Private Monopoly and Restricted Entry – Evidence from the Notary Profession

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Abstract

We study entry restrictions in a private monopoly: the Latin notary system. Under this widespread system, the state appoints notaries and grants them exclusive rights to certify various important economic transactions, including real estate, business registrations, and marriage and inheritance contracts. We develop an empirical entry model to uncover the current policy goals behind the geographic entry restrictions. The entry model incorporates a spatial demand model to infer the extent of market expansion versus business stealing from entry, and a multi-output production model to determine the markups for real estate and other transactions. We find that the entry restrictions primarily serve producer interests, and give only a small weight to consumer surplus, even conditional on the current high markups. We subsequently perform policy counterfactuals with welfare-maximizing and free entry. We show how reform would generate considerable welfare improvements, and imply a substantial redistribution towards consumers without threatening geographic coverage.

JEL: L43, L50, L84, L88

Keywords: entry restrictions, occupational licensing, regulatory policy

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1 Introduction

Entry restrictions, or occupational licensing requirements, are common across many service sectors. They typically consist of general training requirements, but they are often also combined with geographic entry restrictions and price regulation. According to the public interest view, entry restrictions serve to correct market failure such as insufficient quality provision or excessive entry in the presence of market power (e.g. Mankiw and Whinston, 1986). According to the private interest view, entry restrictions are the result of regulatory capture, and primarily serve to maximize industry profits (Stigler, 1971). Although the impact of general occupational requirements has received considerable attention, there is only limited research on the role of geographic entry restrictions combined with price regulation. A notable exception is the work by Seim and Waldfogel (2013), who analyze the effects of such regulation in the context of a public monopoly and find that entry outcomes are more in line with the maximization of industry profits than total welfare[1]. However, the question of how regulation balances consumer and producer interests in the context of a privately regulated industry remains underexplored.

In this paper, we analyze geographic entry restrictions in a private monopoly enforced by the state. We study the Latin notary system, under which the state appoints high-skilled lawyer notaries. It grants them exclusive rights to produce authentic deeds at legally fixed prices to certify various important economic transactions, such as real estate purchases, business registrations, marriage contracts and inheritance matters.[2] The system is very widespread: it exists in 22 out of the 27 EU countries and in 87 countries worldwide. Its size is also economically important: there are 40,000 notaries across the 22 EU countries with a notary system, with a total number of 160,000 employees. In Belgium, the country which we will study in more depth, total sales (net added value) of the sector amounted to approximately 1.6 (1.0) billion Euro in 2016, which is roughly comparable to the total public budget allotted to law enforcement.

In almost all countries the notary profession has two distinct features: high fixed prices (notably on real estate transactions) and regulated entry based on territorial criteria. As

[1] Seim and Waldfogel (2013) study the Pennsylvania Liquor Control Board, a public monopoly tasked with distributing liquor in the state of Pennsylvania. They find that the number of stores is 2.5 times lower than under a free entry regime and 50% lower than under the welfare-maximizing outcome. They conclude that the restrictive policy is rationalizable as profit maximization with profit sharing.

[2] This system of lawyer notaries is very different from the notary public in common law countries, such as the U.S. and Canada. A notary public does not prepare legal documents or provide legal advice, but instead performs more routine tasks at a very low cost (such as administering oaths or authenticating certain documents).

such, the system is explicitly designed to eliminate competition between notaries and other professions (such as real estate agents, banks and lawyers), as well as among the notaries themselves. A common justification of these restrictions has been that they ensure the impartiality of notaries, who are tasked with representing the interests of all parties in a transaction, while also fostering sufficient geographic availability. Because the high fixed prices have been responsible for high profits, the notary profession provides an ideal setting to study the role of geographic entry restrictions.

Our main question is whether the entry restrictions primarily serve industry profits or instead maximize total welfare. To address this question we proceed in several steps. We first develop a spatial demand model to assess how consumers value the proximity of notary services. Our spatial demand model implies that new entry results in considerable business stealing, but also some limited market expansion. Next, we formulate a multi-output production model to measure the firms' markups on real estate and other transactions. Our findings indicate that the current fixed fees are substantially higher than marginal and average variable costs. Markups are especially high for transactions relating to real estate purchase. Finally, we introduce an entry model for the state's objective function. This enables us to uncover both fixed costs of additional entry and the weight which the state attaches to consumer surplus. We find that the state attaches a much higher weight to industry profits than to consumer surplus when issuing new entry licences, even conditional on the granted market power. Intuitively, allowing additional entry would substantially raise overall welfare because the consumers' value from additional notaries outweighs the duplication of fixed costs.

To obtain further insights in the current distortions, we perform various policy counterfactuals. In a simplified first step, we focus on reforming fees to make them more cost-oriented, while keeping the current geographic entry distribution fixed. We find that there is considerable scope for reducing fees, without jeopardizing geographic coverage across the country. These fee reductions would raise welfare to only a limited extent, because demand is relatively inelastic. However, they would entail a substantial transfer from notary firms to consumers.

We then consider deeper reforms that also loosen the entry restrictions. We consider both welfare-maximizing entry and a full liberalization to free entry. Keeping the current fees constant, the welfare maximizing number of firms would be more than twice the current number. This implies substantial potential welfare gains, as the consumer surplus gains significantly outweigh the losses in industry profits. A full liberalization to free entry without adjusting fees would result in an excessive number of firms, but would still imply a welfare improvement compared with the current outcome of insufficient entry. More interestingly, if free entry is accompanied with a 19% fee reduction, this would result in an outcome close to the first-best. In other words, reducing fees serves as an additional regulatory instrument to prevent excessive free entry. Finally, free entry combined with a fee reduction would also entail a substantial redistribution from firms towards consumers, without threatening geographic coverage.

Related literature and outline Our paper contributes to several strands of literature. First, there is a large literature showing that free entry can be excessive when firms have market power. For theoretical analyses, see Spence (1976), Dixit and Stiglitz (1977) and Mankiw and Whinston (1986). Berry and Waldfogel (1999) and Hsieh and Moretti (2003) are early papers with empirical evidence on the welfare distortions from free entry. Literature on restricted entry is more limited, see the above mentioned work by Seim and Waldfogel (2013) for an analysis of a public monopoly. In comparison, we study a private monopoly, where the state explicitly assigns new entry licenses across locations on an annual basis and in coordination with the sector. As such, our analysis can show to what extent the state is subject to regulatory capture, as hypothesized by Stigler (1971).

Second, there is a literature on occupational licensing studying the impact of general training requirements. Recent work suggests that the costs from reduced competition in the labour market outweigh consumer benefits from higher quality; see e.g. Kleiner and Soltas (2019) and Farronato et al. (2020). Pagliero (2011) studies general entry restrictions on lawyers, using panel data for U.S. states. He finds little evidence that more difficult bar exams raise consumer demand, while the entry restrictions result in excessively high salaries, which is more in line with the maximization of lawyer rents than with a total welfare objective. Compared with this literature, we take general educational and experience requirements as given. By taking this approach, we do not call into question the need for quality standards in the context of asymmetric information. We instead study the impact of geographic entry restrictions beyond general entry requirements, and assess to which extent these geographic restrictions take into account consumer benefits.

Third, from a methodological perspective we contribute to the empirical entry literature. This literature has mainly focused on inferring bounds on fixed costs, using moment inequalities from profit maximization conditions, as in Pakes (2010) and Pakes et al. (2015). For recent applications, see for example Eizenberg (2014), Berry et al. (2016), Fan and Yang (2020), Ishii (2005) and Wollmann (2018). Compared with this work, our goal is to use the entry model to infer both fixed costs and the weight attached to consumer surplus in the state's objective function.[3] To accomplish this, we include additional moment inequalities

[3]This problem is analogous to that of using oligopoly models to infer both marginal costs and firm

and confront our obtained bounds with direct fixed cost information from our data set.

The paper is organized as follows. Section 2 provides the institutional background on the notary profession in the European context. Using a rich data set on entry patterns at the local market level, we estimate population-based entry thresholds in the spirit of Bresnahan and Reiss (1991). We show how to interpret these entry thresholds in an environment of restricted entry, as opposed to the free entry setting in which these methods are conventionally used. Our results demonstrate similar monopolistic entry patterns across the countries in our sample, indicating relative uniformity in the application of entry restrictions across jurisdictions.

The next sections then provide a more in-depth analysis based on a detailed firm-level dataset from Belgium, which exhibits similarities to other European countries both in terms of entry patterns and in terms of price regulation. Section 3 describes the dataset used for the analysis, with rich information on all notary office locations, including the number of notaries per office, multiple outputs (real estate and other transactions), employment, intermediate inputs and capital expenditures or fixed costs. Section 4 develops a spatial model of demand for notary services, in the spirit of Holmes (2011) and Ellickson et al. (2019). The model estimates the extent of business stealing and market expansion from additional entry. Section 5 introduces a multi-output production function model, to determine how labor and intermediate inputs are transformed into multiple outputs. A novel feature of this part of our analysis is that we account for correlation between firm-level unobserved productivity and unobserved demand, using local market demographics from the spatial demand model as external instruments. Section 6 introduces our entry model to infer bounds on fixed costs and the weight on consumer surplus in the state's welfare objective function. Finally, Section 7 performs various policy counterfactuals, and Section 8 concludes.

2 The Latin notary system in Europe

Under the Latin notary system, high-skilled notaries produce authentic deeds that certify various important economic transactions: real estate purchases and mortgages, and other transactions such as business registrations, marriage contracts and inheritance matters. The system is very widespread, especially in countries with a civil law system: it exists in 22 out of 27 EU countries (the exceptions being the three Scandinavian countries, Ireland and Cyprus) and in 87 countries worldwide. This includes other European countries such as Russia, most African countries, all of South and Central America, Quebec in North America, and Asian countries such as Japan, Indonesia and China (Weyts and Buyssens,

conduct, as in Bresnahan (1982).

2020). The notaries' decisions have the same legal force as court decisions made by judges. This contrasts dramatically with the Anglo-Saxon system, where these tasks are typically taken on by lawyers, banks, real estate agents or the government. The so-called "notary publics" active within this system fulfill functions of very limited economic importance, and are only required to verify and confirm the identity of the transacting parties.

Within the Latin notary system, the state appoints the notaries and grants them exclusive rights to produce the authentic deeds. In addition, it imposes territorial restrictions and fixes the prices of the notaries' transactions. The exclusive rights eliminate competition with other professions (such as lawyers, banks and real estate agents), while the territorial restrictions and fixed tariffs limit the potential for competition among notaries themselves. The main stated aim of the regulation is to achieve sufficient availability and proximity of services, while ensuring the impartial treatment of consumers.

The rest of this section provides further information on the Latin notary system in Europe. Subsection 2.1 discusses the entry and price regulations in more detail. Subsection 2.2 documents the geographic coverage of notaries across European countries.

2.1 Entry and price regulation

The combination of entry and price regulation is central in almost all European countries with a notary system. The rules regarding the establishment of new notaries, as well as the prices of all types of transactions, are enforced by the respective Ministry of Justice, usually in conjunction with the national Chamber of Notaries.

Entry restrictions Entry into the notary profession is subject to strict academic and experience requirements. This includes the successful completion of a university degree in Law (typically five years), an additional Master degree in Notarial Law (one year), and a minimum number of years of practical experience (comparable to entry requirements to the Bar for lawyers).

In addition, most countries have legislation that restricts the actual number of entrants into the profession. This is usually accompanied by geographic criteria, i.e. a maximum number of notary licenses per geographic area or "judicial district". These geographic criteria are usually defined qualitatively, based on the expected impact of a new entrant on the economic viability of existing notary offices. As such, the legislation puts a high weight on preventing the closure of existing offices.[4]

[4]For instance, in Article 52, III, of the French Law of 6 August, 2015, it is stated that applications by new entrants must be subjected to a risk evaluation in order to determine that they are not "jeopardizing

Table 1: Fees for common transactions

	Real estate purchase + 70 % mortgage			Other transactions	
	Average price of residential property	Notary fee for average property	Property worth €250 000	Inheritance €250 000	Incorporation of LLC €25 000
Austria	214 770	2 835	3 300	4 680	662
Belgium	223 275	3 112	3 311	356	142
France	246 579	3 017	3 050	1 225	-
Germany	174 694	1 470	1 950	535	287
Italy	200 173*	5 100	5 100	1 340	2 105
Portugal	200 173*	608	653	-	-
Spain	141 545	382	458	364	124

Notes: Fees are net compensations to notaries. In addition, notaries may request administrative charges for certain services. The information comes from online price calculators and was collected in August 2018. Further details are available from the authors upon request.
* No average house prices were available for these countries in Eurostat. We therefore calculate an unweighted average of the other countries with a Latin notary system as our baseline.

Beyond these qualitative geographic restrictions, some countries also establish explicit quantitative demand criteria for the entry of a new notary. In Belgium, there cannot be more than one notary per 9 000 inhabitants within each judicial district. A similar law is in force in Italy, where the threshold is set at 7 000 inhabitants.

Price regulation This licensing process is coupled with strict price regulation. The fees for notary services vary substantially across countries and transactions. Table 1 provides an overview of fees for a number of transactions in a selection of Western European countries.[5] Note that the reported fees are net fees only, i.e. a honorarium for the notary services. In addition, notaries typically charge additional fees to compensate for administrative costs related to the transaction, and these may also be quite substantial.

The notaries generate a considerable portion of their revenues from transactions relating to real estate purchases and mortgages. The fees for these services, as a percentage of the

continuity of service by existing offices".

