O-Ring Production Networks

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Abstract

We study a production network where quality choices are interconnected across firms. High-quality firms are skill intensive and disproportionately source inputs from and sell output to other high-quality firms. Consistent with the theory, we document strong assortative matching of skills in the network of Turkish manufacturing firms. In the data, a firm-specific trade shock from a rich country increases the firm's skill intensity and shifts the firm toward skill-intensive domestic partners. We develop a quantitative model with heterogeneous firms, endogenous quality choices, and network formation. Parameter estimates indicate strong complementarity of quality in production. A common export demand shock of 5 percent induces broad quality upgrading among both exporters and domestic firms, raising average wage by 1.2 percent. This effect is about eight times larger than the average effect of a shock that occurs only to a few (zero-measure) firms.

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1 Introduction

The space shuttle *Challenger* exploded because one of its innumerable components, the O-rings, malfunctioned during launch. Using this as a leading example, Kremer (1993) studies production processes, in which the value of output dramatically decreases if a single task fails. In his model, just one mistake of an unskilled worker is enough to destroy a product. So, firms that produce complex, higher-quality products hire skilled workers for all their tasks.

Extending this rationale across firm boundaries, a high-quality, skill-intensive firm sources its inputs from other high-quality firms, and it sells more to high-quality firms that value its output. In addition, a firm's decision to upgrade quality depends critically on the willingness of its trading partners to also upgrade or on its ability to find new higher-quality partners. This mechanism applies to the quality of products as well as to the quality of inventory controls, research and development, and internal communications. Improvements in these areas generally allow for greater product scope and render the firm more flexible to respond to shocks. A firm profits from these improvements if its suppliers also offer scope and flexibility, and if its customers value them.

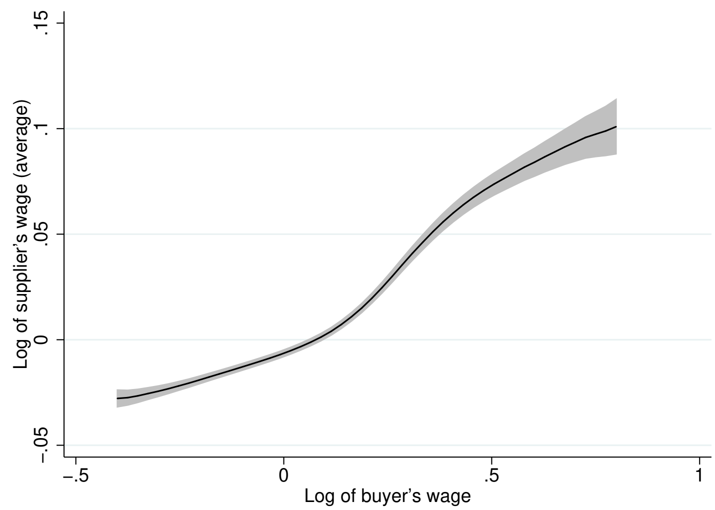
We study this interconnection in firms' quality choices theoretically and empirically. Our data comprise all formal Turkish manufacturing firms from 2011 to 2015. We merge value-added tax (VAT) data with matched employer-employee and customs data. We observe the value of trade for each buyer-seller pair of firms; exports by firm, product and destination, and the occupation and wage of each worker in each firm.

We document a novel, strong assortative matching of skills in the network. As an example, Figure 1 graphs firms' average wage (adjusted for industry-region) against the average wage of their suppliers.[1] A 10 percent increase in a firm's wage is associated with a 2.5 percent increase in its suppliers' wages. This number is large given that firms have on average eleven suppliers. This increasing relationship between buyer and supplier wage may arise from an extensive margin—high-wage firms match more with each other—or from an intensive margin—high-wage firms spend relatively more on their high-wage suppliers. A decomposition indicates that the extensive margin accounts for 59% of the relationship and the intensive margin accounts for 41%.

We use shift-share regressions to evaluate firms' responses to shocks and movements along the schedule in Figure 1. Consider a Turkish firm that in 2011 exported a particular product category to a high-income country, say Germany. An increase in German imports

[1] The figure has only manufacturing firms, later used in our structural estimation, but an equally strong pattern holds if we include all sectors. See Table 1, column (4).

Figure 1: Assortative Matching on Wages



Notes: We define the wage as the firm's wage bill divided by the number of workers. Supplier wage is the average wage across the firm's manufacturing suppliers, weighted by the firm's spending on each supplier. Both x- and y-axis variables are demeaned from 4-digit NACE industry and region. The fitted curve is a local polynomial regression with Epanechnikov kernel. The shaded area shows the 95% confidence intervals. The regression corresponding to this figure is in Table 1, column (2).

of that product category from countries other than Turkey from 2011 to 2015 is associated with an increase in the Turkish firm's wage, and in the average wage of its suppliers and customers. The new employees, suppliers and customers that the firm adds over the period, from 2011 to 2015, had on average higher wages in 2011 than the firms' original employees and partners. Our proposed mechanism combined with evidence from the literature that high-income countries demand relatively more skill-intensive goods explains these patterns:[2] An increase in the relative demand for high-quality goods increases a firm's quality and skill intensity. The firm shifts toward skill-intensive trading partners and may prod its existing partners to upgrade.

The interconnection in firms' quality choices implies that a shock that is common to a significant share of firms may have a larger effect than the sum of idiosyncratic, firm-specific shocks. We develop a model to study these types of shocks. The model is in the spirit of Kremer (1993), but for a quantitative analysis, we base it on Melitz's (2003) model of heterogeneous firms. We add to Melitz (2003) the assumptions on quality from Verhoogen (2008) and Kugler and Verhoogen (2011), and an endogenous network formed through search and matching, similar to models of labor.[3] Firms post costly ads to search

[2]See Hallak (2006), Brambilla et al. (2012), Manova and Zhang (2012), and Bastos et al. (2018).

[3]See Mortensen (1986) and Rogerson et al. (2005) for surveys.

for other firms. More productive firms post more ads and have more customers and suppliers. A firm's quality determines its production function. We assume that higher-quality firms are skill intensive, and we allow them to be intensive in high-quality inputs. When posting ads, firms imperfectly target other firms with similar quality levels.

We estimate the model to Turkish manufacturing firms using the method of simulated moments. We focus on manufacturing firms because the shift-share regressions above apply only to them. The moments describe assortative matching on wages, and the joint distribution of firm revenue, wages, number of customers and suppliers. Targeted search in the model captures differences in matching across firms with different wages (extensive margin of assortative matching). Only about nine percent of the ads posted by buyers in the lowest quintile of wages are directed to suppliers in the highest wage quintile, and vice-versa. Differences in marginal productivity capture the spending patterns (intensive margin). The marginal product of an input in the 90^{th} percentile of the quality distribution is always larger than that of an input in the 10^{th} percentile. But it is 46 percent larger when producing output in the 90^{th} percentile of quality and 10 percent larger when producing output in the 10^{th} percentile.

In the data and in the model, exporters are large, skill intensive, and have many network connections, especially connections to other large, skill-intensive firms. Export intensity generally increases with exporter wage. This pattern holds in the estimated model because the relative demand for higher-quality is higher abroad. A firm-specific export demand shock in the model increases the firm's quality and skill intensity. The responsiveness of firms' quality choices to these idiosyncratic shocks in the model is estimated to match the shift-share regressions. In the data and in the model, a 5 percent increase in export demand increases the firm's wages by 0.21 percent.

We use a counterfactual to study the general equilibrium effect of an export shock of the same magnitude, but applied to all exporters instead of individual firms. The probability that any firm matches with a high-quality firm in the network increases with the shock. Matching with a high-quality supplier decreases the relative cost from producing high-quality output, and matching with a high-quality customer increases the demand for high-quality inputs. This demand effect accounts for about two-thirds of the counterfactual increase in profit from producing high- relative to low-quality goods and the cost effect accounts for one third. Non-exporting firms, not directly impacted by the shock, upgrade quality and increase their wages by 1.0 percent on average. The wages of exporters increase by 1.92 percent, almost an order of magnitude larger than the effect of firm-specific shocks.

To highlight the importance of assortative matching, we consider a special case of the model in which all firms equally value the quality of their inputs. The same counterfactual in this special case increases the wages of exporters by 0.23 percent, almost the same as the 0.21 percent response to the firm-specific shocks. In contrast, the manufacturing output responds similarly in the special case and in the general model. The predicted increase, of about six percent, is larger than Hulten (1978) but in line with Baqaee and Farhi (2019a) due to an elasticity of substitution between varieties larger than one.

The network literature has focused on Hicks-neutral shocks, while quality in our model changes the types of material and labor inputs that firms use. We relax Hicks-neutrality through log-supermodular shifters. We follow Teulings (1995) and Costinot and Vogel (2010) for labor, Fieler et al. (2018) for material inputs and apply it anew to search.[4] Our novel search-and-matching set up is tractable and yields a closed-form solution in the special case of the model with only one quality. We abstract, however, from the following aspects of the network highlighted in the literature: Dynamics in Lim (2018) and Huneeus (2018), asymmetries in network centrality in Acemoglu et al. (2012), and market distortions in Baqaee and Farhi (2019b), Bigio and La'O (2020) and Liu (2019). The model features roundabout production, technologies with constant elasticities of substitution, and each firm has a continuum of suppliers and customers. Some of these theoretical elements and the study of shocks to international trade appear in Lim (2018), Dhyne et al. (2018), Bernard et al. (2019a,b), Eaton et al. (2018), Huneeus (2018), and Lenoir et al. (2019).

The estimated model is consistent with well-established facts in the quality literature. The production of higher-quality is intensive in skilled labor as in Schott (2004), Verhoogen (2008), Khandelwal (2010), and in higher-quality inputs as in Kugler and Verhoogen (2011), Manova and Zhang (2012), and Bastos et al. (2018). Fieler et al. (2018) combine these elements to study, like us, the general equilibrium effect of international trade on demand for skills and quality. These papers all use data on prices. We complement them with direct information on the extent to which skill-intensive firms trade with each other. Our main finding on assortative matching is akin to Voigtländer (2014) who shows that skill-intensive sectors use intensively inputs from other skill-intensive sectors in the United States.[5]

The paper is organized as follows. Section 2 describes the data sources and facts. We present a closed economy version of the model in Section 3 and the small open economy

[4]The production function in Dingel (2017) aggregates workers with heterogeneous skills in the same manner that our production function aggregates material inputs with heterogeneous qualities. See also Milgrom and Roberts (1990) and Costinot (2009) for earlier applications of log-supermodular functions to economics and international trade.

[5]A related finding is in Carvalho and Voigtländer (2014) who show that firms are more likely to match with the suppliers of their suppliers. They interpret the finding in terms of information frictions.

in Section 4. The estimation procedure is in Section 5. Section 6 reports the estimation results and connects them to the empirical facts of Section 2. In Section 7, we experiment with counterfactual export shocks. Alternative counterfactual specifications guide a policy discussion in Section 8. Section 9 concludes.

2 Data and Empirical Facts

2.1 Data Sources

We combine five data sets from Turkey: (1) value added tax (VAT) data on domestic firm-to-firm trade, (2) data on firms' balance sheet and income statement, (3) firm registry, (4) customs data, and (5) linked employer-employee data. These data sets are all maintained by the Ministry of Industry and Technology. They contain the same firm identifier and comprise all formal firms in Turkey from 2011 through 2015.

The VAT data report all domestic firm-to-firm transactions whenever the total value of transactions for a seller-buyer pair exceeds 5,000 Turkish liras (about US$1,800 in 2015) in a given year. From the balance sheet and income statement data, we use information on each firm's gross domestic and foreign sales. From the firm registry, we extract the firm's location (province) and industry. The industry classification is the 4-digit NACE, the standard in the European Union. From the customs data, we use information on annual exports by firm, destination country, and 4-digit Harmonized System product code.

The employer-employee data are collected by the Turkish social security administration. We observe the quarterly wage of each worker in each firm. We also observe the worker's occupation (4-digit ISCO classification), age, and gender. The worker identifier is unique, allowing us to track workers across firms and over time.

We restrict most of the analysis to the more tradable, manufacturing sector. Unless otherwise noted, facts about the network refer to trade between firms within manufacturing. We drop firms that do not report their balance sheet or income statement. These are usually very small firms that use a single-entry bookkeeping system. The cross-sectional facts refer to year 2015. The final sample has 77,418 manufacturing firms in 2015.

Section 2.2 describes the assortative matching in the firm-to-firm network. Section 2.3 associates firm-specific trade shocks to systematic changes in firm outcomes, including wages and network connections. To estimate these trade shocks, we use annual bilateral trade data from BACI, disaggregated at the four-digit Harmonized System product code.[6]

[6]We aggregate these data from 6- to 4-digit HS codes for two reasons. First, it is less likely for any single country to have significant market power in a given destination at the 4-digit product level than at the 6-digit level. Second, the value of trade at the country-product level is too volatile at the 6-digit

Section 2.4 describes other salient features in the data. These features are not novel, but they justify some elements of the model.

2.2 Assortative Matching in the Cross-Section

Kremer (1993)'s O'Ring theory, when applied to inter-firm production chains, yields the prediction that skill-intensive firms disproportionately buy from and sell goods to other skill-intensive firms. We use a firm's average wage as a proxy for its skill intensity, under the assumption that firms observe skills better than us econometricians and that wages reflect differences in skills. We use other measures of skills for robustness in Section 2.2.1.

Define $wage_f$ as firm f's total monthly wage bill divided by its number of workers. Define the wage of firm f's suppliers as:

$$\log wage_f^S = \sum_{\omega \in \Omega_f^S} s_{\omega f} \log wage_\omega, \tag{1}$$

where Ω_f^S is the set of suppliers to firm f, and $s_{\omega f}$ is the share of supplier ω in firm f's total spending on inputs.

Table 1 reports the results from the regression

$$\log wage_f^S = \beta \log wage_f + \gamma X_f + e_f, \tag{2}$$

where e_f is the residual and X_f are control variables that vary across columns. Columns (1) through (3) contain only the manufacturing sub-sample. Column (1) has no control variables. Column (2) includes fixed effects of each industry-province pair. The coefficient decreases from column (1) because firms are more likely to match within province and industry, and some industry-province pairs have higher skill shares. Still, the decrease is small, from 0.294 to 0.259, suggesting that most of the variation across firms occurs within industry-province. A 10 percent increase in average buyer's wage is associated with a 2.5 percent increase in average supplier wages.

Column (3) controls for the buying firm's employment. Since employment and wages are correlated, the coefficient on wages decreases. But its magnitude is comparable to other columns. Column (4) repeats specification (2) with the sample of all firms.[7] The coefficient of 0.241 is similar to specification (2).

product level.

[7] We exclude finance, insurance, utilities and public services.

Table 1: Assortative Matching on Wages

Dependent variable: $\log wage_f^S$

	Manufacturing firms			All firms
	(1)	(2)	(3)	(4)
$\log wage_f$	0.294	0.259	0.188	0.241
	(0.013)	(0.012)	(0.009)	(0.013)
$\log employment_f$			0.044	
			(0.003)	
R^2	0.095	0.173	0.199	0.150
N	77,418	77,418	77,418	410,608
Fixed effects		ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. The suppliers' average wage $\log wage_f^S$ is defined in equation (1). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Decomposition into margins The positive coefficients on Table 1 could arise because high-wage firms have more high-wage suppliers—an extensive margin—or because they spend relatively more on their high-wage suppliers given the same matches—an intensive margin. We decompose the coefficient of our preferred specification (2) into these margins.

Define the extensive margin as the unweighed average of the wage of firm f's suppliers:

$$EM_f^S = \sum_{\omega \in \Omega_f} \frac{1}{|\Omega_f|} \log wage_\omega \tag{3}$$

Define the intensive margin as the difference between $\log wage_f^S$ in (1) and the extensive margin:

$$\begin{aligned} IM_f^S &= \log wage_f^S - EM_f^S \\ &= \sum_{\omega \in \Omega_f} (s_{\omega f} - 1/|\Omega_f|)(\log wage_\omega - \sum_{\omega' \in \Omega_f} (1/|\Omega_f|) \log wage_{\omega'}) \end{aligned} \tag{4}$$

The intensive margin is large if firm f's spending shares $s_{\omega f}$ are particularly large for high-wage suppliers ω.

One at a time, we regress $\log wage_f^S$, EM_f^S and IM_f^S on the wage of firm f and on industry-province fixed effects. The results are in Table 2. The first regression is the same as column (2), Table 1. By construction, the coefficients in the second and third columns add up to the total, 0.259, in the first column. The extensive margin accounts for 59% (= 0.152/0.259) of the partial correlation between the firm's wage and its suppliers' wages, while the intensive margin accounts for 41%. Since these margins are both large, the model will allow for both.

Table 2: Assortative Matching on Wages: Decomposition

	total $\log wage_f^S$ (A)	extensive margin EM_f^S	intensive margin IM_f^S
$\log wage_f$	0.259	0.152	0.107
	(0.012)	(0.007)	(0.007)
coeff. / coeff in (A)		*59%*	*41%*
R^2	0.173	0.150	0.089
N	77,418	77,418	77,418
Fixed effects	ind-prov	ind-prov	ind-prov

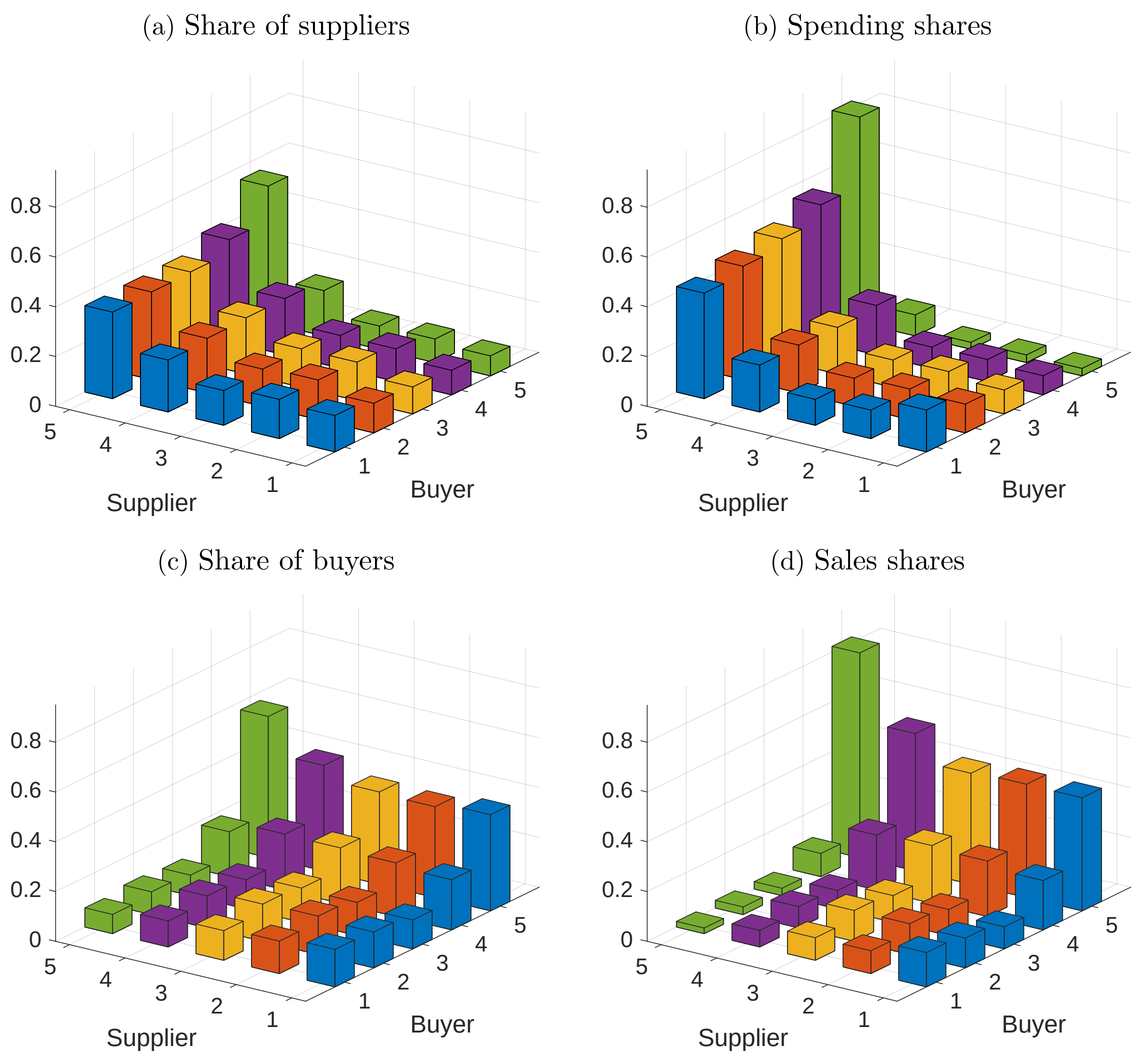
Notes: Wage is defined as the average value of monthly payments per worker. The suppliers' average wage $\log wage_f^S$ is defined in equation (1). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Equations (3) and (4) define the extensive (EM_f^S) and intensive margins (IM_f^S). They capture respectively the extent to which firm f matches with high-wage suppliers or tilts its spending toward high-wage suppliers. Robust standard errors are clustered at 4-digit NACE industry level.

