

# Optimal Monetary Policy in Production Networks\*

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## Abstract

This paper studies the optimal conduct of monetary policy in a multi-sector economy in which firms buy and sell intermediate goods over a production network. We first provide a necessary and sufficient condition for the monetary policy's ability to implement flexible-price equilibria in the presence of nominal rigidities and show that, generically, no monetary policy can implement the first-best allocation. We then characterize the optimal policy in terms of the economy's production network and the extent and nature of nominal rigidities. Our characterization result yields general principles for the optimal conduct of monetary policy in the presence of input-output linkages: it establishes that optimal policy stabilizes a price index with higher weights assigned to larger, stickier, and more upstream industries, as well as industries with less sticky upstream suppliers but stickier downstream customers. In a calibrated version of the model, we find that implementing the optimal policy can result in quantitatively meaningful welfare gains.

*Keywords:* monetary policy, production networks, nominal rigidities, misallocation.

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# 1 Introduction

Optimal monetary policy in the canonical New Keynesian framework is well-known and takes a particularly simple form: as long as there are no missing tax instruments, it is optimal to stabilize the nominal price level. Price stability neutralizes the effects of nominal rigidities, implements flexible-price allocations, and, in the absence of markup shocks, restores productive efficiency. In the language of the New Keynesian literature, the “divine coincidence” holds: price stabilization simultaneously eliminates inflation and the output gap.<sup>1</sup>

The ability of monetary policy to replicate flexible-price allocations in the textbook New Keynesian models, however, relies critically on the assumption that all firms are technologically identical: as long as all firms employ the same production technology, a price stabilization policy that implements zero relative price dispersion across firms equalizes firms’ marginal products and hence achieves productive efficiency, regardless of the extent of nominal rigidities (Correia, Nicolini, and Teles, 2008). But once there are technological differences across firms—say, in a multi-sector economy with input-output linkages—monetary policy may lose its ability to replicate flexible-price allocations: while productive efficiency would dictate movements in relative quantities across different producers in response to producer-specific shocks, monetary policy may not be able to induce the corresponding relative price movements across the various sticky-price producers.

In view of the above, it is not readily obvious what principles should guide the conduct of monetary policy when firms employ heterogeneous production technologies, rely on a host of different intermediate goods and services produced by other firms in the economy, and are subject to various degrees of nominal rigidities.

In this paper, we address these questions by studying the optimal conduct of monetary policy in a multi-sector New Keynesian framework while allowing for inter-sectoral trade over a production network. We first provide a necessary and sufficient condition for the monetary policy’s ability to implement flexible-price equilibria in the presence of nominal rigidities and show that, generically, no monetary policy can implement the first-best allocation. We then characterize the optimal policy in terms of the economy’s production network and the extent of nominal rigidities. Our characterization result yields general principles for the optimal conduct of monetary policy in the presence of input-output linkages.

We develop these results in the context of a static multi-sector general equilibrium model à la Long and Plosser (1983) and Acemoglu et al. (2012) in which firms are linked to one another via input-output linkages and are subject to industry-level productivity shocks.<sup>2</sup> As in the New-Keynesian tradition, we also assume that firms are subject to nominal rigidities. More specifically, we assume that firms make their nominal pricing decisions under incomplete information about the productivity shocks. As a result, nominal prices respond to changes in productivities only to the extent that such changes are reflected in the firms’ information sets. This nominal friction opens the door to potential price

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<sup>1</sup>While divine coincidence holds in the absence of markup shocks or when such shocks are neutralized by tax instruments, it may fail more generally when state-contingent tax instruments that may counteract markup shocks are assumed away. In such a case, monetary policy becomes an imperfect instrument for undoing the impact of markup shocks and as a result, the policymaker faces a trade-off between minimizing the output gap and productive inefficiency due to inflation.

<sup>2</sup>Throughout, we assume that these productivity shocks are the only payoff-relevant shocks in the economy, thus abstracting away from markup, or cost-push, shocks.

distortions throughout the production network, as well as a role for monetary policy in shaping real allocations.

Within this framework, we start by characterizing the entire sets of sticky- and flexible-price equilibria, defined as the sets of allocations that can be implemented as an equilibrium in the presence and absence of nominal rigidities, respectively.<sup>3</sup> We show that while both sets of allocations are characterized by similar sets of conditions relating the marginal rates of substitution between goods and their marginal rates of transformation, the conditions characterizing sticky-price allocations exhibit an additional collection of wedges that depend on the interaction between the conduct of monetary policy and the firms' information sets. Using these characterizations, we then provide the exact conditions on the firms' technologies and information sets under which monetary policy can implement flexible-price equilibria and hence restore productive efficiency. As an important byproduct of this result, we also show that these conditions are violated for a generic set of information structures, thus concluding that, generically, monetary policy cannot achieve productive efficiency.

Having established the failure of monetary policy to implement the first-best allocation, we then turn to the study of optimal monetary policy, i.e., the policy that maximizes household welfare over the set of all possible sticky-price-implementable allocations. In order to obtain closed-form expressions for the optimal policy, we impose a number of functional form assumptions by assuming that all firms employ Cobb-Douglas production technologies and that all signals are normally distributed.

We establish three sets of results. First, we show that firms' optimal pricing decisions can be recast as a generalized version of a "beauty contest" game à la [Morris and Shin \(2002\)](#), in which firms face heterogeneous strategic complementarities in their price-setting decisions due to interdependencies arising from the production network. Second, we demonstrate that monetary policy faces a trade-off between three sources of welfare losses: misallocation due to price dispersion within sectors, misallocation arising from pricing errors across sectors, and output gap volatility. Third, building on our previous results, we derive the monetary policy that optimally trades off these three components.

Our characterization of the optimal policy yields general principles for the conduct of monetary policy in the presence of input-output linkages. In particular, we establish that, all else equal, optimal policy stabilizes a price index with higher weights assigned to (i) larger industries as measured by their Domar weights, (ii) stickier industries, (iii) more upstream industries, and (iv) industries with less sticky upstream suppliers but with stickier downstream customers.

We then use our theoretical results to undertake a quantitative exercise and determine the optimal monetary policy for the U.S. economy as implied by the model. Matching input-output tables constructed by the Bureau of Economic Analysis with industry-level data on nominal rigidities provided to us by [Pastén et al. \(2020b\)](#), we compute the weights corresponding to the optimal price-stabilization index and quantify the resulting welfare loss due to the presence of nominal rigidities. We find that the optimal policy generates a welfare loss equivalent to a 0.65% of quarterly consumption relative to the (unattainable) flexible-price equilibrium, with the overwhelming fraction of this loss arising from misallocation within and across industries. We then provide a comparison between the performance of the optimal policy and four alternative, non-optimal, price-stabilization policies. We

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<sup>3</sup>Our approach is thus similar to [Correia et al. \(2008, 2013\)](#) and [Angeletos and La'O \(2020\)](#), who apply the primal approach to optimal taxation of [Atkinson and Stiglitz \(2015\)](#) and [Diamond and Mirrlees \(1971\)](#) to New Keynesian models.

find that, in our calibration, the welfare differences between the optimal policy and a policy that stabilizes the output gap are minuscule, amounting to roughly 0.02 percentage points of quarterly consumption. In contrast, moving from price stabilization targets based on industries' shares in the household's consumption basket (akin to CPI or PCE), industry size, or sectoral stickiness to the optimal price index would result in quantitatively meaningful welfare gains.

**Related Literature.** Our paper is part of the growing literature that studies the role of production networks in macroeconomics. Building on the multi-sector model of [Long and Plosser \(1983\)](#), [Acemoglu et al. \(2012\)](#) investigate whether input-output linkages can transform microeconomic shocks into aggregate fluctuations.<sup>4</sup> We follow this line of work by focusing on a multi-sector general equilibrium economy with nominal rigidities and investigating the interaction between monetary policy and the economy's production network. Within this literature, our paper builds on the works of [Jones \(2013\)](#), [Bigio and La'O \(2020\)](#), and [Baqae and Farhi \(2020\)](#), who study misallocation in economies with non-trivial production networks. However, in contrast to these papers, which treat markups and wedges as exogenously-given model primitives, we focus on an economy in which wedges are determined endogenously as the result of firms' individually-optimal price-setting decisions and monetary policy. We investigate the monetary authority's ability to shape these wedges using available policy instruments, and use this characterization to derive the optimal policy.

In a series of recent papers, [Pastén, Schoenle, and Weber \(2020a,b\)](#), [Ozdagli and Weber \(2021\)](#), and [Ghassibe \(2021\)](#) study the production network's role as a possible transmission mechanism of monetary policy shocks.<sup>5</sup> We differ from these papers by providing a closed-form characterization of the optimal monetary policy as a function of the economy's underlying production network and the extent of nominal rigidities. Also related is the recent work of [Wei and Xie \(2020\)](#), who study the role of monetary policy in the presence of global value chains. Whereas they focus on an open economy in which production occurs over a single production chain, we provide a characterization of optimal policy for a general production network structure in a closed economy.<sup>6</sup>

More closely related to our work is the independent and contemporaneous work of [Rubbo \(2020\)](#), who also studies the implications of input-output linkages in New Keynesian models. While both papers characterize the optimal policy, our work departs from Rubbo's along two dimensions. First, we obtain necessary and sufficient conditions on the economy's disaggregated structure and the nature of nominal rigidities under which monetary policy can implement a given flexible-price allocation. Second, we provide a series of analytical results that distill the role of the various economic forces that shape the optimal policy. Our normative analysis thereby yields general principles for the optimal conduct of monetary policy in the presence of input-output linkages, establishing that monetary policy

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<sup>4</sup>Other papers in this line of work include [Carvalho \(2010\)](#), [Foerster, Sarte, and Watson \(2011\)](#), [Acemoglu et al. \(2016, 2017\)](#), and [Atalay \(2017\)](#). See [Carvalho \(2014\)](#) and [Carvalho and Tahbaz-Salehi \(2019\)](#) for surveys of the theoretical and empirical literature on production networks.

<sup>5</sup>Also see [Castro \(2019\)](#), who studies the welfare costs of trend inflation in a quantitative New Keynesian model with sectoral heterogeneity and production networks.

<sup>6</sup>While our focus is squarely on monetary policy, a related strand of literature explores fiscal policy in multi-sector economies. [Cox et al. \(2021\)](#) study the transmission mechanism of fiscal policy in a multi-sector New Keynesian model with sectoral government spending, whereas [Flynn, Patterson, and Sturm \(2021\)](#) characterize fiscal multipliers in a heterogenous-agent economy with input-output linkages.

should stabilize a price index with higher weights assigned to larger, stickier, and more upstream industries, as well as industries with less sticky upstream suppliers but stickier downstream customers.

Our paper also belongs to a small strand of the New Keynesian literature that studies optimal monetary policy in multi-sector economies. In one of the earliest examples of this line of work, [Aoki \(2001\)](#) shows that in a two-sector economy with one sticky and one fully flexible industry (but no input-output linkages), a policy that stabilizes the price of the sticky industry implements the first-best allocation. [Mankiw and Reis \(2003\)](#), [Woodford \(2003b, 2010\)](#), [Benigno \(2004\)](#), and [Eusepi, Hobijn, and Tambalotti \(2011\)](#) generalize this insight to multi-sector economies with varying degrees of price stickiness and establish that the monetary authority should stabilize a price index that places greater weights on industries with stickier prices.<sup>7</sup> While most of this literature has abstracted from input-output linkages, [Huang and Liu \(2005\)](#) consider a two-sector economy with a final and an intermediate good and show that it is optimal to stabilize a combination of the price of the two goods. To the best of our knowledge, the prior literature has not studied optimal monetary policy with a general production network.

The importance of strategic complementarities in firms' price-setting behavior in the presence of nominal rigidities has a long history. Investigating "in-line" and "roundabout" production structures, [Blanchard \(1983\)](#) and [Basu \(1995\)](#) emphasize the role of intermediate inputs in creating strategic complementarities in firms' price-setting behavior. We build on these papers by providing a closed-form characterization of how strategic complementarities arising from input-output linkages shape equilibrium nominal prices, quantities, and the optimal conduct of monetary policy.

Finally, our approach in modeling nominal rigidities as an informational constraint on the firms' price-setting decisions follows the extensive literature that proposes informational frictions as an appealing substitute to Calvo frictions and menu costs on theoretical ([Mankiw and Reis, 2002](#); [Woodford, 2003a](#); [Mackowiak and Wiederholt, 2009](#); [Nimark, 2008](#)) and empirical ([Coibion and Gorodnichenko, 2012, 2015](#)) grounds. We follow this approach not only because we find informational frictions to be a priori more plausible than other alternatives, but also because, in the context of our exercise, they lend themselves to a more transparent analysis. Importantly, this modeling feature does not upset the key normative lessons of the New Keynesian paradigm; in particular, price stability remains optimal insofar as monetary policy need not substitute for missing tax instruments ([Angeletos and La'O, 2020](#)).

**Outline.** The rest of the paper is organized as follows. Section 2 sets up the environment and defines the sticky- and flexible-price equilibria in our context. Section 3 characterizes these equilibria and provides necessary and sufficient conditions for the monetary policy's ability to implement the first-best allocation. Section 4 contains our closed-form characterization of the optimal policy in terms of the economy's production network and the extent of nominal rigidities. We present a quantitative analysis of the model in Section 5. The Appendix contains additional theoretical results. All proofs and some additional technical details are presented in an Online Appendix.

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<sup>7</sup>This principle of "sticky-price stabilization" was first proposed by [Goodfriend and King \(1997\)](#) and later formalized in the above-mentioned papers. Also see [Erceg et al. \(2000\)](#), who study an economy with both nominal price and wage rigidities.

## 2 Framework

Consider a static economy consisting of  $n$  industries indexed by  $i \in I = \{1, 2, \dots, n\}$ . Each industry consists of two types of firms: (i) a unit mass of monopolistically-competitive firms, indexed by  $k \in [0, 1]$ , producing differentiated goods and (ii) a competitive producer whose sole purpose is to aggregate the industry's differentiated goods into a single sectoral output. The output of each industry can be either consumed by the households or used as an intermediate input for production by firms in other industries. In addition to the firms, the economy consists of a representative household as well as a government with the ability to levy industry-specific taxes and control nominal aggregate demand.

The monopolistically-competitive firms within each industry use a common constant-returns-to-scale technology to transform labor and intermediate inputs into their differentiated products. More specifically, the production function of firm  $k \in [0, 1]$  in industry  $i$  is given by

$$y_{ik} = z_i F_i(l_{ik}, x_{i1,k}, \dots, x_{in,k}),$$

where  $y_{ik}$  is the firm's output,  $l_{ik}$  is its labor input,  $x_{ij,k}$  is the amount of sectoral commodity  $j$  purchased by the firm,  $z_i$  is an industry-specific productivity shock, and  $F_i$  is a homogenous function of degree one. Throughout, we assume that labor is an essential input for the production technology of all goods, in the sense that  $F_i(0, x_{i1,k}, \dots, x_{in,k}) = 0$  and that  $\partial F_i / \partial l_{ik} > 0$  whenever all other inputs are used in positive amounts.

The nominal profits of the firm are given by

$$\pi_{ik} = (1 - \tau_i) p_{ik} y_{ik} - w l_{ik} - \sum_{j=1}^n p_j x_{ij,k}, \quad (1)$$

where  $p_{ik}$  is the nominal price charged by the firm,  $p_j$  is nominal price of industry  $j$ 's sectoral output,  $w$  denotes the nominal wage, and  $\tau_i$  is an industry-specific revenue tax (or subsidy) levied by the government.

The competitive producer in industry  $i$  transforms the differentiated products produced by the unit mass of firms in that industry into a sectoral good using a constant-elasticity-of-substitution (CES) production technology

$$y_i = \left( \int_0^1 y_{ik}^{(\theta_i-1)/\theta_i} dk \right)^{\theta_i/(\theta_i-1)}$$

with elasticity of substitution  $\theta_i > 1$ . This producer's profits are thus given by  $\pi_i = p_i y_i - \int_0^1 p_{ik} y_{ik} dk$ , where  $p_i$  is the price of the aggregated good produced by industry  $i$ . We include this producer—which has zero value added and makes zero profits in equilibrium—to ensure that a homogenous good is produced by each industry, while at the same time allowing for monopolistic competition among firms within the industry.

The preferences of the representative household are given by

$$W(C, L) = U(C) - V(L), \quad (2)$$

where  $C$  and  $L$  denote the household's final consumption basket and total labor supply, respectively. We impose the typical regularity conditions on  $U$  and  $V$ : they are strictly increasing, twice

differentiable, and satisfy  $U'' < 0$ ,  $V'' > 0$ , and the Inada conditions. The final consumption basket of the household is given by  $C = \mathcal{C}(c_1, \dots, c_n)$ , where  $c_i$  is the household's consumption of the good produced by industry  $i$  and  $\mathcal{C}$  is a homogenous function of degree one. The representative household's budget constraint is thus given by

$$PC = \sum_{j=1}^n p_j c_j \leq wL + \sum_{i=1}^n \int_0^1 \pi_{ik} dk + T,$$

where  $P = \mathcal{P}(p_1, \dots, p_n)$  is the nominal price of the household's consumption bundle. The left-hand side of the above inequality is the household's nominal expenditure, whereas the right-hand side is equal to the household's total nominal income, consisting of labor income, dividends from owning firms, and lump sum transfers from the government.

In addition to the firms and the representative household, the economy also consists of a government with the ability to set fiscal and monetary policies. The government's fiscal instrument is a collection of industry-specific taxes (or subsidies) on the firms, with the resulting revenue then rebated to the household as a lump sum transfer. Therefore, the government's budget constraint is given by

$$T = \sum_{i=1}^n \tau_i \int_0^1 p_{ik} y_{ik} dk,$$

where  $\tau_i$  is the revenue tax imposed on firms in industry  $i$  and  $T$  is the net transfer to the representative household. Finally, to model monetary policy, we sidestep the micro-foundations of money and, instead, impose the following cash-in-advance constraint on the household's total expenditure:

$$PC = m, \tag{3}$$

where we assume that  $m$ —which can be interpreted as either money supply or nominal aggregate demand—is set directly by the monetary authority.

## 2.1 Nominal Rigidities and Information Frictions

We model nominal rigidities by assuming that firms do not observe the realized productivity shocks  $z = (z_1, \dots, z_n)$  and, instead, make their nominal pricing decisions under incomplete information. This assumption implies that nominal prices respond to changes in productivities only to the extent that such changes are reflected in the firms' information sets.

Formally, we assume that each firm  $k$  in industry  $i$  receives a signal  $\omega_{ik} \in \Omega_{ik}$  about the economy's aggregate state. The aggregate state includes not only the vector of realized productivity shocks, but also the realization of all signals, that is,

$$s = (z, \omega),$$

where  $\omega = (\omega_1, \dots, \omega_n) \in \Omega$  denotes the realized cross-sectional distribution of signals in the economy and  $\omega_i = (\omega_{ik})_{k \in [0,1]}$  denotes the realized cross-sectional distribution of signals in industry  $i$ .

Since  $\omega_{ik}$  is the only component of state  $s$  that is observable to firm  $ik$ , the nominal price set by this firm has to be measurable with respect to  $\omega_{ik}$ . We capture this measurability constraint by denoting the

firm's price by  $p_{ik}(\omega_{ik})$ . Similarly, we write  $p_i(\omega_i)$  and  $P(\omega)$  to capture the fact that the nominal prices of sectoral good  $i$  and the consumption bundle have to be measurable with respect to the profile of signals in industry  $i$  and in the entire economy, respectively.<sup>8</sup>

A few remarks are in order. First, note that the above formulation implies that state  $s = (z, \omega)$  not only contains all payoff-relevant shocks, but also contains shocks to the aggregate profile of beliefs. Therefore, our framework can accommodate the possibility of higher-order uncertainty, as  $\omega_{ik}$  may contain information about other firms' (first or higher-order) beliefs, as in [Angeletos and La'O \(2013\)](#). Second, our framework is flexible enough to nest models with "sticky information" ([Mankiw and Reis, 2002](#); [Ball, Mankiw, and Reis, 2005](#)) as a special case by assuming that a fraction of firms in industry  $i$  set their nominal prices under complete information ( $\omega_{ik} = z$ ), whereas the rest of the firms in that industry observe no informative signals ( $\omega_{ik} = \emptyset$ ) and hence set their nominal prices based only on their prior beliefs. Finally, note that the information structure in our framework is exogenous: while firm-level signals can depend on the exogenous productivity shocks  $(z_1, \dots, z_n)$ , they do not depend on the endogenous objects in the economy (such as prices). This means that our formulation rules out the possibility that a firm can set a nominal price that is contingent on prices set by other firms in the economy.

In summary, we can represent the economy's price system by the collection of nominal prices and nominal wage at any given state:<sup>9</sup>

$$\varrho = \left\{ \left( (p_{ik}(\omega_{ik}))_{k \in [0,1]}, p_i(\omega_i) \right)_{i \in I}, P(\omega), w(s) \right\}_{s \in S}.$$

While nominal prices are set under incomplete information, we assume that firms and the household make their quantity decisions *after* observing the prices and the realization of productivities. As a result, quantities may depend on the entire state  $s$ . We thus represent an allocation in this economy by

$$\xi = \left\{ \left( (y_{ik}(s), l_{ik}(s), x_{ik}(s))_{k \in [0,1]}, y_i(s), c_i(s) \right)_{i \in I}, C(s), L(s) \right\}_{s \in S},$$

where  $l_{ik}(s)$ ,  $x_{ik}(s) = (x_{i1,k}(s), \dots, x_{in,k}(s))$ , and  $y_{ik}(s)$  denote, respectively, the labor input, material input, and output of firm  $k$  in industry  $i$ ,  $y_i(s)$  is the output of industry  $i$ ,  $c_i(s)$  is the household's consumption of sectoral good  $i$ , and  $C(s)$  and  $L(s)$  are the household's consumption and labor supply, respectively.

We conclude this discussion by specifying how government policy depends on the economy's aggregate state. Recall from equations (1) and (3) that the fiscal and monetary authorities can, respectively, levy taxes and control the nominal aggregate demand. We assume that while the fiscal authority has the ability to levy industry-specific taxes  $\tau_i$ , these taxes cannot be contingent on the economy's aggregate state. In contrast, the monetary authority can set the nominal demand as an arbitrary function  $m(s)$  of the economy's aggregate state  $s$ . This is equivalent to assuming that the

<sup>8</sup>More specifically, the sectoral good producer's CES technology implies that  $p_i(\omega_i) = (\int_0^1 p_{ik}^{1-\theta_i}(\omega_{ik}) dk)^{1/(1-\theta_i)}$ , whereas the consumption good's price index is given by  $P(\omega) = \mathcal{P}(p_1(\omega_1), \dots, p_n(\omega_n))$ .

<sup>9</sup>This formulation assumes that the nominal wage,  $w$ , can depend on the entire state,  $s$ . While this assumption simplifies the exposition, it is without loss of generality: in our multi-sector framework, one can incorporate nominal wage rigidities by introducing a pseudo-industry that transforms, one-for-one, the labor supplied by the representative household into labor services sold to the rest of the industries in the economy. The information sets of firms in this pseudo-industry then determine the extent and nature of nominal wage rigidity.



monetary authority has the ability to commit, ex ante, to a policy that can, in principle, depend on the realized productivities and the profile of beliefs throughout the economy. Government policy can thus be summarized as

$$v = \{(\tau_1, \dots, \tau_n), m(s)\}_{s \in S}. \quad (4)$$

Note that, as in the standard New Keynesian literature, our formulation of government's policy instruments in (4) allows for non-state-contingent taxes to undo steady-state distortions due to monopolistic markups, while ruling out state-contingent taxes.<sup>10</sup> We also remark that while it may be far-fetched to assume that the monetary authority can commit to a policy that is contingent not just on the payoff-relevant shocks,  $z$ , but also on the entire profile of beliefs,  $\omega$ , we nonetheless make this assumption to illustrate the limit to monetary policy's ability to implement allocations even under maximum policy flexibility.

## 2.2 Equilibrium Definition

We now define our notions of sticky- and flexible-price equilibria. To this end, first note that the market-clearing conditions for labor and commodity markets are given by

$$L(s) = \sum_{i=1}^n \int_0^1 l_{ik}(s) dk \quad (5)$$

$$y_i(s) = c_i(s) + \sum_{j=1}^n \int_0^1 x_{ji,k}(s) dk = \left( \int_0^1 y_{ik}(s)^{(\theta_i-1)/\theta_i} dk \right)^{\theta_i/(\theta_i-1)} \quad (6)$$

for all  $i \in I$  and all  $s \in S$ , whereas the production technology of firm  $k$  in industry  $i$  requires that

$$y_{ik}(s) = z_i F_i(l_{ik}(s), x_{i1,k}(s), \dots, x_{in,k}(s)) \quad (7)$$

for all  $s \in S$ . Given the above, the definition of a sticky-price equilibrium is straightforward:

**Definition 1.** A *sticky-price equilibrium* is a triplet  $(\xi, \varrho, v)$  of allocations, prices, and policies such that

- (i) the monopolistically-competitive firms in each industry set prices  $p_{ik}(\omega_{ik})$  to maximize expected real value of profits given their information and optimally choose inputs to meet realized demand;
- (ii) the competitive producer in each industry chooses inputs to maximize its profits given prices;
- (iii) the representative household maximizes utility subject to its budget constraint;
- (iv) the government budget constraint is satisfied;
- (v) all markets clear.

We next define our notion of flexible-price equilibria by dropping the measurability constraint on prices imposed on the sticky-price equilibrium in Definition 1. More specifically, we assume that, in

<sup>10</sup>It is well-understood that with a sufficiently rich set of state-contingent tax instruments, one can undo the real effects of nominal rigidities and implement the first-best allocation under any monetary policy (see, e.g., [Correia et al. \(2013\)](#)).

contrast to the sticky-price firms, flexible-price firms make their nominal pricing decisions based on complete information of the aggregate state. We can capture this scenario in our framework by simply considering the special case in which all firm-level prices are measurable in the aggregate state,  $p_{ik}(s)$ . Accordingly, we also adjust our notation for the nominal price of sectoral goods and the consumption bundle by expressing them as  $p_i(s)$  and  $P(s)$ , respectively.

**Definition 2.** A *flexible-price equilibrium* is a triplet  $(\xi, \delta, \nu)$  of allocations, prices, and policies that satisfy the same conditions as those stated in Definition 1, except that all prices are measurable with respect to the aggregate state  $s$ .

While not the main focus of our study, the set of flexible-price-implementable allocations serves as a benchmark to which we will contrast equilibria in the presence of nominal rigidities. We conclude with one additional definition, whose meaning is self-evident.

**Definition 3.** An allocation  $\xi$  is *feasible* if it satisfies resource constraints (5), (6), and (7).

### 3 Sticky- and Flexible-Price Equilibria and the Power of Monetary Policy

In this section, we provide a characterization of the set of all allocations that can be implemented as part of flexible- and sticky-price equilibria. We then use our characterization results to establish that, except for a non-generic set of specifications, the two sets of allocations never intersect, thus implying that, in our multi-sector framework, monetary policy cannot undo the effects of nominal rigidities.