[5]The Netherlands is not included in this table, as it no longer imposes restrictions on prices. In Denmark, Finland and Sweden, a number of notary services are performed by licensed real estate agents, which makes a direct comparison of the system with non-Scandinavian countries difficult.

total cost of a real estate purchase, range from 0.27% and 0.30% (in Spain and Portugal) to more than 1% in Austria, Belgium and France, and up to 2.55% (in Italy). Total fees including regulated administrative charges can be much higher, up to 2.30% in Belgium and 1.63% in France. The fees for other transactions tend to be lower, as illustrated for inheritance and business registrations in Table 1. Nevertheless, these transactions may also include considerable administrative fees.[6] These high fees are often motivated as ensuring the economic viability of notaries in (rural) areas with low demand, as well as permitting the cross-subsidization of other, possibly less lucrative services, such as inheritance and business registrations.

Deregulation Over the past two decades, several countries have made reforms to the notary system. According to The Economist, "governments are trying to strike a better balance between caution and carelessness".[7] In Europe, this process was initiated by the 1999 Notary Act in the Netherlands. This Act liberalized the rules for new establishments and allowed competition in prices within the notary sector. Other countries made less drastic reforms. In 1999, Belgium introduced legislation to grant the restricted entry licences in a more objective way, through an annual comparative entry exam. It also enabled new notaries to form associations, in an attempt to exploit more economies of scale. A fee reduction for real estate transactions is currently under debate in the Parliament (as we discuss below in Section 7). France recently passed the 'Macron Law' in 2015. This law adjusted the regulated fees and loosened entry restrictions in 247 'free installation zones', where the current number of notaries was judged as too low.[8] It further allowed the formation of pluri-professional operating companies, as a way to foster economies of scale in the sector.

2.2 Geographic coverage across and within countries

To provide an overview of entry patterns across countries, we assembled a data set on the number of notaries per municipality in various European countries. The information comes from the "European Latin civil law notaries directory" and was collected in November 2017.[9] The full data set contains the names, addresses and other characteristics of 29 864 notaries active in 8 European countries under the Latin notary system. Based on these data

[6]For example, in Belgium a gift of €250 000 involves a notary fee of €903.38, and administrative costs between €800–€1100. Source: www.notaris.be.

[7]See The Economist, Notaries – Breaking the seals, August 11, 2012. Kuijpers et al. (2005) and Schmid et al. (2007) provide early overviews of the policy discussion.

[8]Autorité de la Concurrence (2021) provides an evaluation on the extent to which entry increased.

[9]The download of the directory was done through the designated website https://notaries-europe.com/.

we calculate the total number of notaries per municipality in each country, as well as the number of notary offices, which we define as groups of notaries with the same address.[10] We supplement these data with information on consumer demographics at the municipal level from the European Statistical System 2011 Census database. The source contains population size, as well as the age and gender structure of each municipality. We also collect data on the area of each municipality, together with information on regional income, provided by the University of Groningen.

Figure 1: Number of notaries and notary offices per 100 000 inhabitants

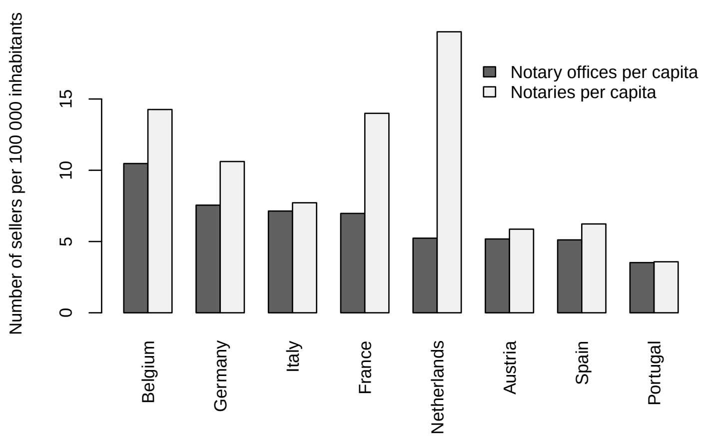


Figure 1 illustrates the cross-country differences in the number of notary offices and individual notaries per capita. In general, there tends to be a positive correlation between the level of fees reported in Table 1 and the number of notary offices per capita. Spain and Portugal exhibit both the lowest fees (in absolute terms and as a percentage of the value of the transaction) and the lowest number of notary offices per inhabitant. At the other end of the spectrum, the high level of tariffs in Belgium coincides with the highest number of notary offices per inhabitant. The Netherlands appears to provide an intermediate case: its deregulated fees fall somewhere in between, and it also has an intermediate number of offices per capita. Interestingly however, the Netherlands shows by far the highest density of *individual* notaries per capita. The relaxation of entry restrictions in the Dutch notary sector thus implied a drastic increase in the number of individual notaries, but it has not necessarily improved geographic coverage because new notaries have largely entered through associations.

[10] A limitation of the data is that some notaries may not sign up to the directory, and hence not all notary offices may be recorded. However, using detailed administrative data, we were able to show that in Belgium the gap in reported numbers is small (with 1101 notary offices found in the directory compared with about 1150 actual notary offices in 2017).

This description reveals some interesting differences in geographic coverage across countries. We now document how geographic coverage differs within countries. We estimate the following ordered probit model for the probability of observing $N = n$ firms in a local municipality, as a function of population size S, and other market demographics X:

$$P(N = n) = \Phi\left(X\beta + \theta_n + \alpha \ln S\right) - \Phi\left(X\beta + \theta_{n+1} + \alpha \ln S\right), \tag{1}$$

where $\Phi\left(.\right)$ denotes the cumulative normal distribution. Bresnahan and Reiss (1991) derive such an ordered probit model based on the assumption of segmented local markets and free entry, where firms enter as long as this is profitable. From the estimated parameters, they construct entry thresholds s_n, i.e. the per-firm population size to support n firms; and entry threshold ratios, $etr_n \equiv s_{n+1}/s_n$. An $etr_n > 1$ shows that the local market size has to increase disproportionately to sustain additional firms. Based on their free entry model, Bresnahan and Reiss interpret this as an indication that additional entry intensifies competition. In our setting with fixed prices and no other forms of competition, an $etr_n > 1$ would be evidence against the assumption of free entry. As we show in Appendix A.1, an $etr_n > 1$ is instead consistent with restricted entry where the number of firms maximizes industry profits, if total local market demand is sufficiently concave in n. An $etr_n > 1$ is also consistent with the maximization of other objective functions such as a weighted sum of industry profits and consumer surplus, reflecting a decreasing marginal value for additional geographic coverage. As such, entry thresholds and entry threshold ratios can also provide useful descriptive information under restricted entry, by indicating a departure from the free entry equilibrium.

Since the interpretation of the ordered probit entry model relies on the assumption of geographically segmented markets, we estimate the model for the subsample of rural areas with a population density of less than 1 200 inhabitants per square kilometer. We censor the number of firms per market at 4, as there are very few markets with a higher number of entrants. Table A1 in the Appendix provides descriptive statistics of the control variables and an overview of the observed market structures by country. Table A2 presents the ordered probit estimates for each country. The results show that population size and area (both in logs) are the main determinants of the number of firms. Income does not appear to have a significant effect.

Table 2 shows how the parameter estimates translate into entry thresholds and entry threshold ratios. Interestingly, the per-firm entry thresholds increase as the number of firms increases, showing the local market size increases disproportionately to support additional firms. This is confirmed by the entry threshold ratios, which tend to be well above one for all countries. This is especially the case for $etr_2 \equiv s_2/s_1$. For example, in Belgium and in

Table 2: Entry thresholds and entry threshold ratios in selected countries

Country	Entry thresholds				Entry threshold ratios		
	s_1	s_2	s_3	s_4	$etr_2 \equiv s_2/s_1$	$etr_3 \equiv s_3/s_2$	$etr_4 \equiv s_4/s_3$
Austria	8797	13863	21899	22648	1.576***	1.580***	1.034
Belgium	4811	7422	9948	14450	1.543***	1.340***	1.453***
France	6505	15902	33027	49361	2.444***	2.077***	1.495***
Germany	12752	15318	15768	16280	1.201***	1.029	1.032
Italy	8680	11060	12570	13609	1.274***	1.137***	1.083**
Netherlands	8983	12853	16460	22622	1.431***	1.281***	1.374***
Portugal	12348	22239	38396	50817	1.801***	1.727*	1.323
Spain	5292	9747	16869	22022	1.842***	1.731***	1.305***

Notes: Wald test ($etr = 1$): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors of the entry threshold ratios range between 0.03 and 0.43.

France the market size per firm should increase by a factor of respectively 1.54 and 2.44 to sustain a second firm. The above-one entry threshold ratios are evidently inconsistent with free entry, because entry does not generate more intense competition under the current regulations. The ratios are instead consistent with an entry process based on industry profit maximization, where firms aim to achieve minimum coverage but constrain further entry to avoid market cannibalization (under concave demand).

While this logic is compelling, we are cautious in interpreting the preliminary findings. Similar entry patterns may be observed under other restricted licensing regimes that put a high weight on having local market coverage of at least one firm. For example, the state may be maximizing overall welfare or follow some more *ad hoc* geographic coverage goals. Furthermore, the interpretations are based on the assumption of geographically segmented markets. In practice, consumers can travel to neighboring local markets if no nearby firm is available.

In the next section we outline a model which can deal with these issues. The data requirements for its estimation are substantially higher and we therefore will constrain our analysis to Belgium, for which we collected detailed firm-level information.

3 Empirical framework and dataset

The previous section reviewed the entry and price regulations in the notary sector across Europe, and documented several suggestive entry patterns. We next aim to understand the entry process in more detail. We study the notary sector in Belgium, which is representative for many of the practices elsewhere.

In this section we first provide a broad overview of the various building blocks of our empirical framework, and then discuss the institutional background and dataset for Belgium.

3.1 Overview of empirical framework

We begin by modeling consumer demand for each firm $i = 1, ..., J$. In the spirit of Holmes (2011) and Ellickson et al. (2019), we adopt a discrete choice spatial demand model for two independent products, $k = 1, 2$, where $k = 1$ refers to real estate and $k = 2$ refers to other transactions. The model incorporates both business stealing and market expansion effects from additional entry in a flexible way. Let a be a $J \times 1$ location vector with a typical element a_i referring to the number of individual notaries active in firm i. The discrete choice model then results in aggregate demand functions for each firm i and product k, $q_i^k = q_i^k(a)$, and total consumer surplus $CS(a)$ across local markets. The demands and consumer surplus depend on each a_i through the distances between firms across markets and the consumers' sensitivity to distance. Total output per firm over both products is $q_i = q_i^1 + q_i^2$. Section 4 outlines the details of the spatial demand model, and the empirical findings.

We then model the firms' input decisions to obtain the various components of variable profits, in particular marginal cost and markup estimates. We introduce a multi-output production model for q_i^1 and q_i^2, where labor l_i and intermediate goods m_i are the variable inputs and capital is a fixed input. This model results in a system of conditional input demand functions, $l_i(q_i^1, q_i^2)$ and $m_i(q_i^1, q_i^2)$, respectively. At a wage w, firm i 's variable costs are $wl_i(q_i^1, q_i^2) + m_i(q_i^1, q_i^2)$. Total revenues for both products are $p^1 q_i^1 + p^2 q_i^2$, where p^k is the price of product k. Section 5 describes the specification and estimation of the system of conditional input demand functions.

Next, we formulate our approach to understand the entry process. The profits of a firm i consist of variable profits π_i^V minus fixed costs f_i per notary, multiplied by the number of active notaries in firm a_i. We write profits explicitly as a function of the location vector a:

$$\begin{aligned}
\pi_i(a) &= \pi_i^V(a) - a_i f_i \\
&= p^1 q_i^1(a) + p^2 q_i^2(a) - w l_i(q_i^1(a), q_i^2(a)) - m_i(q_i^1(a), q_i^2(a)) - a_i f_i.
\end{aligned}$$

We consider the state's objective function as a weighted sum of expected profits of all firms and total consumer surplus. The state chooses the number of notaries at each possible location to maximize this objective function. Section 6 formulates our approach to understand the entry process in more detail. This allows us to perform policy counterfactuals in the final step of our analysis in Section 7.

3.2 Dataset

Notary sector in Belgium Our analysis focuses on the notary sector in Belgium in 2016. As outlined in Section 2, the industry is characterized by explicit entry restrictions, coupled with regulated fees. Compared to the European average, the country has a high number of notaries per capita. There are 1 569 individual notaries, located in 1 152 notary offices (or firms). Hence, the majority of the notary offices consist of a single notary (806). The remaining ones (346) are associations, typically of two notaries (295) and occasionally three (39) or more notaries.

According to the entry regulation, there cannot be more than one notary office per 9 000 inhabitants. This restriction applies separately to each of the ten judicial districts (corresponding to provinces), plus the capital city of Brussels. Since this restriction is binding, we will take the current office locations as given in the empirical analysis. There is flexibility regarding the number of notaries at these office locations. Each year, a maximum number of new candidate-notaries is determined, and selected through a comparative entry exam.[11] The selected candidate-notaries may subsequently become full notaries in two ways: by acquiring a notary office as notary-titularis (replacing a resigning one), or by joining a notary office as a notary-associate.[12]

Dataset Our main dataset consists of information per notary office in 2016 from three sources. Our first source comes from the Ministry of Finance and provides information on output, i.e. the number of transactions ("authentic deeds") by product for each notary office. As mentioned above, we distinguish between two products: real estate transactions (q_i^1), consisting of real estate purchases, new mortgages and mortgage terminations; and other transactions (q_i^2), which include for example business registrations, gifts and inheritance transactions. Our second data source contains balance sheet information from Belfirst (NACE code 69102 "Activities of notaries") and Orbis. It reports on the following variables per notary office: staff costs and hours worked, net added value and profits (EBITDA and EBIT). The balance sheet information appears to be particularly reliable due to specific accounting rules for the notary sector. Intermediate inputs (m_i) are calculated as the difference between turnover and net added value. We do not observe turnover at the firm-level, so we calculate it as $r_i = p^1 q_i^1 + p^2 q_i^2$, based on a measure of average output prices.[13] We measure

[11] Participants must have a five-year law degree, an additional one-year master in notarial law, and a successful completion of three years of internship.