Figure 2 illustrates assortative matching using the raw data. We split firms into quintiles of $wage_f$. Panels (a) and (b) describe firms' upstream links. The height of the bars in panel (a) is the supplier quintile's share in the number of suppliers to firms in each buyer quintile. The height in panel (b) is supplier quintile's share in the spending by firms in each buyer quintile. So, by construction, the sum of bars with the same color, across supplier quintiles, is one for each buyer quintile. Suppliers in the highest quintile of wages generally have larger sales and more buyers. So their shares are larger for all buyer quintiles. But in both panels the difference between sellers in quintiles 1 and 5 is much larger when the buyer has high wage. In addition, due to the intensive margin, these differences are more pronounced in panel (b) than (a). In panel (a), high-wage suppliers account for 35 percent of links to buyers in the lowest quintile of wages and 55 percent of links to buyers in the highest quintile. In panel (b), the corresponding numbers for spending are 43 and 83 percent. Panels (c) and (d) describe the corresponding patterns for firms' downstream links. Shares across buyers now add up to one for each quintile of supplier. Panels (c) and (d) are almost the mirror images of panels (a) and (b).

2.2.1 Robustness of assortative matching

Other measures of skills In addition to differences in skills, wages may contain rents and differences in profit-sharing policies across firms. To address this concern, in Appendix A.1, we decompose the variation in wages into firm and worker components as in Abowd et al. (1999) using our employer-employee data from 2014 to 2016. Following Bombardini et al. (2019), we then take a firm's skill intensity to be the average fixed effect of its

Figure 2: Firm-to-firm Trade Links and Values by Quintile



Notes: Sample includes manufacturing buyers and suppliers. Firms are sorted according to the average value of their monthly payments per worker, and grouped into five equal-sized groups. For each buyer (supplier) quintile, expenditures (revenue) and number of suppliers (buyers) are aggregated at the level of supplier (buyer) quintile. Buyer and supplier quintiles are shown on x- and y-axis while z-axis shows the corresponding shares. For instance, in panel (a), values on the z-axis show for each buyer quintile on the x-axis the share of suppliers that belong to the wage quintiles on the y-axis.

workers. When we repeat the regressions in Table 2 with this measure of skill intensity, the coefficients are about half of the original. This decrease is not surprising since the measure excludes the firm fixed effect and the skills of the workers that never left the firm. Still, the coefficient is highly significant, and the decomposition into extensive and intensive margins remains close to Table 2.

We do not observe worker education in our data. But we observe the share of workers with tertiary education in the EU-15 countries for each one-digit ISCO occupation code. Using this share as a measure of occupational skill-intensity, Appendix A.2 confirms that firms with relatively more workers in skill-intensive occupations buy and sell more inputs to other firms with skill-intensive occupations.

Geography In Appendix Table A3, we investigate whether the positive assortative matching on wages arises because firms trade more with other firms physically close to them and some labor markets are more skill abundant than others. We conduct three exercises. In panel A, we control for firm location at a finer level, i.e., district instead of province in the baseline.[8] In panel B, we construct average supplier wages in equation (1) excluding the suppliers located in the same province as the firm. In panel C, we use a subsample of single establishment firms. Our VAT data aggregate transactions at the firm (instead of establishment) level, limiting our ability to control for the location of firms with establishments in multiple provinces. The positive assortative matching and the decomposition into extensive and intensive margins of Table 2 are robust to all three tests, although the total coefficient decreases from 0.259 in Table 2 to 0.214 in panel B and to 0.161 in panel C of Table A3.

Other firm characteristics Appendix Table A4 repeats the regression of column (2) in Table 1 substituting wages with other firm characteristics. Assortative matching on sales is positive but less pronounced than on wages, and the sorting is insignificant on the number of network links.[9] To evaluate the relative importance of sales vis-à-vis wages in sorting, Appendix A.5 conducts a horse-race between sales and wages following the empirical approach in Johnson and Wichern (1988), in the spirit of Becker (1973). Both wages and sales matter for the positive assortative matching, but wages are about 3 times

[8] Turkey is divided into 81 provinces, which vary by size. Each province is further divided into districts, of which the total number is close to 1000. We use province in our baseline results because a province better represents a local labor market.

[9] Lim (2018) also finds assortative matching on sales using data on large firms in the United States (Compustat). This pattern arises in our estimated model due to a positive correlation between firm sales and wages.

more important than sales for a firm's downstream linkages and 8.5 times more important for its upstream linkages.[10]

2.3 Trade Shocks

We use shift-share regressions to show that firms respond to firm-specific trade shocks by changing their skill intensity and network connections.[11]

Define two shifters associated with country c and product category k:

$$\begin{aligned} Z^u_{ck} &= \Delta \log \text{Imports}_{ck} \\ Z^a_{ck} &= (\Delta \log \text{Imports}_{ck}) * \log(\text{GDP per capita}_{c,2010}) \end{aligned} \tag{5}$$

where $\Delta \log \text{Imports}_{ck}$ is the log change between 2011-2012 and 2014-2015 in total imports of country c in product category k from all countries other than Turkey, and GDP per capita$_{c,2010}$ is the income per capita of country c in 2010.

We measure the export shock to firm f during the period of our data as:

$$\begin{aligned} \text{ExportShock}^u_f &= \sum_{ck} x_{ckf} Z^u_{ck} \\ \text{ExportShock}^a_f &= \sum_{ck} x_{ckf} Z^a_{ck}. \end{aligned} \tag{6}$$

where x_{ckf} is the share of firm f's revenue in 2010 that is exported to country c in product category k. We interpret Z^u_{ck} as a change in the demand for product category k in country c. The underlying assumption is that shocks to imports of product k by country c from countries other than Turkey are uncorrelated to other unobserved shocks to Turkish firms that export k to c. Under this assumption, ExportShock^u_f is a standard shift-share shock that captures the increased demand for firm f's exports. But we are interested in shocks that increase the incentives for firm f to upgrade its quality, and it is well documented that the relative demand for higher-quality, skill-intensive goods is higher in rich countries.[12] Then, export shocks that originate in rich countries should induce larger changes in quality. ExportShock^a_f is an adjusted measure that weights rich countries more.

[10] These numbers are from a canonical correlation analysis. This method is often used in marriage markets to evaluate which individual characteristics are most relevant for matching.

[11] See Bartik (1991) for an early application of these regressions, and Borusyak et al. (2018), Goldsmith-Pinkham et al. (2020), Adão et al. (2019) for statistical properties in general set ups.

[12] See footnote 2 for references.

Table 3: Effects of Export Shock

	$\Delta \log \text{wage}_f$	$\Delta \log \text{wage}_f$ (first stage)	$\Delta \log$ domestic sales$_f$	Δexport intensity$_f$	$\Delta \log wage^S_f$ OLS	$\Delta \log wage^S_f$ IV
	(1)	(2)	(3)	(4)	(5)	(6)
ExportShock^u_f (unadjusted)	0.021 (0.033)					
ExportShock^a_f (adjusted)		0.042 (0.006)	-0.026 (0.022)	0.0146 (0.0023)		
$\Delta \log \text{wage}_f$ (IV = ExportShock^a_f)					0.085 (0.008)	0.434 (0.185)
F-Stat	0.404	43.6	1.409			
N	33,157	33,157	33,157	33,157	33,157	33,157
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Wage$_f$ is the average value of monthly payments per worker in firm f. The suppliers' average wage $\log wage^S_f$ is defined in equation (1). Δ operator denotes changes between 2011-2012 and 2014-2015. ExportShock^u_f is a weighted average of changes in imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. ExportShock^a_f adjusts these shocks by weighting rich destinations more. See equations (6). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

To compare these two measures, we separately use them in the regression:

$$\Delta \log wage_f = \delta \text{ExportShock}_f + \alpha_{sr} + \epsilon_f$$

where α_{sr} are industry-province fixed effects.

Columns (1) and (2) of Table 3 report the results. The unadjusted ExportShock^u_f has an insignificant effect on firm wages, while the adjusted ExportShock^a_f has a positive and significant effect.[13] So as anticipated, an increased demand for a firm's exports only increases a firm's skill intensity if it originates in rich countries.

The mean of ExportShock^a_f is 0.12. To understand the magnitude of the coefficient 0.042 in column (2), consider two firms. They both export a quarter of their sales (the mean export intensity among exporters in the data). One firm exports to a country at the 90$^{\text{th}}$ percentile of per capita GDP distribution (US\$41.3 thousand, France), and the other firm exports to a country at the 10$^{\text{th}}$ percentile (US\$766, Benin). For the average change in imports over the sample period, $Z^u_{ck} = 5\%$, the implied ExportShock^a_f for the two firms is 13.3 percent ($= 0.25 \times 0.05 \times \log(41,300)$) and 8.3 percent, respectively, and the estimated wage increase is 0.56 ($= 0.042 \times 0.133$) and 0.35 percent.

Given these results, we henceforth use the adjusted export shock in all exercises. In column (3), we replace the dependent variable in column (2) with domestic sales. The

[13] These results hold when both shocks are in the same regression, in Appendix Table A7(1).

insignificant coefficient is reassuring, since we assume that ExportShock_f^a is uncorrelated with domestic shocks to firm f. It is also reassuring that the shock is not spurious but associated with an increase in the firm's export intensity (export sales divided by total sales) in column (4).

Columns (5) and (6) regress the change in the wage of firm f's suppliers on the change in firm f's own wage:

$$\Delta \log wage_f^S = \delta \Delta \log wage_f + \alpha_{sr} + \epsilon_f.$$

where α_{sr} are again industry-province fixed effects. In column (6), we instrument the change in the firm's wage $\Delta \log wage_f$ with the export shock.[14] The coefficient is 0.434 with standard error 0.185. The interpretation is that when a firm's average wage increases by one log point relative to other firms, in response to an export shock, then the average wage of its suppliers increase by 0.4 log points. The coefficient in the OLS regression in column (5) is smaller, 0.085. It is difficult to *ex ante* predict the direction of the bias. The OLS coefficient is confounded by unobserved shocks that affect the wage growth of firms in the same industry and province.

In sum, Table 3 suggests that the demand for a firm's exports from rich countries increases the firm's wage and its suppliers' wage. Table 4 shows that these increases, at least in part, arise through new workers and network connections. Recall that the export shock is constructed from changes between 2011-2012 and 2014-2015. Take the workers that a firm f added between 2013 and 2015. Using matched employer-employee data, we regress the log difference between these new workers' wages in 2011-2012 (before they entered the firm) and firm f's average wage in 2011-2012 (before the shock) on the ExportShock_f^a in the first column. The second and third columns repeat the exercise for the firm's new suppliers and new customers. The coefficients on all columns are positive and statistically significant.[15]

Identification and Robustness Checks Recent papers discuss shift-share regressions similar to ours. Borusyak et al. (2018) and Goldsmith-Pinkham et al. (2020) propose

[14]This approach follows Hummels et al. (2014). To study the effect of exports on wages, they use a shift-share variable, similar to ExportShock_f^u, as an instrument for firm exports.

[15]We use the unweighted average in (3) because we cannot measure the weights $s_{\omega f}$ that the firm would have placed on new suppliers in the initial year or the equivalent weights of new customers on initial sales. In Appendix Table A9, we obtain similar results when we compare the the wages of new connections relative to workers, suppliers and customers that left the firm between 2010 and 2015. Appendix Table A8 associates the export shock to the share of newly hired workers after the shock, who received higher monthly wages than the firm's average worker before the shock. So, Table 4 is not driven by a few outliers among new connections.

Table 4: Effects of Export Shock on Composition of Inputs

Log of	Average wage of new workers relative to all workers at $t = 0$	Average wage paid by new suppliers relative to all suppliers at $t = 0$	Average wage paid by new buyers relative to all buyers at $t = 0$
ExportShock_f	0.0189	0.0241	0.0303
	(0.010)	(0.007)	(0.009)
R^2	0.0531	0.0439	0.0434
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. ExportShock_f is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

methods to study, respectively, which shifts or shares matter the most for the consistency. Following the recommendation in Borusyak et al. (2018), we check three key conditions in Appendix B. First, shifts are numerous. To calculate Z^a_{ck}, we use 208 distinct destination countries c and 1,242 4-digit HS codes k, generating 153,186 ck pairs. Second, the shifts are dispersed within industries. The average Herfindal-Hirschman index within industries is 5×10^{-5}. The standard deviation of Z^a_{ck} is 3.26 across all firms and industries, and 3.24 across firms within industries. Third, the shifts are relevant. We obtain a coefficient close to zero when we substitute the $ExportShock^a_z$ with a placebo $ExportShock^{\text{random}}_f$ generated from randomly-drawn shifts Z^a_{ck}.

Appendix Table A7 presents additional checks to Table 3. The results in column (2) do not change when we add the export shares x_{ck} weighted by destination income per capita as a control. This exercise addresses the concern in Adão et al. (2019) that observations with similar shares have correlated residuals. Separately, we add to the same regression, the export shares to address the concern in Borusyak et al. (2018) that shares x_{ck} do not add up to one. Last, we add to column (6) the weighted average of the suppliers' export shock. These shocks have a positive effect on supplier wages (as we would predict), but they do not affect the coefficient of interest on buyer's wage.

2.4 Other Characteristics of the Network

Three other features of the data govern our modelling choices. First, firm sales is the most important indicator of the number of suppliers and customers of a firm. Table 5 reports the endogenous elasticity of number of customers and suppliers with respect to sales.

Table 5: Firm Sales and Network Connections

Number of	Customers			Suppliers		
	(1)	(2)	(3)	(4)	(5)	(6)
$\log Sales_f$	0.440	0.462	0.459	0.577	0.593	0.590
	(0.016)	(0.013)	(0.013)	(0.011)	(0.009)	(0.009)
$\log Wage_f$			0.278			0.208
			(0.211)			(0.175)
R^2	0.328	0.472	0.472	0.609	0.645	0.645
N	77,418	77,418	77,418	77,418	77,418	77,418
Fixed effects		Ind	Ind		Ind	Ind

Notes: Wage is defined as the average value of monthly payments per worker. All variables are in logarithms. Ind refers to 4-digit NACE industries. Robust standard errors are clustered at industry level.

Firm sales explain about a third of variation in the number of buyers, and 60 percent of variation in the number of suppliers (R-squared in columns (1) and (4)). Columns (2) and (5) add industry fixed effects, and columns (3) and (6) add also wages. The coefficients on wages are insignificant and do not change the coefficients on sales or the R-squared.

Second, service firms, mostly wholesalers and retailers, account for almost half of domestic sales and material purchases of manufacturing firms. But we do not observe the skill intensity of the materials purchased through these service intermediaries. So, we introduce to the model a service sector that aggregates manufacturing inputs into a homogeneous good. The service good is used as an input into manufacturing and as a final good.

Third, imports account for only 4 percent of spending on material inputs by a typical manufacturing firm in our data, compared to a 10 percent share of exports in its total sales. Accordingly, in the open economy model of Section 4, we model manufacturing firms' decisions to export, but for simplicity, only service firms import.[16]

We conclude with a brief point on quality measures. Quality in our model is a latent variable that changes the firm's production function, increasing the relative marginal product of skilled workers and of skill-intensive inputs. Kremer (1993) refers to this variable intermittingly as quality or complexity. But our emphasis, like his, is the complementarity between skilled workers in production. Even if we observed unit values in our data, it is not clear that standard measures of quality would be superior to wages in capturing the facts above. Since we cannot answer this question with our data, we leave it for future work. Nevertheless, we do observe unit values for a small subset of the data: Foreign sales

[16] We replicated the moments in Section 2.3 for import shocks, symmetric to export shocks, and found mostly insignificant effects. Possibly, this lack of finding arises because only a small share of manufacturing firms import their inputs directly in our data.

of exporting firms. For this subset, Appendix A.3 confirms the positive relation between wages and the quality measure by Khandelwal et al. (2013) which uses information on unit values and quantities per destination.[17]

3 The Closed-Economy Model

To highlight the novel features of the model, we present first a closed economy. There are two sectors: Services and manufacturing. The service sector is perfectly competitive. It produces a homogeneous good with constant returns to scale using manufacturing inputs. The manufacturing sector has heterogeneous firms.

Each manufacturing firm chooses its quality q from a line segment $Q \subset \mathbb{R}_+$. This choice determines the firm's production function. All tasks performed in a firm of quality $q \in Q$ are also indexed by q, whether the worker is in production or posting ads. Earnings per worker and the marginal product of higher-q inputs may be higher in the production of higher-q output. Firms post ads to find suppliers and customers. The matching of ads form the firm-to-firm network. Like Lim (2018), each firm is matched with a continuum of suppliers and customers, and it charges the monopolistic-competition markup. More productive firms endogenously post more ads and have more customers and suppliers. Firms imperfectly direct their ads to other firms' with similar quality levels.

Differences in input intensity in the production function allow skill-intensive firms to spend more on each others' inputs–the intensive margin of assortative matching. Directed search increases the probability that skill-intensive firms match with each other—the extensive margin.

The manufacturing sector is in Section 3.1. Section 3.1.1 sets up the firm's problem, and Section 3.1.2 aggregates firm choices to form the network. The service sector is in Section 3.2, and the equilibrium is in Section 3.3. Section 3.4 presents key properties of the model. The less-technical reader may skip to Section 3.4. Whenever convenient, we assume functions are continuous, differentiable, and integrable. Parametric assumptions in the estimation ensure these conditions.

[17] In our estimation, we use moments based on quintiles of firm wages, and the appendix documents a significant overlap between the grouping of firms by this quality measure and by wages.

3.1 Manufacturing

3.1.1 The Firm's Problem

The revenue of a firm with quality q, price p and a mass v of ads to find customers (v stands for visibility) is

$$p^{1-\sigma} v D(q) \tag{7}$$

where $\sigma > 1$ is the elasticity of substitution between manufacturing varieties and $D(q)$ is an endogenous demand shifter.

The cost of a bundle of inputs to produce quality q when the firm posts a measure m of ads to find manufacturing suppliers is

$$C(m,q) = w(q)^{1-\alpha_m-\alpha_s} P_s^{\alpha_s} [m^{1/(1-\sigma)} c(q)]^{\alpha_m} \tag{8}$$

where $(\alpha_m, \alpha_s) \gg 0$ are Cobb-Douglas weights with $(\alpha_m + \alpha_s) \in (0,1)$, P_s is the price of the service good, $w(q)$ is the wage rate per efficiency unit of task q, and $c(q)$ is the cost of a bundle of manufacturing inputs when the firm posts a measure one of ads to find suppliers. The marginal cost of the firm is $C(m,q)/z$ where z is her productivity.

The cost of posting v ads to find customers and m ads to find suppliers is respectively

$$\begin{aligned} & w(q) f_v \frac{v^{\beta_v}}{\beta_v} \\ & w(q) f_m \frac{m^{\beta_m}}{\beta_m} \end{aligned} \tag{9}$$

where f_m, f_v, β_m, and β_v are positive parameters with $\beta_m > \alpha_m$ and $\beta_v > \beta_m/(\beta_m - \alpha_m)$.