#### 3.1 First-Best Allocation

We start by focusing on the first-best allocation that maximizes household welfare (2), state-by-state, among all feasible allocations. Note that, by symmetry, a planner who maximizes social welfare dictates that all firms within an industry choose the same intermediate input, labor, and output quantities. We can therefore drop the firm index  $k$ . The equations characterizing the planner's optimum are straightforward and are summarized in the following lemma:

**Lemma 1.** *The first-best optimal allocation is a feasible allocation that satisfies*

$$V'(L(s)) = U'(C(s)) \frac{\partial C}{\partial c_i}(s) z_i \frac{\partial F_i}{\partial l_i}(s) \quad (8)$$

$$\frac{\partial C / \partial c_j}{\partial C / \partial c_i}(s) = z_i \frac{\partial F_i}{\partial x_{ij}}(s) \quad (9)$$

for all pairs of industries  $i$  and  $j$  and all states  $s$ .

Equation (8) states that, for any good, it is optimal to equate the marginal rate of substitution between consumption of that good and labor with the marginal rate of transformation. In particular, the planner equates the household's marginal disutility of labor on the left-hand side of (8) with its marginal social benefit on the right-hand side, which itself consists of three multiplicative components: the marginal product of labor in the production of commodity  $i$ , the marginal product of good  $i$  in the production of the final good, and the marginal utility of consumption of the final good.

The second condition (9) similarly indicates that the planner finds it optimal to equate the marginal rate of substitution between two goods to their marginal rate of transformation. The marginal rate of substitution on the left-hand side of equation (9) is the ratio of marginal utilities from consumption of the two goods, whereas the marginal rate of transformation is simply the marginal product of good  $i$  in the production of good  $j$ , as shown on the right-hand side of this condition.

### 3.2 Flexible-Price Equilibrium

We now turn to the set of allocations that are implementable as flexible-price equilibria. Since the tax instruments  $(\tau_1, \dots, \tau_n)$  are industry-specific and, in a flexible-price equilibrium, all firms in the same industry have identical information sets, we can once again drop the firm index  $k$ .

**Proposition 1.** *A feasible allocation is part of a flexible-price equilibrium if and only if there exists a set of positive scalars  $(\chi_1^f, \dots, \chi_n^f)$  such that*

$$V'(L(s)) = \chi_i^f U'(C(s)) \frac{\partial C}{\partial c_i}(s) z_i \frac{\partial F_i}{\partial l_i}(s) \quad (10)$$

$$\frac{\partial C / \partial c_j}{\partial C / \partial c_i}(s) = \chi_i^f z_i \frac{\partial F_i}{\partial x_{ij}}(s) \quad (11)$$

for all pairs of industries  $i$  and  $j$  and all states  $s$ .

The conditions in Proposition 1 are almost identical to those characterizing the first-best allocation in Lemma 1, aside from the set of scalars  $(\chi_1^f, \dots, \chi_n^f)$ . The first condition (10) indicates that for any good, the marginal rate of substitution between consumption and labor is equal to the marginal rate of transformation, modulo a non-state-contingent wedge  $\chi_i^f$ . Similarly, the second condition equates the marginal rate of substitution between two goods to their marginal rate of transformation, again subject to the wedge  $\chi_i^f$ . This non-stochastic wedge, which is given by

$$\chi_i^f = (1 - \tau_i) \frac{\theta_i - 1}{\theta_i}, \quad (12)$$

consist of two terms: the tax or subsidy levied by the government and the markup that arises due to monopolistic competition among firms within each industry. As a result, the scalars  $(\chi_1^f, \dots, \chi_n^f)$  parameterize the power of the fiscal authority. In particular, with sectoral taxes or subsidies, the fiscal authority can move allocations by inducing wedges in conditions (10) and (11).

Another immediate consequence of Proposition 1 is that the first-best allocation is implementable as a flexible-price equilibrium. This follows from the observation that equations (10) and (11) reduce to (8) and (9) whenever  $\chi_i^f = 1$  for all  $i$ . Consequently, the first-best allocation can be implemented as a flexible-price equilibrium with industry-specific subsidies  $\tau_i = 1/(1 - \theta_i)$ . This, of course, is not surprising: the only distortion in the economy without nominal rigidities arises from monopolistic competition. Therefore, it is optimal for the government to set industry-specific subsidies that are invariant to the economy's aggregate state and undo the monopolistic markups.

### 3.3 Sticky-Price Equilibrium

With the above preliminary results in hand, we are now ready to characterize the set of equilibrium allocations in the presence of nominal rigidities.

**Proposition 2.** *A feasible allocation is implementable as a sticky-price equilibrium if and only if there exist positive scalars  $(\chi_1^s, \dots, \chi_n^s)$ , a policy function  $m(s)$ , and firm-level wedge functions  $\varepsilon_{ik}(s)$  such that*

(i) *the allocation, the scalars  $(\chi_1^s, \dots, \chi_n^s)$ , and the set of wedge functions  $\varepsilon_{ik}(s)$  jointly satisfy*

$$V'(L(s)) = \chi_i^s \varepsilon_{ik}(s) U'(C(s)) \frac{\partial \mathcal{C}}{\partial c_i}(s) z_i \frac{\partial F_i}{\partial l_i}(s) \left( \frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i} \quad (13)$$

$$\frac{\partial \mathcal{C} / \partial c_j}{\partial \mathcal{C} / \partial c_i}(s) = \chi_i^s \varepsilon_{ik}(s) z_i \frac{\partial F_i}{\partial x_{ij}}(s) \left( \frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i} \quad (14)$$

*for all firms  $k$ , all pairs of industries  $i$  and  $j$ , and all states  $s$ ;*

(ii) *the policy function  $m(s)$  induces the wedge functions  $\varepsilon_{ik}(s)$  given by*

$$\varepsilon_{ik}(s) = \frac{\text{mc}_i(s) \mathbb{E}_{ik}[v_{ik}(s)]}{\mathbb{E}_{ik}[\text{mc}_i(s) v_{ik}(s)]} \quad (15)$$

*for all firms  $k$ , all industries  $i$ , and all states  $s$ , where*

$$\text{mc}_i(s) = m(s) \frac{V'(L(s))}{C(s) U'(C(s))} \left( z_i \frac{\partial F_i}{\partial l_i}(s) \right)^{-1} \quad (16)$$

*is the nominal marginal cost of firms in industry  $i$  and*

$$v_{ik}(s) = U'(C(s)) \frac{\partial \mathcal{C}}{\partial c_i}(s) y_i(s) \left( \frac{y_{ik}(s)}{y_i(s)} \right)^{(\theta_i - 1)/\theta_i}. \quad (17)$$

Proposition 2 provides a characterization of the set of sticky-price-implementable allocations in terms of model primitives and the monetary policy instrument  $m(s)$ . It is straightforward to see that conditions (13) and (14) are identical to their flexible-price counterparts (10) and (11) in Proposition 1, except for a new wedge  $\varepsilon_{ik}(s)$ . Also, as in Proposition 1, industry-specific wedges  $(\chi_1^s, \dots, \chi_n^s)$  are given by (12) and capture the fiscal authority's ability to influence allocations via tax instruments.

The new wedge  $\varepsilon_{ik}(s)$  in equations (13) and (14), which is firm-specific and depends on the economy's aggregate state, represents an additional control variable for the government, one that encapsulates the power of monetary policy over real allocations in the presence of nominal rigidities. Similar to the fiscal authority's ability to influence the allocation by setting taxes, the monetary authority can implement different allocations by moving the wedges  $\varepsilon_{ik}(s)$  in (13) and (14). This power is non-trivial, but it is also restrained by conditions (15) and (16): unlike the fiscal authority's full control over  $(\chi_1^s, \dots, \chi_n^s)$ , the monetary authority's choice of the single policy instrument  $m(s)$  pins down all wedges  $\varepsilon_{ik}(s)$  at the same time.

The constraint on the monetary policy's ability in shaping real allocations can also be seen by the fact that equation (15) implies  $\mathbb{E}_{ik}[v_{ik}(s)(\varepsilon_{ik}(s) - 1)] = 0$ . This means the wedge  $\varepsilon_{ik}(s)$  cannot be moved around in an unconstrained manner, as it has to be equal to 1 in expectation irrespective of the policy. This is because these wedges arise only due to "mistakes" by the sticky-price firms in setting their nominal prices. But since firms set their prices optimally given their information sets, they do not make any pricing errors in expectation.

As a final remark, we note that, in a sticky-price equilibrium, firms are uncertain not only about their marginal costs, but also about the demand they face from their downstream customers, as well as

the household's marginal utility of consumption. As a result, firm  $k$  in industry  $i$  sets its nominal price as  $p_{ik}(\omega_i) = (1/\chi_i^s)\mathbb{E}_{ik}[\text{mc}_i(s)v_{ik}(s)]/\mathbb{E}_{ik}[v_{ik}(s)]$ , where  $v_{ik}(s)$  is given by (17) and is proportional to the product of the household's stochastic discount factor and the demand  $y_{ik}(s)$  faced by the firm.<sup>11</sup>

### 3.4 The Power of Monetary Policy

As illustrated in Proposition 2, the monetary authority can use monetary policy to implement different allocations by moving around the wedge  $\varepsilon_{ik}(s)$  as a function of the economy's aggregate state. This leads to the natural question of whether monetary policy can fully undo the effect of nominal rigidities. Our first main result provides an answer to this question by characterizing the set of flexible-price allocations that can be implemented as sticky-price equilibria.

To state this result, let  $\sigma(\omega_{ik})$  denote the  $\sigma$ -field generated by the signal  $\omega_{ik}$ . Also let  $g_i(z_1, \dots, z_n)$  denote the marginal product of labor in the production of commodity  $i$  (as a function of productivity shocks) under the first-best allocation. Note that  $g_i$  only depends on the firms' production technologies and is independent of household preferences, policy, and the economy's information structure. We have the following result:

**Theorem 1.** *A flexible-price allocation indexed by  $(\chi_1^f, \dots, \chi_n^f)$  is implementable as a sticky-price equilibrium if and only if there exists a nominal wage function  $w(s)$  such that*

$$w(s)/g_i(\chi_1^f z_1, \dots, \chi_n^f z_n) \in \sigma(\omega_{ik}) \quad (18)$$

for all firms  $k \in [0, 1]$  in all industries  $i$ , where  $w(s)$  is pinned down by the monetary policy via  $w(s) = m(s)V'(L(s))/C(s)U'(C(s))$ .

This theorem provides a joint restriction on the technology, information structure, and monetary policy under which a flexible-price allocation can be implemented as a sticky-price equilibrium.<sup>12</sup> It establishes that monetary policy can implement a given flexible-price allocation if and only if there exists a policy-induced function  $w(s)$  that can make the expression  $w(s)/g_i(\chi_1^f z_1, \dots, \chi_n^f z_n)$  measurable with respect to the information set of all firms in industry  $i$ , simultaneously for all industries. To see the intuition underlying this result, note that the left-hand side of (18) coincides with the nominal marginal cost of firms in industry  $i$  under complete information. Therefore, Theorem 1 states that the monetary authority can implement flexible-price allocations and neutralize the effect of nominal rigidities if and only if there exists a policy that makes all firms' nominal marginal costs measurable with respect to their corresponding information sets. This guarantees that all firms set their nominal prices as if they had complete information about their nominal marginal costs.

We can now use the characterization in Theorem 1 to obtain the following corollary:

**Corollary 1.** *Let  $\mathcal{X}^f$  and  $\mathcal{X}^s$  denote the set of allocations that are implementable as flexible-price and sticky-price equilibria, respectively. Then,  $\mathcal{X}^f \cap \mathcal{X}^s = \emptyset$  for a generic set of information structures.*

<sup>11</sup>This observation also illustrates the fact that  $\varepsilon_{ik}(s)$  is the reciprocal of the markup induced by the presence of nominal rigidities. Specifically, it implies that  $p_{ik}(\omega_{ik}) = \frac{1}{\chi_i^s \varepsilon_{ik}(s)} \text{mc}_i(s)$ .

<sup>12</sup>Specifically,  $g_i(\chi_1^f z_1, \dots, \chi_n^f z_n)$  only depends on the firms' production technologies and productivity shocks,  $\sigma(\omega_{ik})$  is only a function of the economy's information structure, and  $w(s)$  is controlled by the monetary policy.

That is, in general, any allocation implementable as an equilibrium under flexible prices cannot be implemented as an equilibrium under sticky prices with any monetary policy. Interpreted through the lens of Theorem 1, the above result is an immediate consequence of the observation that, given a generic information structure, it is impossible to satisfy condition (18) for all firms simultaneously. The following result is then immediate.

**Corollary 2.** *In a multi-sector economy with given preferences and technologies, the first-best allocation is not implementable as a sticky-price equilibrium for a generic set of information structures.*

The intuition behind Corollary 2 is straightforward. Consider the planner's optimal allocation which can itself be implemented as a flexible-price equilibrium with appropriate industry-level subsidies. The planner would like relative quantities across industries to move efficiently with productivity shocks, while at the same time ensuring that all firms within each industry produce the same quantity. In order to implement this under flexible prices, relative prices across industries should move with relative productivities, while prices across firms within each industry should be identical. This specific pattern of price movements with productivity shocks is necessary for flexible-price allocations and in particular for the first-best allocation.

However, inducing this pattern of price movements is in general impossible under sticky prices in a multi-sector economy. In order to ensure that prices are uniform within a particular industry, the monetary authority must target price stability for that industry. This is the typical first-best policy in one-sector New Keynesian models as it implements zero price dispersion and hence productive efficiency within that particular industry. But when there are multiple industries, if monetary policy is used to achieve price stability within one particular industry, it cannot, in general, be used to target price stability in any other industry. That is, monetary policy cannot stabilize prices in all industries at once. And even if it could—for example, because the information structure is such that all firms in any given industry set the same exact price—it is still not sufficient for achieving the first best: in general, no monetary policy can induce relative prices of all pairs of industries to move with their corresponding productivity shocks.

Another consequence of condition (18) and Corollaries 1 and 2 is that monetary policy may not be able to implement the first-best allocation even if all firms in the economy have perfect information about their own productivities: incomplete information about productivity shocks to other far away industries generically results in flexible-price allocations that are not sticky-price implementable.<sup>13</sup>

That the single instrument of monetary policy cannot eliminate multiple distortions simultaneously is in line with Erceg, Henderson, and Levin (2000) and Woodford (2003b), who obtain similar results for, respectively, an economy with price and wage rigidities and a two-sector economy with no input-output linkages. Theorem 1 extends these findings and provides a characterization of conditions on model primitives under which monetary policy can neutralize the effect of nominal rigidities and implement flexible-price equilibria.

While Corollaries 1 and 2 illustrate the limitation of monetary policy in a generic multi-sector economy, there are some non-generic, yet important, special cases in which the monetary authority

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<sup>13</sup>However, (18) implies that a sufficient condition for sticky-price implementability of flexible-price allocations is that firms in industry  $i$  have complete information about productivity shocks to all their (direct and indirect) upstream industries.

can implement the first-best allocation.

**Corollary 3.** *If there is a single sticky-price industry  $i$ , any flexible-price allocation can be implemented as a sticky-price equilibrium with a monetary policy that stabilizes industry  $i$ 's price.*

This result is an immediate consequence of Theorem 1: if there is a single sticky-price industry  $i$ , then setting  $w(s) = Mg_i(\chi_1^f z_1, \dots, \chi_n^f z_n)$  for any constant  $M > 0$  ensures that the left-hand side of (18) is measurable with respect to the information set of all firms in industry  $i$ . Importantly, such a policy stabilizes the nominal marginal cost and hence the nominal price of firms in the sticky-price industry:  $mc_i(s) = M$  and  $p_{ik}(\omega_{ik}) = M/\chi_i^f$ , both of which are independent of the economy's aggregate state.

Though focused on a non-generic class of economies, Corollary 3 nests two important economies as special cases. The first special case is the textbook single-sector New Keynesian model with no markup shocks. As is well-known (and in line with Corollary 3), the first-best allocation can always be implemented by a combination of (i) price stabilization and (ii) an industry-level subsidy that eliminates monopolistic markups. Importantly, the above result establishes that such a policy mix is optimal irrespective of the nature and extent of information frictions, thus generalizing the insights of Correia, Nicolini, and Teles (2008) to a broad class of nominal rigidities. The second special case is the two-sector model of Aoki (2001), who considers an economy consisting of one flexible industry and one sticky industry subject to Calvo frictions and shows that stabilizing the price of the sticky industry can implement the first-best allocation. Corollary 3 establishes that, as long as there is a single sticky-price industry, Aoki's result generalizes to a multi-sector economy with input-output linkages and an arbitrary form of pricing friction.

Our next result establishes yet another case in which monetary policy can implement flexible-price allocations.

**Corollary 4.** *If firms have complete information about all shocks except for an aggregate labor-augmenting shock, then any flexible-price allocation can be implemented as a sticky-price equilibrium.*

This result is a consequence of the fact that in an economy with constant returns and a single factor of production, the marginal products of labor of all industries in the first-best allocation move one-for-one with aggregate labor-augmenting shocks. As a result, a monetary policy that implements a nominal wage function,  $w(s)$ , that also moves one-for-one with the shock ensures that the left-hand side of (18) is invariant to the shock, and hence is measurable with respect to all firms' information sets. Importantly, the same argument does not apply to aggregate TFP shocks: in general, monetary policy cannot neutralize the impact of nominal rigidities that are due to incomplete information about aggregate TFP shocks. This disparity highlights the fact that, in our multi-sector economy, whether the single instrument of monetary policy can implement the first-best allocation depends not only on the number of (unobservable) shocks, but also on how relative prices of different industries respond to those shocks.

### 3.5 Cobb-Douglas Economies

We conclude this section by studying the implications of Theorem 1 for the benchmark class of Cobb-Douglas economies. The log-linearity of production technologies in this class of economies allows us

to obtain explicit conditions on the economy's production network and information structure under which monetary policy is able to implement flexible-price equilibria.

Let the production technology of firm  $k$  in industry  $i$  be given by

$$y_{ik} = z_i F_i(l_{ik}, x_{i1,k}, \dots, x_{in,k}) = z_i \zeta_i l_{ik}^{\alpha_i} \prod_{j=1}^n x_{ij,k}^{a_{ij}}, \quad (19)$$

where  $\alpha_i \geq 0$  denotes the share of labor in industry  $i$ 's production technology,  $a_{ij} \geq 0$  parameterizes the importance of good  $j$  in the production technology of firms in industry  $i$ , and  $\zeta_i$  is a normalization constant, the value of which only depends on model parameters and is independent of the shocks.<sup>14</sup> Input-output linkages in this economy can thus be summarized by matrix  $\mathbf{A} = [a_{ij}]$ , which with some abuse of terminology, we refer to as the economy's *input-output matrix*. We also define the economy's *Leontief inverse* as  $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ , whose  $(i, j)$  element captures the role of industry  $j$  as a direct or indirect intermediate input supplier to industry  $i$ .

The consumption basket is also a Cobb-Douglas aggregator of the sectoral goods given by

$$\mathcal{C}(c_1, \dots, c_n) = \prod_{i=1}^n (c_i / \beta_i)^{\beta_i}, \quad (20)$$

where  $c_i$  is the amount of good  $i$  consumed and the constants  $\beta_i \geq 0$  measure various goods' shares in the household's consumption basket, normalized such that  $\sum_{i=1}^n \beta_i = 1$ . We have the following result:

**Proposition 3.** *In a Cobb-Douglas economy with Leontief inverse  $\mathbf{L}$ ,*

- (a) *the monetary authority can implement the first-best allocation using a price-stabilization policy if and only if there exists a substochastic vector  $\psi = (\psi_1, \dots, \psi_n)'$  such that<sup>15</sup>*

$$(\psi' - u'_i) \mathbf{L} \mathbf{V}_{ik} = 0 \quad \text{for all firms } k \in [0, 1] \text{ in all industries } i, \quad (21)$$

where  $u_i$  is the  $i$ -th unit vector and  $\mathbf{V}_{ik} = \text{var}_{ik}(\log z)$  denotes the covariance matrix of the vector of log productivity shocks conditional on the information set of firm  $k$  in industry  $i$ .

- (b) *If (21) is satisfied, the first-best allocation can be implemented by stabilizing a price index that assigns weight  $\psi_i$  to the price of industry  $i$ , that is,  $\sum_{i=1}^n \psi_i \log p_i + (1 - \sum_{i=1}^n \psi_i) \log w = 0$ .*

This result restates the measurability condition in (18) as an explicit algebraic condition on the interaction between the production network (as summarized by the Leontief inverse,  $\mathbf{L}$ ) and the economy's information structure (as captured by conditional covariance matrices,  $\mathbf{V}_{ik}$ ) under which monetary policy can neutralize the impact of nominal rigidities and implement the first-best allocation. It also characterizes the policies that do so.

The characterization in Proposition 3 illustrates our earlier result that the monetary policy's ability to implement the first-best allocation is limited to a non-generic class of information structures: all

<sup>14</sup>In what follows, we set the value of this constant to  $\zeta_i = \alpha_i^{-\alpha_i} \prod_{j=1}^n \alpha_{ij}^{-a_{ij}}$ . This choice has no bearing on the results, as the sole purpose of this constant is to simplify the analytical expressions. Also note that, for all technologies to exhibit constant returns, it must be the case that  $\alpha_i + \sum_{j=1}^n a_{ij} = 1$  for all  $i$ .

<sup>15</sup>Vector  $\psi \in \mathbb{R}^n$  is *substochastic* if  $\psi_i \geq 0$  for all  $i$  and  $\sum_{i=1}^n \psi_i \leq 1$ .



it takes to violate (21) is for a non-zero fraction of firms in two industries to have an arbitrarily small idiosyncratic uncertainty about productivity shocks.<sup>16</sup> At the other extreme, (21) is trivially satisfied when all firms know the exact realizations of all shocks (in which case,  $\mathbf{V}_{ik} = 0$ ).

## 4 Optimal Monetary Policy

Our results in Section 3 establish that, in general, monetary policy cannot implement the first-best allocation as a sticky-price equilibrium. In view of these results, we now turn to the study of optimal monetary policy, i.e., the policy that maximizes household welfare over the set of all possible sticky-price-implementable allocations.

In order to obtain closed-form expressions for the optimal policy, we impose a number of functional form assumptions on preferences, technologies, and the nature of nominal rigidities in the economy. More specifically, we focus on the class of Cobb-Douglas economies in Subsection 3.5 by assuming that the production technology of firms in industry  $i$  and the household consumption bundle are given by (19) and (20), respectively. In addition, we assume that the representative household's preferences (2) are homothetic, with

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad , \quad V(L) = \frac{L^{1+1/\eta}}{1+1/\eta},$$

where  $C$  and  $L$  denote the household's consumption and labor supply, respectively. We also assume that sector-specific taxes/subsidies in (1) are set to  $\tau_i = 1/(1 - \theta_i)$  for all  $i$ . As discussed in Section 3, this choice undoes the effect of monopolistic markups and guarantees that the flexible-price equilibrium is efficient.

To specify firms' information sets and the resulting nominal rigidities, we assume all productivity shocks are drawn independently from a log-normal distribution:

$$\log z_i \sim \mathcal{N}(0, \delta^2 \sigma_z^2). \tag{22}$$

Each firm  $k$  in industry  $i$  then receives a collection of private signals  $\omega_{ik} = (\omega_{i1,k}, \dots, \omega_{in,k})$  about the realized productivities given by

$$\omega_{ij,k} = \log z_j + \epsilon_{ij,k} \quad , \quad \epsilon_{ij,k} \sim \mathcal{N}(0, \delta^2 \sigma_{ik}^2), \tag{23}$$

where the noise terms  $\epsilon_{ij,k}$  are independent from one another and the productivity shocks. In this formulation,  $\delta > 0$  is a normalization constant,  $\sigma_z^2$  measures firms' (common) prior uncertainty about the shocks, and  $\sigma_{ik}^2$  parametrizes the quality of information available to firm  $k$  in industry  $i$ .<sup>17</sup> Hence, an increase in  $\sigma_{ik}^2/\sigma_z^2$  corresponds to an increase in the extent of nominal rigidity faced by firm  $k$  in

<sup>16</sup>More generally, (21) is violated if a non-zero fraction of firms in industries  $i$  and  $j$  face some uncertainty about all linear combinations of log productivity shocks. In that case, the corresponding covariance matrices  $\mathbf{V}_{ik}$  and  $\mathbf{V}_{jk}$  are positive definite for such firms, and as a result, (21) is satisfied only if  $\psi = u_i = u_j$ , which is impossible.

<sup>17</sup>The formulation in (22) assumes that log productivity shocks are independent with identical volatilities. We consider the case with heteroskedastic and correlated shocks in Appendix A.2. Also note that the formulation in (23) allows for potential heterogeneity in the degrees of price flexibility, not only across industries, but also among firms within the same industry. See Weber (2015) and Gorodnichenko and Weber (2016) for evidence of substantial heterogeneity in the frequency of price adjustments within narrowly defined industries.

industry  $i$ . In contrast, the extreme case that  $\sigma_{ik}^2 = 0$  for all  $k$  and all  $i$  corresponds to an economy with fully flexible prices. More generally, it is straightforward to verify that

$$\begin{aligned}\mathbb{E}_{ik}[\log z_j] &= \phi_{ik}\omega_{ij,k} \\ \text{var}_{ik}(\log z_j) &= (1 - \phi_{ik})\text{var}(\log z_j),\end{aligned}\tag{24}$$

where

$$\phi_{ik} = \sigma_z^2 / (\sigma_z^2 + \sigma_{ik}^2).\tag{25}$$

Equation (24) indicates that an increase in  $\phi_{ik}$  corresponds to a reduction in the firm's uncertainty about the payoff-relevant productivity shocks. We thus refer to  $\phi_{ik} \in [0, 1]$  as the *degree of price flexibility of firm  $k$*  in industry  $i$ . Similarly, we define the *degree of price flexibility of industry  $i$*  as

$$\phi_i = \int_0^1 \phi_{ik} dk.$$

Note that in the special case that  $\sigma_{ik}^2 \in \{0, \infty\}$  for all firms,  $\phi_{ik} \in \{1, 0\}$  for all  $i$  and  $k$ , in which case our framework reduces to an economy that is subject to “sticky information” pricing frictions à la [Mankiw and Reis \(2002\)](#): firms in each industry can either set their prices flexibly with no frictions or face full nominal rigidity.<sup>18</sup>

To keep the analysis tractable, we work with the log-linearization of the above economy as  $\delta \rightarrow 0$ , where recall from equations (22) and (23) that  $\delta > 0$  simultaneously parametrizes the firms' prior uncertainty about (log) productivity shocks and the noise in their private signals. This specific parametrization leads to two desirable features. First, the fact that  $\text{var}(\log z_i) = \delta^2 \sigma_z^2$  means that our small- $\delta$  approximation is akin to focusing on small departures from the economy's steady-state, as is typical in the New Keynesian literature. Second, scaling  $\text{var}(\epsilon_{ij,k})$  with  $\delta^2$  ensures that the degree of price flexibility  $\phi_{ik}$  in equation (25) remains independent of  $\delta$ .

