'\Lambda_t - 2D_{k-1}'\Sigma\Lambda_t + \eta_g'\eta_g + D_{k-1}'\Sigma\Sigma' D_{k-1}$$

Using the standard expectation of a log normal random variable formula to compute the expectation, we then get the recursion that

$$C_k + D_k' x_t = -(\delta_0 - \gamma_0 + (\delta_1' - \gamma_1')x_t) + C_{k-1} + D_{k-1}'(\mu + \Phi x_t - \lambda_0 - \lambda_1 x_t) - \eta_g'\Sigma^{-1}(\lambda_0 + \lambda_1 x_t)$$

$$+\frac{1}{2}D_{k-1}'\Sigma\Sigma' D_{k-1}$$

Matching terms on x_t gives the recursion for D_k' and then matching constants gives the recursion for C_k.

B.5 Proof of formula (28)

One can prove this formula using the moment generating function for normal random variables. In particular, we start by computing for a normal random variable

$$\mathbb{E}\exp(atx) = \exp(at\mu + \frac{1}{2}a^2t^2\sigma^2)$$

We then have

$$\mathbb{E}ax\exp(atx) = \mathbb{E}\frac{d}{dt}\exp(atx) = \exp(at\mu + \frac{1}{2}a^2t^2\sigma^2)(a\mu + ta^2\sigma^2)$$

If we evaluate this expression at $t = 1$ with $\mu = 0$ and $\sigma = 1$ for a standard normal, we have

$$\mathbb{E}ax\exp(ax) = \exp(\frac{1}{2}a^2)a^2$$

we multiply by c/a to obtain

$$\mathbb{E}cx\exp(ax) = \exp(\frac{1}{2}a^2)ca$$

We then have

$$\mathbb{E}\exp(ax + by)(cx + dz) = \mathbb{E}\exp(by)\mathbb{E}cx\exp(ax) + \mathbb{E}\exp(by)\mathbb{E}\exp(ax)\mathbb{E}dz$$

by the independence of x, y and z. Finally, since $\mathbb{E}z = 0$ and $\mathbb{E}\exp(by) = \exp(\frac{1}{2}b^2)$ we get equation 28.

B.6 Proof of Formula 31

We solve for the coefficients F_k and G'_k as follows.

We have

$$\frac{P^{(k)}_{Yt}}{Y_t}\left(\mathbb{E}_t\kappa_{t+k} + F_k + G'_k x_t\right) =$$

$$\mathbb{E}_t \exp(m_{t+1} + g_{Yt+1})\frac{P^{(k-1)}_{Yt+1}}{Y_{t+1}}\left(\mathbb{E}_{t+1}\kappa_{t+k} + F_{k-1} + G'_{k-1}x_{t+1}\right) =$$

$$\mathbb{E}_t \exp(m_{t+1} + g_{Yt+1})\frac{P^{(k-1)}_{Yt+1}}{Y_{t+1}}\left(\mathbb{E}_t\kappa_{t+k} + F_{k-1} + G'_{k-1}\mathbb{E}_t x_{t+1}\right) +$$

$$\mathbb{E}_t \exp(m_{t+1} + g_{Yt+1})\frac{P^{(k-1)}_{Yt+1}}{Y_{t+1}}\left(\mathbb{E}_{t+1}\kappa_{t+k} - \mathbb{E}_t\kappa_{t+k} + G'_{k-1}\left(x_{t+1} - \mathbb{E}_t x_{t+1}\right)\right) =$$

$$\frac{P^{(k-1)}_{Yt}}{Y_t}\left(\mathbb{E}_t\kappa_{t+k} + F_{k-1} + G'_{k-1}\left(\mu + \Phi x_t\right)\right) +$$

$$\mathbb{E}_t \exp(m_{t+1} + g_{Yt+1})\frac{P^{(k)}_{Yt+1}}{Y_{t+1}}\left((\eta'_\kappa + \theta'_{x\kappa})\epsilon_{t+1} + G'_{k-1}\Sigma\epsilon_{t+1}\right)$$

where we have use the results that

$$\mathbb{E}_{t+1}\kappa_{t+k} - \mathbb{E}_t\kappa_{t+k} = (\eta'_\kappa + \theta'_{x\kappa})\epsilon_{t+1}$$

and

$$x_{t+1} - \mathbb{E}_t x_{t+1} = \Sigma\epsilon_{t+1}$$

We then can expand terms to get

$$\frac{P^{(k)}_{Yt}}{Y_t}\left(F_k + G'_k x_t\right) =$$

$$\frac{P^{(k)}_{Yt}}{Y_t}\left(F_{k-1} + G'_{k-1}\left(\mu + \Phi x_t\right)\right) +$$