From (7), the firm charges markup $\sigma/(\sigma-1)$ over marginal cost. Given q and z, she chooses v and m to maximize profit:

$$\max_{v,m} \frac{v m^{\alpha_m}}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{C(1,q)}{z} \right]^{1-\sigma} D(q) - w(q) f_v \frac{v^{\beta_v}}{\beta_v} - w(q) f_m \frac{m^{\beta_m}}{\beta_m} \tag{10}$$

Rearranging the first order conditions, the firm's revenue x, mass of ads to find customers

v and to find suppliers m, and price p are functions of productivity z and quality q:

$$
\begin{aligned}
x(z,q) &= \Pi(q) z^{\gamma(\sigma-1)} \\
v(z,q) &= \left(\frac{x(z,q)}{\sigma f_v w(q)}\right)^{1/\beta_v} \\
m(z,q) &= \left(\frac{\alpha_m x(z,q)}{\sigma f_m w(q)}\right)^{1/\beta_m} \\
p(z,q) &= \frac{\sigma}{\sigma-1}\frac{C(m(z,q),q)}{z}
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
\Pi(q) =& [\sigma w(q)]^{1-\gamma}\left[D(q)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \\
\gamma =& \frac{\beta_v\beta_m}{\beta_v(\beta_m-\alpha_m)-\beta_m} > 1.
\end{aligned} \tag{12}
$$

A firm is characterized by a vector $\omega = (\omega_0, \omega_1) \in \mathbb{R}^2$ which determines her productivity for each quality level:

$$z(q,\omega) = \exp\left\{\omega_0 + \omega_1 \log(q) + \overline{\omega}_2[\log(q)]^2\right\} \tag{13}$$

where $\overline{\omega}_2$ is a parameter common to all firms. Parameter ω_0 captures the firm's absolute advantage in production and ω_1 captures her comparative advantage in producing higher quality. These two dimensions of heterogeneity capture the joint distribution of sales and wages in the estimation. Since profit (10) is a share $1/(\gamma\sigma)$ of revenue, firm ω chooses q to maximize revenue:

$$q(\omega) = \arg\max_{q\in Q}\left\{x(z(q,\omega),q)\right\} = \arg\max_{q\in Q}\left\{z(q,\omega)^{\gamma}\Pi(q)\right\}. \tag{14}$$

If wage $w(q)$ is continuous in q, then function $\Pi(q)$ derived below is continuous in q, and (14) is the maximization of a continuous function in a compact set Q. Firms quality choices are interconnected through the endogenous terms in $\Pi(q)$. Manufacturing firm-to-firm trade determines the input cost $c(q)$ and the component of demand $D(q)$ that comes from other firms.

3.1.2 Manufacturing firm-to-firm trade

Production Function The quantity produced by firm ω producing quality q is

$$z(q,\omega)l^{1-\alpha_m-\alpha_s}y_s^{\alpha_s}Y(q)^{\alpha_m}$$

where l are efficiency units of labor, y_s are units of the service good, and $Y(q)$ is an aggregate of manufacturing inputs. This production function yields unit costs in (8). Following Fieler et al. (2018), we assume:

$$Y(q) = \left[\int_{\omega'\in\Omega} y(\omega')^{(\sigma-1)/\sigma}\phi_y(q,q(\omega'))^{1/\sigma}d\omega'\right]^{\sigma/(\sigma-1)} \tag{15}$$

where $y(\omega)$ is the quantity of input ω and function $\phi_y(q,q')$ governs the productivity of an input of quality q' when producing output of quality q. We parameterize

$$\phi_y(q,q') = \frac{\exp(q'-\nu_y q)}{1+\exp(q'-\nu_y q)}, \tag{16}$$

which is increasing in input quality and decreasing in output quality if $\nu_y > 0$. It is also log-supermodular if $\nu_y > 0$. Then, the ratio of the firm's demand for any two inputs 1 and 2 with prices $p(1)$ and $p(2)$ and qualities $q(1) > q(2)$,

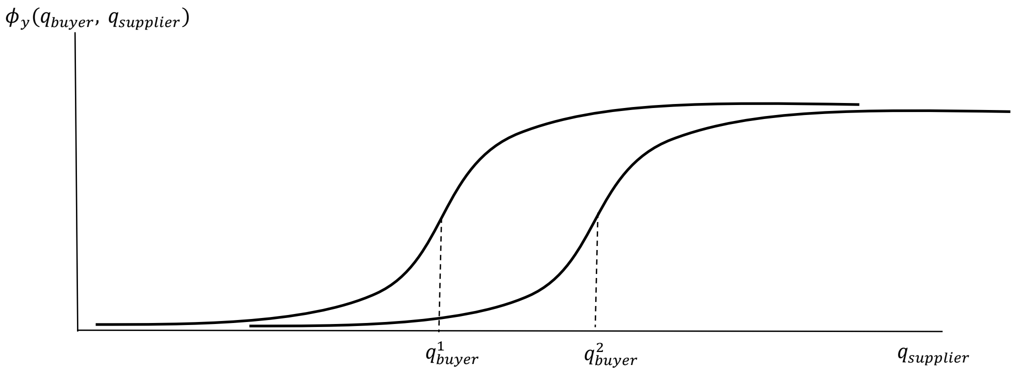
$$\frac{y(1)}{y(2)} = \left(\frac{p(1)}{p(2)}\right)^{-\sigma}\frac{\phi_y(q,q(1))}{\phi_y(q,q(2))}, \tag{17}$$

is strictly increasing in the producing firm's quality q. Figure 3A illustrates ϕ_y as a function of supplier quality for two producing firms (buyers). One can see how, given the same prices and matches, the buyer with higher quality q^2_{buyer} spends relatively more on high-quality input suppliers compared to the buyer with quality q^1_{buyer}.

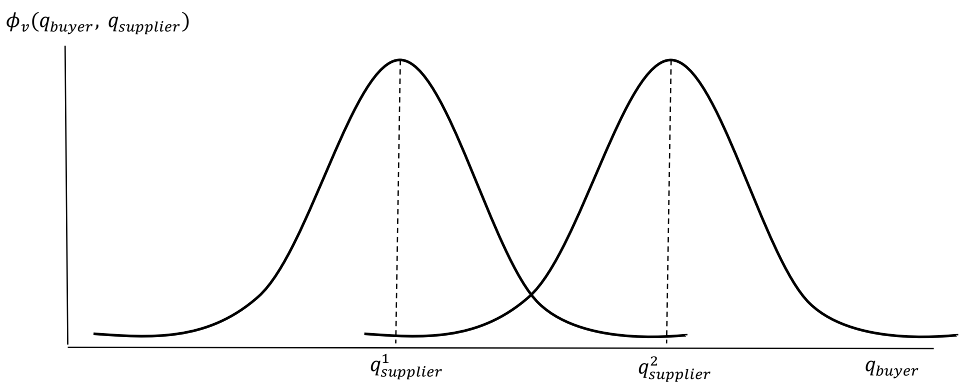
Directed Search Buyers can only see the selling ads that target their own quality. The ads posted by a seller with quality q' are distributed across buyers' qualities $q \in Q$ according to function $\phi_v(q,q')$. We parameterize $\phi_v(q,q')$ as the density of a normal distribution with variance parameter ν_v and mean q', the quality of the seller posting the ads.[18] Figure 3B illustrates the distribution of ads across buyers for two sellers (suppliers). Clearly, the ads posted by the higher-quality supplier q^2_{supplier} is disproportionately targeted at higher-quality buyers. Here, the direction of ads is exogenous for simplicity.

[18] One dimension of directed search, whether from buyers or sellers, is enough to generate assortative matching at the extensive margin.

Figure 3: Assortative Matching on quality in the model



A. Intensive margin of assortative matching: The marginal product of input supplier is $\phi_y(q_{buyer}, q_{supplier})^{1/\sigma}$ and spending given prices is proportional to $\phi_y(q_{buyer}, q_{supplier})$. The figure plots function $\phi_y(q_{buyer}, q_{supplier})$ for two buyers with output qualities q^1_{buyer} and q^2_{buyer}.



B. Extensive margin of assortative matching: The distribution of ads posted by two sellers with output qualities $q^1_{supplier}$ and $q^2_{supplier}$ are targeted at buyers according to function $\phi_v(q_{buyer}, q_{supplier})$.

In Appendix I, we modify the model to allow firms to choose the direction of their search (the mean of ϕ_v), and we obtain similar estimation and counterfactual results.

Aggregation There is a fixed set of firms Ω. Firm choices in (14) above give rise to the measure

$$J(z,q) = |\{\omega \in \Omega : z(q(\omega),\omega) \leq z \quad \text{and} \quad q(\omega) \leq q\}| \,. \tag{18}$$

Assume J has a density, denoted with $j(z,q)$. Directed search implies that there is a continuum of matching submarkets, one for each buyer quality. In the submarket of buyers with quality $q \in Q$, the measure of ads posted by buyers and sellers is respectively:

$$M(q) = \int_Z m(z,q)j(z,q)dz, \tag{19}$$

$$V(q) = \int_Q \phi_v(q,q')\overline{V}(q')dq', \tag{20}$$

where $\overline{V}(q)$ is the measure of ads posted by sellers of quality q:

$$\overline{V}(q) = \int_Z v(z,q)j(z,q)dz.$$

A standard matching function determines measure of matches with buyers of quality q:[19]

$$\tilde{M}(q) = V(q)\left[1 - \exp(-\kappa M(q)/V(q))\right]. \tag{21}$$

where parameter $\kappa > 0$ captures the efficiency in the matching market. The success rate of ads is $\theta_v(q) = \tilde{M}(q)/V(q)$ for sellers and $\theta_m(q) = \tilde{M}(q)/M(q)$ for buyers.

Using (20), for each ad posted by a buyer of quality q, the probability of finding a supplier with productivity-quality (z',q') is

$$\theta_m(q)\frac{\phi_v(q,q')v(z',q')j(z',q')}{V(q)} \tag{22}$$

Combining with the CES price associated with (15), a bundle of manufacturing inputs used by a firm of quality q with a measure one of buying ads costs:

$$c(q) = \left[\frac{\theta_m(q)}{V(q)}\int_Q \phi_y(q,q')\phi_v(q,q')P(q')^{1-\sigma}dq'\right]^{1/(1-\sigma)} \tag{23}$$

[19]See Petrongolo and Pissarides (2001) for a survey on matching functions and their properties.

where

$$P(q) = \left[\int_Z p(z,q)^{1-\sigma} v(z,q) j(z,q) dz \right]^{1/(1-\sigma)} \tag{24}$$

takes into account the greater visibility of firms that post more selling ads $v(z,q)$.

We now turn to demand. A firm with quality q posts price p and v selling ads. From (19), the measure of buyers with (z', q') matched to the firm is

$$v\theta_v(q')\phi_v(q',q)\frac{m(z',q')j(z',q')}{M(q')}$$

Conditional on the match, the firm's sales to a buyer with (z', q') is

$$\phi_y(q',q)\left(\frac{p}{c(q')}\right)^{1-\sigma}\frac{\alpha_m(\sigma-1)}{\sigma}\frac{x(z',q')}{m(z',q')}$$

Multiplying these last two expressions and summing over buyers (z', q'), the sales of the firm to other manufacturing firms is[20]

$$p^{1-\sigma}vD_m(q)$$

where

$$\begin{aligned} D_m(q) &= \frac{\alpha_m(\sigma-1)}{\sigma}\int_Q \frac{\theta_v(q')}{M(q')}\phi_y(q',q)\phi_v(q',q)c(q')^{\sigma-1}X(q)dq, \qquad (25) \\ X(q) &= \int_Z x(z,q)j(z,q)dz. \end{aligned}$$

3.2 Service Sector and Final Demand

Service firms aggregate manufacturing inputs into a homogeneous good sold in a perfectly competitive market. Their production function is given by $Y(0)$ in (15). There's a fixed set of service firms, each endowed with a fixed measure $\overline{m}$ of manufacturing suppliers. The

[20]We may also derive $D_m(q)$ from buyer connections. Using (23), the share of spending on materials by buyers of quality q' allocated to a supplier with price p, quality q, and v ads is

$$\theta_m(q')\frac{\phi_y(q',q)\phi_v(q',q)vp^{1-\sigma}}{V(q)c(q')^{1-\sigma}}.$$

Multiplying by domestic spending on materials $[\alpha_m(\sigma-1)/\sigma]X(q')$ and integrating over buyers q', demand is

$$vp^{1-\sigma}\frac{\alpha_m(\sigma-1)}{\sigma}\int_Q \frac{\theta_m(q')}{V(q')}\phi_y(q',q)\phi_v(q',q)c(q')^{\sigma-1}X(q')dq'$$

which is the expression above since $\theta_m(q)/V(q) = \theta_v(q)/M(q)$.

probability that a service firm matches with a supplier with productivity-quality (z, q) is proportional to the measure of selling ads:

$$\frac{v(z,q)j(z,q)}{V_T}$$
$$\text{where} \quad V_T = \int_Q \overline{V}(q)dq \tag{26}$$

Then, the price index of the service good is

$$P_s = \left[\frac{\overline{m}}{V_T}\int_Q \phi_y(0,q)P(q)^{1-\sigma}dq\right]^{1/(1-\sigma)} \tag{27}$$

Total sales to the service sector by a manufacturing firm with price p, quality q, posting v ads in Home to find customers is:

$$\frac{v}{V_T}\left(\frac{p}{P_s}\right)^{1-\sigma}\overline{m}\phi_y(0,q)X_s$$

where X_s is total absorption of services. Using (27), these sales are

$$p^{1-\sigma}vD_s(q) \tag{28}$$
$$\text{where} \quad D_s(q) = \phi_y(0,q)\left[\int_Q \phi_y(0,q')P(q')^{1-\sigma}dq'\right]^{-1} X_s.$$

They do not depend on $\overline{m}$.

Households consume only the service good. Then service absorption X_s is the share of manufacturing absorption in (10) allocated to service and labor inputs plus profits:

$$X_s = 1 - \frac{(\sigma-1)}{\sigma}\alpha_m.$$

3.3 Equilibrium

The demand shifter faced by a manufacturing firm in (7) is the sum of demand from services (28) and other manufacturing firms (25):

$$D(q) = D_m(q) + D_s(q). \tag{29}$$

We take the supply of efficiency units of labor to produce task q to be an exogenous function $L(q, w)$ where w is the whole wage schedule, $w(q)$ for all $q \in Q$. Labor markets clear if for all q

$$L(q,w) = \frac{1}{w(q)\sigma}\left[(1-\alpha_m-\alpha_s)(\sigma-1)+1-\frac{1}{\gamma}\right]X(q) \tag{30}$$

where the constant is the labor share in manufacturing production in (10).

We have derived aggregate variables as functions of equilibrium wages $w(q)$ and firm outcomes. Measure $J(z,q)$ is in (18). The success rates of ads are $\theta_m(q) = \tilde{M}(q)/M(q)$ and $\theta_v(q) = \tilde{M}(q)/V(q)$ where $M(q)$, $V(q)$ and $\tilde{M}(q)$ are in (19), (20) and (21). Costs $c(q)$ and $C(m,q)$ are in (8) and (23), and demand $D(q)$ is in (29). Firms maximize profits in (10) given wages $w(q)$ and other firms' actions summarized in $c(q)$ and $D(q)$. Denote with Θ a set of firm outcomes, specifying for each $\omega \in \Omega$, its quality, productivity, sales, measures of upstream and downstream ads and price.

An **equilibrium** is a set of wages w and of firm outcomes Θ such that functions $D(q)$ and $C(1,q)$ exist and that satisfy the following conditions:

1. The labor market clears (30).

2. Firms maximize profits. Firm ω chooses $q(\omega)$ in (14) and has productivity $z(\omega) = z(q(\omega),\omega)$ at the optimal. Its sales, measure of ads, and prices are $x(z(\omega),q(\omega))$, $m(z(\omega),q(\omega))$, $v(z(\omega),q(\omega))$, and $p(z(\omega),q(\omega))$ in (11).

3.4 Properties of the Network

The model has two novel features: The use of log-supermodular functions to capture assortative matching, and the search-and-matching set up of network formation. We explain these features in Sections 3.4.1 and 3.4.2 respectively.

3.4.1 Assortative Matching

In the estimation below, we assume that wage per worker is increasing in firm quality. Then assortative matching in wage per worker in the network arises through buyers' and sellers' quality levels.

For a firm with quality q, the measure of its suppliers that have quality q_1 relative to quality q_2 is (integrating (22)):

$$\frac{\phi_v(q,q_1)}{\phi_v(q,q_2)}\frac{\overline{V}(q_1)}{\overline{V}(q_2)} \tag{31}$$

The firm's average spending on its suppliers of quality q_1 relative to its suppliers of quality

q_2 is (integrating (17)):

$$\frac{\phi_y(q,q_1)}{\phi_y(q,q_2)}\left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma}\frac{\overline{V}(q_2)}{\overline{V}(q_1)} \tag{32}$$

Multiplying these expressions (or using equation (23)), the ratio of the firm's total spending on the two qualities is:

$$\frac{\phi_v(q,q_1)}{\phi_v(q,q_2)}\frac{\phi_y(q,q_1)}{\phi_y(q,q_2)}\left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma} \tag{33}$$

These expressions summarize the extensive margin (31), intensive margin (32) and total (33) assortative matching in the network. Since the terms $\overline{V}(q)$ and $P(q)$ are common to all buyers, functions ϕ_y and ϕ_v alone govern assortative matching. By definition, a function ϕ is log-supermodular if $\phi(q,q_1)/\phi(q,q_2)$ is increasing in q whenever $q_1 > q_2$ or equivalently $\partial^2 \log(\phi(q,q'))/\partial q\partial q' > 0$. Function $\phi_v(q,q')$ governs the distribution of selling ads posted by suppliers with quality q' across buyers of quality q. We parameterize ϕ_v as the density of a normal random variable with variance ν_v. Its derivative $\partial^2 \log(\phi_v(q,q'))/\partial q\partial q' = 1/\nu_v$ is positive. Then higher-quality firms have relatively more higher-quality suppliers in (31). Function $\phi_y(q,q')$ governs the marginal product of an input of quality q' in the production of output quality q. It is log-supermodular if $\nu_y > 0$ in (16). Then higher-quality firms spend relatively more on their higher-quality suppliers in (32).

3.4.2 Search and Matching

We consider a special case of the model to highlight its search and matching set up.[21] Assume there is only one quality and $\beta_v = \beta_m \equiv \beta$. Set $\phi_v = \phi_y = 1$ without loss of generality. Take wages as the numeraire, and drop the quality arguments from functions. We refer to a firm by its productivity z, instead of ω. The mass of firms N and the distribution of z are exogenous. Appendix D has the complete, closed-form solution to this special case and analyses its efficiency properties.[22]

With $\beta_v = \beta_m$, the ratio of ads to find suppliers and customers in (11) is $m(z)/v(z) = (\alpha_m f_v/f_m)^{1/\beta}$, independent of firm productivity. Then, the success rates of ads θ_m and

[21] This special case relates to Miyauchi (2019) who incorporates matching frictions in firm-to-firm trade in a version of multi-location multi-sector Melitz (2003) model.

[22] There are two externalities for each ad in the decentralized equilibrium. A positive externality is that more ads increases the total mass of matches $\tilde{M}$. A negative externality is that more ads decreases the probability of matching for firms in the same of side of the market as the ad (sellers for v ads and buyers for m ads). The negative externality is always greater than the positive externality so that the planner posts less ads than the market equilibrium. There is no inefficiency from the allocation of ads across heterogeneous firms. The allocation of labor for production is also efficient. Markups are constant in manufacturing and the service sector has no labor.

θ_v are exogenous functions of parameters. The number of customers and the number of suppliers of firm z are the same:

$$\theta_v \left(\frac{x(z)}{\sigma f_v} \right)^{1/\beta} = \theta_m \left(\frac{\alpha_m x(z)}{\sigma f_m} \right)^{1/\beta}.$$

They increase log-linearly with firm sales, as in Table 5.

The probability that a firm with productivity z is the buyer or the seller in a match is

$$\frac{m(z)}{M} = \frac{v(z)}{V} = \frac{z^{\gamma(\sigma-1)/\beta}}{N\mathbb{E}\left(z^{\gamma(\sigma-1)/\beta}\right)}.$$

It does not depend on the other firm in the match. So, there is no assortative matching in the network: All firms are more likely to match with more productive firms.[23]

The market share of a firm with productivity z in total manufacturing sales is:

$$x(z) = \frac{z^{\gamma(\sigma-1)}}{N\mathbb{E}(z^{\gamma(\sigma-1)})}$$

The expression is the same as Melitz (2003) except for the added parameter $\gamma > 1$. The effect of productivity on sales is augmented because more productive firms post more ads to find suppliers and customers. So, the model needs a smaller dispersion in firms' fundamental productivity z to generate the same distribution of sales as in Melitz (2003).

4 Open Economy

We embed the model above into a small open economy. The price of foreign varieties and foreign demand for domestic goods are exogenous. Manufacturing firms may export by paying a fixed cost, posting ads abroad and facing an exogenous foreign demand. To produce the final service good, service firms combine domestic and foreign varieties with a constant elasticity of substitution σ. We focus here only on the distinctions to the closed economy. Appendix E presents the full model.

The manufacturing firm ω has productivity $z(q,\omega)$ in (13). The firm chooses $q \in Q$ and then draws a random fixed export cost f_E units of the service good from a common distribution. She then decides her export status, posts ads to search for domestic suppliers, for domestic customers, and for foreign customers if exporting. We introduce randomness

[23] Bernard et al. (2019b), Lim (2018), Huneeus (2018) generate an increasing relation between a firm sales and the number of its network connections by imposing a fixed cost for firms to trade. Their setting generates a strong negative assortative matching because only more productive firms pay a fixed cost to trade with less productive firms.

in the fixed cost of exporting because firms in the data with similar size and wages have different export status. The timing simplifies aggregation in the estimation.