As a final remark, we note that, under this information structure, monetary policy can implement the first-best allocation only if there is at most one industry with firms that receive imperfect signals. To see this, note that equation (24) implies that the covariance matrix of the vector of log productivity shocks conditional on the information set of firm  $k$  in industry  $i$  is given by  $\mathbf{V}_{ik} = \delta^2 \sigma_z^2 (1 - \phi_{ik}) \mathbf{I}$ . As a result, it is straightforward to verify that condition (21) in Proposition 3 is violated unless  $\phi_i = 1$  for at least  $n - 1$  industries. This means that monetary policy cannot neutralize the impact of nominal rigidities and implement the first-best allocation as long as  $\phi_i, \phi_j < 1$  for  $i \neq j$ .

## 4.1 Strategic Complementarities and Monetary Non-Neutrality

In this subsection, we use Proposition 2 to obtain a set of preliminary results that will serve as the basis of our characterization of the optimal policy. We first illustrate the central role of strategic complementarities in firms' price-setting decisions in our production network economy. We then show how the interaction of such strategic complementarities with nominal rigidities impacts nominal prices and marginal costs and shapes the extent of monetary non-neutrality.

<sup>18</sup>This special case is also isomorphic to the static variant of the Calvo friction, in which a fraction  $\phi_i$  of firms in industry  $i$  can adjust set their nominal prices with no restrictions, whereas the remaining  $1 - \phi_i$  are subject to full nominal rigidity.

**Lemma 2.** *The nominal price set by firm  $k$  in industry  $i$  satisfies*

$$\log p_{ik} = \mathbb{E}_{ik}[\log mc_i] + o(\delta) \quad (26)$$

$$= \alpha_i \mathbb{E}_{ik}[\log w] - \mathbb{E}_{ik}[\log z_i] + \sum_{j=1}^n a_{ij} \mathbb{E}_{ik}[\log p_j] + o(\delta) \quad (27)$$

to a first-order approximation as  $\delta \rightarrow 0$ .

Equation (26), which is a consequence of Proposition 2, establishes that, to a first-order approximation, each firm sets its log nominal price equal to its expected log marginal cost, given its information set. This is a consequence of monopolistic competition and the assumption that sector-specific taxes/subsidies are set to  $\tau_i = 1/(1 - \theta_i)$  to eliminate monopolistic markups.

Lemma 2 also illustrates that our multi-sector New Keynesian model is isomorphic to a “beauty contest” game over the production network (Bergemann, Heumann, and Morris, 2017). In particular, equation (27) is identical to the first-order conditions of a network game of incomplete information in which firms in industry  $i$  choose their log nominal prices to match an industry-specific “fundamental” (given by  $\alpha_i \log w - \log z_i$ ), while simultaneously coordinating with a linear combination of the (log) prices set by their supplier industries (given by  $\sum_{j=1}^n a_{ij} \log p_j$ ).<sup>19</sup> This coordination motive is the consequence of strategic complementarities in firms’ price-setting behavior in the presence of input-output linkages: all else equal, an increase in the price set by firms in an industry increases the incentive of its downstream customers to also raise their prices.

To capture the implications of the interaction between such strategic complementarities and nominal rigidities, we next define the following concept:

**Definition 4.** The *upstream (price) flexibility* of industry  $i$  is given by

$$\rho_i = \alpha_i + \sum_{j=1}^n a_{ij} \phi_j \rho_j, \quad (28)$$

where  $\phi_j$  is the degree of price flexibility of industry  $j$  and  $\alpha_i$  is the labor share of industry  $i$ .

A few observations are in order. First, note that while the definition in (28) is recursive, the vector of upstream flexibilities has a simple closed-form representation given by  $\rho = (\mathbf{I} - \mathbf{A}\Phi)^{-1}\alpha$ , where  $\Phi = \text{diag}(\phi_1, \dots, \phi_n)$  is the diagonal matrix of (own) price flexibilities and  $\alpha = (\alpha_1, \dots, \alpha_n)'$  is the vector of industry labor shares. Second, it is straightforward to verify that  $\rho_i \in [0, 1]$  for all  $i$ . Third, and more importantly, the recursive nature of (28) implies that upstream flexibility of industry  $i$  is strictly increasing in the own ( $\phi_j$ ) and upstream ( $\rho_j$ ) price flexibilities of any of  $i$ ’s supplier industries  $j$ , with these terms weighted by the importance of  $j$  in  $i$ ’s production technology,  $a_{ij}$ . As a result,  $\rho_i$  decreases if any of  $i$ ’s direct or indirect upstream suppliers are subject to more pricing frictions, whereas  $\rho_i$  takes its maximum value of 1 if none of its direct and indirect upstream suppliers are subject to any pricing friction. This observation clarifies the sense in which  $\rho_i$  serves as a summary statistic for the extent of nominal rigidities in industries upstream to  $i$ .

We now use Definition 4 and the representation in Lemma 2 to characterize equilibrium nominal prices and marginal costs as a function of model primitives and the nominal wage.

<sup>19</sup>More specifically, equation (27) coincides with the first-order conditions of a quadratic-payoff game in which the payoff of firms in industry  $i$  is given by  $u_{ik}(\log p_{ik}, \log p, \log w, \log z) = -(\log p_{ik} - (\alpha_i \log w - \log z_i))^2 - (\log p_{ik} - \sum_{j=1}^n a_{ij} \log p_j)^2$ .

**Proposition 4.** *Equilibrium log nominal marginal costs and prices are, respectively, given by*

$$\log mc_i = \rho_i \log w - \sum_{j=1}^n h_{ij} \log z_j + o(\delta) \quad (29)$$

$$\log p_i = \phi_i \rho_i \log w - \phi_i \sum_{j=1}^n h_{ij} \log z_j + o(\delta) \quad (30)$$

to a first-order approximation as  $\delta \rightarrow 0$ , where  $h_{ij}$  is the  $(i, j)$  element of matrix  $\mathbf{H} = (\mathbf{I} - \mathbf{A}\Phi)^{-1}$ .

Proposition 4 illustrates that the interaction between nominal rigidities and the strategic complementarities that arise from the economy's production network amplifies the sluggishness of the response of nominal variables to real and monetary shocks. In particular, it is immediate from (29) and (30) that the pass-through of changes in the nominal wage to  $i$ 's nominal marginal cost and nominal price are given by

$$\frac{d \log mc_i}{d \log w} = \rho_i \quad \text{and} \quad \frac{d \log p_i}{d \log w} = \phi_i \rho_i, \quad (31)$$

respectively, both of which are increasing in industry  $i$ 's upstream flexibility,  $\rho_i$ . This is in line with Blanchard (1983), Basu (1995), and Christiano (2016) who, in simpler settings, argue that strategic complementarities that arise from the presence of input-output linkages amplify the effect of nominal rigidities and increase the sluggishness of the response of nominal variables to shocks.

We conclude our set of preliminary results by characterizing how the strategic complementarities arising from the economy's production network shape the extent of monetary non-neutrality. To this end, let

$$\rho_0 = \sum_{j=1}^n \beta_j \phi_j \rho_j, \quad (32)$$

where  $\beta_j$  is the share of good  $j$  in the household's consumption basket. In view of (31),  $\rho_0$  captures the pass-through of changes in the log nominal wage to the price of the household consumption basket.<sup>20</sup> Recall from our previous discussion that this quantity has a closed-form representation in terms of model primitives given by  $\rho_0 = \beta' \Phi (\mathbf{I} - \mathbf{A}\Phi)^{-1} \alpha$ . Furthermore, as expected,  $\rho_0$  is increasing in the extent of price flexibilities  $(\phi_1, \dots, \phi_n)$  of all industries in the economy, regardless of whether they sell directly to the household or not. We have the following result:

**Proposition 5.** *The degree of monetary non-neutrality is*

$$\Xi = \frac{d \log C}{d \log m} = \frac{1 - \rho_0}{1 + (\gamma - 1 + 1/\eta) \rho_0}, \quad (33)$$

where  $C$  is the household's consumption and  $m$  is the nominal aggregate demand.

Proposition 5, which generalizes the results of Pastén, Schoenle, and Weber (2020b), characterizes how nominal rigidities interact with the economy's production network to generate monetary non-neutrality. As a first observation, note that  $\Xi$  is monotone decreasing in  $\rho_0$ , which itself is decreasing

<sup>20</sup>Alternatively, in view of (28),  $\rho_0$  can also be interpreted as the degree of (upstream) price flexibility faced by the household.

in the degree of price stickiness of all industries in the economy. Therefore, as expected, an increase in nominal rigidities anywhere in the economy results in a higher degree of monetary non-neutrality. Furthermore, the recursive nature of equation (28) underscores how the strategic complementarities arising from the economy's production network amplify monetary non-neutrality: an increase in the degree of price stickiness of industry  $j$  (i.e., a decrease in  $\phi_j$ ) not only decreases  $\rho_0$  directly as is evident from (32), but also does so indirectly by making the marginal cost of any industry  $i$  that rely on  $j$  more sluggish (thus reducing  $\rho_0$  by reducing  $\rho_i$ ).

## 4.2 Welfare Loss and Policy Objective

In the remainder of this section, we use our preliminary results in Subsection 4.1 to obtain a closed-form expression for the optimal monetary policy, which maximizes the expected welfare of the representative household over the set of all sticky-price equilibrium allocations.

We express the household's welfare relative to a benchmark with no nominal rigidities, which corresponds to the first-best allocation. More specifically, let  $W$  and  $W^*$  denote the representative household's welfare in the presence and absence of nominal rigidities, respectively. Similarly, let  $\varrho = (p_{ik}, p_i, w)$  and  $\varrho^* = (p_{ik}^*, p_i^*, w^*)$  denote the nominal price systems under the two scenarios. Given the indeterminacy of prices in the flexible-price equilibrium, we normalize the nominal wage such that  $w^* = w$ . We also use  $e_{ik} = \log p_{ik} - \log p_{ik}^*$  to denote the "pricing error" of firm  $k$  in industry  $i$  in the sticky-price equilibrium relative to the benchmark with no nominal rigidities. The cross-sectional average and dispersion of pricing errors within industry  $i$  are thus given by

$$\bar{e}_i = \int_0^1 e_{ik} dk \quad (34)$$

$$\vartheta_i = \int_0^1 e_{ik}^2 dk - \left( \int_0^1 e_{ik} dk \right)^2. \quad (35)$$

We have the following result:

**Proposition 6.** *The welfare loss due to the presence of nominal rigidities is given by*

$$W - W^* = -\frac{1}{2} \left[ \sum_{i=1}^n \lambda_i \theta_i \vartheta_i + (\gamma + 1/\eta) \Delta^2 + \sum_{i=1}^n \lambda_i \text{xvar}_i(\bar{e}_1, \dots, \bar{e}_n) + \text{xvar}_0(\bar{e}_1, \dots, \bar{e}_n) \right] + o(\delta^2) \quad (36)$$

to a second-order approximation as  $\delta \rightarrow 0$ , where  $\lambda_i$  is industry  $i$ 's Domar weight,  $\vartheta_i$  is the dispersion of pricing errors in industry  $i$  defined in (35),

$$\Delta^2 = (\log C - \log C^*)^2 = \frac{1}{(\gamma + 1/\eta)^2} \left( \sum_{j=1}^n \beta_j \bar{e}_j \right)^2 + o(\delta^2) \quad (37)$$

is the volatility of output gap, and

$$\begin{aligned} \text{xvar}_i(\bar{e}_1, \dots, \bar{e}_n) &= \sum_{j=1}^n a_{ij} \bar{e}_j^2 - \left( \sum_{j=1}^n a_{ij} \bar{e}_j \right)^2 \\ \text{xvar}_0(\bar{e}_1, \dots, \bar{e}_n) &= \sum_{j=1}^n \beta_j \bar{e}_j^2 - \left( \sum_{j=1}^n \beta_j \bar{e}_j \right)^2 \end{aligned} \quad (38)$$

are the inter-industry cross-sectional dispersions of pricing errors from the perspectives of industry  $i$  and the household, respectively.

Proposition 6 generalizes the well-known expression for welfare loss in single-sector New Keynesian models (e.g., Galí (2008)) as well as the corresponding expressions in Woodford (2003b, 2010) and Huang and Liu (2005) for two-sector economies. In particular, equation (36) illustrates that the loss in welfare due to the presence of nominal rigidities in our multi-sector economy with input-output linkages manifests itself via four separate terms.

The first term,  $\lambda_i \theta_i \vartheta_i$  measures welfare losses due to price dispersion *within* each industry  $i$  and is the counterpart of welfare loss due to inflation in the textbook New Keynesian models: price dispersion  $\vartheta_i$  in industry  $i$  translates into output dispersion and hence misallocation of resources, with the extent of this misallocation increasing in the elasticity of substitution  $\theta_i$  between firms in that industry. This term vanishes if all firms in industry  $i$  make their nominal pricing decisions under the same information. Not surprisingly, the loss due to price dispersion in industry  $i$  is weighted by the industry's Domar weight,  $\lambda_i$ .

The second term on the right-hand side of (36) is proportional to the volatility of output gap  $\Delta^2 = (\log C - \log C^*)^2$ . This term, which is also present in the textbook New Keynesian models and vanishes as the Frisch elasticity of labor supply  $\eta \rightarrow 0$ , captures loss of welfare due to inefficient supply of labor by the household, i.e., the aggregate labor wedge. Equation (37) then characterizes how output gap volatility relates to industry-level pricing errors in our multi-sector economy.

In contrast to the first two terms, the third and fourth terms on the right-hand side of (36) only appear in multi-sector economies and correspond to welfare losses arising from misallocation of resources *across* industries. To see this, consider the expression  $\text{xvar}_i(\bar{e}_1, \dots, \bar{e}_n)$  in equation (38). This term measures the cross-sectional dispersion in the average pricing error of  $i$ 's supplier industries, with higher weights assigned to industries that are more important input-suppliers to  $i$ . To be even more specific, suppose industry  $i$  has two suppliers indexed  $j$  and  $r$  such that  $a_{ij} + a_{ir} = 1$ . In this case, it is immediate that

$$\text{xvar}_i(\bar{e}_j, \bar{e}_r) = a_{ij}a_{ir}(\bar{e}_j - \bar{e}_r)^2 = a_{ij}a_{ir} \left( \log(p_j/p_r) - \log(p_j^*/p_r^*) \right)^2$$

simply measures the extent to which nominal relative prices of  $i$ 's inputs diverge from the relative prices that would have prevailed under the flexible-price (and hence efficient) allocation. Finally, note that  $\text{xvar}_i(\bar{e}_1, \dots, \bar{e}_n) = 0$  whenever industry  $i$  has only a single input supplier  $j$  with  $a_{ij} = 1$ , as this corresponds to a scenario in which there is no room for misallocation between  $i$ 's input suppliers.<sup>21</sup>

In summary, Proposition 6 indicates that, in a multi-sector economy, the monetary authority faces an inherent trade-off between minimizing the various losses captured by equation (36). Importantly, as we already established in Corollaries 1 and 2, this trade-off cannot be circumvented, in the sense that, generically, there is no policy that can simultaneously eliminate all forms of welfare loss. Finally, note that, unlike the textbook New Keynesian models, the various trade-offs faced by the monetary authority arises from the structural properties of the economy's production network as opposed to ad hoc markup shocks.

<sup>21</sup>The interpretation for the term  $\text{xvar}_0(\bar{e}_1, \dots, \bar{e}_n)$  in equation (36) is identical, with the household replacing industry  $i$  as purchaser of various goods.

### 4.3 Optimal Monetary Policy

We are now ready to present our main result of this section, which characterizes the optimal policy as a function of the economy's production network and the extent of nominal rigidities.

**Theorem 2.** *The optimal monetary policy is a price-stabilization policy of the form  $\sum_{i=1}^n \psi_i^* \log p_i = 0$ , with the weight assigned to industry  $i$  in the target price index given by*

$$\psi_i^* = \psi_i^{\text{o.g.}} + \psi_i^{\text{within}} + \psi_i^{\text{across}}, \quad (39)$$

where

$$\psi_i^{\text{o.g.}} = (1/\phi_i - 1)\lambda_i \left( \frac{1 - \rho_0}{\gamma + 1/\eta} \right) \quad (40)$$

$$\psi_i^{\text{within}} = (1 - \phi_i)\lambda_i\theta_i\rho_i \quad (41)$$

$$\psi_i^{\text{across}} = (1/\phi_i - 1) \left[ (\rho_0 - \rho_i)\lambda_i + \sum_{j=1}^n (1 - \phi_j)\lambda_j\rho_j\ell_{ji} \right], \quad (42)$$

$\phi_i$  is degree of price flexibility of industry  $i$ ,  $\rho_i$  is  $i$ 's degree of upstream price flexibility in (28),  $\lambda_i$  is the  $i$ 's Domar weight, and  $\mathbf{L} = [\ell_{ij}]$  is the economy's Leontief inverse.

Theorem 2 provides a characterization of the optimal policy in terms of model primitives, with each term on the right-hand side of (39) aimed at minimizing a specific source of welfare loss in (36). Recall from Subsection 4.2 that the monetary authority faces a trade-off between minimizing the various sources of allocational inefficiencies due to nominal rigidities. Not surprisingly then, the optimal policy in (39) consists of three different terms corresponding to the relative importance of each of these misallocations for household welfare: the first term on the right-hand side of (39) aims to minimize the welfare loss induced by the labor wedge (or equivalently, output gap volatility), the second term arises due to the policymakers' concern about within-industry price dispersion, and the last term is in response to misallocation across industries.<sup>22</sup> Note that the three terms constituting the optimal policy in (39) are in general not proportional to one another, thus indicating that the monetary authority faces a real trade-off between the corresponding misallocation losses.

The above result also illustrates that the optimal policy is shaped by how nominal rigidities interact with the economy's production network. In particular, the weight  $\psi_i^*$  assigned to any given industry  $i$  in the optimal policy depends not just on that industry's size (as measured by its Domar weight) and price flexibility (as parametrized by  $\phi_i$ ), but also on  $i$ 's position in the production network and the nominal rigidities faced by other industries in the economy. This dependence is a consequence of strategic complementarities in firms' price-setting decisions discussed in Subsection 4.1.

As a final remark, we note that in deriving the above result, we did not restrict our attention to the class of price-stabilization policies to begin with. In particular, the monetary authority can choose the nominal aggregate demand,  $m$ , as an arbitrary function of productivities in an unrestricted manner, including policies that do not stabilize any specific price index. Nonetheless, Theorem 2 establishes

<sup>22</sup>Alternatively, the expressions in (40), (41), and (42) correspond to the optimal price-stabilization policies of a planner who only intends to minimize welfare losses arising from output gap volatility, within-industry price dispersion, and inter-industry misallocation, respectively.

that the optimal policy fully stabilizes the price index  $\sum_{s=1}^n \psi_s^* \log p_s$  with weights given by (39). This reflects the fact that the underlying flexible-price economy is efficient: even though our multi-sector economy gives rise to an endogenous output gap, the absence of cost-push shocks means that there are no forces driving optimal policy away from price stabilization.<sup>23</sup>

#### 4.4 Comparative Statics

In what follows, we present a series of comparative static results to distill the role of the various forces that shape the optimal policy (39) in a transparent manner. These results allow us to obtain general insights on how optimal policy depends on the production network and the distribution of pricing frictions throughout the economy. In particular, we establish that, all else equal, optimal policy stabilizes a price index with higher weights assigned to (i) larger industries as measured by their Domar weights, (ii) stickier industries, (iii) more upstream industries, (iv) industries with less sticky upstream suppliers but with stickier downstream customers, and (v) industries whose customers exhibit higher upstream flexibilities.

We start with a definition.

**Definition 5.** Industries  $i$  and  $j$  are *upstream symmetric* if  $a_{ir} = a_{jr}$  for all industries  $r$ . They are *downstream symmetric* if  $a_{ri} = a_{rj}$  for all industries  $r$  and  $\beta_i = \beta_j$ .

While upstream symmetry means that  $i$  and  $j$  have the same production technology, downstream symmetry means that the two industries take identical roles as input suppliers of other firms in the economy as well as in household preferences. Consequently, if  $i$  and  $j$  are upstream symmetric, then they have the same degree of upstream flexibilities ( $\rho_i = \rho_j$ ), whereas if they are downstream symmetric, they have the same steady-state Domar weights ( $\lambda_i = \lambda_j$ ). We have the following result:

**Proposition 7.** *Suppose industries  $i$  and  $j$  are upstream and downstream symmetric. Also suppose  $\theta_i = \theta_j$ . Then,  $\psi_i^* > \psi_j^*$  in the optimal policy if and only if  $\phi_i < \phi_j$ .*

This result thus extends what [Eusepi, Hobijn, and Tambalotti \(2011\)](#) refer to as the “stickiness principle” to our multi-sector economy with input-output linkages: all else equal, the monetary authority should stabilize a price index that places larger weights on producers with stickier prices.

To see the intuition underlying Proposition 7, note that, by the upstream symmetry assumption,  $mc_i$  and  $mc_j$  have the same ex ante distribution. Yet, firms in the stickier of the two industries respond more sluggishly to changes in their realized marginal costs. Therefore, by targeting the industry that is subject to more nominal rigidities for price stabilization, the optimal policy reduces the need for price adjustments by firms in that industry and thus reduces the overall level of within-industry price dispersion. In fact, equation (41) in Theorem 2 implies that, due to upstream symmetry, if  $\phi_i < \phi_j$ , then  $\psi_i^{\text{within}} > \psi_j^{\text{within}}$ . At the same time, the downstream symmetry assumption implies that a policy

<sup>23</sup>The fact that full stabilization of the price index with weights given by (39) is feasible is a consequence of the assumption that the monetary authority can set nominal aggregate demand,  $m$ , as a function of productivity shocks in an unrestricted manner. Fully stabilizing such a price index may no longer be feasible when nominal aggregate demand cannot be contingent on the shocks’ realizations, for example, because the monetary authority is also subject to information frictions. As we show in Appendix A.3, in that case, the optimal policy minimizes the volatility of the target price index  $\sum_{i=1}^n \psi_i^* \log p_i$ , where  $\psi_i^*$  is given by (39).



that targets the stickier industry more also reduces output gap volatility and the inter-industry price dispersion faced by  $i$  and  $j$ 's common customers: from (40) and (42), it is easy to see that whenever  $\phi_i < \phi_j$ , it must be the case that  $\psi_i^{\text{og}} > \psi_j^{\text{og}}$  and  $\psi_i^{\text{across}} > \psi_j^{\text{across}}$ . Putting these inequalities together then guarantees that  $\psi_i^* > \psi_j^*$ .

By assuming that  $i$  and  $j$  take symmetric positions in the production network, Proposition 7 effectively assumes away how differences in input-output linkages may matter for optimal policy. In our subsequent results, we instead assume that the two industries are equally sticky and focus on the role of network connections in shaping the optimal policy.

**Proposition 8.** *Suppose  $i$  and  $j$  are downstream symmetric. Also suppose  $\phi_i = \phi_j < 1$  and  $\theta_i = \theta_j$ . Then,  $\psi_i^* > \psi_j^*$  if and only if  $\rho_i > \rho_j$ .*

This result encapsulates the second general principle that emerges from our characterization of optimal policy: all else equal, the optimal target price index places a larger weight on the industry with less sticky upstream suppliers, as summarized by a higher degree of upstream flexibility  $\rho$ .

The intuition underlying Proposition 8 is straightforward. Since  $i$  and  $j$  are downstream symmetric, stabilizing the price of either industry would have the same exact effect on the labor wedge and the extent of inter-industry relative price distortions perceived by their (direct and indirect) customers. As a result, any differential treatment of  $i$  and  $j$  by the optimal policy is solely driven by concerns for within-industry misallocation. At the same time, the fact that  $\rho_i > \rho_j$  means that industry  $j$ 's marginal cost responds more sluggishly to shocks, making the lack of complete information about the realized shocks less material for price-setting by firms in  $j$  compared to those in  $i$ . As a result, all else equal, the within-industry price dispersion would be lower in  $j$  than in  $i$ , despite the fact that both industries are equally sticky. Not surprisingly then, the optimal price-stabilization target assigns a smaller weight to the industry with stickier suppliers, which has an already more stabilized marginal cost.

**Proposition 9.** *Suppose industries  $i$  and  $j$  are upstream symmetric. Also suppose  $\phi_i = \phi_j < 1$ ,  $\theta_i = \theta_j$ , and  $\lambda_i = \lambda_j$ . Then,  $\psi_i^* > \psi_j^*$  if and only if*

$$\sum_{s=1}^n (1 - \phi_s) \lambda_s \rho_s (\ell_{si} - \ell_{sj}) > 0, \quad (43)$$

where  $\rho$  is the degree of upstream price flexibility in (28) and  $\mathbf{L} = [\ell_{ij}]$  is the economy's Leontief inverse.

The above result highlights yet another important dimension along which the production network structure shapes the optimal policy. In particular, it establishes that, all else equal, industry  $i$  receives a larger weight in the optimal price-stabilization index if (i) it is a more important supplier to stickier industries and (ii) its customers have a higher degree of upstream flexibility. To see these from inequality (43), recall that expression  $\ell_{si} - \ell_{sj}$  captures the differential importance of  $i$  and  $j$  as direct or indirect input suppliers to any industry  $s$ . Therefore, the left-hand side of (43) is positive if, relative to  $j$ , industry  $i$  is a more important supplier of industries with lower degree of price flexibility  $\phi_s$  but higher degree of upstream price flexibility  $\rho_s$ .

Why is it optimal to stabilize the price of the industry with stickier downstream customers? This is because such a policy would reduce the need for the firms in the customer industry to adjust their

nominal price. Therefore, the stickier are those customers, the larger is the welfare gain of stabilizing their marginal cost by assigning a larger weight on their suppliers in the target price-stabilization index.

The argument for why optimal policy places a larger weight on the industry whose customers have a higher degree of upstream flexibility  $\rho$  is also similar. Recall that, all else equal, a higher degree of upstream flexibility  $\rho_s$  means that firms in industry  $s$  face a more volatile nominal marginal cost and hence, on average, have to adjust their nominal price by more. Therefore, stabilizing the price of one of their inputs,  $i$ , would reduce the need for such price adjustment and hence reduce the welfare loss arising from nominal rigidities.