$$\mathbb{E}_t \exp\left(-(\delta_0 - \gamma_0 + (\delta'_1 - \gamma'_1)x_t) - \frac{1}{2}\Lambda'_t\Lambda_t - (\Lambda'_t - \eta'_g)\epsilon_{t+1} + C_{k-1} + D'_{k-1}(\mu + \Phi x_t + \Sigma\epsilon_{t+1})\right) \times$$

$$\left(\eta'_\kappa + \theta'_{x\kappa} + G'_{k-1}\Sigma\right)\epsilon_{t+1} =$$

$$\frac{P^{(k)}_{Yt}}{Y_t}\left(F_{k-1} + G'_{k-1}\left(\mu + \Phi x_t\right)\right) +$$

$$\frac{P^{(k)}_{Yt}}{Y_t}\left(\eta'_\kappa + \theta'_{x\kappa} + G'_{k-1}\Sigma\right)\left(\Sigma' D_{k-1} - \Sigma^{-1}\left(\lambda_0 + \lambda_1 x_t\right) + \eta_g\right)$$

We can use this equality to get formulas for the coefficients G_k and F_k. Matching terms on x_t gives

$$G'_k = G'_{k-1}\left(\Phi - \lambda_1\right) - (\eta'_\kappa + \theta'_{x\kappa})\Sigma^{-1}\lambda_1$$

Matching constant terms gives

$$F_k = F_{k-1} + G'_{k-1}\mu + \left(\eta'_\kappa + \theta'_{x\kappa} + G'_{k-1}\Sigma\right)\left(\Sigma' D_{k-1} - \Sigma^{-1}\lambda_0 + \eta_g\right)$$

B.7 Solution of the Capital Euler Equation:

To prove this result, note that the capital Euler equation can be written as

$$1 = \mathbb{E}_t \exp(m_{t+1})\frac{Y_{t+1}}{Y_t}\alpha(1-\kappa_{t+1})\frac{Y_t}{Q_t K_{t+1}} + \mathbb{E}_t \exp(m_{t+1})(1-\delta_{t+1})\frac{Q_{t+1}}{Q_t}$$

Note that this equation can be written as

$$\frac{Q_t K_{t+1}}{Y_t} = \alpha \frac{\left(\frac{P_{Yt,t+1}}{Y_t} - \frac{P_{\Pi t,t+1}}{Y_t}\right)}{1 - \mathbb{E}_t \exp(m_{t+1})(1-\delta_{t+1})\frac{Q_{t+1}}{Q_t}}$$

Note also that, given our timing assumption regarding the realization of the depreciation rate and the dynamics of Q_t, the term

$$\mathbb{E}_t \exp(m_{t+1})(1-\delta_{t+1})\frac{Q_{t+1}}{Q_t} = (1-\delta_{t+1})\mathbb{E}_t \exp(m_{t+1} + g_{Qt+1})$$

We then have from our assumed dynamics for capital that

$$\mathbb{E}_t \exp(m_{t+1} + g_{Qt+1}) = \exp(H_1 + J'_1 x_t)$$

with

$$J'_1 = -(\delta'_1 - \zeta'_1) - \eta'_Q \Sigma^{-1}\lambda_1$$

and

$$H_1 = -(\delta_0 - \zeta_0) - \eta'_Q \Sigma^{-1}\lambda_0$$

C Comparison to Shiller (1981)

In this document, we compare and old-style Campbell-Shiller excess volatility test with what we do on our paper.

We assume that in the data, we have a time series for the realized share of factorless income denoted by κ_t^D where the superscript D denotes the data and the dates span from $t = 0$ to $T + 1$. We assume that we have data on the ratio of the value of factorless income to output denoted by

$$\frac{V_{\Pi t}^D}{Y_t}$$

where again the superscript D refers to data.

We model the value of factorless income as

$$\frac{V^{\star}_{\Pi t}}{Y_t} = \sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \mathbb{E}_t \kappa^{\star}_{t+k} + F$$

where

$$\frac{P_Y^{(1)}}{Y}$$

is a parameter that we choose related to the price dividend ratio for a claim to output by

$$\frac{V_Y}{Y} = \frac{1}{1 - \frac{P_Y^{(1)}}{Y}}$$

F is a constant risk adjustment, and $\mathbb{E}_t \kappa^{\star}_{t+k}$ is a model of the value of κ_{t+k} expected at t that we impose in our excess volatility calculation.