[12] Specific compensation rules apply. A new notary-titularis pays a compensation no larger than 2.5 times the average, corrected income over the past five years, see Weyts and Buyssens (2020).

[13] More precisely, we measure p^1 (including administrative charges) based on the average house price and assuming a 70% mortgage value. We then compute $p^2 = (R - p^1 Q^1 / Q^2)$, where industry turnover

fixed costs (f_i) in annualized terms as depreciation and amortization expenditures, i.e. the difference between EBITDA and EBIT. Our third data source consists of information from the official Belgian notary website (snapshot in January 2018), which provides additional detailed information for every notary and notary office, including address, starting date of establishment and company type (sole proprietorship or limited liability).

Table 3, Panel (a), presents descriptive statistics of the collected variables. To illustrate, an average notary office certifies 813 authentic deeds per year, of which 465 relate to real estate. To produce this output, an average notary office spends €369 000 on staff costs and €592 000 on intermediate inputs. The latter mainly consists of payments for the notary services (apart from costs of administrating notary transactions). In addition to these variable cost components, the average notary office incurs an annual fixed cost of €116 000 relating to depreciation and amortization. With an average of 1.4 individual notaries per notary office, this amounts to a fixed cost per notary of approximately €84 000. The fixed costs relate to building expenses (depreciation) and goodwill from acquiring the property (amortization). The average notary office engages 9.5 thousand working hours per year, implying an average labor productivity of 12 working hours per transaction and an average hourly staff cost of €39. Net added value (defined as sales minus intermediate input costs) amounts to an average of €877 000 per notary office, or approximately 60% of sales. The average gross profits per notary office (net added value minus staff costs) amounted to €508 000 in 2016, with a large dispersion across firms. The average net profits per notary office (net of depreciation and amortization) amounted to €392 000. Recall that these profit measures already subtract service payments made to notaries in the firm, which are a component of the intermediate input expenses.

We augment our main firm-level dataset with demographic information at the municipality level from census sources. This includes population, per capita income, level of urbanization (population density per square kilometer) and demographic information (share of young and elderly citizens). Table 3, Panel (b), shows summary statistics for these variables.

In a final step, we geocode the addresses of the notaries. We approximate the location of the municipalities by the coordinates of each unit's centroid. Based on this information, we calculate the distances from each notary to all municipalities in its judicial district. In practice, we restrict attention to smaller "catchment areas", i.e. all municipalities located within 15 kilometers of the notary office. This selection rule implies that some municipalities have fewer than 5 notaries in their catchment area. For these municipalities, we include the closest notaries such that they have access to at least 5 notaries. Table 3, Panel (c), provides descriptive statistics of the distribution of the calculated distances of notaries to these mar-

$R = \sum_{i=1}^{J} r_i$ comes from the Ministry of Economic Affairs (FOD Economie, 2021), and $Q^k = \sum_{i=1}^{J} q_i^k$.

Table 3: Summary statistics of Belgian dataset

Variable	Mean	Std. Dev.	10^{th} perc.	90^{th} perc.
Panel (a): variables observed at the notary level ($N = 1152$)				
Transactions	813.096	538.484	327	1405
Real estate transactions	464.931	295.951	187	807
Other transactions	348.165	260.763	127	620
Hours (FTE)	9.542	7.106	3.283	16.465
Staff costs (FTE)	369.410	309.752	112.214	658.274
Intermediate inputs	592.258	478.0314	167.801	1154.25
Depreciation and amortization				
office level	116.350	120.525	17.17	230.54
notary level	83.860	73.721	11.97	171.465
Net added value	877.150	774.942	284.54	1576.92
Notaries per office	1.362	0.639	1	2
Date of establishment (years since 1999)	9.040	5.625	0.299	16.49
Company type = sole proprietorship (SP)	0.234	0.424	0	1
Panel (b): variables observed at the market (tract) level ($N = 589$)				
Income (log)	9.807	0.139	9.639	9.971
Population density	0.758	2.186	0.064	1.195
% young inhabitants	0.172	0.019	0.150	0.196
% elderly inhabitants	0.170	0.026	0.140	0.197
Panel (c): variables observed at the notary-market level ($N = 16353$)				
Distance to market (km)	9.590	3.717	4.214	14.100

Notes: The variables refer to the year 2016. All monetary values are provided in €1 000. Distance information is only provided for markets within a specific notary's range.

kets. Figure A1 in the Appendix provides a map to show where firms are located across the country, and demonstrates the importance of local population density in determining firm locations.

4 Spatial demand

4.1 Specification

We specify the demand of a firm (i.e. notary office) i for its two products ($k = 1, 2$), q_i^k, based on a spatial discrete choice model. We first obtain the demand at the level of the municipal tract t, and then aggregate to the observed total demand across local markets. Product $k = 1$ refers to real estate transactions, and product $k = 2$ to other transactions. Since both products are unrelated, we treat them as independent and effectively specify two spatial demand models, one for each product k.

We first derive the probability that a consumer located in municipal tract t will purchase the product of firm i. Her choice set C_t consists of all firms i within the same judicial district A_t and within a maximum distance of $\bar{d}$ kilometers, $C_t = \{i : i \in A_t \text{ and } d_{it} \leq \bar{d}\}$, and the outside option (good 0). The utility of making a transaction of product k at firm i is

$$\begin{aligned} \widetilde{u}_{it}^k &= u_{it}^k + \varepsilon_{it}^k \\ &= \beta_t^k d_{it} + \gamma_i^k x_i + \gamma_t^k w_t + \varepsilon_{it}^k, \end{aligned}$$

where d_{it} is the distance from the centroid of municipality t to the location of firm i, x_i is a vector of firm characteristics, and w_t is a vector of municipal characteristics. The parameters β_t^k, γ_i^k and γ_t^k are preference parameters and ε_{it}^k is a consumer-specific taste shock. A consumer's utility evidently also depends on the price of the transaction. However, since it is regulated and uniform across all firms and municipalities, it does not enter utility as a variable and we include it as part of the constant term. The utility of the outside option is normalized to $\widetilde{u}_{0t}^k = \varepsilon_{0t}^k$.

We specify the consumer-specific taste shock ε_{it} to follow an extreme value distribution according to a nested logit model with two nests. The first nest is the set of inside products C_t and the second nest is the outside good 0. The nest parameter $\mu_1^k \in [0, 1]$ measures the correlation of preferences for using a notary office, relative to choosing the outside good. A value of $\mu_1^k = 1$ implies that the consumer-specific taste shocks are independent across notary offices (no correlation), whereas $\mu_1^k = 0$ means that these taste shocks are common to all notary offices within the choice set (perfect correlation). Lower values of μ_1^k thus indicate higher substitutability across notary offices, relative to the outside good.

We specified utility at the level of the firm or notary office i, but in practice a consumer also chooses an individual notary (if there are multiple ones at firm i). To account for this, we include the logarithm of the number of notaries in office i, $\ln a_i$, as a variable in x_i. As shown by Ben-Akiva et al. (1985), the parameter of $\ln a_i$ can be interpreted as a second, lower-level nest parameter, labeled as μ_2^k. It measures the extent of taste correlation between "elemental alternatives", i.e. notaries within the same firm i.[14] With this interpretation, we expect $0 \leq \mu_2^k \leq \mu_1^k \leq 1$.

Assume that a consumer in municipality t chooses the option that maximizes utility among all available alternatives in her choice set C_t, and let ι_t^k be the option selected by a consumer in municipality t. Conditional on purchasing (i.e. choosing any notary office rather than the outside good), the probability that a consumer from municipality t chooses a specific notary i is

$$P^k(\iota_t^k = i | \iota_t^k \in C_t) = \frac{\exp(u_{it}^k/\mu_1^k)}{\sum_{i \in C_t} \exp(u_{it}^k/\mu_1^k)}.$$

The probability that a consumer will decide to purchase (rather than choose the outside good) is

$$P^k(\iota_t^k \in C_t) = \frac{\left(\sum_{i \in C_t} \exp(u_{it}^k/\mu^k)\right)^{\mu_1^k}}{1 + \left(\sum_{i \in C_t} \exp(u_{it}^k/\mu^k)\right)^{\mu_1^k}}.$$

The product of these probabilities yields the unconditional probability that a consumer in tract t selects notary i:

$$P_{it}^k = P^k(\iota_t^k \in C_t) P(\iota_t^k = i | \iota_t^k \in C_t).$$

We can now derive the expected demand of a notary i aggregated over all local markets within its catchment area. Let the potential number of transactions for product k in tract t be given by $\alpha^k S_t$, where S_t is population size in tract t and α^k is the potential number of transactions per capita for product k. Predicted demand in tract t for firm i is then $\widehat{q}_{it}^k = P_{it}^k \alpha^k S_t$. Since firm i's demand is not observed at the tract level, we aggregate over all tracts:

$$\widehat{q}_i^k = \sum_{t \in L_i} P_{it}^k \alpha^k S_t,$$

where $L_i = \{t : t \in A_i \text{ and } d_{it} \leq \overline{d}\}$ denotes the catchment area of firm i. Similar to Holmes (2011) and Ellickson et al. (2019), we assume that observed demand of firm i is $q_i^k = \widehat{q}_i^k e^{\eta_i^k}$, where η_i^k captures a firm-specific error term. This may reflect either measurement error or

[14] This can easily be verified from specifying a two-level instead of one-level nested logit, and imposing the utilities for individual notaries within the same office to be the same, except for the consumer-specific taste shock.

a firm-specific demand shock. Given a vector of preference parameters θ, firm i's predicted sales are $\widehat{q}_i^k(\theta)$. The resulting demand model is

$$\ln q_i^k = \ln \widehat{q}_i^k(\theta) + \eta_i^k.$$

The parameters in θ are estimated with an NLS algorithm:

$$\hat{\theta} = \arg\min_{\theta} \sum_i \left(\ln q_i^k - \ln \widehat{q}_i^k(\theta) \right).$$

4.2 Empirical results

As mentioned, a consumer's choice set C_t consists of all firms i within the same judicial district A_t and within a maximum distance of $\bar{d}$. We set $\bar{d}$ to 15 kilometers, which in practical terms implies that consumers effectively consider about half of the firms in their judicial district. We calibrate the potential number of transactions per capita α^k for each product by assuming that the current total demand for the product ($\sum_i q_i^k$) is equal to 10% of potential demand ($\alpha^k \sum_t S_t$). While this calibration includes an element of arbitration, the presence of an intercept and the nest parameter offsets this by flexibly estimating the market share of the outside good for a given α^k.

Table 4 reports the estimated parameters of the spatial demand model. The first column reports the preference coefficients for total output per firm $q_i = q_i^1 + q_i^2$, the two remaining columns show the estimates separately for real estate q_i^1 and other transactions q_i^2.

The distance parameter β_t^k is significant and negative for all products. The coefficient differs marginally between both products: consumers are willing to travel longer distances for real estate transactions than for other services. The nesting parameters are small in magnitude. The estimated values of μ_1^k imply that notaries are perceived as close substitutes, relative to the outside option. This is especially the case for real estate transactions and somewhat less so for other transactions. As we document further below, the relatively low nesting parameters imply that entry mainly involves business stealing and only modest market expansion. The coefficient for $\ln a_i$ is also small and significant. Consistent with its interpretation of a lower level nesting parameter μ_2^k, we find $\mu_2^k < \mu_1^k$ in all cases. Intuitively, consumers view notaries within the same office as closer substitutes than other notaries (which are in turn closer substitutes to each other than the outside good).

Other firm characteristics play a limited role. Several market demographics tend to be important. For example, consumers in high income tracts obtain higher utility for notary services, especially for other services. This may reflect the fact that other services refer to more exclusive transactions such as inheritance matters, gifts and (non-standard) marriage contracts, while real estate transactions are more common across income groups (in part promoted by government policies).

To illustrate what our estimates (in particular the nesting parameters) imply for the relative magnitude of business stealing and market expansion, we calculate the impact on sales after removing each notary from the choice set, and we average this over all 1152 notaries. In single notary offices, we calculate the counterfactual sales in the absence of the entire office, while in associations we calculate the counterfactual sales after eliminating just one of the notaries.

Table 5 reports the results. The addition of a single notary office generates on average about 70 new transactions. Given the average estimated sales per notary of 566 transactions, this represents a market expansion effect of 12% of average notary sales. However, the majority of the sales of the single notary office would have been realized by competitors had it not entered. The fourth column of Table 5 demonstrates that the addition of a single notary office reduces the total sales of other offices by 530 units, which is 94% of the average notary sales.

The addition of a notary at an association yields a smaller average market expansion effect: sales increase by 36 units, amounting to a sales increase of 6% of average notary sales. This is as expected given the demand parameters, which showed that notaries within an office are closer substitutes than notaries from different offices. The extent of business stealing to other notary offices is also smaller when a notary is added to an existing association. We also computed the extent of business stealing within different distance bands, and find that business stealing mainly occurs from nearby notaries, consistent with our finding of a significant distance coefficient.

In summary, our estimates of the spatial model imply that new entry into the market generates limited new demand for notary services. At the same time however, it has a positive effect on consumer surplus by reducing travel distance.

5 Multi-output production and markups

In this section we introduce a multi-output production model to describe the firms' input decisions and obtain marginal cost and markup estimates.