The revenue from foreign sales of an exporter with quality q, price p and v ads to find customers in foreign is

$$p^{1-\sigma} v e^{\sigma} D_F(q) \tag{34}$$

where $D_F(q)$ is an exogenous demand function and e is the exchange rate. The cost of posting v ads in foreign is the same as the domestic cost in (9), $w(q) f_v v^{\beta_v}/\beta_v$. Assuming the same curvature β_v is important to maintain the log linearity in the firm's problem. Cost parameter f_v is the same as for domestic ads only to simplify notation, since we do not observe foreign trading partners.

By backward induction, we start with the problem of the firm after it has chosen its quality and export status. A firm with quality q, productivity z and export status $E \in \{0, 1\}$ chooses the mass of ads to find suppliers m, the mass of ads to find customers v and the share $r_v \in [0, 1]$ of the selling ads that are posted domestically:

$$\begin{aligned}\max_{m,v,r_v} \frac{v m^{\alpha_m}}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{C(1,q)}{z}\right]^{1-\sigma} & [r_v D_H(q) + (1-r_v) E e^{\sigma} D_F(q)] \\ & - w(q) f_v [r_v^{\beta} + (1-r_v)^{\beta}] \frac{v^{\beta_v}}{\beta_v} - w(q) f_m \frac{m^{\beta_m}}{\beta_m}\end{aligned} \tag{35}$$

where $C(1, q)$ is the input cost in (8) and $D_H(q)$ is the endogenous domestic demand shifter, denoted with $D(q)$ in equation (7). The optimal share of ads r_v is a function of quality q and export status E:

$$\frac{1 - r_v(q, E)}{r_v(q, E)} = \left(\frac{E e^{\sigma} D_F(q)}{D_H(q)}\right)^{1/(\beta_v - 1)} \tag{36}$$

Given the optimal r_v, problem (35) differs from the closed economy (10) only in the level of demand and the level of the cost of posting selling ads v. Then, the relationship between sales, ads and prices take the form of (11). Total sales are

$$x(z, q, E) = \Pi(q, E) z^{\gamma(\sigma-1)} \tag{37}$$

where

$$\Pi(q, E) = [\sigma w(q)]^{1-\gamma} \left[D(q, E) \left(\frac{\sigma}{\sigma-1} C(1, q)\right)^{1-\sigma} \left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \tag{38}$$

$$D(q, E) = \left[D_H(q)^{\beta_v/(\beta_v-1)} + E(e^{\sigma} D_F(q))^{\beta_v/(\beta_v-1)}\right]^{(\beta_v-1)/\beta_v}. \tag{39}$$

Exporting increases the firm's profit by more than the sum of the profits from operating separately in each market. The firm uses the same input suppliers for producing all its goods, independent of destination. So, exporting increases the firm's incentives to search for suppliers, which lowers price and increases the firm's incentives to search for customers in both markets. The exponent in the CES term $D(q, E)$ and γ capture these magnification effects.[24]

The firm exports if its fixed exporting cost $f_E \leq \overline{f}_E(z, q)$ where

$$\overline{f}_E(z, q) = \frac{z^{\gamma(\sigma-1)}}{\gamma\sigma P_s}\left[\Pi(q, 1) - \Pi(q, 0)\right]. \tag{40}$$

Denote with Φ the cumulative distribution function of f_E. After observing its productivity $z(q, \omega)$ but before observing f_E, the firm chooses its quality:

$$\begin{aligned} q(\omega) = \arg\max_{q \in Q} \Bigg\{ \frac{z(q,\omega)^{\gamma(\sigma-1)}}{\gamma\sigma} \Bigg[& \Pi(q,1)\Phi\left(\overline{f}_E(z(q,\omega),q)\right) + \Pi(q,0)\Big[1 - \Phi\left(\overline{f}_E(z(q,\omega),q)\right)\Big]\Bigg] \\ & - P_s \mathbb{E}(f_E | f_E \leq \overline{f}_E(z(q,\omega),q))\Bigg\} \end{aligned} \tag{41}$$

Appendix E makes exactly the same assumptions on production and network formation as in the closed economy. The only difference is that, because sales, mass of ads and prices depend on export status, aggregation in the open economy is over two measure functions:

$$\begin{aligned} \tilde{J}(z, q, 1) &= J(z, q)\Phi\left(\overline{f}_E(z, q)\right) \\ \tilde{J}(z, q, 0) &= J(z, q)\left[1 - \Phi\left(\overline{f}_E(z, q)\right)\right] \end{aligned} \tag{42}$$

where $J(z, q)$ is defined in (18). The equilibrium is also similarly defined with the exchange rate e as an additional equilibrium variable and a trade equilibrium condition, in which we allow for an exogenous trade imbalance.

5 Estimation and Identification

The key estimation assumption is that wage per worker ($w(q)\times$ labor endowment per worker) is strictly increasing in q. Using a Roy (1951) model, Teulings (1995) provides a micro foundation for the labor supply function $L(q, w)$ and for this estimation assump-

[24]The interconnection between a firm's decisions on sales, prices and purchases in the domestic market, and its participation in other markets (exporting or not) does not appear in standard models of exporting à la Melitz (2003) but appears in importing models such as Antràs et al. (2017).

tion.[25] We also prove that we can construct a set of labor endowments that exactly match the distribution of wage per worker across firms in the data. See Appendix C for details.

We calibrate some parameters and estimate others using the method of simulated moments. A closed economy is defined by parameters $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, \overline{m}, \kappa, \nu_y, \nu_v, \overline{\omega}_2\}$, the labor supply $L(q, w)$, and the set of firms Ω, itself specified by a mass N and a distribution of firm productivity parameters (ω_0, ω_1). The open economy in addition has the price of the bundle of imported goods P^*, foreign demand $D_F(q)$, and the distribution of fixed costs of exporting f_E.

5.1 Calibrated Parameters and Normalizations

We calibrate production parameters $\{\alpha_m, \alpha_s, \sigma, \beta_v, \beta_m\}$. We set $\alpha_m = 0.33$ and $\alpha_s = 0.38$ in (8) to the cost shares of manufacturing and services in the Turkish manufacturing sector. The elasticity of substitution $\sigma = 5$ is from Broda and Weinstein (2006). We set $\beta_m = 1/0.59$ and $\beta_v = 1/0.46$ to match the endogenous elasticity of number of suppliers and customers with respect to firm sales in Table 5.

We also normalize the mass of firms $N = 1$. We set $f_m = f_v = 1$. Since search efforts are not observable, we cannot separately identify the cost of one ad, f_m and f_v, from the matching efficiency κ in (21). Similarly, parameter $\overline{m}$ is not identified because it governs the theoretical price index P_s in (27) but not the observable sales of manufactures to services in (28). We pick $\overline{m}$ so that $P_s = 1$.

We set equilibrium efficiency wages $w(q) = 1$ for all q and real exchange rates $e = 1$. While these variables endogenously respond to counterfactuals, they may be normalized in the cross-section. We observe wage per worker in the data, but we can always normalize the endowment of efficiency units of labor per worker so that efficiency wage $w(q) = 1$. Similarly, we can set e and adjust foreign demand $D_f(q)$ and price P^* accordingly.

5.2 Parametrization

Assume (ω_0, ω_1) are distributed according to a bivariate normal with standard deviations σ_{ω_0} and σ_{ω_1} and correlation ρ. The fixed export costs f_E are distributed log-normal with mean μ_E and standard deviation σ_E. We parameterize

$$D_F(q) = b_1 q^{b_2}$$

where b_1 and b_2 are parameters.

[25]See also Costinot and Vogel (2010) for an application of Teulings (1995) to international trade.

5.3 Moments and Identification

We use 39 moments to estimate the remaining 11 parameters: $\{\kappa, \nu_y, \nu_v, \bar{\omega}_2, \sigma_{\omega_0}, \sigma_{\omega_1}, \rho, \mu_E, \sigma_E, b_1, b_2\}$. To exploit information on the joint distribution of firm wages, sales, number of network links, and export activities, as well as the novel sorting patterns, we summarize most moments conditional on the 5 quintiles of firm wage per worker:

1. The mean number of suppliers (5 moments) and mean number of customers (5 moments)

2. The share in total network sales (5 moments) and the standard deviation of sales (5 moments).

3. The share of firms exporting (5 moments) and the average export intensity for exporting firms (5 moments).

4. Average of log-wage of suppliers, unweighted (4 moments) and weighted by spending shares (4 moments).[26]

5. The shift-share regression coefficient of wage response to idiosyncratic export demand shock (1 moment).

Although all parameters are estimated jointly, some parameters are associated to some moments more closely. The average number of trading partners per firm identifies κ, the efficiency in transforming ads into matches in (21). Total sales and standard deviation by quintile of wages identify parameters σ_{ω_0}, σ_{ω_1}, and ρ. Parameter μ_E governs the share of firms exporting and σ_E governs how this share changes across quintile of firm wages. If σ_E is large, then the share of firms exporting does not vary much across quintiles because it depends more on firm draws of f_E than on quality choices (wages). Parameter b_1 governs the level of export intensity while b_2 governs how export intensity changes across quintile of firm average wages. If b_2 is large, $D_F(q)/D_H(q)$ is increasing in q and export intensity increases with quintile of wages.

The moments on suppliers' wages summarize the total and extensive margins of assortative matching in the network. As per Section 3.4, parameters ν_y govern the intensive margin in (32), and parameter ν_v governs the extensive margin (31).

Finally, the shift-share coefficient of Table 3 column (2) identifies $\overline{\omega}_2$. Consider a shock that increases a single firm's export demand $D_F(q)$ by 5 percent. If $D_F(q)/D_H(q)$ is increasing in quality as in our estimated model, the firm increases $q(\omega)$. This increase is

[26] This set of moments are only four (and not one per quintile) because we normalize the wages in the lowest quintile to 0 and match the log difference from the lowest quintile in the data and model.

associated with an increase in wage per worker since each quality in the estimated model is associated with an average wage per worker in the data (the ranking is the same). Parameter $\overline{\omega}_2$ governs the concavity of $z(q, \omega)$ in (13). If $\overline{\omega}_2$ is large and negative then $z(q, \omega)$ is very concave, and the firm does not respond much to the export demand shock. If $\overline{\omega}_2$ is small, the response is large.[27]

5.4 Model Computation

We solve the equilibrium of the model for each guess of parameters. We discretize the quality space into a grid of 100 equally spaced choices in [0,8]. Given a guess of $\sigma_{\omega 0}, \sigma_{\omega 1}, \rho$, we sample 50,000 firms from the bivariate distribution of $\omega = (\omega_0, \omega_1)$ and calculate each firm's productivity at each quality, $z(q, \omega)$ in (13).

The solution algorithm, detailed in Appendix G, is composed of two blocks. The inner block takes the equilibrium distribution of productivity-quality $J(z, q)$ as given. It solves for the equilibrium in the matching and product markets given $J(z, q)$, and the optimal export status, search and production decisions for each (z, q). From this inner block, we obtain the aggregate functions $\Pi(q, 0)$ and $\Pi(q, 1)$ that govern each firm's export cutoff $\overline{f}_E(z, q)$ in (40) and quality choice in (41). The outer block solves the optimal quality choice for each firm ω and updates $J(z, q)$ used in the inner block. We iterate over these two blocks until firms do not change their quality choices.[28]

6 Estimation Results

The targeted moments are in Table 7. The estimated parameters in Table 6 are split into three sets. The first set ν_v, ν_y, κ governs network formation. Parameter ν_v is the standard deviation of the distribution of ads ϕ_v in Figure 3B. The estimated value $\nu_v = 3.09$ implies, for example, that 65 percent of the ads posted by sellers in the top quintile of quality go to buyers also in the top quintile and 8 percent go to buyers in the lowest quintile. Parameter $\nu_y = 0.35$ governs the complementarity in production, i.e., the log-supermodularity of function ϕ_y in Figure 3A. Take two suppliers charging the same

[27] In Appendix F, we prove that we can non-parametrically identify the joint distribution of (ω_0, ω_1) using the joint distribution of sales and wages and that $\overline{\omega}_2$ is not identified in the cross-section. We also show its identification through idiosyncratic firm-specific shocks.

To construct the model's response, we sample firms and estimate the expected effect from the idiosyncratic demand shocks as the average change in wages per worker weighted by firms' export probabilities.

[28] The estimated function $\Pi(q, E)$ is concave in q because all buyers' (service and manufacturing firms) valuation of quality, ϕ_y in (15), is concave. Then, the quadratic form of $z(q, \omega)$ in (13) together with $\bar{\omega}_2 < 0$ imply that all firms' problem of choosing quality (14) is concave, and quality choices are bounded even for firms that have a comparative advantage in producing higher quality, $\omega_1 > 0$.

Table 6: Parameter Estimates

	Parameter	Estimate	Standard error
Matching friction	κ	0.00087	(0.00003)
Directed search	ν_v	3.09	(0.06)
Complementarity	ν_y	0.35	(0.03)
Sd of quality capability	σ_{ω_1}	0.116	(0.001)
Sd of efficiency capability	σ_{ω_0}	0.110	(0.000)
Correlation	ρ	0.137	(0.002)
Efficiency cost of quality	$\overline{\omega}_2$	-0.103	(0.001)
Mean of log export cost	μ_E	-3.95	(0.02)
Sd of log export cost	σ_E	1.52	(0.04)
Foreign demand shifter	b_1	93.16	(2.49)
Foreign demand curvature	b_2	0.49	(0.01)

price, one in the highest quintile of quality and one in the lowest quintile. Conditional on matching, a firm in the top quintile of quality spends 12.2 times more in the high-quality supplier than in the low-quality one, while a firm in the lowest quintile of quality spends only 5.5 times more. Parameter $\kappa = 8.7 \times 10^{-4}$ implies a low probability of finding a trading partner per ad. This is not surprising given that the number of partners per firm in the data is a tiny fraction of all manufacturing firms. The average number of suppliers and customers per quintile of wages ranges from 5.6 to 25.8 in Table 7. The model fits these averages well. With only two parameters ν_v and ν_y to govern assortative matching, it also fits reasonably well the increasing relation between buyers' and sellers' wages, weighted and unweighted.

The second set $\sigma_{\omega_0}, \sigma_{\omega_1}, \rho$ are the parameters of the joint distribution of (ω_0, ω_1), where ω_0 determines firms' productivity level and ω_1 their comparative advantage in higher-quality. This distribution governs the predicted joint distribution of wages and sales. There is a large dispersion of sales across quintiles of wages in Table 7. Firms in the highest quintile account for 78 percent of network sales in the data and in the model.

The third set of parameters $\mu_E, \sigma_E, b_1, b_2$ governs exporting patterns. The log of the export cost has mean $\mu_E = -3.95$ and standard deviation $\sigma_E = 1.52$. The share of firms exporting is higher among high-wage firms, but still about 10 percent of low-wage firms export in the data and in the model. Parameters $b_1 = 93$ and $b_2 = 0.49$ govern export intensity by quintile of wage. Conditional on exporting, export intensity is increasing in firm wages in the data. The model captures this pattern with an estimate of $D_F(q)/D_H(q)$

Table 7: Model Fit – Targeted Moments

	Quintiles of average wage per worker				
	1	2	3	4	5 (largest)
Mean number of suppliers					
Data	5.8	6.7	5.8	11.4	25.8
Model	4.7	4.7	6.0	9.1	29.4
Mean number of customers					
Data	5.6	7.0	6.7	11.7	25.1
Model	5.4	5.9	7.6	10.9	23.8
Standard deviation of log sales					
Data	1.37	1.34	1.37	1.52	1.79
Model	1.20	1.18	1.20	1.24	1.55
Share of total network sales					
Data	0.03	0.04	0.04	0.10	0.78
Model	0.04	0.03	0.05	0.11	0.78
Fraction of exporters					
Data	0.08	0.18	0.16	0.34	0.57
Model	0.11	0.13	0.18	0.29	0.60
Export intensity of exporters					
Data	0.24	0.21	0.23	0.23	0.26
Model	0.18	0.21	0.22	0.23	0.25
Unweighted average log wage of suppliers					
Data	-	0.01	0.01	0.04	0.14
Model	-	0.02	0.04	0.07	0.12
Weighted average log wage of suppliers					
Data	-	0.02	0.02	0.07	0.23
Model	-	0.04	0.07	0.11	0.17
Shift-share IV coefficient (5% export shock)					
Data	0.21%				
Model	0.21%				

that is increasing in q.

This increasing ratio $D_F(q)/D_H(q)$ matters because a firm-specific shock that increases $D_F(q)$ leads the firm to upgrade its quality and thereby increase its wage per worker. This prediction is consistent with the shift-share regressions of Table 3. In the data, a 5% export shock on average increases the wage per worker by 0.21% for exporting firms, and we pick $\bar{\omega}_2 = -0.103$ to exactly match this response. In column (6) of Table 3, a one percent increase in firm's wage, in response to the shock, increases its suppliers' wages by 0.434 percent (standard error 0.185 percent). Out-of-sample, this number is 0.219 percent in the model.

Overall, the moments of the model and the data are similar in Table 7. As further validation, Figure 4 illustrates the predictions of the model for the non-parametric patterns of assortative matching of Figure 2 above. These figures are related to targeted moments but they were not directly targeted. The model matches well the extent to which firms with similar wages disproportionately transact with each other, upstream and downstream, in the intensive and extensive margins.

Equipped with these estimates, we investigate how a counterfactual increase in export demand affects firms' quality choices directly and indirectly through the O-Ring production network. Having captured well the joint distribution of wages, sales, number of trading partners and exporting margins matters quantitatively because the shock directly impacts large, high-wage firms with many domestic trading partners.

7 Counterfactual Analysis

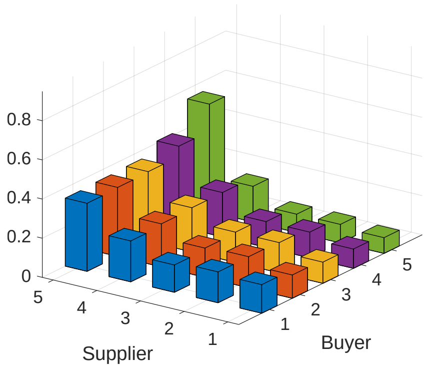
Starting with the equilibrium of the estimated model, our baseline counterfactual increases export demand $D_F(q)$ by 5%. It maintains the efficiency wages $w(q) = 1$ for all q, the exchange rate $e = 1$ and price of services $P_s = 1$. We allow gross manufacturing output and the trade balance to increase with the shock. We choose this as the baseline because it captures the effect of the shock on manufacturing but shuts down the interaction between manufacturing and the rest of the economy by assuming that (i) labor supply in and out of manufacturing is perfectly elastic ($w(q) = 1$), (ii) the export expansion does not lead to a real exchange rate appreciation ($e = 1$), and (iii) the price of the inputs that manufacturing firms use from distributors does not change ($P_s = 1$).[29] Relaxing each of these assumptions, in Section 8, requires out-of-sample assumptions.

Figure 5 plots the density of quality choices. The counterfactual first order stochas-

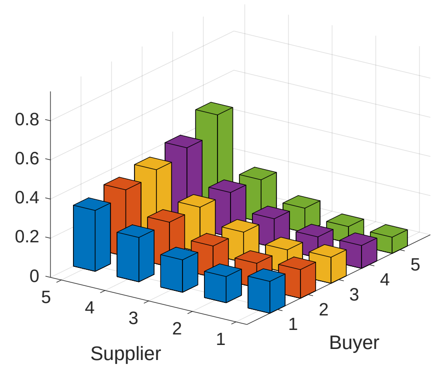
[29]Price stays $P_s = 1$ in a limiting case in which domestic manufacturing is a small share of inputs into services. Other inputs may be imports or other (not modeled) domestic goods or factors.