**Proposition 10.** *Suppose  $j$  is the sole input-supplier of  $i$  and  $i$  is the sole customer of  $j$ . Also, suppose  $\phi_i = \phi_j < 1$  and  $\theta_i = \theta_j$ . Then,  $\psi_i^* < \psi_j^*$ .*

The assumption that  $i$  and  $j$  are, respectively, each other's only customer and supplier and have identical stickiness and substitution elasticities is meant to ensure that the difference between the two industries is solely in their respective positions in the chain of production, with industry  $j$  taking an unambiguously upstream position vis-à-vis industry  $i$ . As such, Proposition 10 establishes that, all else equal, the optimal policy assigns a larger weight to more upstream industries. The differential treatment of the two industries by the optimal policy is driven purely by concerns about within-industry misallocation.

Taken together, our results presented as Propositions 7–10 yield general principles for the optimal conduct of monetary policy in the presence of input-output linkages. In particular, they establish that, all else equal, optimal policy stabilizes a price index with larger weights assigned to industries that are stickier (Proposition 7), have more flexible upstream suppliers (Proposition 8) but more sticky downstream customers (Proposition 9), have downstream customers with higher degrees of upstream flexibility (Proposition 9), and are themselves more upstream (Proposition 10). Last but not least, the characterization in Theorem 2 also implies that the optimal policy assigns a higher weight on larger industries, as measured by their Domar weights.

## 4.5 Examples

We conclude this section by providing a series of examples to further clarify the dependence of optimal policy on model primitives.

**Example 1** (vertical production network). Consider the economy depicted in Figure 1(a), in which each industry  $i \neq n$  depends on the output of a single other industry as its input for production ( $a_{i,i+1} = 1$ ), industry  $n$  only uses labor ( $\alpha_n = 1$ ), and the household, labeled as vertex 0 in the figure, only consumes the good produced by industry 1 ( $\beta_1 = 1$  and  $\beta_i = 0$  for all  $i \neq 1$ ).

Given the vertical nature of production, it is immediate that in this economy—a variant of which is studied by Huang and Liu (2005) and Wei and Xie (2020)—there is no room for inter-industry misallocation. Indeed, the expressions in (38) corresponding to welfare losses arising from inter-industry misallocation are equal to zero, irrespective of the extent of nominal rigidities. It is therefore not surprising that this source of welfare loss is immaterial for the design of optimal policy: equation (42) in Theorem 2 implies that  $\psi_i^{\text{across}} = 0$  for all  $i$ .

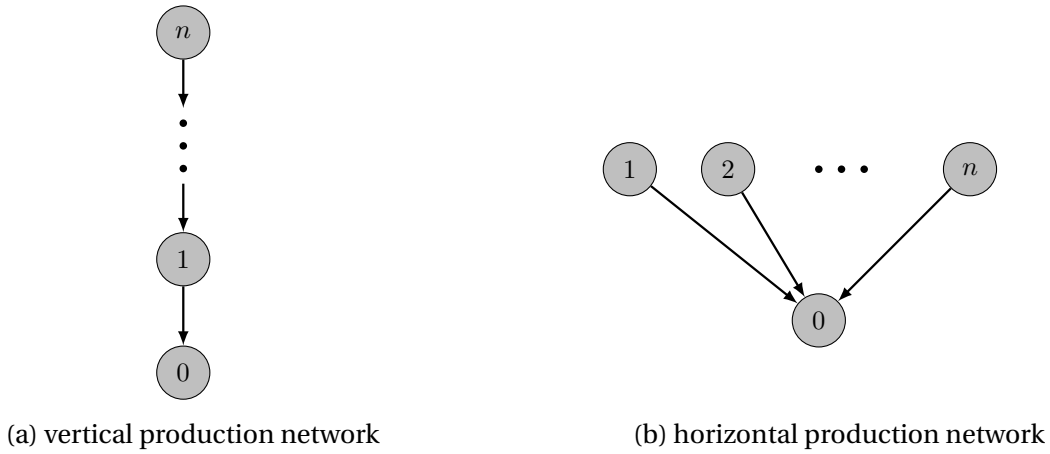


Figure 1. Each vertex corresponds to an industry, with a directed edge present from one vertex to another if the former is an input supplier to the latter. The vertex indexed 0 represents the household.

There is, however, room for within-industry misallocation, as firms' incomplete information about productivity shocks may result in a non-trivial price dispersion within each industry. Indeed, equation (41) implies that the corresponding component of optimal policy is non-zero and is given by

$$\psi_i^{\text{within}} = (1 - \phi_i)\theta_i\rho_i,$$

where  $\rho_i = \phi_{i+1}\phi_{i+2}\dots\phi_n$  is the degree of upstream flexibility of industry  $i$ . Note that due to the presence of strategic complementarities,  $\rho_i$  is smaller for industries further downstream:  $\rho_i \leq \rho_{i+1}$ .

Combining the above with equation (40)—which captures the component of optimal policy that aims at reducing welfare losses arising from the labor wedge—implies that the optimal price-stabilization target is given by

$$\psi_i^* = \psi_i^{\text{within}} + \psi_i^{\text{o.g.}} = (1 - \phi_i)\theta_i\phi_{i+1}\phi_{i+2}\dots\phi_n + \frac{1 - \phi_1\phi_2\dots\phi_n}{\gamma + 1/\eta}.$$

Consequently, the optimal policy assigns a larger weight to (i) industries with higher degrees of price stickiness (i.e., lower  $\phi_i$ ), (ii) those with higher elasticities of substitution  $\theta_i$ , and (iii) more upstream industries. This latter observation is of course consistent with the prescription of Proposition 10.

**Example 2** (horizontal production network). Consider the economy depicted in Figure 1(b), in which all industries only rely on labor as their input for production, i.e.,  $\alpha_i = 1$  for all  $i$ . This economy is therefore similar to the multi-sector economies with no input-output linkages that were studied in the prior literature, such as [Mankiw and Reis \(2003\)](#), [Benigno \(2004\)](#), and [Woodford \(2010\)](#).

Unlike the economy in Example 1, nominal rigidities in the horizontal economy result in misallocation not only within but also across industries: while efficiency requires relative prices across industries to move with corresponding productivities, such movements are in general not possible. This observation means that the component of optimal policy that targets inter-industry misallocation losses is non-zero. In particular, equation (42) in Theorem 2 implies that

$$\psi_i^{\text{across}} = (1/\phi_i - 1)\beta_i \sum_{j=1}^n \beta_j(\phi_j - \phi_i).$$

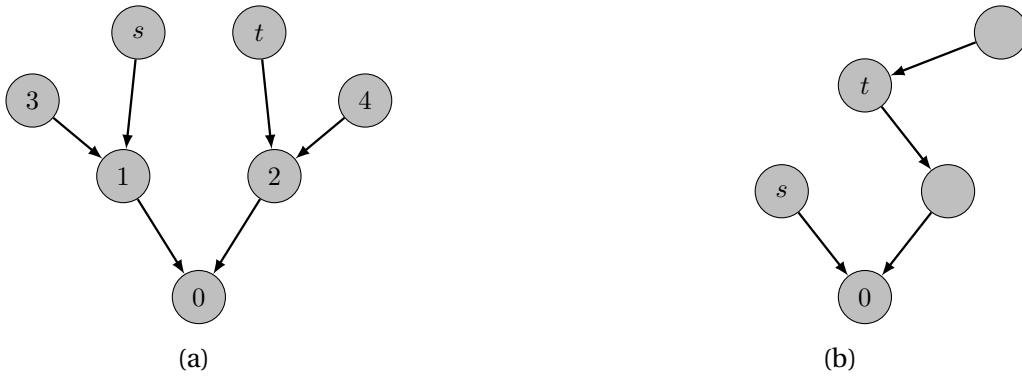


Figure 2. Each vertex corresponds to an industry, with a directed edge present from one vertex to another if the former is an input-supplier to the latter. The vertex indexed 0 represents the household.

Next, note that all industries in this economy are upstream symmetric, and in fact, since no industry has a sticky-price supplier,  $\rho_i = 1$  for all  $i$ . As a result, (41) reduces to

$$\psi_i^{\text{within}} = (1 - \phi_i)\beta_i\theta_i,$$

whereas (40) implies that

$$\psi_i^{\text{o.g.}} = (1/\phi_i - 1)\beta_i \left( \frac{1 - \sum_{j=1}^n \beta_j \phi_j}{\gamma + 1/\eta} \right).$$

Taken together, the above expressions imply that, consistent with the results of Benigno (2004) and Woodford (2010), industries with (i) higher levels of price stickiness and (ii) larger shares in the household's consumption basket receive a larger weight in the optimal price-stabilization policy.

**Example 3.** Next, consider the economy depicted in Figure 2(a), in which industries  $s$  and  $t$  only rely on labor as their input for production ( $\alpha_s = \alpha_t = 1$ ) and are in turn input-suppliers to industries 1 and 2, with  $a_{1s} = a_{2t} = a < 1$ . To isolate the role of the network structure, we in addition impose the symmetry assumptions that  $\phi_s = \phi_t = \phi$  and  $\theta_s = \theta_t = \theta$  and that  $s$  and  $t$  have the same steady-state Domar weight, i.e.,  $\lambda_s = \lambda_t = \lambda$ . Therefore, any heterogeneity across  $s$  and  $t$  only comes from the stickiness of their respective customers 1 and 2 and the latter firms' other suppliers, 3 and 4.

As a first observation, note that since the two industries are identical in size and stickiness and are upstream symmetric, they contribute equally to welfare losses arising from output gap volatility and within-industry misallocation. Not surprisingly then equations (40) and (41) imply that the weights in the optimal policy corresponding to output gap and within-industry misallocation losses are also equal, i.e.,  $\psi_s^{\text{o.g.}} = \psi_t^{\text{o.g.}}$  and  $\psi_s^{\text{within}} = \psi_t^{\text{within}}$ .

The contributions of  $s$  and  $t$  to across-industry misallocation, on the other hand, depend on their downstream supply chains, which are not necessarily identical. In particular, an immediate application of equation (42) in Theorem 2 implies that  $\psi_s^{\text{across}} > \psi_t^{\text{across}}$  if and only if

$$(1 - \phi_1)\rho_1 > (1 - \phi_2)\rho_2. \quad (44)$$

To further clarify the nature of optimal policy, suppose that industries 3 and 4 are equally sticky, so that  $\rho_1 = \rho_2$ . In such a case, the above inequality implies that  $\psi_s^* > \psi_t^*$  if and only if  $\phi_1 < \phi_2$ . In other

words, all else equal, the optimal policy places a larger weight on the industry whose downstream customer is stickier, as prescribed by Proposition 9. This is because by stabilizing the industry with the stickier customer, the policy can reduce the need for the firms in the customer industry to adjust their nominal price, thus effectively reducing the variance of pricing errors by more. A similar argument also illustrates that if firms in industry 1 and 2 are equally sticky (so that  $\phi_1 = \phi_2$ ), then  $\psi_s^* > \psi_t^*$  if and only if  $\phi_3 > \phi_4$ , which guarantees that  $\rho_1 > \rho_2$ .

**Example 4.** Finally, consider the simple economy depicted in Figure 2(b) and suppose that  $s$  and  $t$  are the only industries that are subject to nominal rigidities. To isolate the role of the network structure, we once again impose the symmetry assumptions that  $\phi_s = \phi_t$  and  $\theta_s = \theta_t$ . Additionally, assume that  $s$  only uses labor as an input for production ( $\alpha_s = 1$ ), whereas  $t$  solely relies on intermediate inputs ( $\alpha_t = 0$ ). Under these assumptions, it is immediate that, compared to  $t$ , industry  $s$  has a larger value added share as well as a greater share in the household’s consumption basket. Nonetheless, an immediate application of Theorem 2 implies that industry  $t$  receives a larger weight in the optimal policy whenever it has a larger Domar weight ( $\lambda_t > \lambda_s$ ), highlighting the fact that, as far as size is concerned, what matters for the optimal policy is an industry’s sales share as opposed to its value-added or consumption shares.

## 5 Quantitative Analysis

In this section, we use our results in Section 4 to determine the optimal monetary policy for the U.S. economy as implied by the model.

Our analysis relies on three sources of data. As our first source, we use the 2019 input-output tables constructed by the Bureau of Economic Analysis (BEA) to determine the intermediate input expenditure by various industries. The BEA tables also detail total compensation of employees for each industry as well as each industry’s contribution to final uses.

The second source of data is provided to us by [Pastén, Schoenle, and Weber \(2020b\)](#), who use the confidential micro data underlying the Bureau of Labor Statistics (BLS) producer price index (PPI) to calculate the frequency of price adjustment at the industry level. The PPI measures changes in producers’ selling prices of all goods-producing industries including mining, manufacturing, gas and electricity, as well as retail and service sectors. [Pastén et al. \(2020b\)](#) calculate the frequency of producer price changes as the ratio of the number of price changes to the number of sample months. We use this data to obtain proxies for each industry’s degree of price flexibility.

Our third and final source of data is the March 2021 release of the BEA/BLS Integrated Industry-Level Production Account (ILPA). This dataset contains estimates of industry-level productivities over the 1987–2019 period.

We merge the BEA input-output data at the summary level with price-adjustment data at the 3-digit North American Industry Classification System (NAICS) level, while excluding industries corresponding to federal, state, and local governments. This results in a matched dataset consisting of 66 industries. The mean and median price change frequency across industries are 23.0% and 21.9%, respectively, while the implied cross-sectional mean and median of expected price durations are 5.26

and 4.56 months, respectively. These durations are in line with prior estimates of price frequencies and durations produced by [Bils and Klenow \(2004\)](#) and [Klenow and Kryvtsov \(2008\)](#).<sup>24</sup> We then merge the resulting dataset with the ILPA data to obtain a measure of productivity shocks for each of the 66 industries.<sup>25</sup>

**Calibration.** We interpret each period as a quarter. We calibrate the input-output matrix,  $\mathbf{A}$ , and labor expenditure shares,  $\alpha$ , so as to match the intermediate good expenditure shares and compensation of employees by industry in the BEA input-output data, respectively. We similarly construct the vector of consumption shares,  $\beta$ , to match the share of final uses of each industry’s output.

We then use the ILPA data to calibrate the distribution of productivity shocks in the model. Specifically, we assume that log productivity shocks  $(\log z_1, \dots, \log z_n)$  in the model are jointly normally distributed, with a variance-covariance matrix set equal to the empirical variance-covariance matrix of the detrended productivity series for the 66 industries.

To calibrate the model’s vector of price flexibilities  $(\phi_1, \dots, \phi_n)$ , we assume that firms in each industry are subject to “sticky information” pricing frictions à la [Mankiw and Reis \(2002\)](#). Specifically, we assume that firms in each industry receive a perfectly informative signal about the vector of realized productivity shocks according to a Poisson process with a constant rate. This implies that, in our quarterly calibration, a fraction  $\phi_i = 1 - e^{-3 \times \text{FPA}_i}$  of firms in industry  $i$  receive a perfectly informative signal about the realized productivities, where  $\text{FPA}_i$  is industry  $i$ ’s monthly frequency of price adjustment as measured by [Pastén et al. \(2020b\)](#). On the other hand, a fraction  $1 - \phi_i$  of firms in industry  $i$  receive no signals during a given quarter.<sup>26</sup> To introduce nominal wage rigidities, we add a pseudo-industry that transforms, one-for-one, the labor supplied by the representative household into labor services, which are then sold to the rest of the industries in the economy. We calibrate the extent of nominal wage rigidity according to the estimates of [Beraja, Hurst, and Ospina \(2019\)](#) and set the degree of price flexibility of this pseudo-industry to  $\phi_w = 0.30$ .<sup>27</sup>

We set the within-industry elasticity of substitution,  $\theta$ , equal to 6. This number is consistent with values commonly used in the New Keynesian literature, typically set to match steady-state levels of markups. For example, [Coibion, Gorodnichenko, and Wieland \(2012\)](#) set the elasticity of substitution equal to 7 in order to match steady-state markups of 17% as estimated by [Burnside \(1996\)](#) and [Basu and Fernald \(1997\)](#). Similarly, [McKay, Nakamura, and Steinsson \(2016\)](#) and [Christiano, Eichenbaum, and Rebelo \(2011\)](#) set the elasticity of substitution equal to 6, consistent with steady-state markups of

<sup>24</sup>In particular, [Bils and Klenow \(2004\)](#) report a median duration of prices across all categories to be 4.3 months for posted prices and 5.5 months for regular prices. For posted prices, [Klenow and Kryvtsov \(2008\)](#) report the mean and the median to be 6.8 and 3.7 months, respectively.

<sup>25</sup>The ILPA dataset is slightly more aggregated than the BEA input-output data. Whenever there is a mismatch between the two datasets, we attribute the productivity of the more aggregated industry to its more disaggregated components. Specifically, we attribute the productivity series of ‘Retail trade’ in the ILPA dataset to ‘Motor vehicle and parts dealers’, ‘Food and beverage stores’, ‘General merchandise stores’, and ‘Other retail’; that of ‘Real estate’ to ‘Housing’ and ‘Other real estate’; and that of ‘Hospitals and nursing and residential care facilities’ to ‘Hospitals’ and ‘Nursing and residential care facilities’.

<sup>26</sup>In the notation of Section 4, this is equivalent to assuming that  $\sigma_{ik} = 0$  for a fraction  $\phi_i = 1 - e^{-3 \times \text{FPA}_i}$  of firms industry  $i$  and  $\sigma_{ik} = \infty$  for the rest of the firms in that industry. As discussed in footnote 18, this pricing friction can also be interpreted as the static variant of the Calvo friction.

<sup>27</sup>[Beraja, Hurst, and Ospina \(2019\)](#) estimate that 76 percent of wages adjust during a given year. The implied quarterly degree of wage flexibility is thus  $\phi_w = 1 - (0.24)^{1/4} = 0.30$ .

20%. While we allow for a subsidy in our model that eliminates all steady-state markups, we keep the value of this elasticity in line with the rest of the literature.

Finally, the preference parameters  $\eta$  and  $\gamma$  are chosen as follows. We set the Frisch elasticity of labor supply to  $\eta = 2$ ; this value is consistent with “macro” elasticities of labor supply (Hall, 2009). As for  $\gamma$ , note that intertemporal substitution plays no role in our setting as all choices are static. Nonetheless,  $\gamma$  still controls the household’s wealth effect on labor supply. We thus set  $\gamma = 0.1$ ; this value essentially minimizes the income effect on labor supply, similar to using GHH preferences (Greenwood, Hercowitz, and Huffman, 1988). In Online Appendix C, we provide a series of robustness checks with respect to these parameters values.

**Optimal Monetary Policy.** With the calibrated model in hand, we use equation (39) to obtain the optimal monetary policy implied by the model as well as the associated welfare loss (measured as a percentage of steady-state consumption).<sup>28</sup> We calculate the expected welfare loss due to the presence of nominal rigidities in two different ways. First, we simulate the fully nonlinear model and calculate the resulting average welfare loss relative to the flexible-price economy for 10,000 draws of the vector of productivity shocks. Second, we rely on the closed-form expression in equation (36) to obtain the welfare loss under the model’s quadratic approximation. While the former allows us to calculate the welfare loss taking into account all nonlinearities in the model, the latter provides us with a transparent decomposition of the welfare loss in terms of the various sources of misallocation as well as the labor wedge.

The first column of Table 1 reports the results. We find that the optimal policy generates an expected welfare loss equivalent to a 0.65% loss in quarterly consumption relative to the (unattainable) flexible-price equilibrium. This estimate remains virtually unchanged if instead one uses the quadratic approximation of the model, which predicts a loss equal to 0.67% of consumption under the optimal policy. The largest component of this welfare loss is due to misallocation within industries, accounting for 0.53 percentage points of loss in consumption. The second largest component is due to misallocation across industries, which accounts for another 0.14 percentage points loss in consumption. Finally, as the table indicates, under the optimal policy, there is nearly zero welfare loss due to the third component: volatility of the output gap.

Table 1 also provides a comparison between the performance of the optimal policy and four alternative, non-optimal, price-stabilization policies. The first of these is the policy that minimizes the volatility of output gap. Recall from equation (40) that such a policy weighs industries solely based on their size and stickiness. Specifically, it stabilizes a price index with weights given by  $\psi_i^{o.g.} \propto (1/\phi_i - 1)\lambda_i$  for all  $i$ , where  $\phi_i$  is the degree of price flexibility of industry  $i$  and  $\lambda_i$  is the corresponding Domar weight. As the second column of the table indicates, the policy that stabilizes the output gap generates a welfare loss that is equivalent to a 0.67% fall in quarterly consumption (0.68% under the quadratic approximation). This welfare loss is incredibly similar to that under the optimal policy both in magnitude as well as in decomposition.

<sup>28</sup>While we derived equation (39) in Theorem 2 under the assumption that productivity shocks are independent, we show in Appendix A.2 that the optimal policy’s target price index remains unchanged even if one allows for heteroskedastic and correlated shocks.

Table 1. Expected Welfare Loss under Various Policies

	optimal policy (1)	output-gap stabilization (2)	consumption weighted (3)	Domar weighted (4)	stickiness weighted (5)
Welfare loss (fully nonlinear model)	0.65	0.67	1.95	1.58	3.31
Welfare loss (quadratic approximation)	0.67	0.68	1.08	0.87	1.25
within-industry misallocation	0.53	0.54	0.73	0.64	0.83
across-industry misallocation	0.14	0.14	0.15	0.14	0.14
output gap volatility	$10^{-5}$	0	0.20	0.08	0.27
Cosine similarity to optimal policy	1	0.9931	0.1181	0.1649	0.2749

*Notes:* The table reports the expected welfare loss due to the presence of nominal rigidities under various monetary policies as a percentage of steady-state consumption. The expected welfare loss for the fully nonlinear model is calculated using 10,000 draws. The quadratic approximation of welfare loss and its various components are obtained in accordance with the decomposition in equation (36). The last row reports the cosine similarity between each policy and the optimal policy.

The third and fourth columns of Table 1 present welfare losses corresponding to two policies that weigh sectors by size. The first of these policies, which is akin to targeting CPI or PCE, stabilizes the household’s consumption price index, with weights that are equal to consumption shares:  $\psi_i^{\text{cons}} = \beta_i$ . The second policy weighs sectors not by consumption shares but by their sales shares:  $\psi_i^{\text{Domar}} \propto \lambda_i$ . As is evident from the table, and in contrast to the output-gap-stabilization policy, these policies result in materially larger welfare losses compared to the optimal policy: consumption-weight-based price stabilization leads to a welfare loss equivalent to a 1.95% fall in quarterly consumption, while the Domar-weights-based price stabilization results in a welfare loss equivalent to a 1.58% reduction in quarterly consumption.<sup>29</sup>

Finally, the last column of Table 1 reports the expected welfare loss under a policy that targets a price index with weights proportional to the industries’ degree of price stickiness, i.e.,  $\psi_i^{\text{sticky}} \propto 1 - \phi_i$ , where  $\phi_i$  is the degree of price flexibility of industry  $i$ . We find that such a policy also generates a significantly larger welfare loss compared to the optimal policy.

**Approximate Optimality of Output-Gap Stabilization.** As already discussed, the welfare differences between the optimal and output-gap-stabilization policies in our calibration are minuscule, amounting to roughly 0.02 percentage points of quarterly consumption. Crucially, the industry-specific weights in the implied target price indices of the two policies are also similar. This similarity can be seen from Figure 3, which plots the weights corresponding to the two policies side by side (aggregated to sectoral level and normalized such that the weights in each policy add up to one).

To quantify the similarity between various policies, the last row of Table 1 reports the cosine

<sup>29</sup>Unlike the optimal and output-gap stabilization policies—for which the estimates obtained using the quadratic approximation are very close to the those obtained by simulating the fully nonlinear model—the quadratic approximation underestimates the expected welfare loss under the consumption- and Domar-weights-based stabilization policies.



similarities between the optimal policy and each of the four alternative policies. According to this measure—which is equal to the cosine of the angle between the two vectors representing the weights in the policies’ corresponding target price indices—the optimal and the output-gap-stabilization policies are very similar, with a cosine similarity that exceeds 99%.<sup>30</sup> It is thereby no surprise that the difference in welfare between the two is negligible. For comparison, the cosine similarity between the optimal policy and any of the other three policies never exceeds 28%.

It is important to note that, while the optimal and output-gap-stabilization policies result in nearly identical expected welfare losses, this does not mean that losses due to misallocation are immaterial. In fact, as Table 1 illustrates, the bulk of the loss under either policy is due to price dispersion within industries. Rather, the similarity between the optimal and output-gap-stabilization policies is driven by the fact that, in our calibration, the components of optimal policy aimed at minimizing the losses from output-gap volatility and misallocation are more or less aligned with one another. This, in turn, is due to two features of our calibration. First, the fact that the within-industry elasticity of substitution,  $\theta = 6$ , is larger than the unit elasticity of substitution across industries (because of Cobb-Douglas technologies) means that losses due to across-industry misallocation are quantitatively less important than those due to within-industry misallocation.<sup>31</sup> Second, the fact that all within-industry elasticities of substitution are assumed to be identical implies that the component of optimal policy aimed at minimizing within-industry price dispersion (i.e.,  $\psi^{\text{within}}$  in (41)) does not discriminate across industries based on these elasticities and instead ends up weighing sectors based on their size and stickiness—similar to  $\psi^{\text{o.g.}}$ . As a result, even though  $\psi^{\text{within}}$  and  $\psi^{\text{o.g.}}$  are not identical, they both assign higher weights to larger and stickier industries, making the two major components of optimal policy more or less aligned with one another, with a cosine similarity of  $\cos(\psi^{\text{o.g.}}, \psi^{\text{within}}) = 97\%$ .

We conclude by noting that, as indicated by Figure 3, the target price indices under both the optimal and the output-gap-stabilization policies place the greatest weight on the nominal wage. This reflects the facts that (i) labor has the largest Domar weight (equal to 1) and (ii) in our calibration, nominal wage is stickier than most industries’ prices. The large weight assigned to the nominal wage by the optimal and the output-gap-stabilization policies is one of the key differences between these policies and the policy that stabilizes the household’s consumption price index (which assigns no weight on the nominal wage).<sup>32</sup>

Aside from the nominal wage, the optimal and the output-gap-stabilization policies also assign large weights on certain service industries (‘Miscellaneous professional, scientific, and technical services’ and ‘Administrative and support services’) as well as health care (‘Hospitals’) and real estate (‘Other real estate’). The reason these particular industries command such large weights is due to the fact that they exhibit large Domar weights but low price flexibility. The service and health care sectors

<sup>30</sup>Specifically, if  $\psi$  and  $\hat{\psi}$  denote the vectors representing the weights in two policies’ corresponding target price indices, the cosine similarity between the two policies is given by  $\cos(\psi, \hat{\psi}) = \psi' \hat{\psi} / \|\psi\|_2 \|\hat{\psi}\|_2$ . When  $\psi$  and  $\hat{\psi}$  are element-wise non-negative,  $\cos(\psi, \hat{\psi})$  is always between 0 and 1, and reaches its maximum value of 1 if and only if  $\psi$  and  $\hat{\psi}$  are proportional.