5.1 Specification

Notary firms produce multiple outputs (q_i^1 and q_i^2) using labor and intermediate inputs (l_i and m_i) as variable inputs. Capital (k_i) is fixed, and mainly consists of building expenses. In

Table 4: Spatial demand model estimates

	q_i	q_i^1	q_i^2
Distance parameters (β_t^k)			
Distance to market	-0.019	-0.020	-0.024
	(0.009)	(0.009)	(0.012)
Nesting parameters (μ^k)			
Industry nesting parameter (μ_1^k)	0.106	0.107	0.150
	(0.056)	(0.049)	(0.080)
Office nesting parameter (μ_2^k)	0.061	0.059	0.091
	(0.032)	(0.028)	(0.049)
Notary characteristics (γ_i^k)			
Date of establishment	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)
Company type = SP	-0.045	-0.046	-0.061
	(0.024)	(0.022)	(0.034)
Market characteristics (γ_t^k)			
Population density	-0.160	-0.159	-0.260
	(0.062)	(0.048)	(0.127)
Income (log)	1.445	0.814	2.409
	(0.256)	(0.252)	(0.264)
% young inhabitants	12.349	12.071	14.681
	(3.902)	(3.708)	(3.771)
% elderly inhabitants	8.045	6.859	11.207
	(2.645)	(2.494)	(2.714)
Intercept	-19.950	-13.514	-30.408
	(2.774)	(2.698)	(2.722)
Observations	1152	1152	1152

Note: Standard errors are in parentheses.

Table 5: Implied market expansion and business stealing effects

Office type	Notary sales	Market expansion		Business stealing	
	change (Δq_i)	change (ΔQ)	% of average sales per notary	change (ΔQ_{-i})	% of average sales per notary
Single notary	600	70	12.35%	-530	-93.71%
Association	296	36	6.43%	-260	-45.98%

Notes: The reported sales correspond to the estimated effect of entry in a new (existing) location for single (association) notaries. The values are averaged over all 1152 firms. Calculations are based on the spatial demand model estimates of Table 4. The average predicted sales per notary are 566 transactions.

this section we focus on the modeling of the two variable inputs, and postpone the discussion of the fixed input to Section 6.

Labor refers to administrative and legal services of high-skilled employees, usually with a bachelor or master education. As documented in Table 3, the average yearly staff costs per firm amount to approximately €369 000. Intermediate inputs consist of the services of the self-employed notaries, candidate-notaries and notary-trainees, and to a less important extent of administrative costs such as requesting certain documents at government agencies. The average annual expenditure on such intermediate inputs is €592 000 per office.

Each firm i transforms labor l_i and intermediate inputs m_i into two outputs q_i^1 and q_i^2 according to:

$$T(q_i^1, q_i^2) = F(l_i, m_i), \tag{2}$$

where $T(q_i^1, q_i^2)$ is the transformation function and $F(l_i, m_i)$ is the production function. We observe physical output, as in Foster et al. (2008), so we can distinguish physical productivity from revenue productivity. Furthermore, we separately observe both q_i^1 and q_i^2, in contrast to e.g. De Loecker et al. (2016). We therefore do not need to estimate the technological parameters for a subsample of single-product firms (which would not be feasible in our setting, as all firms produce both outputs).

We assume that the normalized (homogeneous of degree 1) transformation function takes a linear form:

$$T(q_i^1, q_i^2) = \alpha q_i^1 + (1-\alpha) q_i^2, \tag{3}$$

where α is the relative weight on q_i^1 in the transformation of inputs into outputs. If $\alpha = 0.5$, we obtain the special case of a single aggregate output $q_i^1 + q_i^2$.[15] Furthermore, we assume

[15]We also considered a Cobb Douglas transformation function, i.e. $T(q_i^1, q_i^2) = \left(q_i^1\right)^{\alpha} \left(q_i^2\right)^{(1-\alpha)}$. This gave

that the production function takes a Leontief form, i.e. labor and intermediate inputs are non-substitutable but they may be used in variable proportions according to:

$$F(l_i, m_i) = \left\{ \min \left[\exp(\beta^l + \omega_i^l) l_i, \exp(\beta^m + \omega_i^m) m_i^\phi \right] \right\}^\nu, \tag{4}$$

where ν measures the returns to scale, ϕ captures the extent of variable proportions between the two inputs as output grows, and ω_i^l and ω_i^m are unobserved productivity terms. If $\nu = 1$, there are constant returns to scale; if $\nu > 1$, there are increasing returns to scale (and vice versa if $\nu < 1$). If $\phi = 1$, labor and intermediate inputs are used in fixed proportions. If instead $\phi > 1$, labor becomes relatively more important as output grows (and vice versa if $\phi < 1$). Because labor and intermediate inputs are non-substitutable, one may equivalently interpret $\nu > 1$ as increasing returns to labor, and $\phi\nu > 1$ as increasing returns to intermediate inputs.

Substituting (3) and (4) into (2), rearranging and taking logarithms, gives the following system of conditional input demand functions:

$$\begin{aligned} \ln l_i &= -\beta^l + \frac{1}{\nu} \ln \left(\alpha q_i^1 + (1-\alpha) q_i^2 \right) - \omega_i^l, \\ \phi \ln m_i &= -\beta^m + \frac{1}{\nu} \ln \left(\alpha q_i^1 + (1-\alpha) q_i^2 \right) - \omega_i^m. \end{aligned} \tag{5}$$

Due to the regulations, firms do not choose their outputs (directly, or indirectly through price decisions). They passively produce output as demanded by consumers. Nevertheless, estimating (5) with ordinary least squares is not warranted because the unobserved productivity terms ω_i^l and ω_i^m may be correlated with the demand shocks η_i^k $(k = 1, 2)$, and hence with the outputs q_i^k in (5). A negative correlation between the unobserved productivity and demand terms may obtain if firms who require more inputs to complete their services also achieve a higher quality of service, as perceived by consumers. This would mean that firms with a large unobserved productivity shock have both high input demands and high outputs, which would lead to an underestimation of the economies of scale parameter ν. The reverse is also possible. A positive correlation between the unobserved productivity and demand terms would obtain if firms are both efficient in production and in providing a good quality of service, which would lead to an overestimation of scale economies.

To account for this possible endogeneity issue, we use the spatial demand model estimated in Section 4 to compute the predicted sales of both products ($\hat{q}_i^1$ and $\hat{q}_i^2$), which depend solely on the spatial demand determinants. As such, these predictions are correlated with the observed outputs, yet uncorrelated with the unobserved productivity terms, so they serve as natural external instruments to estimate the input demand equations. Put differently, we

very similar empirical results, but does not include the case of a single aggregate input as a special case.

Table 6: Conditional labor and intermediate input demand estimates

	Single product $\ln l_i(q_i)$ & $\ln m_i(q_i)$		Multiple products $\ln l_i(q_i^1, q_i^2)$ & $\ln m_i(q_i^1, q_i^2)$	
	LS	GMM	LS	GMM
	(1)	(2)	(3)	(4)
labor intercept (β^l)	1.357	1.079	0.513	−0.068
	(0.159)	(0.314)	(0.142)	(0.272)
materials intercept (β^m)	0.018	0.329	−0.441	0.577
	(0.209)	(0.355)	(0.204)	(0.309)
returns to scale (ν)	0.936	0.975	0.949	1.054
	(0.021)	(0.045)	(0.022)	(0.051)
materials proportion (ϕ)	1.132	1.037	1.070	0.811
	(0.050)	(0.089)	(0.047)	(0.071)
real estate weight (α)			0.451	0.567
			(0.027)	(0.069)
Observations	1048	1048	1048	1048

Notes: Standard errors are in parentheses.

are using the demand side shifters entering the spatial model as exclusion restrictions in the system of input demand equations. Formally, we use the instruments $z_i^{sd} = (1, \hat{q}_i^1, \hat{q}_i^2)$ in the system of moment conditions $E(\omega_i^l z_i^{sd}) = 0$ and $E(\omega_i^m z_i^{sd}) = 0$, and estimate the parameters in the system (5) jointly using GMM.

5.2 Empirical results

Table 6 reports the results from estimating the input demand equations. Labor is measured by the wage bill instead of physical units of labor. This allows us to account for possible differences in the quality of labor across firms, as variation in hourly wages across firms likely captures differences in labor quality because of training, experience, etc. (see Fox and Smeets, 2011). Intermediate inputs (consisting of services from self-employed suppliers and administrative costs) are also measured in value terms. Columns (1) and (2) show the parameters of the input demand functions for the production of a single aggregate output, $q_i = q_i^1 + q_i^2$, i.e. the total number of transactions of both product types. The single output specification is equivalent to assuming $\alpha = 0.5$. The least squares (LS) estimate of ν is below

Table 7: Marginal costs and firm mark-ups

	p	mc	$p - mc$	$(p - mc)/p$
real estate	€2 053	€1 305	€748	36.70%
other transactions	€1 390	€995	€396	28.46%

Notes: Marginal costs are based on the GMM estimates of the multi-product model in Table 6, evaluated at average output.

1, which would seem to suggest diseconomies of scale; and the variable proportions parameter ϕ is above 1, suggesting labor becomes relatively more important than intermediate inputs as output grows. However, accounting for endogeneity raises ν to 0.975 and reduces ϕ to 1.037, implying close to constant returns to both labor and intermediate inputs.

Columns (3) and (4) of Table 6 report the estimates from the multi-product input demand equations. The estimated returns to scale parameter ν is comparable to the single output model, and not significantly different from 1 under GMM. The variable proportions parameter ϕ is significantly below 1 under GMM, indicating that labor becomes relatively less important as output grows. The product $\phi\nu = 0.85$ is significantly below 1. This points to decreasing returns to intermediate inputs, and is consistent with the presence of capacity constraints in the provision of notary services. Finally, the relative weight on real estate (α) in the transformation function is estimated to be slightly larger than 0.5 under GMM. This suggests that slightly less inputs are required to produce real estate than other transactions, although we cannot reject $\alpha = 0.5$.

Given the conditional input demand functions, we can define the variable cost function as $C_i^V(q_i^1, q_i^2) = wl_i(q_i^1, q_i^2) + m_i(q_i^1, q_i^2)$, where we set $w = 1$, since labor demand is already measured in value terms. We can use the estimated parameters of the input demand system (5) to calculate the variable cost function and also the marginal costs for both products, $\partial C_i^V / \partial q_i^k$, for $k = 1, 2$.

Table 7 shows the marginal cost estimates, along with the prices and the implied markups for both product types. The average price is €2 053 for a real estate transaction, and €1 390 for other transactions. Marginal costs are only slightly higher for real estate than other transactions (respectively €1 305 and €995). This implies that markups are higher for real estate transactions, i.e. €748, compared with €396 for other transactions. These findings are not inconsistent with the sector's claim that real estate transactions serve to "cross-subsidize" other transactions. Nevertheless, even these other transactions still yield positive markups. This may be due to earnings from sufficiently large allowed administrative charges in addition to the legally fixed low fees for other transactions.

The variable profits $\pi_i^V(q_i^1, q_i^2)$ implied by our estimates amount to €496 448 per notary office, which translates to €387 991 per individual notary in an office. Since these variable profits are already net of service payments to notaries (as part of intermediate inputs), they only need to cover fixed costs. We return to that in the next section.

6 Understanding the entry restrictions

We now use the building blocks of the previous sections to study how we can rationalize the observed entry outcomes in the notary sector. We have in mind a two-stage game, as in much of the empirical entry literature (e.g. Eizenberg, 2014; Berry et al., 2016; Fan and Yang, 2020; Ishii, 2005; Wollmann, 2018). In a first stage, the entry decisions take place. The state maximizes a weighted sum of producer and consumer surplus, based on the realized fixed costs and expectations regarding variable profits and consumer surplus. In the second stage, demand and cost shocks are realized, consumers decide whether to use notary services and which notary to take, and firms decide how much labor and intermediate inputs to deploy. As prices have been fixed by law, we treat them as given, set before the entry stage.

The empirical entry literature typically focuses on inferring fixed costs from the observed entry outcomes. A lower bound on fixed costs comes from the inequality condition that it is not profitable to add entry, and a lower bound comes from the condition that it is not profitable to remove entry. Our goals reach further because we also aim to infer the weight the state implicitly attaches to producer and consumer surplus when issuing entry licenses. To accomplish this, we will pursue two approaches to complement the inequality conditions from adding/removing notaries at existing locations. First, we introduce inequality conditions from entry switches between locations. Second, we make use of direct information on fixed costs from our dataset.

Our entry model takes into account the institutional features of the entry process, as discussed earlier in section 2. First, the entry regulations impose a fixed number of notary locations in each of the eleven judicial districts. This number is proportional to the districts' population size and restricts the ability of the state to open new notary offices. Although the regulations may not strictly prevent the possibility of creating new notary locations provided other ones are closed, such closures have been rarely observed in the past decades. Second, the entry regulations allow the state to issue new entry licenses every year (replacements and possibly new licenses). Such an entry license is allocated to one specific location only. Based on these features, we constrain the set of potential entry locations to the addresses of the existing notary offices (of which there are $J = 1152$). We then model the state's decision regarding the number of notaries per office, rather than the location itself.[16] In our counterfactual analysis in the next section, we will study the possibility to open offices at new locations.

Section 6.1 describes the entry model and section 6.2 develops the empirical implementation. Section 6.3 then discusses our empirical findings.

6.1 Entry model

Let a be the $J \times 1$ location vector introduced earlier in subsection 3.1, with elements a_i referring to the number of notaries active in location or notary office i. Furthermore, let f_i be the fixed cost per notary at notary office i. We assume the state selects the number of notaries in each location i to maximize the following objective function:

$$\Gamma(a) = \Pi^V(a) - \sum_{j=1}^{J} f_j a_j + \lambda CS(a). \tag{6}$$

The first term $\Pi^V(a)$ denotes the expected total variable industry profits, the second term consists of total fixed costs, and the third term $CS(a)$ denotes consumer surplus. The parameter λ reveals the weight the state places on consumer surplus when granting entry licenses. This objective function nests various possible cases, such as industry profit maximization or full "regulatory capture" ($\lambda = 0$), total welfare maximization ($\lambda = 1$), intermediate outcomes with $\lambda \in (0, 1)$, or outcomes with a higher weight on consumer than producer surplus ($\lambda > 1$).