Figure 4: Firm-to-firm Trade Links and Values by Quintile

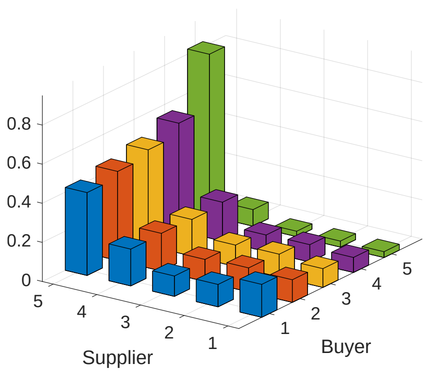
(a) Share of Suppliers (Data)



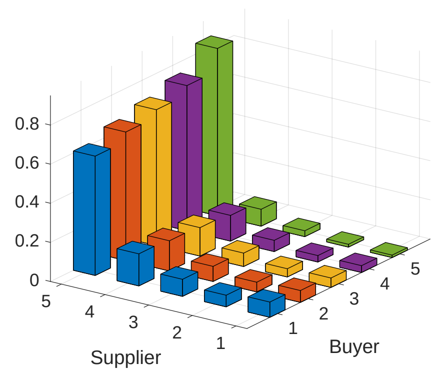
(b) Share of Suppliers (Model)



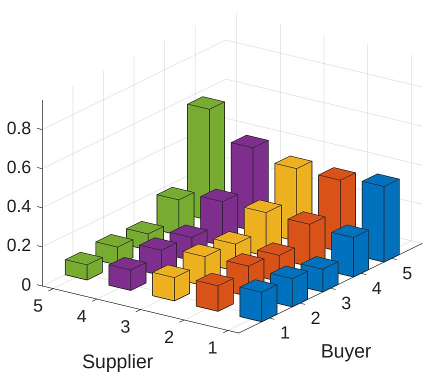
(c) Spending Shares (Data)



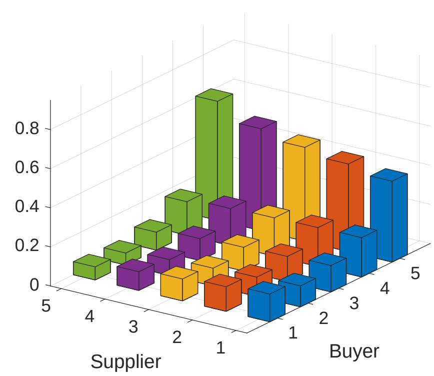
(d) Spending Shares (Model)



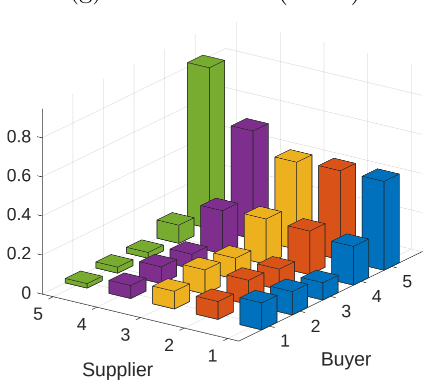
(e) Share of Buyers (Data)



(f) Share of Buyers (Model)



(g) Sales Shares (Data)



(h) Sales Shares (Model)

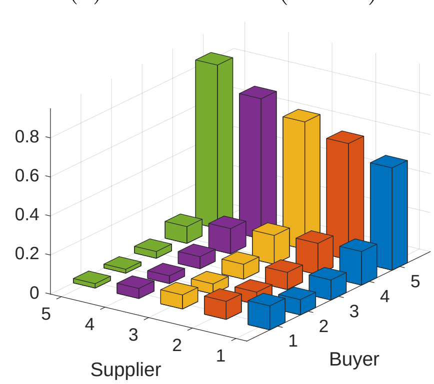
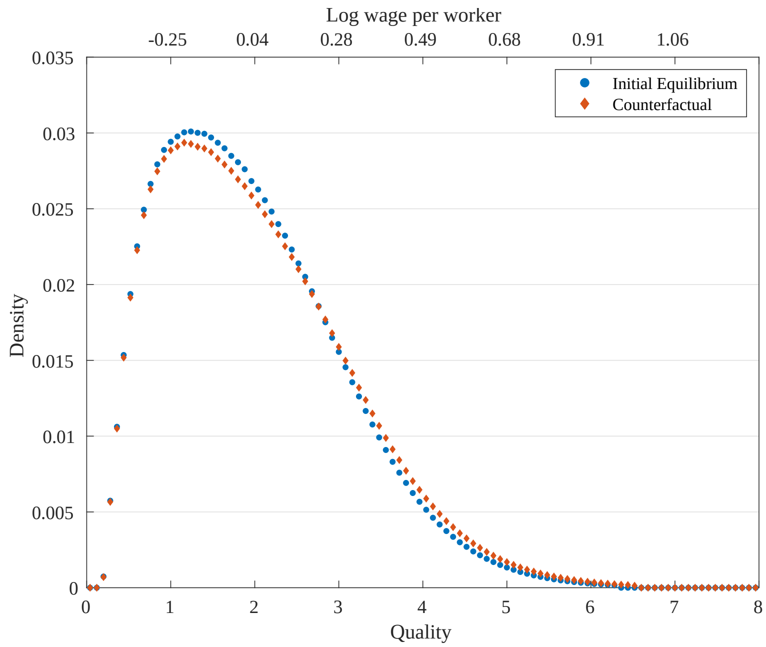


Figure 5: Distribution of Quality Choices



tically dominates the initial equilibrium. By assumption, the rankings of quality and average wage per worker (efficiency wage $w(q)\times$ labor endowment per worker) are the same in the model, and the model exactly matches the distribution of wage per worker across firms in the data. So, wages in the top x-axis of Figure 5 yield an economic interpretation to quality. Since $w(q) = 1$ in the counterfactual, wage changes reflect only quality upgrading (shifting to higher-quality tasks). For example, the log of wages is 0.21 larger in quality 4 than quality 3 ($= 0.49 - 0.28$).

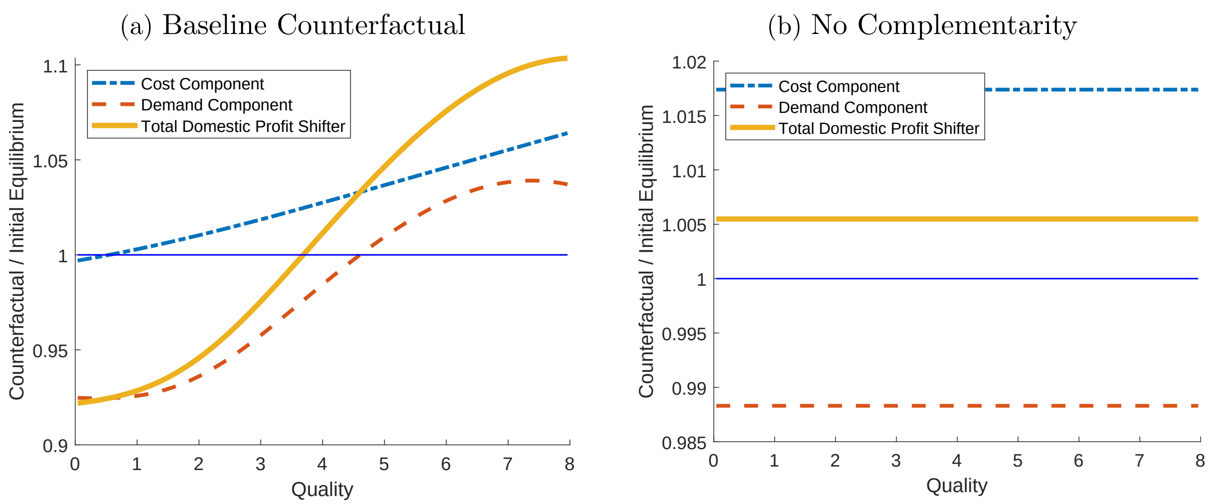
Table 8 reports the changes in wages, sales and number of trading partners for exporters and non-exporters by *ex ante* quintile of the quality distribution. Wage per worker increases in all groups of firms, especially among the *ex ante* high-quality firms. For example, wages in non-exporting, high-quality firms increase by 2.5 log points.

The network propagates the shock from exporting to non-exporting firms. Profit shifter $\Pi(q, 0)$ summarizes the benefit of upgrading quality for non-exporters. As per equations (58) and (8), $\Pi(q, 0)$ is proportional to a demand $D(q, 0)^{\gamma}$ and a cost component $c(q)^{\gamma\alpha_m(1-\sigma)}$. Figure 6(a), plots the counterfactual changes relative to the initial equilibrium of $\Pi(q, 0)$ and each of these components. First, take the demand component $D(q, 0)^{\gamma}$ in the red dotted curve. Exporters upgrade quality and increase their posting of ads. Then, the probability of matching increases for high-quality suppliers who direct their ads toward high-quality market segments. At the intensive margin, conditional on

Table 8: Counterfactual changes by quintile of quality

	Ex-ante quintiles of quality				
	1	2	3	4	5 (largest)
log*(Wage per worker)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	0.31	0.52	0.92	1.66	2.90
Non-exporters	0.23	0.48	0.89	1.61	2.53
All Firms	0.24	0.48	0.90	1.63	2.76
log*(Sales)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	-1.25	0.50	1.48	3.05	6.58
Non-exporters	-7.69	-7.03	-6.03	-4.25	-1.23
All Firms	-6.93	-5.98	-4.58	-2.01	3.60
log*(Number of Suppliers)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	-0.74	0.29	0.88	1.81	3.90
Non-exporters	-4.56	-4.17	-3.58	-2.52	-0.73
All Firms	-4.11	-3.55	-2.71	-1.19	2.14
log*(Number of Customers)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	-2.47	-1.28	-0.12	1.47	3.82
Non-exporters	-3.55	-2.58	-1.43	0.16	2.14
All Firms	-3.42	-2.40	-1.18	0.56	3.18

Figure 6: Decomposition of Changes in Domestic Profit Shifter



Notes: The figure displays the counterfactual changes in the domestic profit shifter. This shifter $\Pi(q, 0)$ is proportional to $D(q, 0)^{\gamma} \cdot c(q)^{\alpha_m(1-\sigma)\gamma}$, and we separately plot these demand and cost components. The baseline counterfactual is in the left panel, and the special case with no complementarity ($\nu_y = 0$, $\nu_v = \infty$) is in the right panel.

the match, exporters increase their spending on high- relative to low-quality domestic suppliers. Second is the cost component $c(q)^{\gamma\alpha_m(1-\sigma)}$ in the blue dotted curve. The increased search effort and quality upgrading among exporters decrease the cost of manufacturing inputs for all firms. This decrease accrues disproportionately to high-quality firms whose production is intensive in high-quality inputs (estimated $\nu_y > 0$). The more firms respond to these shifts by upgrading their qualities, the more they augment the effect of the shock. Overall, the profitability for non-exporters increases by 7 percent in the high-quality segment ($q \approx 6$), and it decreases by about 7 percent in the low-quality segment ($q \approx 1$). Both $c(q)$ and $D(q, 0)$ significantly contribute to these changes.

Exporters (not in the figure) experience similar indirect effects. Their profit shifter $\Pi(q, 1)$ is proportional to the same cost component $c(q)^{\gamma\alpha_m(1-\sigma)}$ and their demand component $D(q, 1)^\gamma$ is a CES aggregate of domestic demand $D(q, 0)$ and foreign demand $D_F(q)$ in equation (39). In all, the average wage increases by 1.0 percent for non-exporters, 1.92 percent for exporters and 1.22 percent for all firms. This increase in exporters' wages is an order of magnitude larger than the increase of 0.21 percent induced by the idiosyncratic export demand shocks of the same magnitude.

The effect of the counterfactual on sales and network connections is more heterogeneous. The domestic market for inputs becomes more competitive ($c(q)$ decreases) and the appeal of low-quality inputs decreases because their marginal product is low in the production of high-quality. As a result, lower-quality, non-exporting firms decrease their sales and search efforts. In Table 8, the number of suppliers and customers decreases by 4 log-points and sales decrease by 7.7 log points for these firms. In spite of the positive cross-sectional correlations, the counterfactual simultaneously predicts reductions in sales and network connections, and increases in quality for domestic manufacturers.

To further probe into these mechanisms, we study a special case of the model without the complementarity in matching ϕ_v and in production ϕ_y. The value of high- and low-quality inputs in production is independent of the output quality ($\nu_y = 0$), and all firms' ads are uniformly distributed across the quality set Q ($\nu_v \to \infty$). We re-estimate the model with these parameter restrictions in Appendix H. By assumption, the special case cannot match the increasing relation between buyer and supplier wage. For all other moments, the fit is similar to the general model. Importantly, the ratio $D_F(q)/D_H(q)$ is increasing in quality so that exporters upgrade quality when $D_F(q)$ increases.

We experiment with the same 5% counterfactual increase in export demand $D_F(q)$ in this special case. The average wage increase for exporters is 0.23%, very close to the average firm response to an idiosyncratic export demand shock 0.21%. Figure 6 panel (b) plots the change of $\Pi(q, 0)$ and of its cost $c(q)^{\gamma\alpha_m(1-\sigma)}$ and demand $D(q, 0)^\gamma$ components.

The shock decreases the price index $P(q)$ in the domestic market. Competition tightens decreasing demand and costs. But these changes are independent of quality. Profit shifter $\Pi(q, 0)$ increases by 0.5% for all non-exporters. In the model, firms choose quality before observing their exporting cost. The flattened $\Pi(q, 0)$ mutes the quality response of all firms in this special case, especially those with a low probability of exporting.

Manufacturing output increases by 6.03% in the general model and 5.78% in the special case. These effect are larger than the classical Hulten (1978) because the positive shock increases exporters' search efforts and leads other firms to tilt their input purchases toward exporters, whose prices decrease. Baqaee and Farhi (2019a) highlight this role of an elasticity of substitution greater than one ($\sigma = 5$ in the estimation). Despite similar predictions on output, the dramatic differences in quality upgrading between the general model and the special case indicate that economies of scale are not sufficient to understand the effect of international trade on developing countries.

8 Alternative Counterfactuals and Discussion

In all of the counterfactuals below, foreign demand $D_F(q)$ increases 5 percent as in the baseline counterfactual. We experiment with four specifications:

1. We allow the real exchange rate e (wages in foreign relative to home) to move to balance trade. In Appendix E, trade balances if

$$\left(\frac{eP_{Fs}}{P_s}\right)^{1-\sigma_s} X_s = \int_{q\in Q} \left(\frac{e^{\sigma} D_F(q)}{D_H(q)}\right)^{\beta_v/(\beta_v-1)} \left[\int_z x(z,q,1)j(z,q,1)dz\right] dq.$$

   Clearly, imports on the left-hand-side decrease with e and exports on the right-hand-side increase. In the baseline we kept $e = 1$ and allowed for the trade surplus to increase with the export shock.

2. Free entry. In the baseline, average profits increase with the foreign demand shock. Here, we allow the mass of firms to increase, which tightens competition and maintains the average profit as in the initial equilibrium.

3. We allow the wages of skilled workers to increase. In the baseline, we maintain the wage schedule $w(q) = 1$ for all q, assuming that the labor supply is perfectly elastic. There, labor demand for high-quality tasks increases. Here, we allow the wage $w(q)$ to go up relative to the initial equilibrium and to go up proportionately with quality. In particular, we assume $w(q)$ is linear, set $w(1) = 1$ and $w(q^{max}) = 1.0079$ so that the average counterfactual quality change across firms is zero.

4. We increase the productivity of the highest-quality firms under the assumption that the agglomeration of skilled workers in manufacturing increases firm productivity, as estimated Diamond (2016). In the baseline, the stock of labor in the *ex ante* top quintile of quality goes up by 0.846 percent. Diamond (2016) estimates that the increase in college graduates by one percent in a location increases their productivity by 0.854 percent. Using these numbers, we increase the productivity $z(q, \omega)$ of firms in the *ex ante* top quintile of quality by 0.72 percent ($= 0.854 \times 0.00846$).[30]

Table 9 summarizes the results. With free entry (2), the number of firms increases by 1.13 percent, but the remaining results are close to the baseline. With balanced trade (1), the real exchange rate appreciates by 1.15 percent, and in counterfactual (3), the wages of skilled workers increase. Both of these price changes decrease the incentives for firms to upgrade quality. Although they are small, they have a significant effect, because both positive and negative effects are magnified in general equilibrium. Average wage per worker increases by 0.20 percent with balanced trade (1) and by 0.16 with the increase in skill premium (3), compared to 1.22 percent in the baseline.

Agglomeration effects increase the productivity of high-quality firms in specification (4). By the same general equilibrium effects, even a small increase, of 0.72 percent, has a large effect: The average wage per worker increases by 3.17 percent and output by 13.7 percent, roughly double the baseline numbers.[31]

Shifter $D_F(q)$ summarizes foreign market size, price index, and frictions in matching with foreign customers. So, the counterfactual increase in $D_F(q)$ may be interpreted as a foreign shock or as a policy to promote exports through decreases in search frictions, e.g., export fairs and conferences.[32] Counterfactuals (1), (3) and (4) highlight critical factors in the effectiveness of these export-promotion policies. In counterfactual (3), the rise in skill premium dampens the incentives for firms to upgrade to skill-intensive qualities. This points to the importance of ensuring an elastic supply of skilled workers into manufacturing, perhaps through education and training.

Counterfactuals (1) and (4) together rationalize the concomitant increases in trade surplus, and in manufacturing production and upgrades commonly observed in fast-growing

[30]In Diamond's (2016) model, the inverse demand function for college graduates is

$$\log w_H = \gamma \log L_H - (1/\sigma) \log L_H$$

where L_H is the supply of college graduates in a location, σ is the elasticity of substitution between skilled and unskilled workers and γ is the external scale parameter. She estimates $\sigma = 1.6$ and $\gamma - 1/\sigma = 0.229$ yielding $\gamma = 0.854$.

[31]This exercise is akin to Jones (2011) who emphasizes the roles of complementarity and economies of scale in economic growth.

[32]Rauch (2001) surveys case studies of this type of export-promotion policies.

Table 9: Summary of Counterfactuals

	Counterfactual Specifications				
	Baseline	Balanced Trade	Free Entry	Δ Skill Premium	Agglomeration
Percentage changes in		(1)	(2)	(3)	(4)
Output (X)	6.03	0.00	6.39	3.07	13.72
Exchange rate (e)	-	-1.15	-	-	-
Mass of firms (N)	-	-	1.13	-	-
Efficiency wage at $w(q^{max})$	-	-	-	0.79	-
Average wage per worker (All)	1.22	0.20	1.30	0.16	3.17
Average wage per worker (Exporters)	1.92	0.32	2.06	0.13	5.01
Average Quality (All)	2.06	0.34	2.19	0.00	5.24

emerging markets, notably in East Asia. In counterfactual (1), the effects of export promotion in quality upgrading are dampened when a real exchange rate appreciation prevents the country from running a trade surplus. And in counterfactual (4), output grows when the agglomeration of skilled workers in manufacturing increases firm productivity.

9 Conclusion

We document novel facts about firm-to-firm trade using data from Turkey. High-wage firms are more likely to match with each other in the network, and the value of transactions is larger when the trading partners' wages are both high. Over time, a firm-specific demand shock from a rich export destination is associated with an increase in the firm's wage and in the average wage of its suppliers.

We rationalize these findings in a model where firms' choices of quality and skill intensity are interconnected through the production network. Higher-quality production is intensive in skilled labor and in higher-quality inputs, and higher-quality firms direct their search toward other higher-quality firms. Counterfactuals show that even a small export shock leads to large and widespread quality upgrades in manufacturing firms, due to the complementarity in their quality choices.

These findings are broadly consistent with Goldberg and Reed (2020) who show that exporting even a small amount of output to developed countries is associated with economic growth in developing countries. Alternative counterfactual scenarios in Section 8 point to other economic factors that interact with the effects of international trade on manufacturing firms: Education, trade imbalances, and agglomeration effects.

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Contents (Appendix)

Online Appendix: Not For Publication

We provide complementary results on the descriptive analysis of the data, details of the model, computation, and counterfactuals, and alternative model specifications. In Appendix A, we document a battery of robustness checks and facts that support the positive sorting of worker skills and firm quality across business partners in our data. Appendix B provides details of the identification and robustness of the shift-share IV regressions. We develop micro-foundation for the wage schedule using a Roy model of labor supply in Appendix C. Appendix D uses a simplified model with homogeneous quality to investigate the efficiency property of endogenous network formation. Details of the open economy model are in Appendix E. Appendix F discusses the identification of parameter $\overline{\omega}_2$. Computation algorithm is described in Appendix G. Finally, Appendix H and Appendix I report estimates and moments of the model extensions with no complementarity and with endogenous targeting of ads, respectively.

A Robustness of Sorting Patterns

We check the robustness of the positive assortative matching of firm skill intensity in the network. In subsection A.1, we decompose firm-worker level wages into firm and worker components and show that the sorting pattern holds for worker components. In subsection A.2, we use occupational categories to measure skill intensity. In subsection A.3, we estimate exporters' quality using information on export destination, prices and quantities. The quality of firms' exports is increasing in the firm's own wage and in its suppliers' wage. Robustness checks in Subsection A.4 confirm that results are not driven by the geographic clustering of similar firms. Finally, subsection A.5 investigates other firm characteristics and finds that wage is the dominant factor in sorting.