<sup>31</sup>This is also reflected in the fact that while, on average, the term corresponding to within-industry misallocation ( $\psi^{\text{within}}$ ) accounts for 36.8% of the optimal weight  $\psi^*$ , the term corresponding to across-industry misallocation ( $\psi^{\text{across}}$ ) is, on average, responsible for only 3.4% of the optimal weight.

<sup>32</sup>That the optimal policy assigns even a larger weight on the nominal wage than the output-gap-stabilization policy is in turn a consequence of the fact that labor is upstream to all other industries (consistent with Proposition 10) and does not rely on any sticky upstream suppliers (Proposition 8).

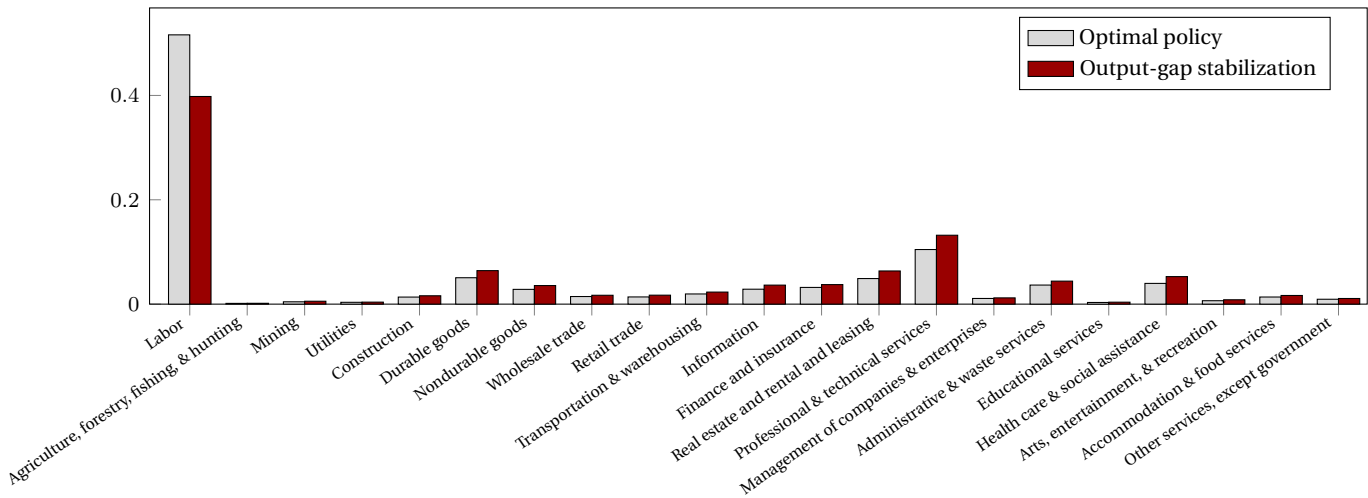


Figure 3. Industry Weights under Optimal and Output-Gap-Stabilization Policies

*Notes:* The figure plots the weights in the price indices corresponding to the optimal and output-gap-stabilization policies. The weights for each policy are aggregated to sectoral level and are normalized such that they add up to 1.

in particular comprise a generous portion of the economy but are relatively sticky.<sup>33</sup> In contrast, the optimal policy assigns relatively small weights to industries such as ‘Apparel’, ‘Printing’, and ‘Waste Management’, which, despite being relatively sticky, do not exhibit large Domar weights, making them less of a priority for stabilization.

## 6 Conclusion

In this paper, we study the optimal conduct of monetary policy in a multi-sector economy in which firms buy and sell intermediate goods over a production network. We introduce nominal rigidities into a rather canonical multi-sector, input-output model along the lines of [Long and Plosser \(1983\)](#) and [Acemoglu et al. \(2012\)](#) by assuming that firms make nominal pricing decisions under incomplete information about the aggregate state.

Within the context of this model, we make two theoretical contributions. First, we obtain necessary and sufficient conditions on the economy’s disaggregated production structure and the nature of nominal rigidities under which monetary policy can implement flexible-price equilibria and hence restore productive efficiency. As an important byproduct of this result, we also show that these conditions are violated for a generic set of information structures, thus concluding that, generically, monetary policy cannot achieve productive efficiency. This is in stark contrast to the canonical one-sector New Keynesian model in which, in the absence of markup shocks, the efficient allocation can be implemented with price stability.

Given that the first-best allocation is generically unattainable, our second theoretical contribution is thereby to characterize the optimal policy and to provide general principles for the optimal conduct of monetary policy in the presence of input-output linkages. In particular, we show that the optimal

<sup>33</sup>See [Bils and Klenow \(2004\)](#), [Dhyne et al. \(2006\)](#), and [Gorodnichenko and Weber \(2016\)](#) for more evidence on the stickiness of health care and service sectors in the cross-section (both in the U.S. and in Europe).

policy faces a trade-off between three components of welfare loss: misallocation across industries, misallocation within industries, and volatility of the output gap. We find that the optimal monetary policy is a price-index-stabilization policy with higher weights assigned to larger, stickier, and more upstream industries, as well as industries with less sticky upstream suppliers but stickier downstream customers.

Finally, in a quantitative application of our framework, we determine the optimal price index for the U.S. economy and find that moving from a policy that targets the household's consumption price index (i.e., a policy that is akin to targeting CPI or PCE) to the optimal price index can result in significant welfare gains. At the same time, we also find that, in our calibration, the difference in welfare loss under the optimal policy and under the policy that stabilizes the output gap is rather negligible.

Our theoretical and quantitative results can inform the policy debate around the appropriate price level the central bank should target (Mishkin, 2007; Bullard, 2011; Thornton, 2011). There are numerous measures of the aggregate price level; the indices most often considered by policymakers are overall measures of consumer prices (the CPI or the PCE), measures of consumer prices that exclude food and energy categories (core CPI or core PCE), as well as measures of producer prices (the PPI). On the theoretical side, our results provide a formal framework to account for the disaggregated nature of production in designing the proper target index. On the quantitative side, the near optimality of the output-gap-stabilization policy indicate that inflation measures that discount flexible-price sectors but also weigh sectors by their sales shares are desirable stabilization targets.

We view our paper as a step towards exploring the implications of the disaggregated nature of production for the transmission and the optimal conduct of monetary policy. Several important issues, however, remain open for future research. First, as emphasized throughout the paper, we assumed that the underlying flexible-price allocation in our economy is efficient. While this was a conscious modeling decision made in order to isolate how the multi-sector, input-output feature of our economy fundamentally changes the policy prescriptions of one-sector New Keynesian models, the role of monetary policy would be more complicated in an economy with an inefficient steady-state, as the monetary policy faces an additional trade-off between stabilizing prices and substituting for missing tax instruments. Exploring the implications of such a trade-off for the conduct of monetary policy would be a natural next step.

Second, a growing empirical literature has documented the propagation of real shocks—such as natural disasters (Carvalho et al., 2021), trade shocks (Huneus, 2019), and demand shocks (Acemoglu et al., 2016)—over input-output linkages. Similar empirical investigations on the production network's role as a monetary transmission mechanism, along the lines of Ozdagli and Weber (2021), would shed further light on how monetary policy can shape real economic outcomes.

## A Appendix

### A.1 Propagation of Productivity and Monetary Shocks

In this appendix, we characterize the propagation of real and monetary shocks over the economy's production network under a general specification of nominal rigidities modeled as information

frictions. As in Section 4, we focus our analysis on the class of Cobb-Douglas economies by assuming that the production technology of firms in industry  $i$  and the household consumption bundle are given by (19) and (20), respectively. We also assume logarithmic utilities ( $\gamma = 1$ ) and a fully elastic labor supply ( $\eta \rightarrow \infty$ ). In contrast to Section 4, however, we do not impose any restrictions on the information structure. Finally, as is standard, we consider the log-linearization of the economy as  $\delta \rightarrow 0$ , where  $\delta$  parameterizes the standard deviation of productivity shocks. Specifically, we assume that  $\text{var}_{ik}(\log z) = O(\delta^2)$  for all firms in the economy.

We start with a definition. Denote the economy's input-output matrix by  $\mathbf{A}$  and let  $\mathbb{E}_{ik}[\cdot]$  denote the expectation with respect to the information set of firm  $k$  in industry  $i$ . For any vector  $t = (t_1, \dots, t_n)'$  and any integer  $r \geq 1$ , define

$$\bar{\mathbb{E}}_i^{(r+1)}[t] = \sum_{j=1}^n a_{ij} \int_0^1 \mathbb{E}_{ik} \bar{\mathbb{E}}_j^{(r)}[t] dk, \quad (\text{A.1})$$

with the initial condition  $\bar{\mathbb{E}}_i^{(1)}[t] = \int_0^1 \mathbb{E}_{ik}[t_i] dk$ . In words,  $\bar{\mathbb{E}}_i^{(r+1)}[\cdot]$  is the cross-sectional average expectation of firms in industry  $i$  of their suppliers' expectations in the previous iteration,  $\bar{\mathbb{E}}_j^{(r)}[\cdot]$ , with weights given by (steady-state) expenditure shares,  $a_{ij}$ . The expression in (A.1), which is similar to the iterated expectations operator of Golub and Morris (2018), captures firms' higher-order average expectations, with a larger  $r$  corresponding to a higher order of iterated expectations. As is evident from (A.1), these expectations depend on the interaction between the production network and the information structure.<sup>34</sup> We have the following result:

**Proposition A.1.** *Aggregate output satisfies*

$$\log C = \sum_{r=1}^{\infty} \sum_{i=1}^n \beta_i \bar{\mathbb{E}}_i^{(r)}[\log z] + \sum_{i=1}^n \beta_i \left( \log m - \sum_{r=1}^{\infty} \bar{\mathbb{E}}_i^{(r)}[\alpha \log m] \right) + o(\delta), \quad (\text{A.2})$$

where  $\bar{\mathbb{E}}_i^{(r)}[\cdot]$  denotes the  $r$ -th order average expectations of firms in industry  $i$  as defined in (A.1).

This result expresses (log) aggregate output in terms of the economy's production network and information structure. It also characterizes the impact of real and monetary shocks on output: the first term on the right-hand side of (A.2) captures the aggregate impact of productivity shocks, whereas the second term captures the impact of monetary shocks (and hence, the degree of monetary non-neutrality). The key observation is that both terms depend not only on firms' first-order expectations, but also on their higher-order expectations (and in particular, on firms' expectations of their suppliers' expectations, firms' expectations of their suppliers' expectations of their suppliers' expectations, and so on). The dependence on these iterated expectations reflects the strategic complementarities in firms' price-setting behavior discussed in Subsection 4.1: firms set their nominal prices based on their expectations of their nominal marginal costs, which in turn depends on their suppliers' expectations.

To see the implications of Proposition A.1, it is instructive to consider the two stylized production networks depicted in Figure 1. For simplicity, we assume that all firms within each industry have access

<sup>34</sup>For example, in the special case that the economy's information structure is given by (22) and (23), the vector of average iterated expectations of order  $r$  of log productivity shocks is equal to  $\mathbb{E}^{(r)}[\log z] = \Phi(\mathbf{A}\Phi)^{r-1} \log z$ , where  $\Phi = \text{diag}(\phi_1, \dots, \phi_n)$  is the diagonal matrix of price flexibilities.

to the same information. Starting with the horizontal production network in Example 2, it follows from (A.2) that aggregate output is given by

$$\log C = \sum_{i=1}^n \beta_i \mathbb{E}_i[\log z_i] + \sum_{i=1}^n \beta_i (\log m - \mathbb{E}_i[\log m]) + o(\delta), \quad (\text{A.3})$$

where  $\beta_i$  is the steady-state share of good  $i$  in the household's consumption bundle (which is also equal to  $i$ 's steady-state Domar weight). A few observations are immediate. First, since there are no input-output linkages in the horizontal economy, aggregate output only depends on the firms' first-order expectations. Second, the impact of idiosyncratic productivity shocks to industry  $i$  depends not only on  $i$ 's Domar weight—as would have been the case in the absence of nominal rigidities—but also on  $i$ 's estimate of the shocks' realizations.<sup>35</sup> Finally, (A.3) indicates that a monetary shock would have a greater impact on aggregate output the more uncertain the firms are about its realization. This can be seen by regressing  $\log C$  on  $\log m$ , which results in a slope coefficient that is equal to  $\sum_{i=1}^n \beta_i \mathbb{E}[\text{var}_i(\log m)] / \text{var}(\log m)$ .<sup>36</sup>

Next, consider the vertical economy in Example 1. In this case, (A.2) implies that

$$\log C = \sum_{i=1}^n \mathbb{E}_1 \mathbb{E}_2 \dots \mathbb{E}_i[\log z_i] + (\log m - \mathbb{E}_1 \mathbb{E}_2 \dots \mathbb{E}_n[\log m]) + o(\delta).$$

In contrast to the horizontal economy, the aggregate impacts of productivity and monetary shocks in this economy also depend on firms' higher-order expectations. This, of course, is a consequence of the presence of input-output linkages in the vertical economy and the resulting strategic complementarities in firms' price-setting behavior. Furthermore, as discussed in Subsection 4.1, they translate into more sluggish adjustments in nominal prices and increase the degree of monetary non-neutrality.

**Proposition A.2.** *The vector of industry-level log outputs is given by*

$$\begin{aligned} \log y = \log y^* - (\mathbf{A} + \mathbf{\Lambda}^{-1} \mathbf{L}' \mathbf{\Lambda} (\mathbf{I} - \mathbf{A})) & \left( \mathbf{L} \log z - \sum_{r=1}^{\infty} \bar{\mathbb{E}}^{(r)}[\log z] \right) \\ & + (\mathbf{A} + \mathbf{\Lambda}^{-1} \mathbf{L}' \mathbf{\Lambda} (\mathbf{I} - \mathbf{A})) \left( \mathbf{1} \log m - \sum_{r=1}^{\infty} \bar{\mathbb{E}}^{(r)}[\alpha \log m] \right) + o(\delta). \end{aligned} \quad (\text{A.4})$$

where  $y^*$  denotes the vector of outputs under flexible prices,  $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_n)$  is the diagonal matrix of Domar weights, and  $\bar{\mathbb{E}}^{(r)}[t]$  is a vector whose  $i$ -th element is given by  $\bar{\mathbb{E}}_i^{(r)}[t]$ .

This result, which is the industry-level counterpart to Proposition A.1, characterizes the impact of real and monetary shocks on sectoral output in the presence of nominal rigidities. Specifically, the second term on the right-hand side of (A.4) captures how nominal rigidities distort the propagation

<sup>35</sup>More specifically, regressing  $\log C$  on  $\log z_i$  results in a slope coefficient that is equal to  $\beta_i \text{var}[\mathbb{E}_i(\log z_i)] / \text{var}(\log z_i)$ . This coefficient is always less than or equal to industry  $i$ 's Domar weight ( $\beta_i$ ) and is equal to it only when firms in industry  $i$  face no uncertainty about the realization of shocks to  $i$  (i.e., when  $\text{var}_i(\log z_i) = 0$ ).

<sup>36</sup>This coefficient is always in the unit interval, and is equal to zero if and only if all firms have perfect information about the realization of the monetary shock. More generally, the slope coefficient of the regression of  $\log C$  on  $\log m$  extends the measure of monetary non-neutrality in (33) to economies with arbitrary information structures.

of productivity shocks compared to the flexible-price economy, whereas the third term captures the extent to which monetary shocks propagate over the production network. As in (A.2), both terms in general depend not only on firms' first-order expectations, but also on their expectations of higher orders.

## A.2 Heteroskedastic and Correlated Shocks

In Section 4, we assumed that productivity shocks  $(z_1, \dots, z_n)$  are independent and identically distributed. In this appendix, we derive the optimal monetary policy while allowing for heteroskedastic and correlated shocks. In particular, we generalize (22) by assuming that log productivity shocks are jointly normally distributed according to

$$\log z \sim \mathcal{N}(0, \delta^2 \Sigma), \quad (\text{A.5})$$

where  $\delta > 0$  is a normalization constant and  $\Sigma$  is an  $n \times n$  positive definite matrix that parameterizes the variance-covariance matrix of the shocks. The assumption that  $\Sigma$  is positive definite is trivially satisfied if the productivity shock to each industry has some idiosyncratic component. In order to keep the analysis tractable, we focus on “sticky information” pricing frictions à la Mankiw and Reis (2002), according to which firms in each industry can either set their prices flexibly with no frictions or face full nominal rigidity. More specifically, we assume that fraction  $\phi_i$  of firms in industry  $i$  receive perfectly informative signals about the realized productivity shocks, while the remaining  $1 - \phi_i$  fraction of firms receive no information.<sup>37</sup> Thus, as in Section 4,  $\phi_i$  captures the degree of price flexibility of industry  $i$ . We have the following result:

**Proposition A.3.** *Suppose productivity shocks are distributed according to (A.5). The optimal monetary policy is a price-stabilization policy of the form  $\sum_{s=1}^n \psi_s^* \log p_s = 0$ , where  $\psi_s^*$  is given by (39).*

Proposition A.3 establishes that optimal monetary policy is invariant to the distribution of shocks, as the target price-stabilization index  $\sum_{s=1}^n \psi_s^* \log p_s$  remains the same as the one in Theorem 2. This is a consequence of the assumption that the monetary authority can set nominal aggregate demand as a function of the realized productivities,  $(z_1, \dots, z_n)$ . That the policy can be indexed to the aggregate state of the economy means that the monetary authority can minimize the welfare loss due to the presence of nominal rigidities state-by-state, irrespective of the ex ante distribution of shocks.

## A.3 Information Frictions on the Monetary Authority

The characterization of the optimal policy in Theorem 2 relies on the assumption that the monetary authority can set nominal aggregate demand as a function of the realized productivity shocks. In this appendix, we relax this assumption by assuming that the monetary authority only has imperfect information about the shocks' realizations.

Specially, we assume that the monetary authority observes a collection of signal  $(\hat{\omega}_1, \dots, \hat{\omega}_n)$  given by

$$\hat{\omega}_i = \log z_i + \hat{\epsilon}_i \quad , \quad \hat{\epsilon}_i \sim \mathcal{N}(0, \delta^2 \hat{\sigma}^2), \quad (\text{A.6})$$

<sup>37</sup>In the notation of Section 4, this is equivalent to assuming that  $\sigma_{ik} = 0$  for a fraction  $\phi_i$  of firms industry  $i$  and  $\sigma_{ik} = \infty$  for the rest of the firms in that industry. This is the pricing friction we assume in our quantitative analysis in Section 5.

where  $\hat{\sigma}$  parameterizes the monetary authority’s uncertainty about the shocks’ realizations and the noise terms  $(\hat{\epsilon}_1, \dots, \hat{\epsilon}_n)$  are independent from one another and the productivity shocks. As in Section 4, we assume that all productivity shocks are drawn independently according to (22), and we characterize the optimal policy to a first-order approximation as  $\delta \rightarrow 0$ . In order to keep the analysis tractable, we focus on “sticky information” pricing frictions à la Mankiw and Reis (2002), according to which fraction  $\phi_i$  of firms in each industry  $i$  have complete information about the realization of all shocks—and hence can set their prices flexibly with no frictions—while the remaining  $1 - \phi_i$  fraction face full nominal rigidity. We also assume logarithmic utilities ( $\gamma = 1$ ) and a fully elastic labor supply ( $\eta \rightarrow \infty$ ).

**Proposition A.4.** *Suppose the monetary authority is subject to information frictions and has access to signals given by (A.6). The optimal monetary policy minimizes the volatility of the price index  $\sum_{s=1}^n \psi_s^* \log p_s$ , where  $\psi_s^*$  is given by (39).*

When the monetary authority cannot set nominal aggregate demand as a function of the realized productivity shocks, it is no longer able to implement the price-stabilization policy in Theorem 2. Nonetheless, Proposition A.4 establishes that the nature of optimal policy remains unchanged: (i) instead of fully stabilizing a target price index, the optimal policy minimizes the volatility of a target price index and (ii) this target price index is the same as the target in Theorem 2.

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## B Online Appendix: Proofs and Derivations

This appendix contains the proofs and derivations omitted from the main body of the paper. Throughout, with some abuse of notation, we write  $\text{diag}(x)$  to denote a diagonal matrix whose entries are equal to vector  $x$ , while we use  $\text{diag}(\mathbf{X})$  to denote a (column) vector whose elements are equal to the diagonal elements of matrix  $\mathbf{X}$ .

### Proof of Proposition 2

We start by establishing equation (16). As a first observation, note that the optimality conditions of the representative household's problem are given by

$$V'(L(s)) = \mu(s)w(s) \quad (\text{B.1})$$

$$U'(C(s)) \frac{\partial \mathcal{C}}{\partial c_i}(s) = \mu(s)p_i(\omega_i) \quad \text{for all } i, \quad (\text{B.2})$$

where  $\mu(s)$  is the Lagrange multiplier corresponding to the household's budget constraint. As a result,

$$V'(L(s)) = \frac{w(s)}{p_i(\omega_i)} U'(C(s)) \frac{\partial \mathcal{C}}{\partial c_i}(s). \quad (\text{B.3})$$

Multiplying both sides of the above equation by  $p_i(\omega_i)c_i(s)$  and summing over all  $i$ , we obtain

$$w(s) = m(s) \frac{V'(L(s))}{C(s)U'(C(s))}, \quad (\text{B.4})$$

where we are using the assumption that the consumption aggregator  $\mathcal{C}(\cdot)$  is a homogenous function of degree 1 and the fact that  $\sum_{i=1}^n p_i(\omega_i)c_i(s) = m(s)$ . Next, note that cost minimization by firms in industry  $i$  implies that<sup>38</sup>

$$mc_i(s) = w(s) (z_i \cdot \partial F_i / \partial l_i(s))^{-1}. \quad (\text{B.5})$$

Replacing  $w(s)$  from (B.4) into the above equation establishes (16).

We next show that if a feasible allocation is implementable as a sticky-price equilibrium, it must satisfy (13). Consider the price-setting problem of firm  $k$  in industry  $i$ , which sets its nominal price to maximize the expected value of its real profits:

$$\max_{p_{ik}(\omega_{ik})} \mathbb{E}_{ik} \left[ \frac{U'(C(s))}{P(s)} \left( (1 - \tau_i) p_{ik}(\omega_{ik}) y_{ik}(s) - mc_i(s) y_{ik}(s) \right) \right] \quad (\text{B.6})$$

$$\text{s.t.} \quad y_{ik}(s) = (p_{ik}(\omega_{ik}) / p_i(\omega_i))^{-\theta_i} y_i(s). \quad (\text{B.7})$$

In the above problem,  $U'(C(s))$  is the representative household's marginal utility of consumption and  $P(s)$  is the nominal price of the consumption good bundle. Consequently, the nominal price set by the firm satisfies the following first-order condition:

$$\mathbb{E}_{ik} \left[ U'(C(s)) \frac{\partial \mathcal{C}}{\partial c_i}(s) y_i(s) \left( \frac{p_{ik}(\omega_{ik})}{p_i(\omega_i)} \right)^{1-\theta_i} \left( (1 - \tau_i) \left( \frac{\theta_i - 1}{\theta_i} \right) - \frac{mc_i(s)}{p_{ik}(\omega_{ik})} \right) \right] = 0,$$

<sup>38</sup>The assumption that labor is an essential input in the production technology of all goods guarantees that  $\partial F_i / \partial l_i(s) > 0$  for all  $i$ . Furthermore, note that the realized marginal cost of all firms in the same industry are identical, i.e.,  $mc_{ik}(s) = mc_i(s)$  for all  $s \in S$  and all firms  $k$  in industry  $i$ . This latter observation implies that  $\partial F_i / \partial l_{ik}(s)$  is the same for all firms  $k$  in industry  $i$ . We therefore drop the firm index  $k$  in (B.5).

where we are using the fact that  $\partial\mathcal{C}/\partial c_i(s) = p_i(\omega_i)/P(s)$ .<sup>39</sup> Using (B.7) to express  $p_{ik}(\omega_{ik})/p_i(\omega_i)$  in terms of quantities then implies that

$$p_{ik}(\omega_{ik}) = \left[ (1 - \tau_i) \left( \frac{\theta_i - 1}{\theta_i} \right) \right]^{-1} \frac{\mathbb{E}_{ik}[v_{ik}(s) mc_i(s)]}{\mathbb{E}_{ik}[v_{ik}(s)]}, \quad (\text{B.8})$$

where  $v_{ik}(s)$  is given by (17). Consequently, the nominal price set by the firm satisfies

$$p_{ik}(\omega_{ik}) = \frac{1}{\chi_i^s \varepsilon_{ik}(s)} mc_i(s), \quad (\text{B.9})$$

where  $\chi_i^s = (1 - \tau_i)(\theta_i - 1)/\theta_i$  is a wedge that arises due to government taxes/subsidies and monopolistic markups, whereas  $\varepsilon_{ik}(s)$  is given by (15) and represents a wedge that arises due to the presence of nominal rigidities. Combining (B.9) with (B.3) and (B.7), we obtain

$$V'(L(s)) = \chi_i^s \varepsilon_{ik}(s) U'(C(s)) \frac{\partial\mathcal{C}}{\partial c_i}(s) \frac{w(s)}{mc_i(s)} \left( \frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i}.$$

Replacing for  $mc_i(s)$  from (B.5) into the above equation then establishes (13).

Finally, to establish (14), recall that the representative household's first-order optimality condition requires that (B.2) is satisfied for all  $i$ . As a result,

$$\frac{\partial\mathcal{C}}{\partial c_j}(s) = \frac{p_j(\omega_j)}{p_i(\omega_i)} \frac{\partial\mathcal{C}}{\partial c_i}(s) = \chi_i^s \varepsilon_{ik}(s) \frac{\partial\mathcal{C}}{\partial c_i}(s) \frac{p_j(\omega_j)}{mc_i(s)} \left( \frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i},$$

for all pairs of industries  $i$  and  $j$ , where once again we are using (B.7) and (B.9). Furthermore, whenever industry  $j$  is an input-supplier of industry  $i$ , cost minimization by firm  $k$  in industry  $i$  implies that

$$mc_i(s) = p_j(\omega_j) (z_i \cdot \partial F_i / \partial x_{ij,k}(s))^{-1}.$$

The juxtaposition of the last two equations establishes (14). □

## Proof of Proposition 1

We establish this result as a special case of Proposition 2. Recall from Definitions 1 and 2 that a flexible-price equilibrium can be cast as a sticky-price equilibrium with an information structure under which the state  $s = (z, \omega)$  is measurable with respect to the information set of all firms. As a result, in any flexible-price equilibrium, the right-hand side of (16) is measurable with respect to the information set of firm  $k$  in industry  $i$ , which in turn implies that  $\mathbb{E}_{ik}[mc_i(s)] = mc_i(s)$ . Consequently, (15) implies that  $\varepsilon_{ik}(s) = 1$  for all  $k \in [0, 1]$ , all  $i \in I$ , and all  $s \in S$ . Plugging this into (13) and (14) and using the fact that all firms in the same industry set identical prices—and hence have identical outputs—then establishes (10) and (11). □

<sup>39</sup>This is a consequence of the household's optimization problem. Specifically, multiplying both sides of (B.2) by  $c_i(s)$  and summing over all  $i$  implies that  $\mu(s) = U'(C(s))/P(s)$ , where we are using the assumption that  $\mathcal{C}(\cdot)$  is a homogenous function of degree 1. Plugging this back into (B.2) then implies that  $\partial\mathcal{C}/\partial c_i(s) = p_i(\omega_i)/P(s)$ .