Expected variable industry profits in (6) are:

$$\Pi^V(a) \equiv E_{\eta,\omega}\left[\sum_{j=1}^{J} \pi_j^V(a; \eta_j, \omega_j)\right],$$

where $\eta_j = (\eta_j^1, \eta_j^2)$ and $\omega_j = (\omega_j^l, \omega_j^m)$ are the demand and productivity shocks. Variable profits of firm i are revenues minus variable costs, and depend on the number of entrants through the demand function $q_i^k(a)$, $k = 1, 2$, from the spatial model:

$$\pi_i^V(a; \eta_j, \omega_j) = p^1 q_i^1(a) + p^2 q_i^2(a) - w l_i(q_i^1(a), q_i^2(a)) - m_i(q_i^1(a), q_i^2(a)).$$

We specify fixed costs at notary office i as

$$f_i = \theta^d + \upsilon_i, \tag{7}$$

[16] Note that the approach of modeling the number of entrants at a given number of locations is similar to Berry et al. (2016) (in a different context).

where θ^d is the mean district-specific fixed cost per notary, υ_i is a mean zero unobserved error term, $E[\upsilon_i|z_i] = 0$, and z_i is a vector of dummy variables for each district d. The error term υ_i may have two interpretations. First, it can reflect expectational error, unobserved by both the state and the econometrician. This assumption appears to be reasonable here, because we condition on district-specific fixed costs (accounting for regional factors such as varying house prices), and the state likely does not have detailed fixed cost information when granting entry licenses to new candidate notaries. Second, υ_i may capture "structural error", observed by the state but not by the econometrician. As we discuss below, the second interpretation is sufficient if our analysis only relies on inequality conditions from adding/removing firms, but the first interpretation is required when we also rely on inequality conditions from switches.

Finally, total consumer surplus is given by the well-known "log-sum" formula for the nested logit spatial demand model outlined in Section 4. A challenge in our setting is that prices are regulated to be uniform across notaries. So consumers do not use prices as a decision criterion to select a notary, and we cannot identify a price coefficient. We therefore monetize consumer utility by scaling the parameter β_t of the distance variable d_{it} (in km) by a travel cost parameter t_d.[17] The expected consumer surplus is then $CS(a) = \sum_t CS_t(a)$, where consumer surplus in tract t is

$$CS_t(A) = -\frac{t_d}{\beta_t} \ln \left(1 + \left(\sum_{i \in C_t(A)} \exp(u_{it}/\mu) \right)^{\mu} \right) \alpha S_t.$$

The consumer surplus in each tract thus implicitly depends on the distribution of notaries through both the distance variable d_{it} (entering u_{it}) and the nesting parameters of the demand equation.

In sum, the state chooses the number notaries a_i in each location i to maximize the objective function $\Gamma(a)$, as given by (6). At the same time, we take into account the constraint that each firm's variable profits is above the fixed cost of operating a notary office (i.e. $E_{\eta,\omega}\left[\pi_i^V(A); \eta_i, \omega_i)\right] \geq f_i a_i$), since in a private industry context the state cannot force entry.

[17]We calibrate $t_d = 100$€. This is in line with estimated consumer travel costs in the same country by Nurski and Verboven (2016). Furthermore, we verified that this value implies a price elasticity of demand for real estate transactions of around -0.45, which is consistent with external empirical evidence on real estate demand. Best and Kleven (2018) estimate a price elasticity of real estate transactions (at the extensive margin) of -20. As outlined in subsection 2.1, in Belgium notary fees comprise 2.2% of the total price of property. This would imply a notary fee elasticity of demand of -0.44.

6.2 Inferring fixed costs and consumer surplus weight

Our goal is to infer both the fixed cost parameters θ^d and the consumer surplus weight λ from necessary conditions on the maximization of the objective function $\Gamma(a)$. First, following the prior entry literature on inferring fixed costs, we use inequalities from adding and removing entrants at the set of potential locations. This provides bounds on fixed costs θ^d, as a function of λ. Second, we include additional necessary conditions, i.e. switches between location pairs. This gives bounds on λ. Taken together, we obtain bounds on (θ^d, λ) instead of just on θ^d. Finally, after obtaining the bounds we will also confront them with direct information on fixed costs from our data set.

To obtain the necessary conditions, we define the difference operator $\Delta\Gamma(a, a') \equiv \Gamma(a) - \Gamma(a')$, i.e. the difference in the objective function Γ under the current location vector a and a new location vector a'. We similarly define $\Delta\Pi^V(a, a') \equiv \Pi^V(a) - \Pi^V(a')$ and $\Delta CS(a, a') \equiv CS(a) - CS(a')$.

Additions and removals According to the first set of necessary conditions, the objective function $\Gamma(a)$ cannot improve by adding or removing a notary at any location i. Formally, these revealed preference conditions are $\Delta\Gamma(a, a + 1_i) \geq 0$ and $\Delta\Gamma(a, a - 1_i) \geq 0$, where 1_i is a $J \times 1$ vector with the i-th element equal to 1, and all remaining elements set to 0. Using (6), we can write the inequality conditions as:

$$\begin{aligned} \Delta\Pi^V(a, a - 1_i) - f_i + \lambda\Delta CS(a, a - 1_i) &\geq 0 \\ \Delta\Pi^V(a, a + 1_i) + f_i + \lambda\Delta CS(a, a + 1_i) &\geq 0. \end{aligned}$$

We can rearrange this to obtain the following bounds:

$$\begin{aligned} f_i &\leq \Delta\Pi^V(a, a - 1_i) + \lambda\Delta CS(a, a - 1_i) \\ f_i &\geq -\Delta\Pi^V(a, a + 1_i) - \lambda\Delta CS(a, a + 1_i). \end{aligned}$$

Taking expectations across notary offices i and using (7) gives:

$$\begin{aligned} \theta^d &\leq E\left[\Delta\Pi^V(a, a - 1_i)|z_i\right] + \lambda E\left[\Delta CS(a, a - 1_i)|z_i\right] \equiv \overline{\theta}^d(\lambda) \\ \theta^d &\geq -E\left[\Delta\Pi^V(a, a + 1_i)|z_i\right] - \lambda E\left[\Delta CS(a, a + 1_i)|z_i\right] \equiv \underline{\theta}^d(\lambda) \end{aligned} \tag{8}$$

because $E[\upsilon_i|z_i] = 0$. The right hand side terms $\overline{\theta}^d(\lambda)$ and $\underline{\theta}^d(\lambda)$ are upper and lower bounds for the fixed cost parameters θ^d, which are linearly increasing functions of the consumer surplus weight λ. We can rearrange (8) to obtain equivalent bounds on the consumer surplus weight as linearly increasing functions of the fixed cost parameters:

$$\begin{aligned}\lambda &\geq \frac{\theta^d - E\left[\Delta\Pi^V(a, a-1_i)|z_i\right]}{E\left[\Delta CS(a, a-1_i)|z_i\right]} \equiv \underline{\lambda}(\theta^d) \\ \lambda &\leq \frac{\theta^d + E\left[\Delta\Pi^V(a, a+1_i)|z_i\right]}{-E\left[\Delta CS(a, a+1_i)|z_i\right]} \equiv \overline{\lambda}(\theta^d)\end{aligned} \tag{9}$$

because $\Delta CS(a, a-1_i) > 0$ and $\Delta CS(a, a+1_i) < 0$. The lower bound $\underline{\lambda}(\theta^d)$ is the average total profit gain relative to the average consumer surplus loss from removing a notary at each location; the upper bound $\overline{\lambda}(\theta^d)$ is the average total profit loss relative to the average consumer surplus gain from adding a notary at each location.

To implement inequalities (8) or (9), we replace the conditional expectations by sample moments. In our set-up, this amounts to averaging across observations i by district d. This approach can be applied under the interpretation that the fixed cost error υ_i is expectational error. Under the alternative interpretation of υ_i as structural error (unobserved by the econometrician), a potential selection issue may arise when evaluating infeasible entry counterfactuals (Pakes et al., 2015). For example, Eizenberg (2014) cannot evaluate the impact of removing non-existing products, or of introducing already existing products, and proposes a solution to deal with this. We instead have an ordered choice structure, as in Ishii (2005) or Berry et al. (2016). It is then always feasible to add one more notary to a notary location. Conversely, it is also always feasible to remove a notary from a location, since we take the set of potential locations to be equal to the actual ones. As such, we can avoid the selection issue also under the interpretation that υ_i is structural error. This is based on our ordered structure that fixed costs (including υ_i) are common to each notary at notary location i, and that no new locations have been under consideration by the state for several decades.

Switches According to the second set of necessary conditions, the objective function $\Gamma(a)$ cannot improve from shifting a notary out of any location i to some other location k. More specifically, for each location i we consider two types of switches within the same district: CS-decreasing switches from each i to some location k ($\Delta CS(a, a-1_i+1_k) > 0$), and CS-increasing switches from each i to some other location l ($\Delta CS(a, a-1_i+1_l) < 0$). Since there can be multiple feasible switches, for each i we randomly select one k and one l. The use of two kinds of switches is analogous to Katz (2007), who obtains bounds on travel costs from shoppers' switches to more distant and more nearby stores.

The revealed preference conditions are $\Delta\Gamma(a, a-1_i+1_k) \geq 0$ and $\Delta\Gamma(a, a-1_i+1_l) \geq 0$.

Using (6), we can write these as

$$\begin{aligned}\Delta\Pi^V(a, a-1_i+1_k) - f_i + f_k + \lambda\Delta CS(a, a-1_i+1_k) &\geq 0\\ \Delta\Pi^V(a, a-1_i+1_l) - f_i + f_l + \lambda\Delta CS(a, a-1_i+1_l) &\geq 0.\end{aligned}$$

Taking expectations across notary offices i gives

$$\begin{aligned}E\left[\Delta\Pi^V(a, a-1_i+1_k)|z_i\right] + \lambda E\left[\Delta CS(a, a-1_i+1_k)|z_i\right] &\geq 0\\ E\left[\Delta\Pi^V(a, a-1_i+1_l)|z_i\right] + \lambda E\left[\Delta CS(a, a-1_i+1_l)|z_i\right] &\geq 0.\end{aligned}$$

where we make use of $E\left[-f_i + f_k|z_i\right] = E\left[-\theta_d + \theta_d - \upsilon_j + \upsilon_k|z_i\right] = 0$ for CS-decreasing switches from i to k within the same district, and similarly for CS-increasing switches from i to l. Rearranging the inequalities, we obtain the following upper and lower bounds on λ:

$$\begin{aligned}\lambda &\geq \frac{-E\left[\Delta\Pi^V(a, a-1_i+1_k)|z_i\right]}{E\left[\Delta CS(a, a-1_i+1_k)|z_i\right]} \equiv \underline{\lambda}\\ \lambda &\leq \frac{E\left[\Delta\Pi^V(a, a-1_i+1_l)|z_i\right]}{-E\left[\Delta CS(a, a-1_i+1_l)|z_i\right]} \equiv \overline{\lambda}\end{aligned} \tag{10}$$

As discussed by Pakes (2010) and Kline et al. (2021) (based on the application of Katz, 2007), these bounds are essentially ratios of averages. In our setting the bounds are ratios of expected producer surplus changes over expected consumer surplus changes; we provide further intuition when discussing our empirical results.

To implement (10), we again replace the conditional expectations by sample averages across observations i. Since there may not always be CS-increasing or CS-decreasing switches at all locations i, we rely on the interpretation that υ_i reflects expectational error. An alternative approach would be to omit the inequalities from switches and instead rely on direct cost information, to which we turn next.

Direct fixed cost information The previous discussion relied solely on moment inequalities to infer both the firms' fixed costs and the state's weight to consumer surplus. Alternatively, we can incorporate direct information from our dataset on the fixed costs f_i for each notary office i. We can then simply replace $E\left[f_i|z_i\right] = \theta^d$ by the district-level sample averages $\hat{f}^d$, and substitute this in the inequalities from additions and removals (9). This provides bounds $\overline{\lambda}(\hat{f}^d)$ and $\underline{\lambda}(\hat{f}^d)$. We will use this information as a way to validate our model, and assess how this may tighten the bounds on the consumer surplus weight.

6.3 Empirical findings

Our entry model provides bounds for both the fixed entry costs and the weight λ attached by the state to consumer surplus.

Table 8: Fixed cost bounds conditional on consumer surplus weights λ

Objective function	Bounds on f	95% CI
Joint profit maximization ($\lambda = 0$)	[14.302 , 82.262]	[13.856 , 85.133]
Standard welfare maximization ($\lambda = 1$)	[174.022 , 387.129]	[171.220 , 397.193]

Notes: Fixed cost bounds are reported in €1 000. Confidence intervals are calculated using 1 000 bootstrap draws with replacement.