A.1 Alternative measure of worker skills

Our measure of firm skill intensity in the main text, wages, contains information about worker skills as well as firm rents. Here, we use an alternative measure of firm skill intensity, proposed by Bombardini et al. (2019) that extracts firm rents.

First, using Turkish linked employer-employee data for the 2014-2016 period, we decompose the variation in firm-worker level wages into firm and worker components as in Abowd et al. (1999). We estimate the following specification for worker earnings:

$$\ln wage_{eft} = \Gamma X_{eft} + \theta_e + \psi_f + e_{eft}, \tag{43}$$

where θ_e and ψ_f are worker and firm fixed effects respectively, and X_{eft} is a vector of controls. For workers, the controls are age (squared) and dummies for 1-digit ISCO occupation codes. For firms, the controls are dummies for each industry-region-time triplet and size (proxied by gross sales).

Our sample includes more than 3.2 million firm-worker-year observations. It is well known that the fixed effects in equation (43) are identified from workers moving between jobs. As with our baseline results, we estimate (43) using only manufacturing firms. Given this industry restriction and the short time span (i.e. 3 years), this sample corresponds to about 65% of all workers.

We measure firm f's slkill intensity using the workers' fixed effects:

$$\theta_f = \frac{1}{N_f} \sum_{e \in E_f} \hat{\theta}_e, \tag{44}$$

where N_f denotes the number of workers of firm f, and E_f the set of workers employed by the firm in the year 2015.

There is significant overlap between the quintiles of firm average wage and the measure θ_f: 62% (42%) of firms in the highest (lowest) quintile based on wages are also in the highest (lowest) two quintiles constructed based on average worker skills. When the middle quintile is included, the respective shares go up to 85% and 62%.

Table A1: Assortative Matching on Worker Fixed Effects

	total (1)	extensive (2)	intensive (3)
θ_f	0.120	0.080	0.040
	(0.006)	(0.005)	(0.007)
R^2	0.095	0.104	0.045
N	53,601	53,601	53,601
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: θ_f denotes average worker skills for firm f in (44). The dependent variable, suppliers' skills are constructed as a weighted average of θ_ω, where weights are the share of supplier ω in firm f's total spending on inputs, in equation (1). Ind and prov refer to 4-digit NACE industries and provinces, respectively. The extensive and intensive margins are defined in (3) and (4). Robust standard errors are clustered at 4-digit NACE industry level.

Table A1 presents the results of our sorting regressions using θ_f as a proxy for firm skill intensity. The coefficient is economically and statistically significant. It is about half of our baseline estimate, even though the measure θ_f does not include the firm fixed effect ψ_f in (43) or the skills of workers that never left the firm. The decomposition into extensive and intensive margins remain close to the baseline.

A.2 Wages and occupational categories

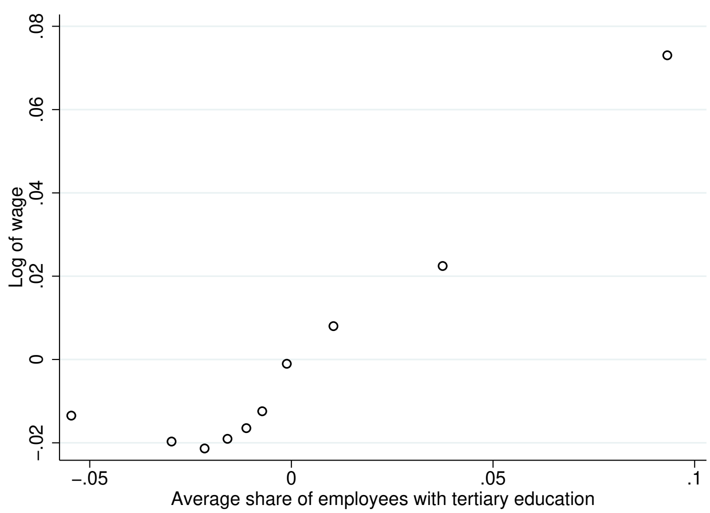
In our data, we observe workers' occupations but not their educational level. From the EUROSTAT dataset, we obtain information on the share of employees with tertiary education for each 1-digit ISCO occupation codes for the EU-15 countries.[33] Assuming the ranking of skill intensity across occupations is similar in EU-15 and in Turkey, we use these EU-15 shares as a measure of occupational skill intensity. We measure firm f's skill intensity as

$$E_f^{\text{occupation}} = \sum_{o=1}^{9} \omega_{of} Educ_o^{\text{EU-15}}, \tag{45}$$

where ω_{of} is the employment share of occupation code o for firm f and $Educ_o^{\text{EU-15}}$ denotes the share of employees with tertiary education for the same occupation code in EU-15. Measure $E_f^{\text{occupation}}$ is the expected share of workers with tertiary education of a firm in Europe with the same distribution of workers across occupations as firm f.

Figure A1 plots firm wages and occupational skill intensity $E_f^{\text{occupation}}$ with both variables adjusted for province-industry fixed effects. The relationship is positive and tight.

Figure A1: Wages and Measure of Skill Intensity based on Occupations



Notes: We define the wage of a firm as the firm's wage bill divided by the number of workers. The occupational measure of skill intensity is defined in equation (45). Both x- and y-axis variables are demeaned from 4-digit NACE industry.

Table A2 presents the main assortative matching regressions replacing wages with

[33]The shares are quite stable across years, and we used data from 2015.

occupational skill intensity $E_f^{\text{occupation}}$. The coefficient is positive and significant but much smaller than the wage regressions. This is not surprising since occupation is measured at the one-digit level and educational shares are based on European data, potentially masking large cross-firm heterogeneity. Yet it is still reassuring to observe a clear positive sorting relationship. Because $E_f^{\text{occupation}}$ is in shares, not logs, it is not amenable to the decomposition of assortative matching into extensive and intensive margins.

Table A2: Assortative Matching on Occupational Measure of Skill Intensity

	Supplier $E_f^{\text{occupation}}$ (1)	Supplier $E_f^{\text{occupation}}$ (2)
$E_f^{\text{occupation}}$	0.0274	0.0274
	(0.0038)	(0.0038)
Employment$_f$		0.0044
		(0.00086)
R^2	0.049	0.051
N	70,967	70,967
Fixed effects	ind-prov	ind-prov

Notes: We measure a firm's skill intensity $E_f^{\text{occupation}}$ in (45). It is the expected share of workers with tertiary education of a firm in the EU-15 with the same occupational mix as firm f. The dependent variable, Supplier $E_f^{\text{occupation}}$ is defined analogously to supplier wages. It is the weighted average of firm f's suppliers $E_\omega^{\text{occupation}}$, where the weights are firm f's spending on each supplier as a share of its spending on manufacturing inputs. All specifications include industry-province fixed effects (ind-prov). Robust standard errors are clustered at 4-digit NACE industry level.

A.3 Wages and quality of exports

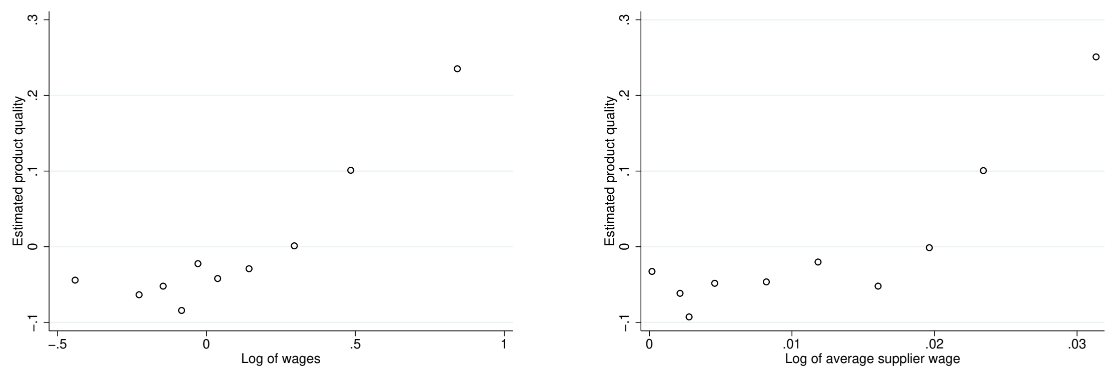
Like Kremer (1993), the focus of our paper is the complementarity in firms' skill intensity. Quality in the model is a latent variable that captures the type of labor and material inputs that a firm uses. A firm's quality varies one-to-one with its wage per worker in the estimation. Here we check the relationship between wage per worker and the measure of quality proposed by Khandelwal et al. (2013). Since this quality measure uses prices, we can only contruct it for exporting firms. We estimate the following regression:

$$\ln X_{fpc} + \sigma \ln UV_{fpc} = \alpha_c + \alpha_p + \epsilon_{fpc}, \tag{46}$$

where X_{fpc} is the quantity of exports of product p by firm f to country c, and UV_{fpc} is its unit value. We set $\sigma = 5$. Estimated (logarithm of) quality is given by $\hat{\epsilon}_{fpc}/(\sigma - 1)$. We aggregate it to the firm level by taking its simple average across all varieties (product-country pairs) exported by the firm.

Figure A2 plots this measure of firm quality against average firm wages (left panel) and against the wages of the firm's suppliers (right panel). In both plots, each circle represents the average value of the variables on the axes for each bin, where the bins are constructed from the x-axis. All variables are adjusted for their industry averages (4-digit NACE level). The relationship is positive, especially in the upper deciles. There is also a considerable overlap in the classification of firms by quintile of wages, used in the estimation. When the quintiles are constructed based on wages, almost half of the firms (45%) in the lowest (highest) quintile fall into the lowest (highest) two quintiles of firm quality. When the middle quintile is included, both shares go up to 65%.

Figure A2: Wages and Product Quality



Notes: We define the wage of a firm as the firm's wage bill divided by the number of workers. Quality is estimated from equation (46). Both x- and y-axis variables are demeaned from 4-digit NACE industry.

A.4 Geographic clustering of business partners

We confirm that the baseline results of assortative matching in Table 2 are not driven by the geographic clustering of similar firms. In the baseline, we control for the firm's province, because each province in Turkey roughly reflects a labor market. Turkey has 81 provinces, each divided into districts. The total number of districts is close to 1,000. In Panel A of Table A3, we add to our baseline estimates district fixed effects. The coefficients on the total, extensive and intensive margins of assortative matching are all very close to the baseline.

An additional concern is that labor market shocks may affect both a firm's average wage and its suppliers wages if firms are more likely to match within provinces. To address this concern, we construct a firm's suppliers wages by excluding suppliers in the province of the firm. We repeat the assortative matching regressions and present the results in Panel B of Table A3. The results are again close to the baseline.[34] We obtain similar

[34]The sample size is smaller than the baseline because we drop firms that source all their inputs locally.

estimates from this sample to he baseline estimates. This tells us that our results are not driven by local trade links or common local labor market conditions.

The VAT dataset that we use to identify domestic buyer-supplier links aggregates transactions at the firm (instead of establishment) level. We investigate whether the positive assortative matching on wages is driven by firms with establishments in more than one province. In panel C of Table A3 we repeat the assortative matching regressions excluding these firms as buyers and suppliers. The estimates again indicate a strong positive assortative matching on wages and the coefficient on the extensive margin is close to the original. The coefficient on the intensive margin is smaller than the baseline. Single-establishment firms generally have few trading partners, and so it is more difficult in this subset to establish the extent to which skill-intensive firms spend relatively more on their skill-intensive suppliers.

Table A3: Assortative Matching on Wages: Controlling for geographic clustering

	total (1)	extensive (2)	intensive (3)
Panel A: Control for district fixed effects			
$\log wage_f$	0.245	0.141	0.104
	(0.011)	(0.006)	(0.007)
R^2	0.185	0.162	0.099
N	77,418	77,418	77,418
Fixed effects	ind-prov,distr.	ind-prov,distr.	ind-prov,distr.
Panel B: Exclude trade partners located in the same province			
$\log wage_f$	0.214	0.130	0.0844
	(0.011)	(0.007)	(0.006)
R^2	0.144	0.127	0.0760
N	66,590	66,590	66,590
Fixed effects	ind-prov	ind-prov	ind-prov
Panel C: Exclude multi-establishment firms			
$\log wage_f$	0.161	0.116	0.0448
	(0.008)	(0.006)	(0.003)
R^2	0.121	0.115	0.0404
N	60,517	60,517	60,517
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. The suppliers' average wage $\log wage_f^S$ is defined in equation (1). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Equations (3) and (4) define the extensive and intensive margins. They capture respectively the extent to which firm f matches with high-wage firm or tilts its spending toward high-wage suppliers. All specifications include industry-province (ind-prov) level fixed effects. In Panel A, district-level (geographic units within provinces) fixed effects are also included. Robust standard errors are clustered at 4-digit NACE industry level.

A.5 Other characteristics and canonical correlation analysis

Appendix Table A4 repeats the regression of column (2) in Table 1 substituting wages with other firm characteristics. Assortative matching on sales is positive but less pronounced than on wages, and it is driven by the intensive margin. Sorting on the number of firm's network links is insignificant.

Table A4: Assortative Matching on Other Variables

	$\log \text{market share}^S_f$		$\log \text{outdegree}^S_f$	
	manuf	all	manuf	all
	(1)	(2)	(3)	(4)
Panel A: Total				
$\log \text{market share}_f$	0.175	0.154		
	(0.013)	(0.029)		
$\log \text{indegree}_f$			0.0985	-0.034
			(0.012)	(0.063)
R^2	0.11	0.14	0.09	0.14
N	77,418	410,608	77,418	410,608
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov
Panel B: Extensive margin				
$\log \text{market share}_f$	0.042	0.009		
	(0.009)	(0.025)		
$\log \text{indegree}_f$			0.009	-0.131
			(0.009)	(0.060)
R^2	0.07	0.12	0.08	0.13
N	77,418	410,608	77,418	410,608
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Market share is the share of a firm's sales in total sales of its 4-digit NACE industry, and indegree is the number of domestic suppliers of a firm. Both variables are in logarithms. Denoting the set of suppliers of firm f by Ω^S_f, average supplier market share in Panel A is defined as follows: $\log \text{market share}^S_f = \sum_{\omega \in \Omega^S_f} \log \text{market share}_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. $\log \text{outdegree}^S_f$ is defined similarly using the number of buyers (outdegree) of firm f's each supplier. The extensive margin in Panel B is the simple average across a firm's suppliers. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

We conduct a cannonical correlation analysis to gauge the relative importance of firm sales and wages in driving assortative matching in Tables 1 and A4. This approach was first proposed by Becker (1973) to evaluate the attractiveness of suitors in marriage markets when multiple dimensions of individual characteristics are observed. We follow the method in Johnson and Wichern (1988).

We construct indices that summarize the attractiveness of buyers and suppliers, A_b

and A_s, as linear combinations of sales and quality:

$$\begin{aligned} A_b &= k_1^b \log sales_b + k_2^b \log wage_b \\ A_s &= k_1^s \log sales_s + k_2^s \log wage_s \end{aligned} \tag{47}$$

Since the number of variables is equal to two in both A_b and A_s, the maximum number of (independent) canonical variate pairs is two. The coefficients on sales and wages are estimated by maximizing the correlation between the two attractiveness indices, subject to two normalization restrictions.

More formally, let $X_b = (\log sales_b, \log wage_b)$ and $X_s = (\log sales_s, \log wage_s)$ denote the vectors of buyer and supplier characteristics, and $k^b = (k_1^b, k_2^b)$ and $k^s = (k_1^s, k_2^s)$. The estimated weights k^b and k^s solve:

$$\begin{aligned} & \max \ k^{b'} E[X_b X_s^{'}] \ k^s \\ \text{subject to} \quad & k^{b'} E[X_b X_b^{'}] \ k^b = 1, \ k^{s'} E[X_s X_s^{'}] \ k^s = 1 \end{aligned}$$

If the buyer and supplier characteristics have Gaussian distributions, the estimated weights are consistent.[35]

Table A5: Results from the Canonical Correlation Analysis

	Canonical coefficients	p-value
$\log sales_b (k_1^b)$	0.29	0.00
$\log wage_b (k_2^b)$	0.80	0.00
$\log sales_s (k_1^s)$	0.11	0.00
$\log wage_s (k_2^s)$	0.94	0.00
First canonical correlation	0.15	0.00
Second canonical correlation	0.04	0.00

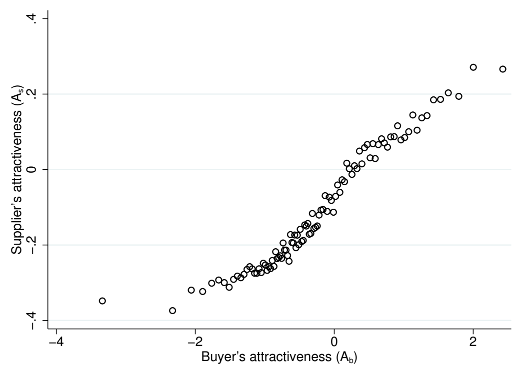
Notes: Wage is defined as the average value of monthly payments per worker.

To carry out the analysis, we first demean wage and sales variables from their 4-digit NACE industry averages, and then standardize them so that all four variables ($\ln sales_b$, $\ln wage_b$, $\ln sales_s$, and $\ln wage_s$) have zero mean and unit variance. Therefore, the estimated weights for different variables are directly comparable. Table A5 presents the results. All canonical coefficients are positive and statistically significant at a 1% level. For buyers, the weight of the wage variable is 2.8 times larger than the weight of the sales variable, and for suppliers, it is 8.5 times larger. This preeminence of firm wages

[35] See Dupuy and Galichon (2015) for a detailed discussion.

in matching is consistent with the bivariate correlations in the raw data: The bivariate correlation between wages of buyers and suppliers is 0.15, which compares to a correlation of 0.08 between their sales. Figure A3 shows a strong positive correlation between the predicted buyer and supplier attractiveness indices.

Figure A3: Predicted Attractiveness of Buyers and Suppliers



Notes: Sample includes manufacturing firms on both sides of the transaction. A_b and A_s denote the attractiveness indices of buyers and suppliers as defined in (47). Each circle represents the average value of the predicted A_b and A_s within a percentile of A_b.

B Identification and Robustness of Shift-share IV Regressions

Our empirical strategy relies on exogenous variation in import demand shocks for the consistency of the estimates on Table 3. To validate this assumption, we follow Borusyak et al. (2018) and verify that shocks (shifts) are numerous, sufficiently dispersed, and relevant. First, our shift-share design relies on a large number of shocks. To calculate Z^a_{ck}, we use 208 distinct destination countries c and 1,242 4-digit HS codes k, generating 153,186 ck pairs.

Second, as presented in Table A6, our shocks are highly dispersed. The average shock is 0.30, with a standard deviation of 3.26 and an interquartile range of 2.52. More importantly, the observed dispersion cannot be explained by firms' industry of operation. In column (2), when the shocks are residualized on 4-digit NACE industry codes, their standard deviation and interquartile range are almost unchanged. In addition, we have a large number of "uncorrelated" shocks. To show this, we construct, as suggested by Borusyak et al. (2018), a measure of shock importance, $x_{ck} = \sum_f (1/N) x_{ckf}$. This measure aggregates shares at the level of shocks and captures the average of importance of a shock for a firm. It is reassuring that even the largest value of x_{ck} in the data is tiny (0.003). For consistency, shocks should not be highly concentrated. The inverse of the Herfindahl–Hirschman index is informative about the effective number of shocks. As reported in Table A6, the effective number of shocks in our data is close to 20,000, implying that distribution of export sales is highly dispersed across a large number of country-product markets.

Table A6: Summary Statistics for Import Demand Shocks

	(1)	(2)
Mean	0.30	0
Standard deviation	3.26	3.24
Interquartile range	2.52	2.55
Number of countries c	208	208
Number of products (k)	1,242	1,242
Number of ck pairs	153,186	153,186
Largest value of x_{ck}		0.003
Effective sample size (inverse of Herfindahl–Hirschman Index of x_{ck})		19,949
Adjusted for 4-digit NACE industry codes	No	Yes

Third, we check for relevance. As a placebo test, we construct firm-level export demand shocks using randomly generated "shifts", drawn independently for each destination-product pair from a Normal distribution that has the same mean and standard deviation as the actual distribution of $\Delta \log \text{Imports}_{ck}$. Then we substitute them into equation (5) to construct our firm-level placebo export demand shocks: $\text{ExportShock}_f^{random}$. The results are in Appendix Table A7 column (2). The coefficient is quantitatively and statistically insignificant. In addition, we confirm in column (1) that putting the adjusted and unadjusted export shocks together in the first stage yields coefficients of similar magnitudes as presented in columns (1) and (2) of Table 3. Since the unadjusted shock is a weak instrument, the F-statistic decreases from 44 in our baseline regression to 13. This result reinforces our focus on the adjusted shock, i.e., weighting shocks by the income per capita of the destination country.