## Proof of Theorem 1

We prove the necessity claim as the converse claim is straightforward. Suppose there exists a flexible-price allocation indexed by  $(\chi_1^f, \dots, \chi_n^f)$  that is also implementable as a sticky-price equilibrium. By Propositions 1 and 2, such an allocation simultaneously satisfies equations (10)–(11) and (13)–(14). As a result,

$$\chi_i^f = \chi_i^s \varepsilon_{ik}(s) \left( \frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i}$$

for all  $(i, k) \in I \times [0, 1]$  and all  $s \in S$ , where the wedge function  $\varepsilon_{ik}(s)$  is given by (15). In any flexible-price allocation, the outputs of all firms in the same industry are identical, i.e.,  $y_{ik}(s) = y_i(s)$  for all  $k \in [0, 1]$ . Therefore,

$$\chi_i^f = \chi_i^s \varepsilon_{ik}(s)$$

for all  $(i, k) \in I \times [0, 1]$  and all  $s \in S$ . Note that, by assumption, scalars  $\chi_i^f$  and  $\chi_i^s$ , which are determined by fiscal policy, are invariant to the state  $s$  and do not depend on the firm index  $k$ . Therefore, the above equation holds only if  $\varepsilon_{ik}(s)$  is also independent of  $s$  and  $k$  for all  $i$ , i.e.,  $\varepsilon_{ik}(s) = \varepsilon_i$ .

Next, note that (15) guarantees that  $\mathbb{E}_{ik}[v_{ik}(s)(\varepsilon_{ik}(s) - 1)] = 0$ , where  $v_{ik}(s)$  is given by (17). Since  $\varepsilon_{ik}(s) = \varepsilon_i$ , this implies that

$$(\varepsilon_i - 1)\mathbb{E}_{ik}[v_{ik}(s)] = 0$$

for all  $i$ . But since  $v_{ik}(s) > 0$  for all  $s \in S$  in any feasible allocation, it must be the case that  $\varepsilon_i = 1$  for all  $i$ . As a result,  $\chi_i^s = \chi_i^f$  for all  $i$ . Furthermore, recall from the proof of Proposition 2 that, in any sticky-price equilibrium, each firm's price and marginal cost are related to one another via (B.9). Therefore,

$$p_{ik}(\omega_{ik}) = \frac{1}{\chi_i^f} \text{mc}_i(s). \quad (\text{B.10})$$

Equation (B.10) has three implications. First, given that its right-hand side is independent of  $k$ , it implies that all firms within the same industry set the same nominal price. Thus, we can write  $p_{ik}(\omega_{ik}) = p_i(\omega_i)$ , with the understanding that  $p_i(\omega_i)$  is measurable with respect to the information set of all firms  $k$  in industry  $i$ . Second, (B.10) also implies that the marginal cost of industry  $i$  is measurable with respect to the information set of all firms in that industry. Finally, it establishes that, whenever an allocation can be implemented as both a sticky- and a flexible-price equilibrium, all firms employ constant markups to set their nominal prices. Consequently, we can write  $i$ 's nominal price as a function of industry  $i$ 's nominal input prices as

$$p_i(\omega_i) = \frac{1}{\chi_i^f z_i} K_i(w(s), p_1(\omega_1), \dots, p_n(\omega_n)), \quad (\text{B.11})$$

where  $K_i(\cdot)$  is a homogenous function of degree 1 and represents the cost function of firms in industry  $i$ . Dividing both sides of the above equation by the nominal wage leads to

$$p_i(\omega_i)/w(s) = \frac{1}{\chi_i^f z_i} K_i(1, p_1(\omega_1)/w(s), \dots, p_n(\omega_n)/w(s)). \quad (\text{B.12})$$

We thus obtain a system of  $n$  equations and  $n$  unknowns that relates all industries' nominal prices relative to the nominal wage to productivity shocks  $(z_1, \dots, z_n)$  and fiscal policy wedges  $(\chi_1^f, \dots, \chi_n^f)$ . Since, by assumption, labor is an essential input for the production technology of all industries, Theorem 1 of [Stiglitz \(1970\)](#) guarantees that there exists a unique non-zero collection of relative prices that solves the system of equations in (B.12). In particular, for any industry  $i$ , there exists a unique function  $h_i : \mathbb{R}_+^n \rightarrow \mathbb{R}_+$  such that

$$p_i(\omega_i) = w(s)h_i(\chi_1^f z_1, \dots, \chi_n^f z_n), \quad (\text{B.13})$$

where  $(h_1, \dots, h_n)$  solves the following system of equations:

$$h_i(z) = \frac{1}{z_i} K_i(1, h_1(z), \dots, h_n(z)) \quad \text{for all } i. \quad (\text{B.14})$$

As we already established, the left-hand side of the (B.13) is measurable with respect to the information set of all firms in industry  $i$ . Therefore, a feasible allocation is implementable as an equilibrium under both flexible and sticky prices only if there exists a nominal function  $w(s)$  such that

$$w(s)h_i(\chi_1^f z_1, \dots, \chi_n^f z_n) \in \sigma(\omega_{ik})$$

simultaneously for all firms  $k$  in all industries  $i$ .

The proof is therefore complete once we show that  $h_i(z) = 1/g_i(z)$ , where  $g_i(z)$  is the marginal product of labor in the production of good  $i$  (as a function of realized productivity shocks) under the first-best allocation. To establish this, we make two observations. First, note that the system of equations in (B.14) is identical to marginal-cost pricing conditions in the economy's competitive equilibrium. Since, as we already discussed, this system of equations has a unique non-zero solution,  $h_i$  has to be equal to the marginal cost of good  $i$  relative to the wage in the economy's competitive equilibrium. Second, cost minimization implies that  $i$ 's marginal product of labor is equal to the wage divided by the marginal cost of  $i$ . Putting these two observations together implies that  $h_i$  is equal to the reciprocal of  $i$ 's marginal product of labor in the economy's competitive equilibrium. Finally, the fact that the economy's competitive equilibrium coincides with the first-best allocation (by the first welfare theorem) guarantees that  $h_i(z) = 1/g_i(z)$ , where  $g_i(z)$  is the marginal product of labor as a function of productivity shocks in the first-best allocation.  $\square$

### Proof of Corollary 3

Suppose all firms in all industries  $j \neq i$  set their prices under complete information. This means that state  $s$  is measurable with respect to  $\omega_{jk}$  for all  $k \in [0, 1]$  and all  $j \neq i$ . Therefore, (15) implies that

$$\varepsilon_{jk}(s) = 1 \quad \text{for all } j \neq i. \quad (\text{B.15})$$

Let the monetary policy function  $m(s)$  be given by

$$m(s) = M z_i \frac{U'(C(s))C'(s)}{V'(L(s))} \frac{\partial F_i}{\partial l_i}(s), \quad (\text{B.16})$$

for some constant  $M$  that does not depend on  $s$ . By (16), such a policy induces  $\text{mc}_i(s) = M$  for all  $s$ . Thus, (15) guarantees that  $\varepsilon_{ik}(s) = 1$  for all firms  $k \in [0, 1]$ . This, together with (B.15), implies that the

policy in (B.16) can eliminate all wedges arising from nominal rigidities, thus reducing equations (13)–(14) to (10)–(11). In other words, any flexible-price-implementable allocation can be implemented as part of a sticky-price equilibrium.

The proof is therefore complete once we show that the policy in (B.16) stabilizes the price of industry  $i$ . As we already established, such a policy induces  $mc_i(s) = M$  for all  $s$ . Thus, by equation (B.9),  $p_{ik}(\omega_{ik}) = M/\chi_i^s$ , which means that the nominal price set by the firms in industry  $i$  is invariant to the economy's aggregate state.  $\square$

### Proof of Corollary 4

First, note that an aggregate labor-augmenting shock can be incorporated to our model in Section 2 by introducing an extra industry, labeled industry 0, that transforms household's labor supply into labor services sold to other industries. A TFP shock to this industry is identical to an aggregate labor-augmenting shock.

Next, recall from the proof of Theorem 1 that the marginal product of labor of industry  $i$  as a function of productivity shocks  $(z_0, z_1, \dots, z_n)$  in the first-best allocation is given by  $g_i(z) = 1/h_i(z)$ , where  $(h_0, h_1, \dots, h_n)$  satisfies (B.14) for all  $i \in \{0, 1, \dots, n\}$ . As a result,

$$h_i(z) = \frac{1}{z_i} K_i(1/z_0, h_1(z), \dots, h_n(z)) \quad \text{for all } i \in \{1, \dots, n\}.$$

Since the marginal cost function  $K_i(\cdot)$  is homogenous of degree 1 for all  $i$ , it is immediate that the unique solution to the above system of equations is given by  $h_i(z) = \frac{1}{z_0} \hat{h}_i(z_1, \dots, z_n)$ , where  $\hat{h}_i(\cdot)$  does not depend on  $z_0$ . Consequently, the marginal product of labor of industry  $i$  in the first-best allocation is equal to  $g_i(z) = z_0/\hat{h}_i(z_1, \dots, z_n)$ . Setting the nominal wage function  $w(s) = z_0 \hat{w}(z_1, \dots, z_n)$  for a function  $\hat{w}(z_1, \dots, z_n)$  that does not depend on  $z_0$  then implies that the left-hand side of (18) is

$$w(s)/g_i(\chi_1^f z_1, \dots, \chi_n^f z_n) = \hat{w}(z_1, \dots, z_n)/\hat{h}_i(\chi_1^f z_1, \dots, \chi_n^f z_n),$$

which is, by assumption, measurable with respect to the information sets of all firms in industry  $i$ . Thus, by Theorem 1, any flexible-price allocation is implementable as a sticky-price equilibrium.  $\square$

### Proof of Proposition 3

Suppose the flexible-price allocation indexed by  $(\chi_1^f, \dots, \chi_n^f)$  is implementable as a sticky-price equilibrium using a price stabilization policy that assigns weight  $\psi_i$  to the price of industry  $i$ , i.e.,

$$\sum_{i=1}^n \psi_i \log p_i(\omega_i) + \left(1 - \sum_{i=1}^n \psi_i\right) \log w(s) = 0. \quad (\text{B.17})$$

In the proof of Theorem 1 we established that a flexible-price allocation is implementable as a sticky-price equilibrium only if condition (B.11) is satisfied for all industries  $i$ . Under the assumption that the production technology of firms in industry  $i$  is given by (19), this condition is equivalent to  $\log p_i(\omega_i) = -\log \chi_i^f - \log z_i + \alpha_i \log w(s) + \sum_{j=1}^n a_{ij} \log p_j(\omega_j)$ . Solving for log nominal prices, we obtain

$$\log p_i(\omega_i) = \log w(s) - \sum_{j=1}^n \ell_{ij} (\log z_j + \log \chi_j^f), \quad (\text{B.18})$$



where  $\ell_{ij}$  denotes the  $(i, j)$  element of the Leontief inverse,  $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ . Multiplying both sides of the above equation by  $\psi_i$ , summing over all  $i$ , and using (B.17), we obtain

$$\log w(s) = \sum_{i=1}^n \sum_{j=1}^n \psi_i \ell_{ij} (\log z_j + \log \chi_j^f).$$

Replacing the above into (B.18) implies that

$$\log p_{ik}(\omega_{ik}) = \sum_{r=1}^n \sum_{j=1}^n \psi_r \ell_{rj} (\log z_j + \log \chi_j^f) - \sum_{j=1}^n \ell_{ij} (\log z_j + \log \chi_j^f) \quad (\text{B.19})$$

for all firms  $k \in [0, 1]$  in industry  $i$ , where we are also using the fact that since the allocation is both flexible- and sticky-price implementable, all firms within the same industry set the same nominal price, i.e.,  $p_{ik}(\omega_{ik}) = p_i(\omega_i)$ . Since  $\log p_{ik}(\omega_{ik})$  is measurable with respect to the information set of firm  $k$  in industry  $i$ , taking conditional expectations from both sides of (B.19) and subtracting the resulting equation from (B.19) implies that

$$\sum_{r=1}^n \sum_{j=1}^n \psi_r \ell_{rj} (\mathbb{E}_{ik}[\log z_j] - \log z_j) - \sum_{j=1}^n \ell_{ij} (\mathbb{E}_{ik}[\log z_j] - \log z_j) = 0.$$

Let  $u_i$  denote the  $i$ -th unit vector. We can rewrite the above equation as  $(\psi' - u_i') \mathbf{L} (\mathbb{E}_{ik}[\log z] - \log z) = 0$ , where  $\psi = (\psi_1, \dots, \psi_n)'$ . Multiplying both sides of this equation by  $\log z'$  from the right and taking expectations with respect to the information set of firm  $k$  in industry  $i$  then establishes (21).  $\square$

## Proof of Lemma 2

By equation (B.8), firm  $k$  in industry  $i$  sets a nominal price equal to  $p_{ik} = \mathbb{E}_{ik}[\text{mc}_i \cdot v_{ik}] / \mathbb{E}_{ik}[v_{ik}]$ , where  $v_{ik}$  is given by (17) and we are using the assumption that  $\tau_i = 1/(1 - \theta_i)$ . Consequently,

$$\log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] = \log \mathbb{E}_{ik} \left[ e^{\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] + \log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i]} \right] - \log \mathbb{E}_{ik} \left[ e^{\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}]} \right].$$

Since the standard deviations of log productivity shocks in (22) and noise shocks in (23) scale linearly in  $\delta$ , it follows that  $\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] = o(1)$  and  $\log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i] = o(1)$  as  $\delta \rightarrow 0$ . As a result,

$$\begin{aligned} \log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] &= \log \left( 1 + \mathbb{E}_{ik} \left[ \log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] + \log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i] \right] + o(\delta) \right) \\ &\quad - \log \left( 1 + \mathbb{E}_{ik} \left[ \log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] \right] + o(\delta) \right), \end{aligned}$$

which in turn implies that  $\log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] = \log(1 + o(\delta)) - \log(1 + o(\delta)) = o(\delta)$ . This establishes (26). To establish (27), recall that the production function of firms in industry  $i$  is given by (19), which implies that  $\log \text{mc}_i = \alpha_i \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j$ . This, together with (26), establishes (27).  $\square$

## Proof of Proposition 4

Since noise shocks  $\epsilon_{ij,k}$  in firms' private signals in (23) are idiosyncratic and of order  $\delta$ , the log-linearization (as  $\delta \rightarrow 0$ ) of any industry-level or aggregate variable only depends on the productivity

shocks. We thus let

$$\log w = \sum_{j=1}^n \kappa_j \log z_j + o(\delta) \quad (\text{B.20})$$

$$\log p_i = \sum_{j=1}^n b_{ij} \log z_j + o(\delta) \quad (\text{B.21})$$

denote, respectively, the log-linearization of the nominal wage and the nominal price of sectoral good  $i$  as  $\delta \rightarrow 0$ , where vector  $\kappa$  and matrix  $\mathbf{B}$  are to be determined. Furthermore, recall from Lemma 2 that, to a first-order approximation as  $\delta \rightarrow 0$ , the nominal price set by firm  $k$  in industry  $i$  is given by (27). Therefore,

$$\begin{aligned} \log p_{ik} &= \alpha_i \sum_{j=1}^n \kappa_j \mathbb{E}_{ik}[\log z_j] - \mathbb{E}_{ik}[\log z_i] + \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \mathbb{E}_{ik}[\log z_r] + o(\delta) \\ &= \alpha_i \sum_{j=1}^n \kappa_j \phi_{ik} \omega_{ij,k} - \phi_{ik} \omega_{ii,k} + \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \phi_{ik} \omega_{ir,k} + o(\delta), \end{aligned} \quad (\text{B.22})$$

where  $\phi_{ik}$  is the degree of price flexibility of firm  $k$  in industry  $i$  given by (25). Integrating both sides of the above equation over all firms  $k$  in industry  $i$  implies that

$$\log p_i = \phi_i \alpha_i \sum_{j=1}^n \kappa_j \log z_j - \phi_i \log z_i + \phi_i \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \log z_r + o(\delta),$$

where  $\phi_i = \int_0^1 \phi_{ik} dk$  is the degree of price flexibility of industry  $i$  and we are using the fact that  $\log p_i = \int_0^1 \log p_{ik} dk + o(\delta)$ . The juxtaposition of the above equation with equation (B.21) therefore implies that  $\mathbf{B} = \mathbf{\Phi} \alpha \kappa' - \mathbf{\Phi} + \mathbf{\Phi} \mathbf{A} \mathbf{B}$ , where  $\mathbf{\Phi} = \text{diag}(\phi)$ . As a result,

$$\mathbf{B} = (\mathbf{I} - \mathbf{\Phi} \mathbf{A})^{-1} \mathbf{\Phi} (\alpha \kappa' - \mathbf{I}) = \mathbf{\Phi} (\mathbf{I} - \mathbf{A} \mathbf{\Phi})^{-1} (\alpha \kappa' - \mathbf{I}). \quad (\text{B.23})$$

Multiplying both sides by  $\log z$  and using equations (B.20) and (B.21) then establishes that industry-level prices satisfy (30). To establish (29), note that the vector of (log) nominal marginal costs is equal to  $\log mc = \alpha \log w - \log z + \mathbf{A} \log p$ . Therefore, by (30),  $\log mc = (\mathbf{I} + \mathbf{A} \mathbf{\Phi} (\mathbf{I} - \mathbf{A} \mathbf{\Phi})^{-1}) (\alpha \log w - \log z)$ , which reduces to (29).  $\square$

## Proof of Proposition 6

We prove this result in three steps. First, we solve for household welfare in terms of nominal prices and the nominal wage. We then compare the result to welfare under the first-best allocation to obtain an expression for welfare loss, taking nominal prices as given. Finally, we provide a quadratic log-approximation to the welfare loss in terms of the cross-sectional mean and variance of firm-level pricing errors in (34) and (35). Given the indeterminacy of prices in the flexible-price equilibrium, we express the pricing errors under the normalization that the nominal wage is the same in the sticky- and flexible-price equilibria.

**Expressing welfare in terms of nominal prices:** As our first step, we obtain an expression for welfare as a function of all nominal prices and the nominal wage.

Recall from equation (B.7) that the output of firm  $k$  in industry  $i$  is given by  $y_{ik} = y_i(p_{ik}/p_i)^{-\theta_i}$ , whereas cost minimization implies that the firm's demand for the good produced by industry  $j$  is equal to  $x_{ij,k} = a_{ij}y_{ik} mc_i/p_j$ . Therefore, total demand for the good produced by industry  $j$  by firms in industry  $i$  is  $\int_0^1 x_{ij,k} dk = a_{ij}p_i y_i \varepsilon_i/p_j$ , where

$$\varepsilon_i = \frac{mc_i}{p_i} \int_0^1 (p_{ik}/p_i)^{-\theta_i} dk. \quad (\text{B.24})$$

Hence, market clearing for sectoral good  $i$  implies that  $p_i y_i = p_i c_i + \sum_{j=1}^n a_{ji} p_j y_j \varepsilon_j$ . Dividing both sides by nominal aggregate demand,  $PC$ , and using the fact that  $p_i c_i = \beta_i PC$ , we obtain

$$\lambda_i = \beta_i + \sum_{j=1}^n a_{ji} \varepsilon_j \lambda_j, \quad (\text{B.25})$$

where  $\lambda_i = p_i y_i/PC$  is the Domar weight of industry  $i$ .

Next, note that the representative household's budget constraint is given by

$$PC = wL + \sum_{i=1}^n \int_0^1 \pi_{ik} dk = wL + \sum_{i=1}^n \left( p_i y_i - mc_i \int_0^1 y_{ik} dk \right) = wL + \sum_{i=1}^n (1 - \varepsilon_i) p_i y_i,$$

which implies that  $PC = wL/(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i))$ . Furthermore, the household's optimal labor supply requires that  $L^{1/\eta} = C^{-\gamma} w/P$ . Therefore, solving for household's aggregate consumption and aggregate labor supply from the last two equations, we obtain

$$\begin{aligned} C &= (w/P)^{\frac{1+1/\eta}{\gamma+1/\eta}} \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{1/\eta}{\gamma+1/\eta}} \\ L &= (w/P)^{\frac{1-\gamma}{\gamma+1/\eta}} \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{\frac{\gamma}{\gamma+1/\eta}}. \end{aligned} \quad (\text{B.26})$$

Plugging the above into (2), we can express the representative household's welfare as a function of nominal prices and the nominal wage as

$$W = \frac{1}{1-\gamma} (w/P)^{\frac{(1-\gamma)(1+1/\eta)}{\gamma+1/\eta}} \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{(1-\gamma)/\eta}{\gamma+1/\eta}} \left( 1 - \frac{1-\gamma}{1+1/\eta} \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) \right), \quad (\text{B.27})$$

where  $\varepsilon_i$  is given by (B.24) and the Domar weights solve the system of equations in (B.25).

**Welfare loss:** We now use the above expression to determine the first-best welfare  $W^*$  under flexible prices, which we then use to calculate the welfare loss arising from nominal rigidities.

Recall that, in the flexible-price equilibrium, all firms in industry  $i$  set identical prices and charge no markups, i.e.,  $mc_i^* = p_{ik}^* = p_i^*$ . Therefore, equation (B.24) implies that  $\varepsilon_i^* = 1$  for all  $i$ . Plugging this into (B.27) implies that  $W^* = \frac{\gamma+1/\eta}{(1-\gamma)(1+1/\eta)} (w/P^*)^{\frac{(1-\gamma)(1+1/\eta)}{\gamma+1/\eta}}$ , where recall that, by assumption,  $w = w^*$ . Hence, we can rewrite (B.27) as

$$W = W^* (P/P^*)^{\frac{(\gamma-1)(1+1/\eta)}{\gamma+1/\eta}} \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{(1-\gamma)/\eta}{\gamma+1/\eta}} \left( 1 + \frac{1-\gamma}{\gamma+1/\eta} \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right).$$

Similarly, we can use (B.26) to relate aggregate output in the sticky-price equilibrium to that in the flexible-price equilibrium:

$$C = C^* (P/P^*)^{-\frac{1+1/\eta}{\gamma+1/\eta}} \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{1/\eta}{\gamma+1/\eta}}. \quad (\text{B.28})$$

The juxtaposition of the last two equations then implies that welfare in the sticky- and flexible-price equilibria are related to one another as follows:

$$W = W^* (C/C^*)^{1-\gamma} \left( 1 + \frac{1-\gamma}{\gamma+1/\eta} \left( 1 - (C/C^*)^{-(1+\eta\gamma)} (P/P^*)^{-(1+\eta)} \right) \right). \quad (\text{B.29})$$

**Second-order approximations:** We next derive log-quadratic approximations to equations (B.24), (B.28), and (B.29) around the economy's steady-state as  $\delta \rightarrow 0$ .

First, consider equation (B.24). Taking logarithms from both sides and using the fact that  $\log mc_i = \alpha \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j$  implies that

$$\log \varepsilon_i = \sum_{j=1}^n a_{ij} (\log p_j - \log p_j^*) + (\theta_i - 1) (\log p_i - \log p_i^*) + \log \int_0^1 (p_{ik}/p_i^*)^{-\theta_i} dk.$$

Consequently, to a second-order approximation,

$$\log \varepsilon_i = \sum_{j=1}^n a_{ij} \bar{e}_j - \bar{e}_i + \frac{1}{2} \sum_{j=1}^n a_{ij} (1 - \theta_j) \vartheta_j + \frac{1}{2} (2\theta_i - 1) \vartheta_i + o(\delta^2), \quad (\text{B.30})$$

where  $\bar{e}_i$  and  $\vartheta_i$  are the cross-sectional average and dispersion of pricing errors in industry  $i$  defined in (34) and (35), respectively, and we are using the fact that  $\log p_j - \log p_j^* = \bar{e}_j + \frac{1}{2} (1 - \theta_j) \vartheta_j + o(\delta)$ .