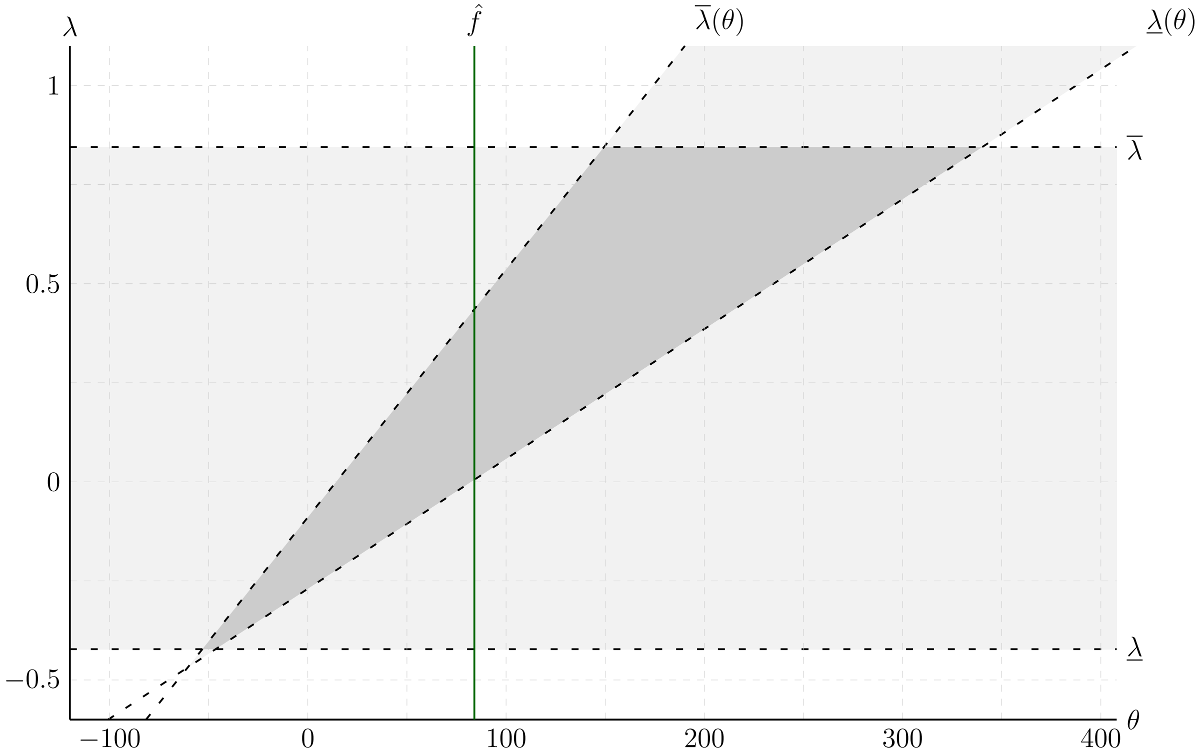
Fixed cost bounds for given λ: To obtain a first idea of the relationship between fixed costs and the consumer weight, we compute fixed costs bounds for given values of λ, i.e. $\overline{\theta}^d(\lambda)$ and $\underline{\theta}^d(\lambda)$, based on the typical inequalities from adding and removing products (8). To simplify the exposition, we first assume fixed costs do not vary across districts ($\theta^d = \theta$); we later confirm that the broad conclusions are comparable for district-specific fixed costs.

Table 8 shows our findings. Suppose that the entry licencing process serves to maximize total industry profits as under full "regulatory capture", i.e. $\lambda = 0$. The current distribution of notaries across office locations can then be rationalized under rather low fixed costs, between €14 302 and €82 262 per notary. Intuitively, adding notaries involves a trade-off between market expansion and duplicated fixed costs. Since market expansion is relatively low, the fixed costs which would rationalize the current entry outcome are also relatively low if the state is only interested in firm profits. Conversely, suppose that the state grants entry licenses to maximize overall welfare with a full account of consumer surplus, i.e. $\lambda = 1$. In this case, the current entry distribution can be rationalized only under very high fixed costs, between €174 022 and €387 129 per notary. Intuitively, under welfare maximization the gains from additional entry are high because of the benefits from increased availability to consumers. As a result, the current (restrictive) entry outcome can be rationalized only under very high fixed costs.

Bounds on (λ, θ): We now combine the moment inequalities from adding and removing notaries with those from switching notaries to other locations. On Figure 2, the two upward sloping lines $\underline{\lambda}(\theta)$ and $\overline{\lambda}(\theta)$ represent the lower and upper bounds from adding and removing notaries, as given by (9) (or equivalently by (8)). The shaded area between these two lines then shows the set of (λ, θ) that rationalizes the state's choice not to add or remove notaries at existing locations. Note that the fixed cost numbers of Table 8 obtain where these lines intersect at $\lambda = 0$ and $\lambda = 1$.

The two horizontal lines on Figure 2 represent the lower and upper bounds $\underline{\lambda}$ and $\overline{\lambda}$ from the CS-decreasing and CS-increasing switches. The shaded area between these lines is the

Figure 2: Set of rationalizable consumer surplus weights and fixed costs (λ, θ)



Notes: Fixed costs θ are reported in €1 000. The horizontal lines $\overline{\lambda}$ and $\underline{\lambda}$ represent the upper and lower bound of λ calculated from CS-increasing and CS-decreasing location switches, respectively. The increasing lines $\overline{\lambda}(\theta)$ and $\underline{\lambda}(\theta)$ represent the upper and lower bounds of λ conditional on fixed costs θ, derived from notary additions and removals. The dark-gray shaded area represents combinations of consumer surplus weights and fixed costs that can rationalize the current distribution of firms. The vertical line $\hat{f}$ shows the direct fixed cost estimate based on our dataset.

set of (λ, θ) that rationalizes the state's choice not to re-distribute notaries across locations. The horizontal upper bound is $\overline{\lambda} = 0.845$, showing that the observed entry outcome cannot be rationalized for any consumer surplus weight higher that 0.845. Intuitively, $\overline{\lambda}$ is the ratio of the average profit decrease over the average consumer surplus increase after CS-increasing switches across locations. A ratio of 0.845 means the state is not willing to sacrifice more than on average €0.845 producer surplus losses for an average €1 consumer surplus increase when considering CS-increasing switches. This upper bound thus rules out that the entry restrictions are consistent with total welfare maximization, even taking the current fixed prices as given. The horizontal lower bound (obtained from CS-decreasing switches) turns out to be negative, i.e. $\underline{\lambda} = -0.422$. The negative value means that the CS-decreasing

Table 9: Estimated λ bounds

	Bound range $[\lambda^L, \lambda^U]$	95% CI
All observations ($f = \hat{f}$)*:*	[0.006 , 0.436]	[−0.004 , 0.445]
By district ($f = \hat{f}^d$)*:*		
Antwerp	[−0.006 , 0.406]	[−0.028 , 0.422]
Brussels	[0.131 , 0.677]	[0.102 , 0.714]
East Flanders	[−0.004 , 0.413]	[−0.025 , 0.430]
Hainaut	[0.010 , 0.479]	[−0.010 , 0.499]
Leuven	[−0.066 , 0.221]	[−0.117 , 0.246]
Liège (incl. Eupen)	[0.041 , 0.477]	[0.010 , 0.500]
Limburg	[−0.026 , 0.403]	[−0.053 , 0.428]
Luxembourg	[−0.027 , 0.461]	[−0.064 , 0.532]
Namur	[−0.026 , 0.391]	[−0.063 , 0.427]
Walloon Brabant	[−0.067 , 0.297]	[−0.107 , 0.332]
West Flanders	[−0.030 , 0.361]	[−0.053 , 0.385]

Note: Confidence intervals are calculated using 1 000 bootstrap draws with replacement.

switches on average also reduce industry profits. This may reflect the friction that prevents the state from removing single notary offices and shifting them to new locations that may be more profitable. As a result, the CS-decreasing switches are essentially uninformative, as they do not rule out any positive weight on consumer surplus.

Taken together, the lines $\underline{\lambda}(\theta)$, $\overline{\lambda}(\theta)$, $\underline{\lambda}$ and $\overline{\lambda}$ delineate the set of (λ, θ) that rationalizes the state's decision to not adjust the total number of notaries, nor the distribution of notaries across locations. This is shown by the dark-gray shaded area on Figure 2, which further shrinks if we limit attention to combinations such that $\lambda > 0$ and $\theta > 0$.

We next incorporate the fixed cost information from our data set. As reported in section 3.2, we estimate the average fixed cost per notary to be approximately $\hat{f} =$€84 000. This is represented by the vertical line $\hat{f}$ in Figure 2, which intersects with the dark-shaded area. This provides support for the entry model and its implied bounds on (λ, θ). Furthermore, the direct fixed cost information yields tighter bounds for the consumer surplus weight λ, which falls between $\underline{\lambda}(\hat{f}) = 0.006$ and $\overline{\lambda}(\hat{f}) = 0.436$.

District-level analysis: To obtain further insights on the state's weight attached to consumer surplus, we extend our analysis to the level of the eleven judicial districts. More specifically, we estimate fixed costs per notary for each judicial district to obtain $\hat{f}^d$, and then compute district-level bounds on the consumer surplus weight from adding and removing notaries, i.e. $\underline{\lambda}^d(\hat{f}^d)$ and $\overline{\lambda}^d(\hat{f}^d)$. As such, we evaluate whether the state attaches different weights to consumers across districts, after accounting for possible differences in fixed costs per notary. Table 9 reports the estimated bounds, including confidence intervals.[18]

The first row of Table 9 simply shows the earlier reported bounds on the consumer surplus weight λ at the country level, now including also the confidence intervals. The subsequent rows show the bounds on λ^d per judicial district. For each district, we reject the hypothesis of full welfare maximzation ($\lambda = 1$) and for nine out of eleven districts we cannot rule out pure industry profit maximization ($\lambda = 0$). Interestingly, the bounds on consumer surplus weights do not reveal notable differences across districts. The bounds in the most rural districts (Luxembourg and Namur) are comparable to those in the most urbanized districts (Antwerp and Brussels). More generally, the confidence bounds overlap across all districts: the highest lower bound (0.131) is below the lowest upper bound (0.221).

In sum, our empirical findings indicate that the current entry restrictions cannot be rationalized as the result of welfare maximization (even conditional on the current high fixed prices). Instead, the entry restrictions appear to be more in line with industry profit maximization, where the state attaches a rather low weight to consumer surplus across all judicial districts.

7 Policy counterfactuals

Given our previous findings that the current entry restrictions mainly serve the industry's interests, we now ask how policy reform can improve welfare and affect the distribution of surplus between consumers and firms. There is scope for policy reform in two directions: (i) reforming the notary fees which have been fixed by law, and (ii) liberalizing the entry process. In fact, a revision of the fee structure is under debate in the Parliament. The fee structure has not been revised since 1950 (except for a general price increase by 20% in 1978 and a reduction by 5% in 1980). The public concerns relate mainly to the notary fees for real estate transactions. These fees are a fixed percentage of the house value, and they have dramatically increased over the past decades as a consequence of rising house prices. The most recent proposal would reduce the notary fees for real estate transactions by up

[18] We construct the $(1-\alpha)$-level confidence interval by respectively taking lower and upper one-sided confidence intervals of the sample averages of the lower and upper bounds. This follows Imbens and Manski (2004) as applied by Eizenberg (2014), except that we do not rely on the standard errors of the means but we instead calculate the distribution of the means using bootstrapping with replacement.

to 15% (for transactions in the lowest house value bracket), while leaving the fees for other transactions unchanged. The proposal does not consider the impact of adjusting fees on the sector's profitability, nor does it consider the possibility of liberalizing the entry process.

We first evaluate the impact of reforming fees, without altering the entry regulations (subsection 7.1). This merely serves to highlight the changes in consumer surplus and total welfare under alternative fee structures, and the implications for the profitability of the individual firms and the sector as a whole. In a second step, we conduct a more complete analysis and consider the impact of liberalizing entry, possibly combined with reforming the fees (subsection 7.2). We consider both welfare maximizing entry regulation and a reform that entirely lifts the entry restrictions, so that entry becomes free. We also allow for the possibility that entry takes place at entirely new locations.

7.1 Reform of the fee policy

We first assess the impact of reforming fees, taking the current entry outcome as given. Both optimal uniform fee changes and differentiated fee changes for real estate and other transactions are considered. We calculate the first-best fee changes, but also fee changes that achieve a target level of industry profits, as in Ramsey pricing. In the Ramsey pricing scenario, a target profit level can serve to cover fixed costs in the sector, and/or implement an internal compensation scheme among notary firms in case individual profitability is under threat.

As shown in Table 10, the welfare maximizing uniform price drop is 32%, while the welfare-maximizing differentiated price drop amounts to 35% for real estate and 26% for other transactions. This corresponds to price drops to approximately marginal costs.[19] Such drastic fee drops imply that almost half of all notary offices would become unprofitable, as they can no longer recoup their fixed costs. We next look at the second-best price changes, which aim for a minimum industry profitability. In this case the reduction in fees is lower, as expected, but the differentiated fees keep the same structure, with a higher price drop for real estate transactions. For example, with a €100 million profit standard, the uniform price drop is 23%, and the differentiated prices drops are 28% for real estate and 14% for other transactions. Despite the relatively large price drops, only a small number of firms is no longer able to cover fixed costs (and these could, in principle, be compensated through transfers by other firms). These unprofitable firms are also distributed relatively uniformly across space, so that service provision does not seem to be threatened by a potential decrease

[19] These do not coincide with current marginal costs, because marginal costs depend on output according to our production model.

Table 10: Counterfactual equilibria under alternative fee structures

minimum Π	price change	Δp_1	Δp_2	ΔCS_1	ΔCS_2	$\Delta\Pi$	ΔW	$\sum_i \mathbf{I}(\pi_i < 0)$
no minimum profit	uniform	-32%	-32%	338,177	169,076	-478,670	28,583	711
	differentiated	-35%	-26%	371,952	136,143	-479,106	28,989	730
100 million	uniform	-23%	-23%	239,033	119,893	-332,843	26,083	22
	differentiated	-28%	-14%	293,711	72,002	-338,898	26,815	26
200 million	uniform	-16%	-16%	164,140	82,534	-225,632	21,042	2
	differentiated	-23%	-4%	239,033	20,267	-237,093	22,207	3

Notes: Fixed costs per notary are evaluated at €84 000. Price reductions are based on a grid search with 1% intervals. Δp_1 and Δp_2 report the optimal price decrease for real estate and other transactions, respectively. Consumer surplus (ΔCS_1 for real-estate, and ΔCS_2 for other transactions), producer surplus ($\Delta\Pi$) and welfare (ΔW) changes are relative to the status quo and are reported in €1 000. $\mathbf{I}(\pi_i < 0)$ is an indicator variable equal to 1 if firm profits in location i are negative.

in margins. To illustrate this, Figure 3 shows the profitability levels under the current fees (left panel), and under fee reductions that guarantee industry profits of €200 million (middle panel) and €100 million (right panel). Recall that the profitability levels refer to net profits, i.e. after deducting intermediate input expenses that include payments to the self-employed notaries.