Appendix Table A7 contains additional exercises. In column (3), we add a weighted average of destination GDP per capita measured as of 2010, where the weights are x_{ckf} (without the shocks). As discussed by Adão et al. (2019), observations with similar shares may have correlated residuals, resulting in invalid standard errors. Therefore, adding this

Table A7: Effects of Export Shock: Robustness checks

	Δ log wage$_f$	Δ log wage$_f$	Δ log wage$_f$	Δ log wage$_f$	$\Delta \log wage_f^S$
	(1)	(2)	(3)	(4)	(5)
ExportShock$_f^u$	0.01				
(unadjusted)	(0.068)				
ExportShock$_f^a$	0.041		0.028	0.028	
(adjusted)	(0.007)		(0.008)	(0.008)	
ExportShock$_f^{random}$		0.0003			
		(0.004)			
Weighted GDP per capita$_f$			0.007		
			(0.001)		
Export share$_f$				0.039	
				(0.008)	
Δ log wage$_f$					0.451
(IV = ExportShock$_f$)					(0.224)
ExportShock$_f^{S,a}$					0.181
(adjusted)					0.050
F-Stat	13.3	0.005	37.6	30.2	
N	33,157	33,157	33,157	33,157	33,157
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Wage$_f$ is the average value of monthly payments per worker in firm f. The suppliers' average wage $\log wage_f^S$ is a weighted average of the wages of firm f's suppliers, equation (1). ExportShock$_f^u$ is a weighted average of changes in imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. ExportShock$_f^a$ adjusts these shocks by weighting destinations by their income per capita (see equations (6)). ExportShock$_f^{random}$ uses randomly generated shocks in the construction of the export demand shock. Export share$_f$ denotes the initial share of foreign sales in total sales of firm f. Weighted GDP per capita$_f$ is the weighted average of GDP per capita of firm's destinations in 2010, where the weights are defined as above. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

variable is a useful robustness check for both the consistency of δ and for inference. With the additional control, the estimated coefficient on the adjusted export shock is reduced slightly compared to the baseline, but it is still economically and statistically significant.

Next, column (4) adds the initial share of exports in total sales as a control for "incomplete shares" in Borusyak et al. (2018). Since we use total sales, rather than total export sales, in the denominator of x_{ckf}, the shares do not add up to one when aggregated at the firm level. This implicitly assigns a value of zero for demand shocks in the domestic market. However, we account for shocks in the domestic market by including industry-region level fixed effects in the baseline specification. Results show that both the size and standard error of the estimated coefficient on the adjusted export shock remain almost unchanged compared to column (3).

Finally, we add the weighted average of export shocks directly faced by firm's suppliers to column (6) of Table 3. This exercise checks if the results are driven by a correlation between the foreign demand shocks faced by firms and faced by its suppliers. If they were, then the exclusion restriction on our instrument would be violated. As expected, the foreign demand shocks faced by firm's suppliers raise their wages. But more importantly, the coefficient on the instrumented variable, buyer's wage, is very close to the baseline thus raising our confidence on the instrument.

Export Shocks and New Connections Table A8 verifies that the results in Table 4 are not driven by a few outliers in firms' new connections. We regress the *share* of newly hired (i.e. after the shock) workers, who received higher monthly wages than the firm's average worker before the shock, on the export shock. The second and third columns have the corresponding shares for the firm's new suppliers and new customers. The coefficients are all positive and statistically significant. That is, the shares of new connections with wages higher than the existing workers, suppliers and customers are positively associated with the export shock, after controlling for industry-province fixed effects.

Table A9 relies on an alternative reference level for firm-level wages to investigate the changes in the composition of inputs due to the export shock. It replaces the outcome of interest in the first column of Table 4 with the average wage of new workers *relative to workers that left the firm* (instead of all workers in the initial year) after the shock. The positive and statistically significant coefficient conveys a similar message to the one from Table 4: A positive export shock is associated with a higher skill intensity of the firm's new connections relative to its previous connections. Columns (2) and (3) present results for the average wages of firm's new suppliers and buyers defined relative to the average wages of the firm's former business connections.

Table A8: Effects of Export Shock on Composition of Inputs: Additional evidence

Share of new	Workers with wages higher than f's avg. wage at $t = 0$	Suppliers with wages higher than f's avg. supplier wage at $t = 0$	Buyers with wages higher than f's avg. buyer wage at $t = 0$
ExportShock$_f$	0.421	0.152	0.169
	(0.154)	(0.0690)	(0.0657)
R^2	0.167	0.0403	0.0394
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. ExportShock$_f$ is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Table A9: Effects of Export Shock on Composition of Inputs

Log of	Average wage of new workers relative to former workers at $t = 0$	Average wage paid by new suppliers relative to former suppliers at $t = 0$	Average wage paid by new buyers relative to former buyers at $t = 0$
ExportShock$_f$	0.0247	0.0220	0.0305
	(0.009)	(0.012)	(0.009)
R^2	0.0542	0.0662	0.0683
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. ExportShock$_f$ is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

C Roy Model of Labor Supply

In the main text, the supply of efficiency units of labor of task q is $L(q, w)$, an exogenous function of the task quality q and the full equilibrium wage schedule $w(q')$ for all $q' \in Q$. This appendix provides a micro foundation for labor supply based on the Roy model in Teulings (1995). It provides sufficient conditions for the ranking of average earnings per firm to equal the ranking of task quality q (also in Teulings (1995)), and it shows that we can construct a set of worker endowments such that labor markets clear and the distribution of earnings per worker across firms exactly matches the data. These claims hold for any fixed continuous and differentiable w – assumptions which hold in the estimation where $w(q) = 1$ for all $q \in Q$.

A measure H of workers have heterogeneous skills indexed with $s \in [0, 1]$. The distribution of workers across skills has density $h(s)$. A worker with skill s is endowed with $e(q, s)$ efficiency units of labor if she works at a firm of quality q. She observes the wage schedule $w(q)$ and chooses task quality q to maximize earnings:

$$\max_{q \in Q} \{w(q)e(q, s)\} \tag{48}$$

Let $s^*(q)$ be the set of skills that choose quality q. To ease notation, assume that $s^*(q)$ is a function or the empty set.[36] The mass of workers supplying task q is $h(s^*(q))$ where we define $h(\emptyset) = 0$.

Then, the supply of efficiency units of labor of task q is

$$L(q, w) = Hh(s^*(q))e(q, s^*(q))$$

where we can define $e(q, s^*(q)) = 0$ if $s^*(q) = \emptyset$. Earnings per worker in firms of task q is $w(q)e(q, s^*(q))$.

In the estimation, we assume that earnings per worker is strictly increasing in q. This assumption holds if $e(q, s)$ is increasing in s and strictly log-supermodular. That is, skilled workers have larger effective endowments of labor and a comparative advantage in higher quality.

Given these assumptions, each q in the model is associated with an earnings per worker y in the data where y is such that the share of firms with qualities smaller than or equal to q in the model is equal to the share of firms with earnings per worker less than or equal to y in the data. To show that we can construct a set of endowments $e(q, s)$ that clear

[36] Correspondence $s^*(q)$ is a function in the interior or Q assuming that functions $w(q)$ and $h(q)$ are continuous and differentiable, and that $e(q, s)$ is continuous, differentiable and strictly log supermodular.

the labor market and that deliver the data's distribution of average earnings across firms, it suffices to show that for any quality-earnings pair $(q^*, y^*) \in Q \times \mathbb{R}_{++}$, we can find an endowment function $e(q, s^*)$ such that q^* is the choice and y^* is the maximum in problem (48) when the worker skill is s^*. We parameterize

$$e(q, s^*) = \exp(s_0^* + s_1^* \log(q) + \overline{s}_2[\log(q)]^2)$$

where $\overline{s}_2$ and $(s_0^*, s_1^*) \in \mathbb{R}^2$ are specific to skill s^*. Sufficient conditions for $e(q, s^*)$ are:

$$y^* = w(q^*) \exp(s_0^* + s_1^* \log(q^*) + \overline{s}_2[\log(q^*)]^2) \tag{49}$$

$$0 = \frac{d \log[w(q^*)]}{d \log(q)} + s_1^* + 2\overline{s}_2[\log(q^*)] \tag{50}$$

$$0 > \frac{d^2 \log[w(q)]}{d[\log(q)]^2} + 2\overline{s}_2 \qquad \text{for all } q \in Q. \tag{51}$$

Parameter $\overline{s}_2$ is not identified for the same rationale as the lack of identification of $\overline{\omega}_2$ in the firm's problem (see Appendix F). For any value sufficiently small (possibly large and negative) that satisfies (51) we can find s_1^* and s_0^* that satisfy (49) and (50). Equation (50) implies that the worker chooses q^* and (49) implies that her earnings is y^* as we wanted to prove.

D Special Case: One quality, $\beta_v = \beta_m$

We solve the special case of the model in Section 3.4.2. Assume there is only one quality and $\beta_v = \beta_m \equiv \beta$. We set $\phi_v = \phi_y = 1$ without loss of generality and drop the quality arguments from functions. We take wages to be the numeraire. Labor income is:

$$L = \frac{1}{\sigma}\left[(1 - \alpha_m - \alpha_s)(\sigma - 1) + \frac{1 + \alpha}{\beta}\right] X$$

where X is aggregate manufacturing absorption and L is total labor force. We normalize the size of the labor force so that $X = 1$.

With $\beta_v = \beta_m$, the ratio of ads to find suppliers and customers in (11) is the same for all firms. Then, the probabilities of success of ads to find suppliers and customers reduce to functions of exogenous variables:

$$\theta_m = \left(\frac{f_m}{\alpha_m f_v}\right)^{1/\beta}\left[1-\exp\left(-\kappa\left(\frac{\alpha_m f_v}{f_m}\right)^{1/\beta}\right)\right]$$
$$\theta_v = \left[1-\exp\left(-\kappa\left(\frac{\alpha_m f_v}{f_m}\right)^{1/\beta}\right)\right].$$

With only one quality, price indices c in (23) and P_s in (27) are

$$c = \left(\frac{\theta_m}{V}\right)^{1/(1-\sigma)} P$$
$$P_s = \left(\frac{\overline{m}}{V}\right)^{1/(1-\sigma)} P \tag{52}$$

Demand functions D_m in (25) and D_s in (28) become

$$D_m = P^{\sigma-1}\frac{\alpha_m(\sigma-1)}{\sigma}$$
$$D_s = P^{\sigma-1}\left[1-\frac{\alpha_m(\sigma-1)}{\sigma}\right]$$

So that $D = P^{\sigma-1}$, as in Melitz (2003). Combining this expression with (7) and (24),

$$P = \left(\frac{\Pi}{D}N\mathbb{E}(z^{\gamma(\sigma-1)})\right)^{1/(1-\sigma)}$$
$$\Rightarrow \quad \Pi = \left[N\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{-1} \tag{53}$$

This yields the expression for sales in the main text.

To get the price index, we write V as a function of price, and substitute it in the definitions of $C(1)$ and P. Using (11) and (20), we have

$$V = (\sigma f_v)^{-1/\beta} N^{(\beta-1)/\beta}\frac{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})}{\left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\beta}} \tag{54}$$

The fraction of expectations is less than one and it is an inverse measure of dispersion. If firm productivity is dispersed firms the total mass of ads V decreases because the mass of ads is a concave function of firm sales, i.e., $1/\beta$. We substitute V, P_s and c in (52) into

$C(1)$ in (8):

$$\begin{aligned}
C(1) &= P_s^{\alpha_s} c^{\alpha_m} \\
&= P^{\alpha_s+\alpha_m} (\overline{m}^{\alpha_s}\theta_m^{\alpha_m})^{1/(1-\sigma)} V^{(\alpha_s+\alpha_m)/(\sigma-1)} \\
&= P^{\alpha_s+\alpha_m} (\overline{m}^{\alpha_s}\theta_m^{\alpha_m})^{1/(1-\sigma)} (\sigma f_v)^{(\alpha_m+\alpha_s)/[\beta(1-\sigma)]} N^{(\beta-1)(\alpha_m+\alpha_s)/[\beta(\sigma-1)]} \\
&\quad \times \left(\frac{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})}{\left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\beta}} \right)^{(\alpha_m+\alpha_s)/(\sigma-1)}
\end{aligned}$$

Substituting $C(1)$ above, $D = P^{\sigma-1}$, Π from (53) into the original expression for Π in (12), we get

$$\begin{aligned}
\Pi =& (\sigma w)^{1-\gamma} \left[D \left(\frac{\sigma}{\sigma-1} C(1) \right)^{1-\sigma} \left(\frac{f_m}{\alpha_m} \right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v} \right]^{\gamma} \\
(N\mathbb{E}(z^{\gamma(\sigma-1)}))^{-1/\gamma} =& \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} P^{(\sigma-1)(1-\alpha_m-\alpha_s)} (\theta_m^{\alpha_m}\overline{m}^{\alpha_s}) N^{-\frac{(\beta-1)}{\beta}(\alpha_m+\alpha_s)} \\
& \times (\sigma f_v)^{(\alpha_m+\alpha_s-1)/\beta} \left(\frac{f_m}{\alpha_m} \right)^{-\alpha_m/\beta} \left(\frac{\left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\beta}}{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})} \right)^{\alpha_m+\alpha_s}
\end{aligned}$$

Rearranging,

$$\begin{aligned}
P =& \left(\frac{\sigma}{\sigma-1} \right)^{1/(1-\alpha_m-\alpha_s)} (\sigma f_v)^{1/[\beta(\sigma-1)]} N^{\frac{1}{1-\sigma} - \frac{1-\alpha_s}{\beta(1-\sigma)(1-\alpha_m-\alpha_s)}} \\
& \left\{ \left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\gamma} \left(\frac{\left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\beta}}{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})} \right)^{\alpha_m+\alpha_s} (\theta_m^{\alpha_m}\overline{m}^{\alpha_s}) \left(\frac{f_m}{\alpha_m} \right)^{-\alpha_m/\beta} \right\}^{1/[(1-\sigma)(1-\alpha_s-\alpha_m)]}
\end{aligned}$$

Real wages is

$$\begin{aligned}
P_s^{-1} =& \left(\frac{\overline{m}}{V} \right)^{1/(\sigma-1)} \frac{w}{P} \\
=& \left\{ \left(\frac{\sigma-1}{\sigma} \right) \left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/[\gamma(\sigma-1)]} \left[\frac{\left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\beta}}{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})} \left(\frac{N f_m}{\alpha_m} \right)^{-\alpha_m/\beta} \theta_m^{\alpha_m}\overline{m}^{1-\alpha_m} \right]^{1/(\sigma-1)} \right\}^{1/(1-\alpha_m-\alpha_s)}
\end{aligned}$$

The first two terms are standard: The markup $\sigma/(\sigma-1)$ decreases real wages and expected productivity $\mathbb{E}(z^{\gamma(\sigma-1)})$ increases real wages, where productivity is adjusted for the elasticity of sales with respect to productivity. The fraction in expectations, $[\mathbb{E}(z^{\gamma(\sigma-1)})]^{1/\beta}/\mathbb{E}(z^{\gamma(\sigma-1)/\beta}) > 1$, is a measure productivity dispersion. Dispersion increases real wages because the variety gain from having more suppliers and customers accrues disproportionately to more productive firms. With search frictions, variety gains depend on the number of sellers per buyer not on the total sellers in the market. An increase in N decreases sales per firm and decreases variety per buyer. Hence, it decreases welfare. This result arises because we assumed constant returns to scale in the matching function $\tilde{M}$. Variety gains increase in N with sufficiently increasing returns to scale in $\tilde{M}$. Estimating such returns to scale is beyond the scope of this paper. We refer the reader to Miyauchi (2020), who provides evidence and estimates increasing returns in matching.

D.1 Efficiency in the special case

We consider the problem of a planner investing in ads $m(z)$ and $v(z)$ to maximize consumer welfare. Since markups are constant, there is no distortion from the allocation of labor across production given network links.[37]

Input cost as a function of consumer prices is

$$c = \left(\frac{\tilde{M}}{MV}\right)^{1/(1-\sigma)} P = \left(\frac{\tilde{M}}{\overline{m}M}\right)^{1/(1-\sigma)} P_s$$

Without markups, consumer price is

$$\begin{aligned} P_s &= \left(\frac{\overline{m}}{V}\right)^{1/(1-\sigma)} \left[\int p(z)^{1-\sigma} v(z) dJ(z)\right]^{1/(1-\sigma)} \\ &= \left(\frac{\overline{m}}{V}\right)^{1/(1-\sigma)} P_s^{\alpha_s+\alpha_m} \left(\frac{\tilde{M}}{\overline{m}M}\right)^{\alpha_m/(1-\sigma)} \left[\int z^{\sigma-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{1/(1-\sigma)} \\ P_s^{1-\alpha_s-\alpha_m} &= \overline{m}^{(1-\alpha_m)/(1-\sigma)} V^{1/(\sigma-1)} \left(\frac{\tilde{M}}{M}\right)^{\alpha_m/(1-\sigma)} \left[\int z^{\sigma-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{1/(1-\sigma)} \end{aligned}$$

The planner chooses ads for all firms $m(z)$ and $v(z)$ to minimize the price index subject

[37] The service sector has no labor. So, although it does not have markups, the planner cannot reallocate labor between manufacturing and services.

to the cost of labor used to produce ads.

$$\min_{m(z),v(z)} \left\{ \overline{m}^{(1-\alpha_m)/(1-\sigma)} V^{1/(\sigma-1)} \left(\frac{\tilde{M}}{M}\right)^{\alpha_m/(1-\sigma)} \left[\int z^{-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{1/(1-\sigma)} \right\}^{1/(1-\alpha_s-\alpha_m)}$$
$$+ \lambda \int \left[f_m \frac{m(z)^\beta}{\beta} + f_v \frac{v(z)^\beta}{\beta} \right] dJ(z)$$

subject to

$$V = \int v(z) dJ(z)$$
$$M = \int m(z) dJ(z)$$
$$\tilde{M} = V(q) \left[1 - \exp(-\kappa M(q)/V(q))\right].$$

where λ is the marginal cost of labor. The first order conditions with respect to $m(z)$ is

$$\frac{\alpha_m}{(1-\sigma)(1-\alpha_m-\alpha_s)} P_s^{\alpha_m+\alpha_s} \left[\int z^{\sigma-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{\sigma/(1-\sigma)} m(z)^{\alpha_m-1} z^{\sigma-1} v(z) + \lambda f_m m(z)^{\beta-1}$$
$$+ \frac{\alpha_m}{(1-\sigma)(1-\alpha_m-\alpha_s)} P_s^{\alpha_m+\alpha_s} \frac{1}{M} \left(\frac{M}{\tilde{M}} \frac{d\tilde{M}}{dM} - 1 \right) = 0 \tag{55}$$

The first order conditions with respect to $v(z)$ is

$$\frac{1}{(1-\sigma)(1-\alpha_m-\alpha_s)} P_s^{\alpha_m+\alpha_s} \left[\int z^{\sigma-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{\sigma/(1-\sigma)} m(z)^{\alpha_m} z^{\sigma-1} + \lambda f_v v(z)^{\beta-1}$$
$$+ \frac{1}{(1-\sigma)(1-\alpha_m-\alpha_s)} P_s^{\alpha_m+\alpha_s} \frac{1}{V} \left(\alpha_m \frac{V}{\tilde{M}} \frac{d\tilde{M}}{dV} - 1 \right) = 0 \tag{56}$$

The first lines of (55) and of (56) are equal at the market solution, from the first order conditions of the firm. Since these are the only terms with firm-specific productivity z, there is no missallocation of ads across firms.

There are four externalities. The first two are the elasticity of $\tilde{M}$ with respect to M in (55) and with respect to V in (56). They both imply a positive externality of ads on the mass of matches, which increase welfare. But ads also create competition. More ads decrease the probability of success of competing ads. This negative externality is the negative one terms subtracting the elasticities. One can easily show that the two elasticities $\frac{M}{\tilde{M}} \frac{d\tilde{M}}{dM}$ and $\frac{V}{\tilde{M}} \frac{d\tilde{M}}{dV}$ are in (0,1). So, the negative externality is always larger than the positive, which push the planner to posting fewer ads than the market.