We next derive a log-quadratic approximation to (B.28). Start with the observation that

$$\log \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) = \sum_{i=1}^n \lambda_i \log \varepsilon_i + \frac{1}{2} \sum_{i=1}^n \lambda_i \log^2 \varepsilon_i - \frac{1}{2} \left( \sum_{i=1}^n \lambda_i \log \varepsilon_i \right)^2 + o(\delta^2). \quad (\text{B.31})$$

By (B.25), the vector of industry Domar weights is given by  $\lambda = (\mathbf{I} - \mathbf{A}' \text{diag}(\varepsilon))^{-1} \beta$ . As a result,

$$\begin{aligned} \lambda - \lambda^* &= (\mathbf{I} - \mathbf{A}' \text{diag}(\varepsilon))^{-1} \beta - (\mathbf{I} - \mathbf{A}')^{-1} \beta = (\mathbf{I} - \mathbf{A}' \text{diag}(\varepsilon))^{-1} \mathbf{A}' (\text{diag}(\varepsilon) - \mathbf{I}) (\mathbf{I} - \mathbf{A}')^{-1} \beta \\ &= \mathbf{L}' \mathbf{A}' \text{diag}(\log \varepsilon) \lambda^* + o(\delta) = (\mathbf{L}' - \mathbf{I}) \text{diag}(\log \varepsilon) \lambda^* + o(\delta), \end{aligned}$$

where  $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$  is the economy's Leontief inverse and we are using the fact that the vector of Domar weights under flexible prices is given by  $\lambda^* = \mathbf{L}' \beta$ . The above equation implies that, to a first-order approximation, the Domar weight of industry  $i$  in the sticky-price equilibrium is given by

$$\lambda_i = (1 - \log \varepsilon_i) \lambda_i^* + \sum_{j=1}^n \ell_{ji} \lambda_j^* \log \varepsilon_j + o(\delta). \quad (\text{B.32})$$

Putting the above equation together with (B.31) leads to

$$\begin{aligned} \log \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) &= \sum_{i=1}^n \lambda_i^* \log \varepsilon_i + \sum_{i=1}^n \sum_{j=1}^n \lambda_j^* \ell_{ji} \log \varepsilon_j \log \varepsilon_i \\ &\quad - \frac{1}{2} \sum_{i=1}^n \lambda_i^* \log^2 \varepsilon_i - \frac{1}{2} \left( \sum_{i=1}^n \lambda_i^* \log \varepsilon_i \right)^2 + o(\delta^2). \end{aligned}$$

Replacing  $\log \varepsilon_i$  by its second-order approximation in (B.30) and simplifying the result, we obtain

$$\begin{aligned} \log \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) &= -(\log P - \log P^*) + \frac{1}{2} \sum_{i=1}^n \lambda_i^* \theta_i \vartheta_i + \frac{1}{2} \sum_{i=1}^n \lambda_i^* \bar{e}_i^2 \\ &\quad - \frac{1}{2} \sum_{i=1}^n \lambda_i^* \left( \sum_{j=1}^n a_{ij} \bar{e}_j \right)^2 - \frac{1}{2} \left( \sum_{i=1}^n \beta_i \bar{e}_i \right)^2 + o(\delta^2), \end{aligned}$$

where we are using the fact that  $\log P - \log P^* = \sum_{i=1}^n \beta_i \bar{e}_i + \frac{1}{2} \sum_{i=1}^n \beta_i (1 - \theta_i) \vartheta_i + o(\delta^2)$ . Consequently, we obtain the following second-order approximation to (B.28):

$$\begin{aligned} \log C - \log C^* &= -\frac{1/\eta}{2(\gamma + 1/\eta)} \left[ \sum_{i=1}^n \lambda_i^* \theta_i \vartheta_i + \sum_{i=1}^n \lambda_i^* \left( \bar{e}_i^2 - \left( \sum_{j=1}^n a_{ij} \bar{e}_j \right)^2 \right) - \left( \sum_{i=1}^n \beta_i \bar{e}_i \right)^2 \right] \\ &\quad - \frac{1}{\gamma + 1/\eta} (\log P - \log P^*) + o(\delta^2). \end{aligned} \quad (\text{B.33})$$

The above expression also implies that, to a first-order approximation, the output gap is equal to

$$\log C - \log C^* = -\frac{1}{\gamma + 1/\eta} (\log P - \log P^*) + o(\delta). \quad (\text{B.34})$$

Finally, we derive a log-quadratic approximation to equation (B.29) as  $\delta \rightarrow 0$ . We have

$$\begin{aligned} \log(W/W^*) &= (1 - \gamma)(1 + \eta) \left[ (\log C - \log C^*) + \frac{1}{\gamma + 1/\eta} (\log P - \log P^*) - \frac{1}{2} \eta (\log C - \log C^*)^2 \right. \\ &\quad \left. - \frac{1}{2} \eta \frac{(1 + \eta)^2}{(1 + \eta\gamma)^2} (\log P - \log P^*)^2 - \frac{1 + \eta}{\gamma + 1/\eta} (\log C - \log C^*) (\log P - \log P^*) \right] + o(\delta^2). \end{aligned}$$

Replace for  $\log P - \log P^*$  in the last two terms from its first-order approximation in (B.34) to obtain

$$\log(W/W^*) = (1 - \gamma)(1 + \eta) \left( \log C - \log C^* + \frac{1}{\gamma + 1/\eta} (\log P - \log P^*) - \frac{1}{2\eta} \Delta^2 \right) + o(\delta^2),$$

where  $\Delta^2 = (\log C - \log C^*)^2$  is the volatility of output gap. Using (B.33) to replace for the first term on the right-hand side above and using the fact that  $\lambda_i^* = \beta_i + \sum_{j=1}^n \lambda_j^* a_{ji}$  implies that

$$\begin{aligned} \log(W/W^*) &= -\frac{1}{2} \frac{(1 - \gamma)(1 + 1/\eta)}{(\gamma + 1/\eta)} \left[ \sum_{i=1}^n \lambda_i^* \theta_i \vartheta_i + (\gamma + 1/\eta) \Delta^2 \right. \\ &\quad \left. + \sum_{i=1}^n \lambda_i^* \text{xvar}_i(\bar{e}_1, \dots, \bar{e}_n) + \text{xvar}_0(\bar{e}_1, \dots, \bar{e}_n) \right] + o(\delta^2), \end{aligned}$$

where  $\text{xvar}_i(\bar{e}_1, \dots, \bar{e}_n)$  is the inter-industry cross-sectional dispersion of pricing errors of inputs from the point of view of industry  $i$  defined in (38). We make two final observations. First, the fact that  $\lambda_i = \lambda_i^* + o(1)$  as  $\delta \rightarrow 0$  implies that we can replace  $\lambda_i^*$  by  $\lambda_i$  in the above equation. Second, note that  $W - W^* = W^* \log(W/W^*) + o(\delta^2)$  and  $W^* = \frac{\gamma + 1/\eta}{(1 - \gamma)(1 + 1/\eta)} + O(\delta)$ . The juxtaposition of these observations with the above equation then establishes (36).  $\square$

## Proof of Proposition 5

We start by stating and proving a lemma, which we will also use in proof in the proof of Theorem 2. Statement (a) of the lemma establishes that even though in our model the monetary policy instrument is the nominal aggregate demand  $m(z)$ , as long as no industry is perfectly sticky, there is an isomorphism between setting the nominal aggregate demand and the nominal wage  $w(z)$ . Statement (b) of the lemma then provides conditions under which a policy can be implemented as a price-stabilization policy.

**Lemma B.1.** *Suppose  $\phi_i > 0$  for all  $i$ . Then, to a first-order approximation,*

(a) *an allocation is implementable by setting the nominal demand if and only if it is implementable by setting the nominal wage;*

(b) *if vector  $\kappa$  satisfies  $\kappa' \alpha = 1$ , then the nominal wage  $\log w(z) = \sum_{i=1}^n \kappa_i \log z_i$  can be implemented by a price-stabilization policy of the form  $\sum_{i=1}^n \psi_i \log z_i = 0$  for some vector  $\psi = (\psi_1, \dots, \psi_n)'$ .*

**Proof of part (a)** It is sufficient to show that, as long as  $\phi_i > 0$  for all industries  $i$ , there is a one-to-one correspondence between the nominal wage  $w(z)$  and nominal aggregate demand  $m(z)$  for all realizations of  $z$ . Let  $P^*$  and  $C^*$  denote, respectively, the consumption price index and aggregate output in the flexible-price equilibrium. Since  $m = PC$ , it is immediate that

$$\log m = \left(1 - \frac{1}{\gamma + 1/\eta}\right) (\log P - \log P^*) + \log P^* + \log C^* + o(\delta),$$

where we are using (B.34) to express the output gap in terms of distortion in the consumption price index. Next, recall from Proposition 4 that industry-level nominal prices satisfy (30). Therefore, the vector of average pricing errors defined in (34) is given by

$$\bar{e} = \Phi(\mathbf{I} - \mathbf{A}\Phi)^{-1} (\alpha \log w - \log z) - (\mathbf{I} - \mathbf{A})^{-1} (\alpha \log w - \log z) + o(\delta),$$

or equivalently,

$$\bar{e} = \mathbf{Q}(\mathbf{L} \log z - \mathbf{1} \log w) + o(\delta), \tag{B.35}$$

where  $\mathbf{Q} = (\mathbf{I} - \Phi\mathbf{A})^{-1}(\mathbf{I} - \Phi)$ . Therefore,

$$\log m = \left(1 - \frac{1}{\gamma + 1/\eta}\right) \beta' \mathbf{Q}(\mathbf{L} \log z - \mathbf{1} \log w) + \log P^* + \log C^* + o(\delta),$$

where note that the distortion in the consumption price index is  $\log P - \log P^* = \beta' \bar{e} + o(\delta)$ . It is also immediate to verify that, in the flexible-price equilibrium, the consumption price index and aggregate output are given by  $\log P^* = \log w - \beta' \mathbf{L} \log z$  and  $\log C^* = \frac{1+1/\eta}{\gamma+1/\eta} \beta' \mathbf{L} \log z$ , respectively. As a result, to a first-order approximation, nominal wage and nominal aggregate demand are related to one another via the following relationship:

$$\log m = \left[1 - \left(1 - \frac{1}{\gamma + 1/\eta}\right) \beta' \mathbf{Q}\mathbf{1}\right] \log w + \left[\left(1 - \frac{1}{\gamma + 1/\eta}\right) \beta' \mathbf{Q}\mathbf{L} - \frac{\gamma - 1}{\gamma + 1/\eta} \beta' \mathbf{L}\right] \log z + o(\delta). \tag{B.36}$$

The above equation establishes a one-to-one correspondence between  $w(z)$  and  $m(z)$  as long as  $\left(1 - \frac{1}{\gamma+1/\eta}\right) \beta' \mathbf{Q} \mathbf{1} \neq 1$ . The proof is therefore complete once we show that this condition is indeed satisfied. To this end, note that it is sufficient to show that  $0 \leq \beta' \mathbf{Q} \mathbf{1} < 1$ . The fact that  $\beta' \mathbf{Q} \mathbf{1} \geq 0$  is a straightforward implication of the fact that  $(\mathbf{I} - \Phi \mathbf{A})^{-1}$  is an inverse M-matrix and hence is element-wise non-negative. To show that  $\beta' \mathbf{Q} \mathbf{1} < 1$ , note that

$$1 - \beta' \mathbf{Q} \mathbf{1} = \beta' (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi (\mathbf{I} - \mathbf{A}) \mathbf{1} = \sum_{i,j=1}^n \beta_i h_{ij} \phi_j \alpha_j,$$

where  $\mathbf{H} = (\mathbf{I} - \Phi \mathbf{A})^{-1}$ . Once again, the fact that  $(\mathbf{I} - \Phi \mathbf{A})^{-1}$  is an inverse M-matrix implies that the right-hand side of the above equation is non-negative. To show that it is in fact strictly positive, suppose to the contrary that it is equal to zero. This means that

$$\beta_i h_{ij} \phi_j \alpha_j = 0$$

for all pairs of industries  $i$  and  $j$ . But for any industry  $i$ , there exists at least one industry  $j$  (which may coincide with  $i$ ) such that  $\alpha_j > 0$  and  $h_{ij} > 0$ . This coupled with the fact that  $\sum_{i=1}^n \beta_i = 1$  and the assumption that  $\phi_j > 0$  for all  $j$  leads to a contradiction. Therefore, it must be the case that  $\beta' \mathbf{Q} \mathbf{1} < 1$ , which completes the proof.  $\square$

**Proof of part (b)** Let vector  $\kappa = (\kappa_1, \dots, \kappa_n)'$  satisfy  $\kappa' \alpha = 1$ . We show that stabilizing the price index  $\sum_{i=1}^n \psi_i \log p_i$  with weights given by

$$\psi' = \kappa' \Phi^{-1} (\mathbf{I} - \Phi \mathbf{A}) \tag{B.37}$$

induces a nominal wage given by  $\log w = \sum_{i=1}^n \kappa_i \log z_i + o(\delta)$ . Start by noting that the juxtaposition of  $\psi' \log p = 0$  and equation (30) implies that  $\psi' (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi (\mathbf{I} - \mathbf{A}) (\mathbf{1} \log w - \mathbf{L} \log z) = o(\delta)$ . Consequently,

$$\log w = \frac{1}{\psi' (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi (\mathbf{I} - \mathbf{A}) \mathbf{1}} \psi' (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi \log z + o(\delta).$$

Replacing for  $\psi$  from (B.37) into the above implies that  $\log w = \frac{1}{\kappa' \alpha} \kappa' \log z + o(\delta)$ . Using the assumption that  $\kappa' \alpha = 1$  then implies that  $\log w = \sum_{i=1}^n \kappa_i \log z_i + o(\delta)$ .  $\square$

**Proof of Proposition 5:** With Lemma B.1 in hand, we are now ready to prove the proposition. First, note that the identity  $PC = m$  implies that degree of monetary non-neutrality satisfies

$$\Xi = 1 - \frac{d \log P}{d \log m} = 1 - \frac{d \log P}{d \log w} \frac{d \log w}{d \log m}.$$

Since  $\log P = \sum_{i=1}^n \beta_i \log p_i$ , equation (30) implies that  $d \log P / d \log w = \sum_{i=1}^n \beta_i \phi_i \rho_i = \rho_0$ , where the second equality follows from the definition of  $\rho_0$  in (32). Thus, using (B.36), we obtain

$$\Xi = 1 - \rho_0 \left[ 1 - \left( 1 - \frac{1}{\gamma + 1/\eta} \right) \beta' \mathbf{Q} \mathbf{1} \right]^{-1},$$

where  $\mathbf{Q} = (\mathbf{I} - \Phi \mathbf{A})^{-1} (\mathbf{I} - \Phi)$ . Finally, noting that  $\beta' \mathbf{Q} \mathbf{1} = \beta' (\mathbf{1} - \Phi \rho) = 1 - \rho_0$  establishes (33).  $\square$

## Proof of Theorem 2

As a first observation, we note that the independence assumption imposed on the noise shocks  $\epsilon_{ij,k}$  implies that aggregate uncertainty in this economy is solely driven by the productivity shocks  $z = (z_1, \dots, z_n)$ . As a result, without loss of generality, we can restrict our attention to monetary policies of the form  $m(z)$  that only depend on the productivity shocks, as opposed to the entire state of the economy  $s = (z, \omega)$ .

By Lemma B.1, as long as  $\phi_i > 0$  for all  $i$ , any allocation that is implementable by setting nominal aggregate demand,  $m(z)$ , is also implementable by setting the nominal wage,  $w(z)$ . Therefore, to determine the optimal policy, we first characterize how the wage should optimally respond to productivity shocks. Specifically, we characterize the vector of optimal weights  $\kappa = (\kappa_1, \dots, \kappa_n)'$  in  $\log w = \sum_{j=1}^n \kappa_j \log z_j$  that minimizes the expected welfare loss in (36). We then show that the price-stabilization policy with weights given by (39) implements such a nominal wage.

To calculate the expected welfare loss in (36), we first determine the cross-sectional average and dispersion of pricing errors in each industry, defined in equations (34) and (35), respectively. In the proof of Lemma B.1, we already established that the vector of average pricing errors is given by (B.35). To determine the within-industry dispersion of pricing errors, note that all firm-level nominal prices within the same industry coincide with one another in the flexible-price equilibrium. Therefore, the dispersion of pricing errors in industry  $i$  satisfies

$$\vartheta_i = \int_0^1 e_{ik}^2 dk - \left( \int_0^1 e_{ik} dk \right)^2 = \int_0^1 (\log p_{ik} - \log p_i)^2 dk.$$

Furthermore, recall from the proof of Proposition 4 that the nominal price of firm  $k$  in industry  $i$  in the sticky-price equilibrium is given by (B.22), which implies that

$$\log p_{ik} = \frac{\phi_{ik}}{\phi_i} \sum_{j=1}^n b_{ij} \omega_{ij,k} + o(\delta),$$

where matrix  $\mathbf{B}$  is given by (B.23). Consequently,

$$\log p_{ik} - \log p_i = \left( \frac{\phi_{ik}}{\phi_i} - 1 \right) \sum_{j=1}^n b_{ij} \log z_i + \frac{\phi_{ik}}{\phi_i} \sum_{j=1}^n b_{ij} \epsilon_{ij,k} + o(\delta).$$

Therefore, the expected dispersion of pricing errors in industry  $i$  is given by

$$\begin{aligned} \mathbb{E}[\vartheta_i] &= \frac{\delta^2}{\phi_i^2} \sum_{j=1}^n b_{ij}^2 \left( \sigma_z^2 \int_0^1 (\phi_{ik} - \phi_i)^2 dk + \int_0^1 \phi_{ik}^2 \sigma_{ik}^2 dk \right) + o(\delta^2) \\ &= \sigma_z^2 \frac{\delta^2}{\phi_i^2} \sum_{j=1}^n b_{ij}^2 \left( \int_0^1 (\phi_{ik} - \phi_i)^2 dk + \int_0^1 \phi_{ik} (1 - \phi_{ik}) dk \right) + o(\delta^2), \end{aligned}$$

where the second equality is a simple consequence of the definition of  $\phi_{ik}$  in (25). Hence,

$$\mathbb{E}[\vartheta_i] = \sigma_z^2 \delta^2 \left( \frac{1 - \phi_i}{\phi_i} \right) \sum_{j=1}^n b_{ij}^2 + o(\delta^2).$$



With the expected within-industry price dispersion and the mean pricing errors in (B.35) in hand, we now minimize the expected welfare loss by optimizing over the vector  $\kappa = (\kappa_1, \dots, \kappa_n)$ , where recall that  $\log w = \sum_{j=1}^n \kappa_j \log z_j$ . Taking expectations from both sides of (36), differentiating it with respect to  $\kappa_s$ , and setting it equal to zero, we obtain

$$\begin{aligned} & \sigma_z^2 \delta^2 \sum_{i=1}^n \lambda_i \theta_i \left( \frac{1 - \phi_i}{\phi_i} \right) \sum_{j=1}^n b_{ij} \frac{db_{ij}}{d\kappa_s} + \frac{1}{\gamma + 1/\eta} \sum_{i=1}^n \sum_{j=1}^n \beta_i \beta_j \mathbb{E} \left[ \bar{e}_i \frac{d\bar{e}_j}{d\kappa_s} \right] \\ & + \sum_{i=0}^n \lambda_i \left( \sum_{j=1}^n a_{ij} \mathbb{E} \left[ \bar{e}_j \frac{d\bar{e}_j}{d\kappa_s} \right] - \sum_{j=1}^n \sum_{r=1}^n a_{ij} a_{ir} \mathbb{E} \left[ \bar{e}_r \frac{d\bar{e}_j}{d\kappa_s} \right] \right) = 0, \end{aligned}$$

with the convention that  $\lambda_0 = 1$  and  $a_{0j} = \beta_j$  for all  $j$ . To simplify the above, note that (B.35) implies that  $d\bar{e}_j/d\kappa_s = -\log z_s \sum_{r=1}^n q_{jr}$ , while equation (B.23) implies that  $db_{ij}/d\kappa_s = 0$  if  $j \neq s$ . As a result,

$$\begin{aligned} & \sum_{i=1}^n \lambda_i \theta_i \left( \frac{1 - \phi_i}{\phi_i} \right) \frac{db_{is}}{d\kappa_s} b_{is} - \frac{1}{\gamma + 1/\eta} \left( \sum_{i=1}^n \sum_{j=1}^n \beta_i q_{ij} (\ell_{js} - \kappa_s) \right) \left( \sum_{i=1}^n \sum_{j=1}^n \beta_i q_{ij} \right) \\ & - \sum_{i=0}^n \sum_{j=1}^n \lambda_i a_{ij} \left( \sum_{r=1}^n q_{jr} (\ell_{rs} - \kappa_s) \right) \left( \sum_{m=1}^n q_{jm} \right) + \sum_{i=0}^n \lambda_i \left( \sum_{j=1}^n \sum_{r=1}^n a_{ij} q_{jr} \right) \left( \sum_{j=1}^n \sum_{r=1}^n a_{ij} q_{jr} (\ell_{rs} - \kappa_s) \right) = 0. \end{aligned}$$

Since the above first-order condition has to hold for all  $s$ , it can be rewritten in matrix form as

$$\begin{aligned} & \left[ \lambda' \text{diag}(\theta)(\mathbf{I} - \Phi)\Phi^{-1}(\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1}))(\mathbf{I} - \mathbf{Q}) + \frac{1}{\gamma + 1/\eta} (\beta' \mathbf{Q}\mathbf{1})\beta' \mathbf{Q} \right. \\ & \left. + (\lambda' \mathbf{A} + \beta') \text{diag}(\mathbf{Q}\mathbf{1})\mathbf{Q} - \lambda' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1})\mathbf{A}\mathbf{Q} - (\beta' \mathbf{Q}\mathbf{1})\beta' \mathbf{Q} \right] (\mathbf{L} - \mathbf{1}\kappa') = 0, \end{aligned}$$

where we are using the fact that matrix  $\mathbf{B}$  defined in (B.23) satisfies  $\mathbf{B} = (\mathbf{I} - \mathbf{Q})(\mathbf{1}\kappa' - \mathbf{L})$ . Solving for  $\kappa$  then implies that the vector of weights  $\kappa$  in  $\log w = \sum_{i=1}^n \kappa_i \log z_i$  that minimizes the expected welfare loss is given by  $\kappa' = \iota' / (\iota' \alpha)$ , where

$$\begin{aligned} \iota' &= \lambda' \text{diag}(\theta)(\mathbf{I} - \Phi)\Phi^{-1}(\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1}))(\mathbf{I} - \mathbf{Q})\mathbf{L} + \frac{1}{\gamma + 1/\eta} (\beta' \mathbf{Q}\mathbf{1})\beta' \mathbf{Q}\mathbf{L} \\ & + \lambda' \mathbf{A} \text{diag}(\mathbf{Q}\mathbf{1})\mathbf{Q}\mathbf{L} - \lambda' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1})\mathbf{A}\mathbf{Q}\mathbf{L} + \beta' \text{diag}(\mathbf{Q}\mathbf{1})\mathbf{Q}\mathbf{L} - (\beta' \mathbf{Q}\mathbf{1})\beta' \mathbf{Q}\mathbf{L}. \end{aligned} \quad (\text{B.38})$$

Having determined the nominal wage that minimizes expected welfare loss, we next determine the price-stabilization policy that implements such a nominal wage. First, note that since  $\kappa' = \iota' / (\iota' \alpha)$ , we have  $\kappa' \alpha = 1$ . As a result, we can apply statement (b) of Lemma B.1, which guarantees that the optimal nominal wage  $\log w = \sum_{i=1}^n \kappa_i \log z_i$  can be implemented by stabilizing the price index  $\sum_{i=1}^n \psi_i^* \log p_i$ , where the industry weights are given by (B.37), i.e.,  $\psi^{*'} = \kappa' \Phi^{-1}(\mathbf{I} - \Phi \mathbf{A})$ . Hence,

$$\begin{aligned} \psi^{*'} &= \lambda' \text{diag}(\theta)(\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1}))(\Phi^{-1} - \mathbf{I}) + \frac{1}{\gamma + 1/\eta} (\beta' \mathbf{Q}\mathbf{1})\beta' \mathbf{L}(\Phi^{-1} - \mathbf{I}) \\ & + (\lambda' \mathbf{A} \text{diag}(\mathbf{Q}\mathbf{1}) - \lambda' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1})\mathbf{A}) \mathbf{L}(\Phi^{-1} - \mathbf{I}) + (\beta' \text{diag}(\mathbf{Q}\mathbf{1}) - (\beta' \mathbf{Q}\mathbf{1})\beta') \mathbf{L}(\Phi^{-1} - \mathbf{I}), \end{aligned} \quad (\text{B.39})$$

where we are using matrix identities  $\mathbf{Q}\mathbf{L}\Phi^{-1}(\mathbf{I} - \Phi \mathbf{A}) = \mathbf{L}(\Phi^{-1} - \mathbf{I})$  and  $(\mathbf{I} - \mathbf{Q})\mathbf{L}\Phi^{-1}(\mathbf{I} - \Phi \mathbf{A}) = \mathbf{I}$ . Next, note that the definition of  $\rho$  in (28) implies that  $\mathbf{Q}\mathbf{1} = \mathbf{1} - \Phi \rho$ . Therefore,

$$\begin{aligned} \psi^{*'} &= \lambda'(\mathbf{I} - \Phi) \text{diag}(\theta) \text{diag}(\rho) + \left( \frac{1 - \beta' \Phi \rho}{\gamma + 1/\eta} \right) \lambda'(\Phi^{-1} - \mathbf{I}) \\ & + \lambda'(\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho) - \text{diag}(\mathbf{A}\mathbf{1} - \mathbf{A}\Phi \rho)\mathbf{A}) \mathbf{L}(\Phi^{-1} - \mathbf{I}) + ((\beta' \Phi \rho)\beta' - \beta' \Phi \text{diag}(\rho)) \mathbf{L}(\Phi^{-1} - \mathbf{I}). \end{aligned}$$

Since  $\mathbf{A}\Phi\rho = \rho - (\mathbf{I} - \mathbf{A})\mathbf{1}$ , we have  $\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho) - \text{diag}(\mathbf{A}\mathbf{1})\mathbf{A} + \text{diag}(\mathbf{A}\Phi\rho)\mathbf{A} = \text{diag}(\rho)\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho)$ . As a result,

$$\begin{aligned}\psi^{*'} &= \lambda'(\mathbf{I} - \Phi) \text{diag}(\theta) \text{diag}(\rho) + \left( \frac{1 - \beta'\Phi\rho}{\gamma + 1/\eta} \right) \lambda'(\Phi^{-1} - \mathbf{I}) \\ &\quad + \lambda'(\text{diag}(\rho)\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho)) \mathbf{L}(\Phi^{-1} - \mathbf{I}) + ((\beta'\Phi\rho)\beta' - \beta'\Phi \text{diag}(\rho)) \mathbf{L}(\Phi^{-1} - \mathbf{I}).\end{aligned}$$

Noting that  $\lambda'\mathbf{A} = \lambda' - \beta'$  and  $\rho_0 = \beta'\Phi\rho$ , we can simplify the above equation as

$$\psi^{*'} = \lambda'(\mathbf{I} - \Phi) \text{diag}(\theta) \text{diag}(\rho) + \left( \frac{1 - \rho_0}{\gamma + 1/\eta} \right) \lambda'(\Phi^{-1} - \mathbf{I}) + \lambda'((\mathbf{I} - \Phi) \text{diag}(\rho)\mathbf{L} + \rho_0\mathbf{I} - \text{diag}(\rho))(\Phi^{-1} - \mathbf{I}).$$

Consequently, the weight on the price of industry  $s$  in the optimal price-stabilization policy satisfies

$$\psi_s^* = (1/\phi_s - 1) \left[ \lambda_s \phi_s \theta_s \rho_s + \lambda_s \left( \frac{1 - \rho_0}{\gamma + 1/\eta} \right) + \sum_{i=1}^n (1 - \phi_i) \lambda_i \rho_i \ell_{is} + (\rho_0 - \rho_s) \lambda_s \right],$$

an expression that coincides with (39).  $\square$

### Proof of Proposition 7

Since  $i$  and  $j$  are upstream symmetric, it is immediate that  $\rho_i = \rho_j$ . Furthermore, the fact that they are downstream symmetric implies that they have identical Domar weights, i.e.,  $\lambda_i = \lambda_j$ . As a result, equations (40)–(42) in Theorem 2 imply that if  $\phi_i < \phi_j$ , then  $\psi_i^{\text{O.G.}} > \psi_j^{\text{O.G.}}$ ,  $\psi_i^{\text{across}} > \psi_j^{\text{across}}$ , and  $\psi_i^{\text{within}} > \psi_j^{\text{within}}$ . Putting the three inequalities together then guarantees that  $\psi_i^* > \psi_j^*$ .  $\square$

### Proof of Proposition 8

Since  $i$  and  $j$  are downstream symmetric, they have identical Domar weights, i.e.,  $\lambda_i = \lambda_j = \lambda$ . In addition, recall that, by assumption,  $\phi_i = \phi_j = \phi$ . Therefore, equation (40) implies that  $\psi_i^{\text{O.G.}} = \psi_j^{\text{O.G.}}$ . Furthermore, equation (42) implies

$$\psi_i^{\text{across}} - \psi_j^{\text{across}} = (1/\phi - 1) \sum_{s=1}^n (1 - \phi_s) \lambda_s \rho_s (\ell_{si} - \ell_{sj}) = 0,$$

where once again we are using the assumptions that  $i$  and  $j$  are downstream symmetric and that  $\phi_i = \phi_j = \phi$ . Finally, equation (41) and the assumption that  $\theta_i = \theta_j = \theta$  implies

$$\psi_i^{\text{within}} - \psi_j^{\text{within}} = (1 - \phi) \lambda \theta (\rho_i - \rho_j).$$

Thus, if  $\rho_i > \rho_j$ , then  $\psi_i^{\text{within}} > \psi_j^{\text{within}}$  and hence,  $\psi_i^* > \psi_j^*$ .  $\square$

### Proof of Proposition 9

By assumption,  $\theta_i = \theta_j$ ,  $\phi_i = \phi_j$ , and  $\lambda_i = \lambda_j$ . Furthermore, the assumption that  $i$  and  $j$  are upstream symmetric implies that  $\rho_i = \rho_j$ . Therefore, equations (40) and (41) in Theorem 2 imply that  $\psi_i^{\text{O.G.}} = \psi_j^{\text{O.G.}}$  and  $\psi_i^{\text{within}} = \psi_j^{\text{within}}$ . Turning to the dimension of policy targeting across-industry misallocation, equation (42) implies

$$\psi_i^{\text{across}} - \psi_j^{\text{across}} = (1/\phi - 1) \sum_{s=1}^n (1 - \phi_s) \lambda_s \rho_s (\ell_{si} - \ell_{sj}),$$

where once again we are using the fact that  $\phi_i = \phi_j = \phi$ . It is now immediate that  $\psi_i^* > \psi_j^*$  if and only if inequality (43) is satisfied.  $\square$

### Proof of Proposition 10

Suppose  $\theta_i = \theta_j = \theta$  and  $\phi_i = \phi_j = \phi < 1$ . Also suppose industry  $j$  is the sole input-supplier of industry  $i$  and  $i$  is the sole customer of  $j$ . This implies that both industries have identical steady-state Domar weights. Therefore, by (40),  $\psi_i^{\text{o.g.}} = \psi_j^{\text{o.g.}}$ .