Given this valuation model, we have innovations to valuation from period t to $t+1$ given by

$$\frac{V^{\star}_{\Pi t+1}}{Y_{t+1}} - \frac{V^{\star}_{\Pi t}}{Y_t} = \sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \mathbb{E}_{t+1} \kappa^{\star}_{t+1+k} - \left(\frac{P_Y^{(1)}}{Y}\right) \sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \mathbb{E}_t \kappa^{\star}_{t+1+k} - \left(\frac{P_Y^{(1)}}{Y}\right) \mathbb{E}_t \kappa^{\star}_{t+1} =$$

$$\sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \left[\mathbb{E}_{t+1} \kappa^{\star}_{t+1+k} - \left(\frac{P_Y^{(1)}}{Y}\right) \mathbb{E}_t \kappa^{\star}_{t+1+k}\right] - \left(\frac{P_Y^{(1)}}{Y}\right) \mathbb{E}_t \kappa^{\star}_{t+1} =$$

$$\sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \left[\mathbb{E}_{t+1} \kappa^{\star}_{t+1+k} - \mathbb{E}_t \kappa^{\star}_{t+1+k} + \left(1 - \left(\frac{P_Y^{(1)}}{Y}\right)\right) \mathbb{E}_t \kappa^{\star}_{t+1+k}\right] - \left(\frac{P_Y^{(1)}}{Y}\right) \mathbb{E}_t \kappa^{\star}_{t+1} =$$

$$\sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \left[\mathbb{E}_{t+1} \kappa^{\star}_{t+1+k} - \mathbb{E}_t \kappa^{\star}_{t+1+k}\right] + \left(\frac{1}{\frac{P_Y^{(1)}}{Y}} - 1\right) \frac{V^{\star}_{\Pi t}}{Y_t} - \mathbb{E}_t \kappa^{\star}_{t+1} =$$

$$\sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \left[\mathbb{E}_{t+1} \kappa^{\star}_{t+1+k} - \mathbb{E}_t \kappa^{\star}_{t+1+k}\right] + \left(1 - \frac{P_Y^{(1)}}{Y}\right) \sum_{k=0}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \mathbb{E}_t \kappa^{\star}_{t+1+k} - \mathbb{E}_t \kappa^{\star}_{t+1}$$

To summarize, innovations to the value of factorless income are given by

$$\frac{V^{\star}_{\Pi t+1}}{Y_{t+1}} - \frac{V^{\star}_{\Pi t}}{Y_t} = \sum_{k=1}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \left[\mathbb{E}_{t+1} \kappa^{\star}_{t+1+k} - \mathbb{E}_t \kappa^{\star}_{t+1+k}\right] + \tag{34}$$

$$\left(1 - \frac{P_Y^{(1)}}{Y}\right) \sum_{k=0}^{\infty} \left(\frac{P_Y^{(1)}}{Y}\right)^k \mathbb{E}_t \kappa^{\star}_{t+1+k} - \mathbb{E}_t \kappa^{\star}_{t+1}$$

where

$$\sum_{k=1}^{\infty}\left(\frac{P_Y^{(1)}}{Y}\right)^k\left[\mathbb{E}_{t+1}\kappa^\star_{t+1+k}-\mathbb{E}_t\kappa^\star_{t+1+k}\right]$$

is a weighted sum of innovations to the expected value of future κ_{t+1+k} implied by a model of expectations, and

$$\left(1-\frac{P_Y^{(1)}}{Y}\right)\sum_{k=0}^{\infty}\left(\frac{P_Y^{(1)}}{Y}\right)^k\mathbb{E}_t\kappa^\star_{t+1+k}-\mathbb{E}_t\kappa^\star_{t+1}$$

is a comparison of a weighted average of expectations of future κ_{t+k+1} and the expectation at t of κ_{t+1}.

C.1 What Shiller (1981) Did

Shiller used realized values of dividends in his calculation. Specifically, let us set

$$\mathbb{E}_t\kappa^\star_{t+1+k}=\kappa^D_{t+1+k}$$

for $k \geq 1$. This implies that the terms

$$\mathbb{E}_{t+1}\kappa^\star_{t+1+k}-\mathbb{E}_t\kappa^\star_{t+1+k}=0$$

and we are left with model-implied volatility of valuations as

$$\frac{V^\star_{\Pi t+1}}{Y_{t+1}}-\frac{V^\star_{\Pi t}}{Y_t}=\left(1-\frac{P_Y^{(1)}}{Y}\right)\sum_{k=0}^{\infty}\left(\frac{P_Y^{(1)}}{Y}\right)^k\kappa^D_{t+1+k}-\kappa^D_{t+1}$$

C.2 What we do

We assume a model for expectations based on

$$\kappa_{t+1}-x^\star_{\kappa_{t+1}}=\rho_\kappa(\kappa_t-x^\star_{\kappa_t})+\epsilon_{\kappa t+1} \tag{35}$$

$$x^\star_{\kappa_{t+1}}=x^\star_{\kappa_t}+\epsilon_{x_\kappa t+1} \tag{36}$$

where $x^\star_{\kappa_t}$ is an unobserved variable that we choose as part of the model. We assume that κ^D_t is taken from the data. This implies that we compute innovations

$$\epsilon_{x_\kappa t+1}=x^\star_{\kappa_{t+1}}-x^\star_{\kappa_t}$$

from the model and then compute innovations

$$\epsilon_{\kappa t+1}=\kappa^D_{t+1}-x^\star_{\kappa_{t+1}}-\rho_\kappa(\kappa^D_t-x^\star_{\kappa_t})$$

from the data given the model specification of $x^\star_{\kappa_t}$ and ρ_κ.