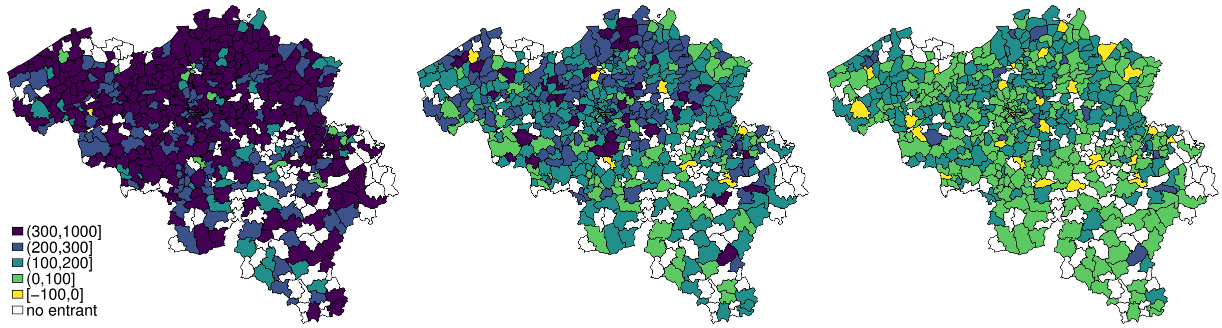
The results reported in Table 10 suggest that the optimal fees have only a small impact on total welfare (a net gain of approximately €30 million). The small scope for total welfare improvements through a revision of the fee structure is due to the low price elasticity of demand for notary services. Hence, the fee reductions mainly serve to redistribute surplus from firms to consumers. In the case of differentiated fee drops, consumers mainly benefit from reduced real estate fees.

7.2 Reforming the entry process

We now consider the impact of reforming the entry process. Our search for counterfactual equilibria starts with the current distribution of firms and implements a sequential entry and exit algorithm, similar to Seim and Waldfogel (2013). Details regarding the procedure are reported in section A.3 in the Appendix.

Table 11 reports the results. Panel A considers the impact of welfare-maximizing entry regulation (subject to a non-negativity constraint for individual firm profitability); Panel B considers a full liberalization to free entry. Under both reforms, we also consider the accompanying welfare-maximizing uniform fee reduction (based on a grid search with 1%

Figure 3: Average office-level profits under alternative fee structures



Notes: The left panel of Figure 3 presents the average office-level profits across Belgian municipalities in €1 000. The middle and right panels plot the same variable after fee reductions which guarantee industry profits of respectively €200 million and €100 million. While per-firm profits decrease, geographic coverage does not seem to be substantially jeopardized by the reforms.

price drop increments). We begin with a base scenario where entry of new notaries remains limited to the current 1 152 notary office locations (as has been the case for the past decades), and then consider an alternative scenario that also allows entry at new locations. We use the address data of all 2 413 post offices in Belgium to proxy for the set of potential new entry locations. Post offices often play an important role in administrative matters, which raises the likelihood that their surroundings are comparable to those selected by notaries, in terms of potential profitability and/or social desirability of additional entry.

7.2.1 Welfare maximization

Our analysis in section 6 showed that the current entry restrictions put a low weight on consumer surplus at best. We thus ask what is the potential for welfare improvement through a major reform of the entry licensing process itself. We assume that the state has full control over the number of notaries in each location, and uses this control to assign notaries to maximize total welfare. We account for a constraint that the individual firms remain profitable.

First, consider an entry process that maximizes welfare, assuming unchanged prices and no entry at new locations (first row of Panel A). This would result in a more than doubling of the number of notaries (from 1 569 to 3 943), which in turn induces an increase in demand of 6.7%. There is a substantial redistribution of surplus to consumers, who gain an amount of €291 million. Industry profits decrease by approximately €171 million, but the sector remains highly profitable, as this decrease is relative to the current profitability of approxi-

Table 11: Counterfactual equilibria under alternative regulatory regimes

	notaries ($\sum_i a_i$)	notary offices	uncovered markets	%Δ output	Δ consumer surplus	Δ producer surplus	Δ welfare
Current entry outcome							
	1,569	1,152	104	-	-	-	-
Panel A: total welfare maximization							
	current locations						
- at current prices	3,943	1,152	104	6.71	290,661	-170,983	119,678
- 17% price reduction	3,712	1,149	107	12.49	549,543	-408,752	140,791
	current and new locations						
- at current prices	4,439	3,522	5	12.64	552,022	-89,140	462,882
- 17% price reduction	4,307	3,306	22	18.93	835,267	-346,970	488,297
Panel B: free entry							
	current locations						
- at current prices	7,046	1,152	104	10.79	468,139	-414,225	53,914
- 17% price reduction	3,754	1,149	107	12.56	552,863	-412,171	140,692
	current and new locations						
- at current prices	8,066	3,434	9	17.15	749,180	-372,809	376,371
- 19% price reduction	4,194	3,195	33	19.45	859,384	-374,056	485,328

Notes: Fixed costs per notary are evaluated at €84 000. Price reductions refer to uniform welfare-maximizing price reductions, based on a grid search with 1% intervals. Surplus and welfare changes are in €1 000. Output changes are reported in percentage terms.

mately €440 million. The number of uncovered markets (municipalities) remains unchanged at 104 (out of 589). On balance, there is a substantial potential for welfare improvement by €120 million. This is in sharp contrast with the small total welfare improvement from a pure price reform found earlier. It follows from the low weight assigned to consumer surplus under the current entry regulations, implying insufficient entry.

Now reconsider the welfare-maximizing entry process when prices are also optimally adjusted, holding constant the possible location choices (second row of Panel A). The welfare-maximizing price drop is 17%.[20] Total welfare increases by an additional €21 million compared to the scenario in which only the number of notaries per office is adjusted. The modest extra welfare gain is in line with our earlier findings on the optimal fees. Note, however, that the redistributive effects are substantial. Consumer surplus increases by €550 million, due to both additional availability and reduced prices. The notary sector would experience a profit drop of €408 million, but still retains a total net profit of €31 million (above the fixed costs and the salary payments to self-employed notaries as part of the intermediate inputs). Compared to the status quo, this equates to welfare gains of €141 million per year.

Finally, consider a welfare-maximizing process that allows for entry at new locations. Without a price drop, the number of notaries would further increase by 496 (from 3 943 to 4 439). Behind this net entry, there is also reallocation, as 14 offices are closed in favour of 2 384 new locations. The number of uncovered markets falls to 5 (down from 104). Total welfare gains more than triple to €463 million. If we additionally allow for a welfare-maximizing price drop, there is strong additional redistribution to consumers but only a modest additional welfare gain (and extra €25 million to €488 million, in line with our earlier findings).

7.2.2 Free entry

A more drastic entry reform would entirely remove the entry restrictions and allow all entry to become free. One of the key concerns with such a reform would be the possibility of excessive entry. As shown by e.g. Mankiw and Whinston (1986), firms do not internalize the business stealing effect of their entry, which diminishes the overall efficiency of the industry. The excessive entry problem is potentially exacerbated by the current price regulation and the implied high markups, but it may be reduced if consumers value differentiated services from individual notaries. To evaluate the extent of excess entry in the industry, we calculate a free entry equilibrium where entry occurs as long as the office locations make non-negative

[20] For the sake of brevity, we will focus on uniform price decreases in the counterfactual analysis. The optimal differentiated price decreases are reported in Table A3. The additional welfare gains from introducing a differentiated price reform rather than a uniform price decrease are relatively small for all counterfactual scenarios (less than €1.5 million), but there are interesting additional distributional effects.

profits.

First, consider free entry under the current prices and no entry at new locations (first row of Panel B in Table 11). The number of notaries under free entry is 1.8 times higher than under welfare maximization (an increase from 3 943 to 7 046). This is in line with the predictions of Mankiw and Whinston (1986) on excessive entry in homogeneous product markets. Intuitively, entrants do not internalize the negative business stealing effects on incumbents. While excess entry is to be expected at the current high markups, it is interesting to point out that the free entry outcome would still be socially preferable to the current outcome with considerable insufficient entry.[21] Finally, we note that free entry generates substantial consumer surplus gains (€468 million) at the expense of industry rents (-€414 million).

Now consider the scope for further welfare improvements when the notaries' fees are optimally reduced by 17% under free entry (second row of Panel B). Reducing fees has a two-fold positive effect. First, it raises consumer welfare through lower prices and thus higher demand. Second, it reduces the level of excess entry into the industry. Free entry with a 17% fee reduction results in an increase in overall welfare of €140 million. Interestingly, this can thus achieve 99% of the potential welfare gains (shown earlier in Panel A). Consistent with this, note that excess entry becomes limited: the number of notaries is just 1.1% higher than under the first-best outcome.[22]

Finally, consider a process of free entry that also includes additional locations. At current prices, this would entail further excess entry of 1 020 notaries (8 066 notaries compared to 7 046 without allowing for entry at additional locations). Only nine markets would not be covered. Consumers would gain €749 million relative to the status quo and total welfare would increase by €376 million. If combined with an optimal price reduction of 19%, the benefits from free entry would again be especially large, and would approach the welfare maximizing outcome. Figure A2 in the Appendix demonstrates that the two outcomes are similar not only in aggregate, but also on a more local level, with entry patterns being similar across geographic regions. Crucially, neither the welfare-maximizing reform, nor the free entry reform, appear to jeopardize coverage in rural areas.

[21] Compared with the welfare optimum, total welfare is €120 million lower in the current outcome, and €120 - €54 = €66 million lower under free entry.

[22] It would in principle also be interesting to conduct a counterfactual with freely set prices instead of a welfare-maximizing price reduction. However, this would require strong confidence in the model of price competition after liberalization. Instead, we simply note that the welfare-maximizing price drop of 17% is equivalent to a markup drop from 37% to 27% for real estate and from 28% to 18% for other transactions. A competitive outcome would thus differ from our results depending on the difference between these values and competitive markups under freely set prices.

7.3 Summary

In sum, our policy counterfactuals show that entry is currently insufficient even conditional on the high fixed prices at the status quo. The reason is that the state puts a reduced weight on consumer gains when granting entry licenses. Free entry without adjusting prices would be excessive, but it would nonetheless imply an improvement over the current outcome. If accompanied with a suitable price drop, it would result in an outcome close to the first-best. Finally, liberalizing entry and reducing fees would entail a substantial redistribution to consumers, without threatening industry profitability.

8 Conclusion

The Latin notary system provides an interesting opportunity to evaluate the role played by entry and price restrictions in a private monopoly. We developed an empirical entry model to uncover the current policy goals behind the imposed restrictions and thus evaluate how close these measures come to welfare maximization. We first specified a spatial demand model, and found that entry mainly entails business stealing and only limited market expansion. Next, we estimated a multi-output production model, which indicated approximately constant returns to scale from variable inputs and high markups over marginal costs, especially for real estate transactions.

We then incorporated the empirical findings on demand and markups into an entry model. This enabled us to evaluate the regulator's implicit objective function when restricting entry, given the high prices as fixed by law for decades. Granting an additional entry license affects welfare through several channels. It raises consumer surplus through increased variety due to more spatial availability. In addition, it may raise industry profits by increasing total demand, but it also raises industry costs, due to duplicated fixed entry costs. Our empirical findings indicate that the regulator places a disproportionately high weight on producer surplus in the licensing process. Conditional on the already high prices, the regulator appears to weigh consumer surplus by on average between 0% and 40% of producer surplus when granting new licenses.

We subsequently performed various policy counterfactuals, and obtained the following results. In a first step, we focused on reforming fees, while keeping the current geographic entry distribution fixed. We found that the welfare maximizing fees should be substantially lower, especially for real estate transactions. However, a pure fee reform would raise welfare to a limited extent. It would mainly imply a substantial transfer of surplus from firms to consumers, putting pressure on maintaining sufficient geographic coverage.

In a next step, we considered deeper reforms that also loosen the entry restrictions. Here, we found that entry reform not only leads to substantial redistribution from firms to consumers, but also results in substantial changes in total welfare. The welfare maximizing number of firms is more than twice the current number, and implies large welfare and consumer surplus gains. A full liberalization to free entry without adjusting fees would result in an excessive number of firms, but it is still preferable over the current outcome of insufficient entry. If free entry is accompanied with a reduction of the regulated fees by 19%, free entry achieves almost all of the potential total welfare gains in the first-best (and in addition creates further redistribution towards consumers).

In sum, across the considered policy counterfactuals, fee reductions imply a substantial redistribution of surplus from firms to consumers. If accompanied with a loosening of the entry restrictions they also imply substantial gains in total welfare. The gains to consumers substantially outweigh the losses to producers, and do not affect geographic coverage of the notary firms to a significant extent.

Our findings raise questions about the functioning of the notary system. This system is very widespread in Europe and worldwide, even though it does not exist in common law countries such as the U.S., parts of Canada, and the U.K. We show that there is considerable room for welfare improving reform, both by liberalizing entry and reducing prices. More broadly, our findings provide a warning on the extent to which entry restrictions can reflect regulatory capture by the state, in particular when these entry restrictions are of a geographic nature. In future research, it would therefore be interesting to conduct additional work on other sectors with strong geographic entry barriers.