The fact that  $j$  is the sole supplier of  $i$  also implies that  $\rho_i = \phi_j \rho_j < \rho_j$ , where we are using the definition of upstream flexibility in (28). As a result, equation (41) implies that, in the optimal policy,  $\psi_i^{\text{within}} < \psi_j^{\text{within}}$ .

Finally, note that the proposition's main assumption guarantees that  $\ell_{sj} = \ell_{si} + \mathbb{I}_{\{s=j\}}$  for all  $s$ . As a result, equation (42) implies

$$\psi_i^{\text{across}} - \psi_j^{\text{across}} = (1/\phi - 1) \sum_{t=0}^n \lambda_t \sum_{s=1}^n a_{ts} (\rho_t - \phi_s \rho_s) (\ell_{si} - \ell_{sj}) = (1/\phi - 1) \sum_{t=0}^n \lambda_t a_{tj} (\phi_j \rho_j - \rho_t).$$

Note that, by assumption,  $a_{tj} = \mathbb{I}_{\{t=j\}}$ . As a result, the right-hand side of the above equation is equal to  $(1/\phi - 1)(\phi_j \rho_j - \rho_i)$ . Now the fact that  $\rho_i = \phi_j \rho_j$  guarantees that  $\psi_i^{\text{across}} = \psi_j^{\text{across}}$ .  $\square$

### Proof of Proposition A.1

Recall from the proof of Lemma 2 that the nominal price set by firm  $k$  in industry  $i$  is given by  $\log p_{ik} = \mathbb{E}_{ik}[\log mc_i] + o(\delta)$  as  $\delta \rightarrow 0$ . Therefore, when the production function of firms in industry  $i$  is given by (19),

$$\log p_{ik} = \alpha_i \mathbb{E}_{ik}[\log w] - \mathbb{E}_{ik}[\log z_i] + \sum_{j=1}^n a_{ij} \mathbb{E}_{ik}[\log p_j] + o(\delta),$$

where  $p_j$  is the nominal price of the sectoral good produced by industry  $j$ . Integrating both sides of this equation over the set of all firms in industry  $i$  implies that

$$\log p_i = \alpha_i \int_0^1 \mathbb{E}_{ik}[\log w] dk - \int_0^1 \mathbb{E}_{ik}[\log z_i] dk + \sum_{j=1}^n a_{ij} \int_0^1 \mathbb{E}_{ik}[\log p_j] dk + o(\delta).$$

Since a similar expression has to hold for the nominal price of all industries  $j$ , iterating on the above equation implies that

$$\log p_i = \sum_{r=1}^{\infty} \bar{\mathbb{E}}_i^{(r)}[\alpha \log w] - \sum_{r=1}^{\infty} \bar{\mathbb{E}}_i^{(r)}[\log z] + o(\delta), \quad (\text{B.40})$$

where  $\bar{\mathbb{E}}_i^{(r)}[\cdot]$  is defined recursively in (A.1). This, coupled with the fact that  $PC = m$  and  $\log P = \sum_{i=1}^n \beta_i \log p_i$ , implies that  $\log$  aggregate output is given by

$$\log C = \sum_{r=1}^{\infty} \sum_{i=1}^n \beta_i \bar{\mathbb{E}}_i^{(r)}[\log z] + \left( \log m - \sum_{r=1}^{\infty} \sum_{i=1}^n \beta_i \bar{\mathbb{E}}_i^{(r)}[\alpha \log w] \right) + o(\delta).$$

Noting that  $m = w$  when  $\gamma = 1$  and  $\eta \rightarrow \infty$  then establishes (A.2).  $\square$

## Proof of Proposition A.2

As a first observation, note that the log output of industry  $i$  is equal to  $\log y_i = \log m - \log p_i + \log \lambda_i$ , where  $\lambda_i$  is  $i$ 's Domar weight. Using (B.32) to obtain a first-order approximation for  $\log \lambda_i$  leads to

$$\log y_i = \log m - \log p_i + \log \lambda_i^* - \log \varepsilon_i + \frac{1}{\lambda_i^*} \sum_{j=1}^n \ell_{ji} \lambda_j^* \log \varepsilon_j + o(\delta).$$

Rewriting the above in vector form, we get

$$\log y = \mathbf{1} \log w - \log p + \log \lambda^* - (\mathbf{I} - \mathbf{\Lambda}^{*-1} \mathbf{L}' \mathbf{\Lambda}^*) \log \varepsilon + o(\delta),$$

where  $\mathbf{\Lambda}^* = \text{diag}(\lambda_1^*, \dots, \lambda_n^*)$  is a diagonal matrix with diagonal elements given by industries' flexible-price Domar weights and we are using the fact that  $m = w$  when  $\gamma = 1$  and  $\eta \rightarrow \infty$ . From the above equation, it is immediate that the vector of industry-level log output under flexible-prices is given by  $\log y^* = \log w \mathbf{1} - \log p^* + \log \lambda^* + o(\delta)$ . Consequently,

$$\log y = \log y^* - (\log p - \log p^*) - (\mathbf{I} - \mathbf{\Lambda}^{-1} \mathbf{L}' \mathbf{\Lambda}) \log \varepsilon + o(\delta),$$

where we are using the fact that  $\lambda_i = \lambda_i^* + o(1)$ . Equation (B.30) implies that  $\log \varepsilon = -(\mathbf{I} - \mathbf{A})(\log p - \log p^*) + o(\delta)$ , or equivalently,  $\log p - \log p^* = -\mathbf{L} \log \varepsilon + o(\delta)$ . Therefore,

$$\log y = \log y^* + (\mathbf{A} + \mathbf{\Lambda}^{-1} \mathbf{L}' \mathbf{\Lambda} (\mathbf{I} - \mathbf{A})) \mathbf{L} \log \varepsilon + o(\delta). \quad (\text{B.41})$$

It is therefore sufficient to characterize  $\log \varepsilon$  in terms of model primitives. To this end, first recall that  $\log \varepsilon = -(\mathbf{I} - \mathbf{A})(\log p - \log p^*) + o(\delta)$ . Furthermore, note that log nominal prices in the sticky and flexible-price equilibria are given by (B.40) and  $\log p_i^* = \log m - \sum_{j=1}^n \ell_{ij} \log z_j$ , respectively. Consequently,

$$\log \varepsilon = (\mathbf{I} - \mathbf{A}) \left( \sum_{r=1}^{\infty} \bar{\mathbb{E}}^{(r)}[\log z] - \mathbf{L} \log z \right) + (\mathbf{I} - \mathbf{A}) \left( \mathbf{1} \log m - \sum_{r=1}^{\infty} \bar{\mathbb{E}}^{(r)}[\alpha \log m] \right) + o(\delta), \quad (\text{B.42})$$

where  $\bar{\mathbb{E}}^{(r)}[t]$  is a vector whose  $i$ -th element is given by  $\bar{\mathbb{E}}_i^{(r)}[t]$ . Plugging the above into (B.41) completes the proof.  $\square$

## Proof of Proposition A.3

As in the proof of Theorem 2, we first determine the optimal policy by characterizing how nominal wage should optimally respond to productivity shocks. We then determine the price-stabilization policy that implements such a nominal wage.

Recall from Proposition 6 that nominal rigidities result in a welfare loss that can be approximated by (36) to a second-order approximation as  $\delta \rightarrow 0$ , where  $\bar{e}_i$  and  $\vartheta_i$  are defined in (34) and (35) and denote the cross-sectional average and cross-sectional dispersion of pricing errors in industry  $i$ , respectively. Therefore, as a first step, we determine  $\bar{e}_i$  and  $\vartheta_i$  in terms of the realized productivity shocks and the nominal wage. To this end, as in (B.20) and (B.21), let  $\log w = \kappa' \log z + o(\delta)$  and  $\log p = \mathbf{B} \log z + o(\delta)$  denote the log-linearization of, respectively, the nominal wage and the vector of nominal prices of

sectoral goods as  $\delta \rightarrow 0$ . Furthermore, recall from Lemma 2 that, to a first-order approximation, the nominal price set by firm  $k$  in industry  $i$  is given by equation (27). Integrating both sides of (27) over all firms  $k$  in industry  $i$  implies that

$$\log p_i = \phi_i \alpha_i \log w - \phi_i \log z_i + \phi_i \sum_{j=1}^n a_{ij} \log p_j + o(\delta),$$

where we are using the assumption that a fraction  $\phi_i$  of firms industry  $i$  receive perfectly informative signals about the realization of the shocks, while the remainder  $1 - \phi_i$  fraction receive no information at all. Writing the above equation in matrix form and using (B.20) and (B.21) leads to  $\mathbf{B} = \Phi(\alpha\kappa' - \mathbf{I} + \mathbf{A}\mathbf{B})$ . Solving for matrix  $\mathbf{B}$  therefore implies that the vector of log nominal prices is given by

$$\log p = \mathbf{B} \log z + o(\delta), \quad (\text{B.43})$$

where

$$\mathbf{B} = (\mathbf{I} - \Phi\mathbf{A})^{-1} \Phi(\mathbf{I} - \mathbf{A})(\mathbf{1}\kappa' - \mathbf{L}). \quad (\text{B.44})$$

Equations (B.43) and (B.44) additionally imply that, in the absence of nominal rigidities, the vector of log nominal prices is  $\log p^* = (\mathbf{1}\kappa' - \mathbf{L}) \log z + o(\delta)$ . Therefore, the vector of cross-sectional average of pricing errors, defined in (34), is given by

$$\bar{e} = \log p - \log p^* = \mathbf{Q}(\mathbf{L} - \mathbf{1}\kappa') \log z + o(\delta), \quad (\text{B.45})$$

where  $\mathbf{Q} = (\mathbf{I} - \Phi\mathbf{A})^{-1}(\mathbf{I} - \Phi)$ . Next, we obtain the expression for cross-sectional dispersion of pricing errors within each industry. Since the marginal cost of firms in industry  $i$  is given by  $\log mc_i = \alpha_i \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j$ , equation (B.43) implies that

$$\log mc = (\alpha\kappa' - \mathbf{I} + \mathbf{A}\mathbf{B}) \log z + o(\delta) = \Phi^{-1} \log p + o(\delta). \quad (\text{B.46})$$

Furthermore, the assumption that a fraction  $\phi_i$  of firms in industry  $i$  can set their prices flexibly implies that the cross-sectional dispersion of pricing errors in industry  $i$  is equal to  $\vartheta_i = \phi_i(\log mc_i - \log p_i)^2 + (1 - \phi_i)(\log p_i)^2 = (1/\phi_i - 1)(\log p_i)^2 + o(\delta^2)$ , where the second equality follows from (B.46). Consequently, using (B.43), we obtain

$$\mathbb{E}[\vartheta_i] = \delta^2(1/\phi_i - 1)b'_i \Sigma b_i + o(\delta^2), \quad (\text{B.47})$$

where  $b'_i$  is the  $i$ -th row of matrix  $\mathbf{B}$  in (B.43) and  $\Sigma$  denotes the variance-covariance matrix of log productivity shocks.

With the expressions for the cross-sectional average and dispersion of pricing errors in hand, we next turn to determining the expected welfare loss under an arbitrary policy. Recall from Proposition 6 that, to a second-order approximation as  $\delta \rightarrow 0$ , welfare loss is given by (36). Therefore, using (B.47), we can write the expected welfare loss as

$$\begin{aligned} \mathbb{E}[W - W^*] = & -\frac{1}{2} \left[ \delta^2 \lambda' \text{diag}(\theta) (\Phi^{-1} - \mathbf{I}) \text{diag}(\mathbf{B}\Sigma\mathbf{B}') + \frac{1}{\gamma + 1/\eta} \beta' \mathbb{E}[\bar{e}\bar{e}'] \beta + (\lambda' \mathbf{A} + \beta') \text{diag}(\mathbb{E}[\bar{e}\bar{e}']) \right. \\ & \left. - \lambda' \text{diag}(\mathbf{A}\mathbb{E}[\bar{e}\bar{e}']\mathbf{A}') - \beta' \mathbb{E}[\bar{e}\bar{e}'] \beta \right]. \end{aligned}$$

On the other hand, equation (B.45) implies that  $d\bar{e}/d\kappa_s = -\mathbf{Q}\mathbf{1} \log z_s$ . Therefore, differentiating the expected welfare loss with respect to vector  $\kappa$  and setting it equal to zero implies that

$$\begin{aligned} & \Sigma \mathbf{B}'(\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1}))(\Phi^{-1} - \mathbf{I}) \text{diag}(\theta)\lambda - \frac{1}{\gamma + 1/\eta} \Sigma(\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}'\beta\beta'\mathbf{Q}\mathbf{1} \\ & - \Sigma(\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}' \text{diag}(\mathbf{Q}\mathbf{1})(\mathbf{A}'\lambda + \beta) + \Sigma(\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}'\mathbf{A}' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1})\lambda + \Sigma(\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}'\beta\beta'\mathbf{Q}\mathbf{1} = 0. \end{aligned}$$

Multiplying both sides of the above equation by  $\Sigma^{-1}$  from the left and replacing for  $\mathbf{B}'$  in the first term from (B.44) implies that

$$\begin{aligned} & -(\mathbf{L}' - \kappa\mathbf{1}')(\mathbf{I} - \mathbf{Q}')(\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1}))(\Phi^{-1} - \mathbf{I}) \text{diag}(\theta)\lambda - \frac{1}{\gamma + 1/\eta} (\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}'\beta\beta'\mathbf{Q}\mathbf{1} \\ & - (\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}' \text{diag}(\mathbf{Q}\mathbf{1})(\mathbf{A}'\lambda + \beta) + (\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}'\mathbf{A}' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1})\lambda + (\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}'\beta\beta'\mathbf{Q}\mathbf{1} = 0. \end{aligned}$$

Solving for  $\kappa$  then implies that the vector of weights in the optimal wage-setting policy is given by  $\kappa' = \iota' / (\iota'\alpha)$ , where

$$\begin{aligned} \iota' &= \lambda' \text{diag}(\theta)(\mathbf{I} - \Phi)\Phi^{-1}(\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1}))(\mathbf{I} - \mathbf{Q})\mathbf{L} + \frac{1}{\gamma + 1/\eta} (\beta'\mathbf{Q}\mathbf{1})\beta'\mathbf{Q}\mathbf{L} \\ &+ \lambda'\mathbf{A} \text{diag}(\mathbf{Q}\mathbf{1})\mathbf{Q}\mathbf{L} - \lambda' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1})\mathbf{A}\mathbf{Q}\mathbf{L} + \beta' \text{diag}(\mathbf{Q}\mathbf{1})\mathbf{Q}\mathbf{L} - (\beta'\mathbf{Q}\mathbf{1})\beta'\mathbf{Q}\mathbf{L}. \end{aligned}$$

The above expression is identical to equation (B.38). Therefore, the optimal price-stabilization policy coincides with the policy in (39).  $\square$

### Proof of Proposition A.4

We prove this result by establishing that the policy that minimizes the welfare loss (36) (i.e., the optimal policy) coincides with the policy that minimizes the volatility of the target price index  $\sum_{i=1}^n \psi_i^* \log p_i$ , with weights given by (39). Note that since  $\gamma = 1$  and  $\eta \rightarrow \infty$ , the nominal aggregate demand,  $m$ , is equal to the nominal wage,  $w$ . As a result, without loss of generality, we can parameterize the set of policies by vector  $\kappa = (\kappa_1, \dots, \kappa_n)'$ , where  $\log w = \sum_{i=1}^n \kappa_i \hat{\omega}_i$  and  $\hat{\omega}_i$  is monetary authority's signal given by (A.6).

**Minimizing the volatility of target price index:** We first characterize the policy that minimizes the volatility of the target price index  $\sum_{i=1}^n \psi_i^* \log p_i$ .

By assumption, fraction  $\phi_i$  of firms in industry  $i$  can set their prices flexibly, while the remaining  $1 - \phi_i$  fraction are subject to full nominal rigidities. As a result, the log nominal price of industry  $i$  satisfies  $\log p_i = \phi_i(\alpha_i \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j)$ . Writing this equation in matrix form and solving for the vector of log nominal prices, we get

$$\log p = (\mathbf{I} - \Phi\mathbf{A})^{-1}\Phi(\mathbf{I} - \mathbf{A})(\mathbf{1} \log w - \mathbf{L} \log z). \quad (\text{B.48})$$

Next, recall that the vector of industry weights  $\psi^*$  satisfies (B.39). As a result,

$$\sum_{i=1}^n \psi_i^* \log p_i = v'(\mathbf{1} \log w - \mathbf{L} \log z),$$

where

$$v' = \lambda' [\text{diag}(\theta)(\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1}))(\mathbf{I} - \mathbf{A}) + \text{diag}(\mathbf{Q}\mathbf{1}) - \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1})\mathbf{A}] \mathbf{Q} \quad (\text{B.49})$$

and we are using the following identity:  $\mathbf{L}(\mathbf{I} - \Phi)\Phi^{-1}(\mathbf{I} - \Phi\mathbf{A})^{-1}\Phi(\mathbf{I} - \mathbf{A}) = \mathbf{Q}$ . Since  $\log w = \kappa'\hat{\omega} = \kappa'(\log z + \hat{\epsilon})$ , the volatility of the target price index is equal to

$$\text{var} \left( \sum_{s=1}^n \psi_s^* \log p_s \right) = \delta^2 \sigma_z^2 \|v'(\mathbf{L} - \mathbf{1}\kappa')\|_2^2 + \delta^2 \hat{\sigma}^2 \|v'\mathbf{1}\kappa'\|_2^2.$$

Optimizing the above with respect to  $\kappa$  implies that the policy that minimizes the volatility of the target price index is given by

$$\kappa' = \frac{\sigma_z^2}{\sigma_z^2 + \hat{\sigma}^2} \frac{v'\mathbf{L}}{v'\mathbf{1}}, \quad (\text{B.50})$$

where  $v$  is given by (B.49).

**Optimal policy:** We next characterize the optimal policy and show that it coincides with policy (B.50), which minimizes the volatility of the target price index.

By Proposition 6, welfare loss due to the presence of nominal rigidities is given by (36). Therefore, when  $\gamma = 1$  and  $\eta \rightarrow \infty$ , the expected welfare loss is equal to

$$\mathbb{E}[W - W^*] = -\frac{1}{2} \lambda' (\text{diag}(\theta)\mathbb{E}[\vartheta] + \text{diag}(\mathbb{E}[\bar{e}\bar{e}']) - \text{diag}(\mathbf{A}\mathbb{E}[\bar{e}\bar{e}']\mathbf{A}')) + o(\delta^2), \quad (\text{B.51})$$

where  $\bar{e}$  and  $\vartheta$  denote the vectors of cross-sectional mean and dispersion of pricing errors defined in (34) and (35), respectively.

To determine the right-hand side of (B.51), we make two observations. First, note that  $\bar{e} = \mathbf{Q}(\mathbf{L} \log z - \mathbf{1} \log w)$ , where  $\mathbf{Q} = (\mathbf{I} - \Phi\mathbf{A})^{-1}(\mathbf{I} - \Phi)$ . This follows from the fact that the vectors of log nominal prices in the sticky- and flexible-price equilibria are given by (B.48) and  $\log p^* = \mathbf{1} \log w - \mathbf{L} \log z$ , respectively. As a result,

$$\mathbb{E}[\bar{e}\bar{e}'] = \delta^2 \sigma_z^2 \mathbf{Q}(\mathbf{L} - \mathbf{1}\kappa')(\mathbf{L}' - \kappa\mathbf{1}')\mathbf{Q}' + \delta^2 \hat{\sigma}^2 \mathbf{Q}\mathbf{1}\kappa'\kappa\mathbf{1}'\mathbf{Q}', \quad (\text{B.52})$$

where we are using the fact that  $\log w = \sum_{i=1}^n \kappa_i \hat{\omega}_i$ . Second, since a fraction  $\phi_i$  of firms in industry  $i$  can set their prices flexibly, while the remaining  $1 - \phi_i$  fraction face full nominal rigidities, price dispersion in industry  $i$  is equal to  $\vartheta_i = (1/\phi_i - 1) \log^2 p_i$ , where the log nominal price of industry  $i$  in the sticky-price equilibrium satisfies (B.48). As a result,

$$\mathbb{E}[\vartheta] = \delta^2 (\Phi^{-1} - \mathbf{I}) \text{diag} \left( (\mathbf{I} - \mathbf{Q}) [\sigma_z^2 (\mathbf{L} - \mathbf{1}\kappa')(\mathbf{L}' - \kappa\mathbf{1}') + \hat{\sigma}^2 \mathbf{1}\kappa'\kappa\mathbf{1}'] (\mathbf{I} - \mathbf{Q}') \right). \quad (\text{B.53})$$

With the expressions in (B.52) and (B.53) in hand, we then minimize the expected welfare loss (B.51) with respect to  $\kappa$ , leading to the corresponding first-order condition:  $\sigma_z^2 v'(\mathbf{1}\kappa' - \mathbf{L}) + \hat{\sigma}^2 v'\mathbf{1}\kappa' = 0$ , where  $v$  is defined in (B.49). This, in turn implies that the policy that minimizes the expected welfare loss is given by  $\kappa' = \frac{\sigma_z^2}{(\sigma_z^2 + \hat{\sigma}^2)v'\mathbf{1}} v'\mathbf{L}$ , which coincides with policy (B.50), which minimizes the volatility of target price index  $\sum_{i=1}^n \psi_i^* \log p_i$ .  $\square$

## C Online Appendix: Robustness Checks

This appendix contains some robustness checks for our quantitative analysis, where we report the expected welfare loss under the optimal policy and the four alternative policies considered in Section 5 for different parameter values. We calculate the expected welfare loss of each of the five policies (relative to the flexible-price equilibrium and measured as a fraction of steady-state consumption) by simulating the fully nonlinear model for 10,000 draws of the vector of productivity shocks. We consider parameter values on a grid with  $\eta \in \{0.5, 1, 2\}$ ,  $\gamma \in \{0.1, 1, 2\}$ , and  $\phi_w \in \{0.25, 0.30\}$ , where  $\eta$ ,  $\gamma$ , and  $\phi_w$  denote the Frisch elasticity of labor supply, the household's coefficient of relative risk aversion, and the degree of wage flexibility, respectively. Aside from the values discussed in the main body of the paper, we choose the remaining parameter values as follows. We consider  $\eta = 0.5$  and  $\gamma = 2$  as in McKay, Nakamura, and Steinsson (2016) and  $\phi_w = 0.25$  as in Galí (2008). We also consider the case of  $\eta = 1$  and  $\gamma = 1$  as a natural benchmark.

Table C.1 reports the results. Three observations are immediate. First, the expected welfare loss under the optimal policy is fairly robust to the choice of parameter values, taking a value between 0.63% and 0.66% of steady-state consumption. Second, irrespective of the calibration, the policy that stabilizes the output gap is nearly optimal: among all specifications, the wedge between the optimal and the output-gap-stabilization policies never exceeds 0.03 percentage points. Finally, while the expected welfare loss under the policy that targets the consumer price index is more sensitive to the choice of parameters, this policy significantly underperforms the optimal policy irrespective of the specification.



Table C.1. Expected Welfare Loss under Various Policies

$(\eta, \gamma, \phi_w)$	optimal policy	output-gap stabilization	consumption weighted	Domar weighted	stickiness weighted
(0.5, 0.1, 0.25)	0.63	0.65	1.44	1.30	1.87
(0.5, 0.1, 0.30)	0.63	0.65	1.18	1.10	1.54
(0.5, 1, 0.25)	0.64	0.65	1.29	1.17	1.61
(0.5, 1, 0.30)	0.64	0.65	1.14	1.04	1.45
(0.5, 2, 0.25)	0.63	0.65	1.20	1.11	1.44
(0.5, 2, 0.30)	0.63	0.65	1.07	0.98	1.29
(1, 0.1, 0.25)	0.64	0.66	1.91	1.74	2.76
(1, 0.1, 0.30)	0.65	0.67	1.44	1.33	2.09
(1, 1, 0.25)	0.64	0.65	1.37	1.30	1.86
(1, 1, 0.30)	0.64	0.65	1.22	1.08	1.53
(1, 2, 0.25)	0.65	0.65	1.29	1.13	1.49
(1, 2, 0.30)	0.64	0.65	1.12	1.01	1.33
(2, 0.1, 0.25)	0.66	0.67	3.10	2.31	7.93
(2, 0.1, 0.30)	0.65	0.67	1.95	1.58	3.31
(2, 1, 0.25)	0.65	0.66	1.64	1.38	2.06
(2, 1, 0.30)	0.64	0.66	1.26	1.15	1.67
(2, 2, 0.25)	0.64	0.65	1.31	1.18	1.61
(2, 2, 0.30)	0.64	0.65	1.11	1.05	1.43

*Notes:* The table reports the expected welfare loss due to the presence of nominal rigidities under various monetary policies as a percentage of steady-state consumption. The expected welfare loss for the fully nonlinear model is calculated using 10,000 draws.  $\eta$ ,  $\gamma$ , and  $\phi_w$  denote the Frisch elasticity of labor supply, the household's coefficient of relative risk aversion, and the degree of wage flexibility, respectively.