These equations imply that

$$\mathbb{E}_t \kappa^\star_{t+k} = \rho^k_\kappa(\kappa^D_t - x^\star_{\kappa t}) + x^\star_{\kappa t}$$

and

$$\mathbb{E}_t \kappa^\star_{t+\infty} = x^\star_{\kappa t}$$

With these dynamics of the factorless income share, we have for $k \geq 0$, innovations to expectations of the factorless income share at horizon k are given by

$$\mathbb{E}_{t+1}\kappa^\star_{t+k+1} - \mathbb{E}_t\kappa^\star_{t+k+1} = \rho^k_\kappa(\kappa^D_{t+1} - x^\star_{\kappa_{t+1}}) - \rho^{k+1}_\kappa(\kappa^D_t - x^\star_{\kappa_t}) + x^\star_{\kappa_{t+1}} - x^\star_{\kappa_t} = \rho^k_\kappa \epsilon_{\kappa t+1} + \epsilon_{x_\kappa t+1}$$

Using these results with our equation 34 gives us that our model implied changes in valuation are given by

$$\frac{V^\star_{\Pi t+1}}{Y_{t+1}} - \frac{V^\star_{\Pi t}}{Y_t} = \sum_{k=1}^{\infty}\left(\frac{P^{(1)}_Y}{Y}\right)^k \left[\rho^k_\kappa \epsilon_{\kappa t+1} + \epsilon_{x_\kappa t+1}\right] +$$

$$\left(1 - \frac{P^{(1)}_Y}{Y}\right)\sum_{k=0}^{\infty}\left(\frac{P^{(1)}_Y}{Y}\right)^k \rho^{k+1}_\kappa(\kappa^D_t - x^\star_{\kappa_t}) - \rho_\kappa(\kappa^D_t - x^\star_{\kappa_t}) =$$

$$\frac{\rho_\kappa \frac{P^{(1)}_Y}{Y}}{1 - \rho_\kappa \frac{P^{(1)}_Y}{Y}}\epsilon_{\kappa t+1} + \frac{\frac{P^{(1)}_Y}{Y}}{1 - \frac{P^{(1)}_Y}{Y}}\epsilon_{x_\kappa t+1} - \left(\frac{(1-\rho_\kappa)\frac{P^{(1)}_Y}{Y}}{1 - \rho_\kappa \frac{P^{(1)}_Y}{Y}}\right)\rho_\kappa(\kappa^D_t - x^\star_{\kappa_t}) =$$

$$\frac{\rho_\kappa \frac{P^{(1)}_Y}{Y}}{1 - \rho_\kappa \frac{P^{(1)}_Y}{Y}}\left(\kappa^D_{t+1} - \kappa^D_t\right) + \left(\frac{\frac{P^{(1)}_Y}{Y}}{1 - \frac{P^{(1)}_Y}{Y}} - \frac{\rho_\kappa \frac{P^{(1)}_Y}{Y}}{1 - \rho_\kappa \frac{P^{(1)}_Y}{Y}}\right)\left(x^\star_{\kappa_{t+1}} - x^\star_{\kappa_t}\right)$$

We can draw a number of conclusions from this formula.

First consider the case in which we assume that $x^\star_{\kappa t}$ is fixed over time. Then the terms $x^\star_{\kappa_{t+1}} - x^\star_{\kappa_t} = 0$. Then the volatility of values implied by the model are pinned down from parameter choices ρ_κ, $\frac{P^{(1)}_Y}{Y}$, and the data on $\kappa^D_{t+1} - \kappa^D_t$.

Second, if we add variation over time in $x^\star_{\kappa_{t+1}} - x^\star_{\kappa_t}$, then this adds to model-implied volatility to the extent that we assume $\rho_\kappa < 1$. If we set $\rho_\kappa = 1$, then the terms connected to $x^\star_{\kappa_{t+1}} - x^\star_{\kappa_t}$ disappears.

Third, one can interpret what we are doing as follows. We have

$$\frac{V^\star_{\Pi t+1}}{Y_{t+1}} - \frac{V^\star_{\Pi t}}{Y_t}$$

and

$$\kappa^D_{t+1} - \kappa^D_t$$

from the data. Then, given a choice of parameters ρ_κ and $\frac{P^{(1)}_Y}{Y}$ we are solving for $x^\star_{\kappa_{t+1}} - x^\star_{\kappa_t}$ to rationalize these data.

D Data Appendix