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Appendix

A.1 Ordered probit entry model

This Appendix derives the ordered probit entry model under two scenarios: the standard free entry scenario of Bresnahan and Reiss (1991) and a restricted entry scenario under industry profit maximization. In both cases, we assume fixed prices. Entry threshold ratios above one measure the extent to which market size has to increase disproportionately to sustain additional firms. We show that entry threshold ratios above one are inconsistent with free entry (under fixed prices), while they may be consistent with industry profit maximization, provided that demand is sufficiently concave in the number of firms.

Our analysis is based on the model of entry proposed by Bresnahan and Reiss (1991). However, we deviate from the standard approach by assuming that firms charge a regulated price p, which is independent of the number of firms. Additionally, firms incur a constant marginal cost c and a fixed cost f. Firm profits are

$$\pi(N) = (p-c)\, q(N)S - f,$$

where $q(N)$ is demand per capita and per firm and S denotes total market size.

Under free entry, observing N firms implies non-negative profits for the N^{th} incumbent and losses for the $N+1^{\text{st}}$ potential entrant:

$$(p-c)\, q(N+1)S - f < 0 \leq (p-c)\, q(N)S - f. \tag{11}$$

The entry threshold market size per firm on a market with N firms is given by

$$s^F(N) = \frac{S^F(N)}{N} = \frac{f}{(p-c)q(N)N}.$$

The ratio of consecutive thresholds reveals the scope of output expansion:

$$ETR^F(N) = \frac{s^F(N+1)}{s^F(N)} = \frac{Q(N)}{Q(N+1)},$$

where $Q(N) = q(N)N$ is industry demand per capita. If additional entry expands the market ($Q' > 0$), we expect to observe ratios below 1 ($ETR^F(N) < 1$).

Under restricted entry with joint profit maximization, firms enter as long as total industry profits $\Pi(N) = \sum \pi(N)$ increase. Observing N firms implies that the marginal industry profits are positive for the N^{th} entrant and negative thereafter. If we denote the marginal effect of entry on sales as $Q'(N) \equiv Q(N) - Q(N-1)$, this yields the following entry condition:

$$(p-c)Q'(N+1)S - f < 0 \leq (p-c)Q'(N)S - f. \tag{12}$$

The per-firm entry threshold ratio becomes

$$ETR^R(N) = \frac{Q'(N)N}{Q'(N+1)(N+1)}.$$

If firms maximize joint profits, observing $ETR^R(N) > 1$ is possible if $Q'(N) + Q''(N)N < 0$, i.e. demand is sufficiently concave in N.

We estimate the threshold ratios using a general framework, which does not require an apriori belief regarding the nature of the licensing process. To cover both cases, define $g(N) = (p - c)q(N)$ under free entry, and $g(N) = (p - c)Q'(N)$ under restricted entry. The inequality conditions for either free entry (11) or restricted entry (12) can then be written as:

$$\ln \frac{g(N+1)}{f} + \ln S < 0 \leq \ln \frac{g(N)}{f} + \ln S.$$

Consider the following specification for the ratio of $g(N)$ over fixed costs:

$$\ln \frac{g(N)}{f} = X\beta^* + \theta_N^* - \varepsilon,$$

where X is a vector of market characteristics, θ_N^* is a fixed effect for the impact of the N^{th} firm, and ε is an unobserved error term affecting the variable profit to fixed cost ratio, with a normal distribution $\mathcal{N}(0, \sigma^2)$. Assume that $\theta_{N+1}^* < \theta_N^*$, as implied by the assumption that $q'(N) < 0$ under free entry and $Q''(N) < 0$ under restricted entry. The probability of observing N firms is then given by the following ordered probit expression:

$$P(N) = \Phi\left(X\beta + \theta_N + \alpha \ln S\right) - \Phi\left(X\beta + \theta_{N+1} + \alpha \ln S\right), \tag{13}$$

where $\Phi\left(.\right)$ denotes the cumulative normal distribution, $\beta \equiv \beta^*/\sigma$, $\theta_N \equiv \theta_N^*/\sigma$ and $\alpha \equiv 1/\sigma$.

A.2 Supplementary Tables and Figures

Table A1: Summary statistics of European dataset

Panel A: control variables ($N = 69832$)

Variable	Mean	Std. Dev.	10^{th} perc.	90^{th} perc.
Population (log)	6.669	1.51	4.812	8.711
Mean income in 1000 €	23.063	6.964	11.789	30.3
% male inhabitants	0.502	0.03	0.475	0.532
% young inhabitants	0.158	0.053	0.096	0.224
% elderly inhabitants	0.209	0.084	0.121	0.31
Area (log)	2.758	1.003	1.532	4.083

Panel B: market structure by country

Country	Number of offices on the market					Total
	0	1	2	3	4+	
Austria	2106	173	33	5	9	2326
Belgium	106	236	109	53	29	533
Germany	9580	928	283	141	246	11178
Spain	6920	690	189	43	40	7882
France	33426	2080	320	56	32	35914
Italy	6477	820	198	79	148	7722
Netherlands	45	156	87	36	15	339
Portugal	3773	141	20	3	1	3938

Table A2: Estimation results from ordered probit entry model

	Austria	Belgium	France	Germany	Italy	Netherlands	Portugal	Spain
Population (log)	1.309	1.663	0.965	1.324	1.718	2.408	0.998	1.409
	(0.076)	(0.112)	(0.017)	(0.034)	(0.044)	(0.198)	(0.074)	(0.045)
Area (log)	0.052	0.349	0.113	0.114	0.166	0.145	0.048	0.258
	(0.058)	(0.096)	(0.019)	(0.032)	(0.030)	(0.122)	(0.061)	(0.031)
Mean income	−0.033	−0.032	−0.000	−0.006	0.005	0.011	0.012	0.020
	(0.007)	(0.009)	(0.004)	(0.003)	(0.003)	(0.008)	(0.009)	(0.006)
% male	−38.141	−7.208	−10.278	−12.509	−19.993	−12.375	−25.207	0.400
	(5.081)	(7.496)	(1.005)	(2.703)	(3.142)	(12.271)	(3.353)	(2.159)
% young	3.891	3.688	1.193	20.181	−7.364	4.034	3.651	−1.870
	(4.108)	(4.807)	(0.687)	(1.769)	(2.182)	(4.925)	(3.261)	(1.579)
% elderly	6.409	7.463	5.921	9.641	4.689	3.069	1.210	3.128
	(2.423)	(3.373)	(0.420)	(1.088)	(1.158)	(4.251)	(1.651)	(0.900)
θ_1	−6.175	12.881	4.914	11.200	6.448	17.952	−1.744	14.370
	(2.912)	(4.737)	(0.630)	(1.645)	(1.812)	(7.645)	(2.021)	(1.410)
θ_2	−4.673	14.755	6.446	12.361	8.054	20.483	−0.466	16.207
	(2.905)	(4.744)	(0.631)	(1.646)	(1.814)	(7.669)	(2.021)	(1.422)
θ_3	−3.544	15.916	7.542	12.936	8.971	22.055	0.484	17.551
	(2.906)	(4.750)	(0.633)	(1.648)	(1.815)	(7.678)	(2.023)	(1.432)
θ_4	−3.123	17.015	8.208	13.359	9.601	23.513	1.050	18.332
	(2.913)	(4.759)	(0.637)	(1.648)	(1.817)	(7.690)	(2.034)	(1.438)
Observations	2326	533	35914	11178	7722	339	3938	7882

Table A3: Counterfactual equilibria with optimal differentiated price decreases

	price decrease	notaries ($\sum_i a_i$)	notary offices	uncovered markets	%Δ output	Δ consumer surplus	Δ producer surplus	Δ welfare
Current entry outcome								
	1,569	1,152	104	-	-	-	-	
Panel A: total welfare maximization								
	current locations							
real estate	-21%	3,746	1,150	106	12.37	552,405	-410,356	142,049
other transactions	-9%							
	current and new locations							
real estate	-21%	4,309	3,320	22	18.58	828,753	-339,448	489,305
other transactions	-8%							
Panel B: free entry								
	current locations							
real estate	-22%	3,707	1,149	107	12.39	554,808	-412,782	142,026
other transactions	-8%							
	current and new locations							
real estate	-23%	4,299	3,210	31	19.30	861,605	-374,779	486,826
other transactions	-10%							

Notes: Fixed costs per notary are evaluated at €84 000. Price reductions refer to differentiated welfare-maximizing price reductions, based on a grid search with 1% intervals. Surplus and welfare changes are reported in €1 000. Output changes are reported in percentage terms.

Figure A1: Firm locations and population density (inhabitants per km^2)

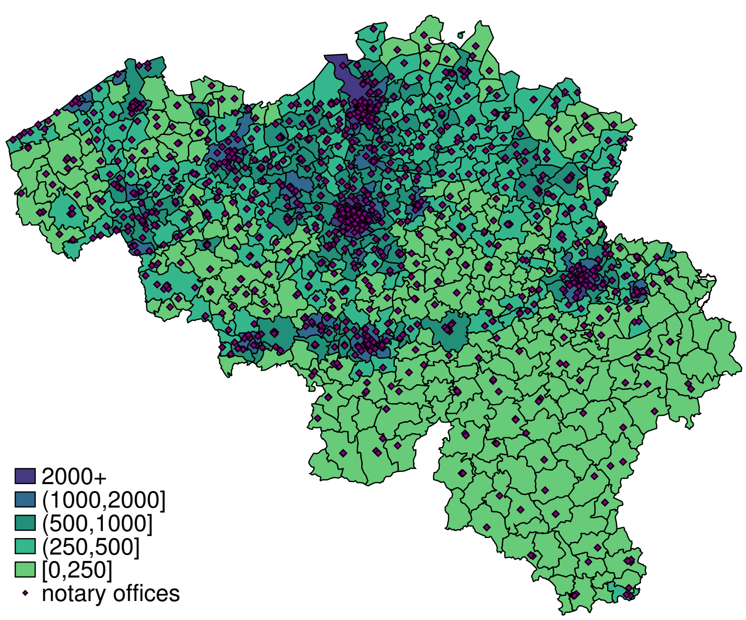
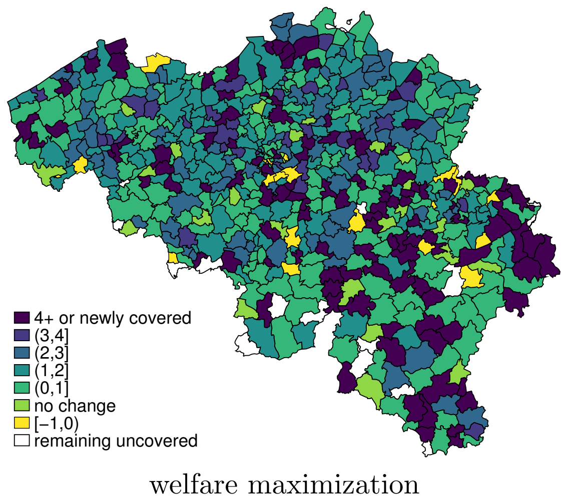


Figure A2: Relative change in the number of notaries under counterfactual scenarios



Notes: Figure A2 presents the relative increase in the number of notaries in each municipality (e.g. a value of 2 implies that the number of notaries has doubled). The left panel of the figure presents the change in coverage moving from the status quo to welfare maximization with an optimal price decrease of 17% and potential new offices in post office locations. The right panel plots analogous information under free entry with an optimal price decrease of 19%. While there are subtle differences across the two graphs, the overall patterns of entry are similar.

A.3 Calculation of counterfactual entry equilibria

A common problem in the calculation of counterfactual entry outcomes is the potential for multiple (local) equilibria and the computational feasibility of comparing all possible entry configurations. The predictions regarding the counterfactual distribution of firms may depend on our assumptions regarding the initially occupied locations, as well as on the ordering of moves during the search for an equilibrium. We check the robustness of our results by adjusting each of these elements. The results are not substantially affected by the aforementioned choices. We report the equilibria derived from starting values corresponding to the current entry distribution, as this is reasonably the starting point of policy reform.

More specifically, we predict entry outcomes by implementing a sequential entry and exit algorithm, in the spirit of Seim and Waldfogel (2013). Starting from the current entry outcome, the new entry equilibrium is derived through the following step-wise process:

1. Calculate the change in the criterion function resulting from removing a notary from each location i.[23]

2. If removing a notary increases the criterion function, remove a notary from the location

[23] Under welfare maximization, this corresponds to $\Gamma(a-1_i)-\Gamma(a)$, where a is the entry outcome at this stage of the loop. Under free entry, notaries are removed if the office is unprofitable, i.e. $\pi_i(a)<0$.

with the highest marginal change across i, update entry outcome a, and repeat Step 1.

3. Calculate the change in the criterion function resulting from adding a notary to each location i.[24]

4. If adding a notary increases the criterion function, add a notary to the location with the highest marginal effect across i,[25] update entry outcome a, and repeat Step 3.

5. Repeat Steps 1-4, as long as the number of entrants within at least one location changes in each loop.

Since the algorithm does not optimize all locations simultaneously, but rather considers a sequential entry process, we can guarantee only the local optimality of the final distribution.[26]

[24]Under welfare maximization, the change in the criterion function corresponds to the change in welfare $\Gamma(a+1_i)-\Gamma(a)$, where a is the entry outcome at this stage of the loop. Under free entry, the algorithm checks whether profits would remain non-negative after the addition of a notary to the office, i.e. $\pi_i(a+1_i) \geq 0$.

[25]We impose the constraint that all active notaries make non-negative profits. Negative profit outcomes may occur under welfare maximization without this constraint, as losses to the notary may be offset by gains to consumers. However, the private nature of the notary profession prohibits this outcome in equilibrium.

[26]Seim and Waldfogel (2013) present evidence on the difference between the sequential outcome and a global optimal configuration of firms for a subset of geographic markets. In their application, the maximum welfare calculated based on the sequential entry algorithm is within 0.5 percent of the exact maximum welfare computed with an integer programming approach.