E Open Economy Model

We present the parts of the model that were missing from Section 4. A manufacturing firm with productivity z, quality q and export status E has the following sales x, a measure of ads v to find customers (domestic and abroad) and m to find suppliers, and price:

$$\begin{aligned}
x(z,q,E) &= \Pi(q,E) z^{\gamma(\sigma-1)} \\
v(z,q,E) &= \left(\frac{x(z,q,E)}{\sigma f_v w(q)}\right)^{1/\beta_v} \\
m(z,q,E) &= \left(\frac{x(z,q,E)}{\sigma f_m w(q)/\alpha_m}\right)^{1/\beta_m} \\
p(z,q,E) &= \frac{\sigma}{\sigma-1}\frac{C(m(z,q,E),q)}{z}
\end{aligned} \tag{57}$$

where

$$\begin{aligned}
\Pi(q,E) &= [\sigma w(q)]^{1-\gamma}\left[D(q,E)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \\
D(q,E) &= \left[D_H(q)^{\beta_v/(\beta_v-1)} + E(e^{\sigma} D_F(q))^{\beta_v/(\beta_v-1)}\right]^{(\beta_v-1)/\beta_v}.
\end{aligned} \tag{58}$$

With the fixed exporting cost, profit is no longer a constant share of revenue. The expected profit of a firm that draws a productivity parameter ω upon entry is (equation (41)):

$$\begin{aligned}
\pi(\omega) = \max_{q\in Q}\Bigg\{ &\frac{z(q,\omega)^{\gamma(\sigma-1)}}{\gamma\sigma}\left[\Pi(q,1)\Phi\left(\overline{f}_E(z(q,\omega),q)\right) + \Pi(q,0)\left[1-\Phi\left(\overline{f}_E(z(q,\omega),q)\right)\right]\right] \\
&- P_s\mathbb{E}(f_E | f_E \le \overline{f}_E(z(q,\omega),q))\Bigg\}
\end{aligned}$$

Free entry implies

$$P_s f = \mathbb{E}_\omega(\pi(\omega)) \tag{59}$$

The firm choices give rise to the measure functions:

$$\begin{aligned}
\tilde{J}(z,q) &= N\text{Prob}\left\{\omega : z(q(\omega),\omega) \le z \quad \text{and} \quad q(\omega) \le q\right\} \\
J(z,q,1) &= \tilde{J}(z,q)\Phi\left(\overline{f}_E(z,q)\right) \\
J(z,q,0) &= \tilde{J}(z,q)\left[1-\Phi\left(\overline{f}_E(z,q)\right)\right]
\end{aligned} \tag{60}$$

$J(z,q,E)$ is the measure of functions with export status $E \in \{0,1\}$ and productivity-quality pairs less than or equal to (z,q). Denote the density of J as $j(z,q,E)$ for $E=0,1$.

The production function (15) and network formation are the same as in the closed economy, only expressions for some aggregate variables change. The mass of ads posted by firms of quality q to find suppliers and sellers is respectively:

$$M(q) = \sum_{E=0,1} \int_Z m(z,q,E)j(z,q,E)dz \tag{61}$$

$$\overline{V}(q) = \sum_{E=0,1} r_v(q,E) \int_Z v(z,q,E)j(z,q,E)dz \tag{62}$$

The mass of ads directed at buyers of quality q, $V(q)$, and the mass of matches $\tilde{M}(q)$ are in (20) and (21). The success rate of ads is $\theta_v(q) = \tilde{M}(q)/V(q)$ for sellers and $\theta_m(q) = \tilde{M}(q)/M(q)$ for buyers, as before.

Cost function $c(q)$ and demand function and $D_m(q)$ are in equations (23) and (25) respectively, where now the price index $P(q)$ and total sales $X(q)$ are:

$$P(q) = \left[\sum_{E=0,1} r_v(q,E) \int_Z p(z,q,E)^{1-\sigma} v(z,q,E)j(z,q,E)dz\right]^{1/(1-\sigma)} \tag{63}$$

$$X(q) = \sum_{E=0,1} \int_Z x(z,q,E)j(z,q,E)dz. \tag{64}$$

The cost of domestic services is defined as before:

$$P_{Hs} = \left[\frac{\overline{m}}{V_T} \int_Q \phi_y(0,q)P(q)^{1-\sigma}dq\right]^{1/(1-\sigma)}$$

where

$$V_T = \int_Q \overline{V}(q)dq.$$

The bundle of services is a combination of domestic and foreign services. It costs:

$$P_s = \left[P_{Hs}^{1-\sigma_s} + (eP_{Fs})^{1-\sigma_s}\right]^{1/(1-\sigma_s)}. \tag{65}$$

We experiment with different assumptions on the response of the trade balance and exchange rate adjustment in our counterfactual. So, we close the equilibrium here in a generic way. Let B be the exogenous trade deficit, i.e., the difference between consumer spending and income. Then, total spending on services is

$$X_s = 1 - \frac{\alpha_m(\sigma - 1)}{\sigma} + B \tag{66}$$

where we have taken gross manufacturing output again as the numeraire. Similar to the closed economy, the revenue from sales to service firms of a domestic manufacturing firm posting v ads and price p is

$$p^{1-\sigma} v D_s(q)$$

where

$$\begin{aligned} D_s(q) &= \phi_y(0, q) \left[\int_Q \phi_y(0, q') P(q')^{1-\sigma} dq' \right]^{-1} X_{Hs} \\ X_{Hs} &= \left(\frac{P_{Hs}}{P_s} \right)^{1-\sigma_s} X_s \end{aligned} \tag{67}$$

X_{Hs} is spending on domestic services. Total demand shifter $D(q) = D_m(q) + D_s(q)$ as in (29).

Home's exports of manufacturing to Foreign is

$$X^* = \int_{q \in Q} (1 - r_v(q, 1)) e^\sigma D_F(q) \left[\int_z p(z, q, 1)^{1-\sigma} v(z, q, 1) j(z, q, 1) dz \right] dq.$$

Trade equilibrium implies that the difference between imports of services and exports of manufacturing equals the exogenous trade deficit B (consumer demand for savings):

$$B = \left(\frac{e P_{Fs}}{P_s} \right)^{1-\sigma_s} X_s - X^*. \tag{68}$$

So from (66), independently of the trade deficit, spending on domestic services is

$$X_s = 1 - \frac{\alpha_m(\sigma - 1)}{\sigma} - X^*.$$

This equation confirms that the market for manufacturing goods clears: Gross manufacturing absorption (normalized to one) equals spending on services, plus manufacturing inputs into manufacturing plus manufacturing exports.

Labor markets clear if

$$L(q,w)=\frac{1}{w(q)\sigma}\left[(1-\alpha_m-\alpha_s)(\sigma-1)+1-\frac{1}{\gamma}\right]\left[\sum_{E=0,1}\int_Z x(z,q,E)j(z,q,E)dz\right] \quad (69)$$

As in the main text, aggregate functions are functions of wages $w(q)$, real exchange rate e and firm outcomes. The success rate of ads $\theta_m(q)=\tilde{M}(q)/M(q)$ and $\theta_v(q)=\tilde{M}(q)/V(q)$ where $\tilde{M}(q)$ is in (21), $V(q)$ is in (20), and $M(q)$ and $\overline{V}(q)$ are in (61) and (62). Cost $c(q)$ satisfies (23) and $D(q)$ satisfies (29), where $P(q)$ and $X(q)$ are in (63) and (64). Firms again best respond to each others' actions through demand and cost aggregators $D(q)$ and $c(q)$.

An **equilibrium** is a set of wages w and exchange rate e and of firm outcomes Θ such that functions $D(q)$ and $C(1,q)$ exist and that satisfy the following conditions:

1. Labor market clears (69).

2. Firms maximize profits. Firm ω chooses $q(\omega)$ in (41) and has productivity $z(\omega)=z(q(\omega),\omega)$ at the optimal. The firm export status is $E=1$ if its fixed cost of exporting is less than $\overline{f}_E(q(\omega),z(q,\omega))$, and $E=0$ otherwise. Its sales, measure of ads, and prices are $x(z(\omega),q(\omega),E)$, $m(z(\omega),q(\omega),E)$, $v(z(\omega),q(\omega),E)$, and $p(z(\omega),q(\omega),E)$ in (57). The direction of selling ads $\mu(q(\omega))$ solves (25).

3. Trade is in equilibrium (68).

F Identification of $\overline{\omega}_2$

The key parameter $\overline{\omega}_2$ governs the efficiency-quality trade-off in firm's quality choice. We discuss the identification of $\overline{\omega}_2$ below.

Recall that we parameterize firm productivity in equation (13) as

$$\log z(q,\omega)=\omega_0+\omega_1\log(q)+\overline{\omega}_2[\log(q)]^2$$

where ω_0 and ω_1 are firm-specific and $\overline{\omega}_2$ is common to all firms. Substituting $z(q,\omega)$ into the firm's quality choice in (14), we have

$$q(\omega)=\arg\max_{q\in Q}\left\{\gamma(\sigma-1)\left[\omega_0+\omega_1\log(q)+\overline{\omega}_2[\log(q)]^2\right]+\log\Pi(q)\right\}$$

Consider any productivity-quality pair (z^*,q^*) with q^* in the interior of Q. The firm ω^*

that corresponds to such pair satisfies $z(q^*, \omega^*) = z^*$ and the first order condition:

$$\exp\left[\omega_0^* + \omega_1^* \log(q^*) + \overline{\omega}_2[\log(q^*)]^2\right] = z^* \tag{70}$$

$$\gamma(\sigma - 1)\left[\omega_1^* + 2\overline{\omega}_2 \log(q^*)\right] + \frac{\partial \log \Pi(q^*)}{\partial \log(q^*)} = 0 \tag{71}$$

The second order sufficient conditions are

$$2\gamma(\sigma - 1)\overline{\omega}_2 + \frac{\partial^2 \log \Pi(q)}{\partial (\log(q))^2} \leq 0 \qquad \text{for all } q. \tag{72}$$

For any $\overline{\omega}_2$ satisfying (72) and any (z^*, q^*), we can find (ω_0^*, ω_1^*) that satisfies (70) and (71). So, firm ω^* produces output of quality q^* with efficiency z^* in equilibrium.

Two points are in order. First, parameter ω_1 governs the firm's quality choice in (71), and ω_0 governs its productivity at the chosen quality in (70). So, these two dimensions of firm heterogeneity allows us to non-parametrically fully match the joint distribution of wages (quality rank) and sales in the data.

Second, parameter $\overline{\omega}_2$ is not identified with the cross-sectional distribution of sales and wages. We identify it with the elasticity of firms choices of q with respect to idyosyncratic shocks to the economy. Denote the model fundamentals of the economy as Θ and consider a shock that affects an element Θ_i for a single firm ω. The first order condition (71) implicitly defines the firm's optimal choice $q(\omega)$ as a function of parameter Θ_i:

$$\frac{\partial \log q(\omega)}{\partial \Theta_i} = -\frac{\frac{\partial^2 \log \Pi(q(\omega))}{\partial \log q \partial \Theta_i}}{2\gamma(\sigma - 1)\overline{\omega}_2 + \frac{\partial^2 \log \Pi(q(\omega))}{\partial (\log(q))^2}} \tag{73}$$

where the denominator is the second order condition (72) evaluated at the optimal $q(\omega)$. The firm is infinitely elastic to the shock if the second order condition holds with equality and infinitely inelastic as it approaches negative infinity. In the open economy, we interpret the export shocks in Table 3 as such idiosyncratic shocks. Our regression coefficients of how exporter wage responded to the export shocks can be mapped into $\partial \log q(\omega)/\partial \Theta_i$. We can also use our model-based economy to compute the derivatives of $\Pi(q)$. We can then apply (73) to estimate $\overline{\omega}_2$. A key assumption is that the shock does not affect other firms. Otherwise, it would affect Π not only directly in the firm's problem, but through other firm's choices in measure J.

G Computation Algorithm

G.1 Outer Loop Iteration: $\Pi(q,0)$, $\Pi(q,1)$, $q(\omega)$

1. New guesses of $\Pi(q,0)^{(n)}$, $\Pi(q,1)^{(n)}$ for each $q \in Q$, and $q(\omega)^{(n)}$ for each firm type ω

2. Calculate export probability for each type ω as $\Phi(\overline{F}_E(q,\omega))$ where $\Phi(\cdot)$ is the normal CDF, and $\overline{F}_E(q,\omega)$ is the normalized fixed cost cutoff of exporting:

$$\overline{F}_E(q,\omega) \equiv \frac{\ln Z(q,\omega) + \ln[\Pi(q,1) - \Pi(q,0)] - \mu_E}{\sigma_E}$$

   where $Z(q,\omega) \equiv \frac{[z(q,\omega)]^{(\sigma-1)\gamma}}{\sigma\gamma}$.

3. Given the mass of type ω firm $n(\omega)$, we calculate $J(z,q,1) = \int_{\omega:q(\omega)=q,z(q,\omega)=z} \Phi(\overline{F}_E(q,\omega))n(\omega)d\omega$ and $J(z,q,0) = \int_{\omega:q(\omega)=q,z(q,\omega)=z}(1-\Phi(\overline{F}_E(q,\omega)))n(\omega)d\omega$

4. Define and evaluate three useful integrals for the inner loop:

$$E_{zm}(q,E) \equiv \int_z z^t j(z,q,E)dz \text{ where } t = \frac{(\sigma-1)\gamma}{\beta_m}$$
$$E_{zv}(q,E) \equiv \int_z z^t j(z,q,E)dz \text{ where } t = \frac{(\sigma-1)\gamma}{\beta_v}$$
$$E_{zx}(q,E) \equiv \int_z z^t j(z,q,E)dz \text{ where } t = (\sigma-1)\gamma$$

5. Solve the **inner loop** and update $\Pi(q,0)^{(n+1)}$, $\Pi(q,1)^{(n+1)}$

6. Grid search to update quality choice $q(\omega)^{(n+1)}$ that maximizes expected profit:

$$q(\omega)^{(n+1)} = \arg\max_{q\in Q} \ln \mathbb{E}[\pi(q,\omega)] = \arg\max_{q\in Q} \ln Z(q,\omega) + \ln E\Pi(q,\omega)$$
$$\text{where } E\Pi(q,\omega) \equiv \Pi(q,0) + Z(q,\omega)[\Pi(q,1) - \Pi(q,0)]\Phi\left(\overline{F}_E(q,\omega)\right)$$
$$- \frac{\exp\left(\mu_E + \frac{1}{2}\sigma_E^2\right)}{Z(q,\omega)}\Phi\left(\overline{F}_E(q,\omega) - \sigma_E\right)$$

7. Iterate until outer loop converges

G.2 Inner Loop Iteration: $D_H(q)$, $c(q)$

1. New guesses of $D_H(q)^{(n)}$, $c(q)^{(n)}$ for each $q \in Q$

2. Calculate the demand shifter for non-exporters and exporters:

$$D(q,0) = D_H(q)$$
$$D(q,1) = \left[D_H(q)^{\frac{\beta_v}{\beta_v-1}} + [e^\sigma D_F(q)]^{\frac{\beta_v}{\beta_v-1}}\right]^{\frac{\beta_v-1}{\beta_v}}$$
the share of ads to domestic market: $r_v^{ads}(q,1) = \dfrac{[D_H(q)]^{\frac{1}{\beta_v-1}}}{[D_H(q)]^{\frac{1}{\beta_v-1}} + [e^\sigma D_F(q)]^{\frac{1}{\beta_v-1}}}$

3. Calculate the profit function for non-exporters and exporters:

$$\Pi(q,E) = D(q,E)^\gamma [c(q)^{\alpha_m} P_s^{\alpha_s}]^{(1-\sigma)\gamma} C_x(q,0) \quad E = 0,1$$
where $\gamma = \dfrac{\beta_v \beta_m}{\beta_v(\beta_m - \alpha_m) - \beta_m}$
and $C_x(q,0) = \left[\dfrac{\sigma w(q)^{1-\alpha_m-\alpha_s}}{\sigma - 1}\right]^{(1-\sigma)\gamma} \left[\dfrac{\alpha_m}{\sigma f_m w(q)}\right]^{\frac{\alpha_m}{\beta_m}\cdot\gamma} \left[\dfrac{1}{\sigma f_{vH} w(q)}\right]^{\frac{1}{\beta_v}\cdot\gamma}$

4. Calculate the mass of buyer and seller ads in each quality segment:

$$\begin{aligned} M(q) &= \sum_{E\in\{0,1\}} \int_Z m(z,q,E) j(z,q,E) dz \\ &= C_m(q) \sum_{E\in\{0,1\}} \int_Z x(z,q,E)^{\frac{1}{\beta_m}} j(z,q,E) dz \\ &= C_m(q) \sum_{E\in\{0,1\}} \Pi(q,E)^{\frac{1}{\beta_m}} \int_Z z^{\frac{(\sigma-1)\gamma}{\beta_m}} j(z,q,E) dz \\ &= C_m(q) \left[\Pi(q,0)^{\frac{1}{\beta_m}} E_{zm}(q,0) + \Pi(q,1)^{\frac{1}{\beta_m}} E_{zm}(q,1)\right] \end{aligned}$$
where $C_m(q) = \left[\dfrac{\alpha_m}{\sigma f_m w(q)}\right]^{\frac{1}{\beta_m}}$

$$
\begin{aligned}
V(q) &= \int_Q \phi_v(q,q') \sum_{E=0,1} r_v^{ads}(q',E) \int_Z v(z,q',E) j(z,q',E) dz dq' \\
&= \int_Q \phi_v(q,q') \sum_{E=0,1} r_v^{ads}(q',E) \int_Z \left[x(z,q',E)^{\frac{1}{\beta_v}} C_v(q',E) \right] j(z,q',E) dz dq' \\
&= \int_Q \phi_v(q,q') \sum_{E=0,1} r_v^{ads}(q',E) C_v(q',E) \Pi(q',E)^{\frac{1}{\beta_v}} \int_Z z^{\frac{(\sigma-1)\gamma}{\beta_v}} j(z,q',E) dz dq' \\
&= \int_Q \phi_v(q,q') \left[C_v(q',0)\Pi(q',0)^{\frac{1}{\beta_v}} E_{zv}(q',0) + r_v^{ads}(q',1) C_v(q',1)\Pi(q',1)^{\frac{1}{\beta_v}} E_{zv}(q',1) \right] dq'
\end{aligned}
$$

where $C_v(q,0) = [\sigma f_v w(q)]^{-\frac{1}{\beta_v}}$, $\quad C_v(q,1) = C_v(q,0) \left[r_v(q,1)^{\beta_v} + (1 - r_v(q,1)^{\beta_v} \right]^{-\frac{1}{\beta_v}}$

5. Calculate the tightness and match rates of seller and buyer ads in each quality segment:

$$
\begin{aligned}
\xi(q) &= \frac{M(q)}{V(q)} \\
\theta_v(q) &= 1 - e^{-\kappa \cdot \xi(q)} \\
\theta_m(q) &= \frac{1 - e^{-\kappa \cdot \xi(q)}}{\xi(q)}
\end{aligned}
$$

6. Calculate the total sales for exporters and non-exporters:

$$
\begin{aligned}
X(q,E) &\equiv \int_Z x(z,q,E) j(z,q,E) dz \\
&= \Pi(q,E) E_{zx}(q,E)
\end{aligned}
$$

7. Calculate the price index:

$$
P(q) = \left[\frac{X(q,0)}{D_H(q)} + \frac{D_H(q)^{\frac{1}{\beta_v - 1}} X(q,1)}{D_H(q)^{\frac{\beta_v}{\beta_v - 1}} + [e^{\sigma} D_F(q)]^{\frac{\beta_v}{\beta_v - 1}}} \right]^{\frac{1}{1-\sigma}}
$$

8. Calculate the demand from manufacturing firms:

$$
D_m(q) = \int_Q \frac{\theta_v(q')}{M(q')} \phi_y(q',q) \phi_v(q',q) c(q')^{\sigma-1} X_m(q') dq'
$$

where $X_m(q) \equiv \dfrac{\alpha_m(\sigma-1)}{\sigma} [X(q,0) + X(q,1)]$

9. Calculate the spending on services:

$$X_s = \left[1 - \frac{(\sigma-1)\alpha_m}{\sigma}\right] X - B_1$$

$$\text{where } B_1 = \int_Q \left[1 - r_x(q,1)\right] X(q,1) dq$$

$$\text{and home sales share } \; r_x(q,1) = \frac{D_H(q)^{\frac{\beta_v}{\beta_v-1}}}{D_H(q)^{\frac{\beta_v}{\beta_v-1}} + \left[e^{\sigma} D_F(q)\right]^{\frac{\beta_v}{\beta_v-1}}}$$

10. Calculate the total demand from home:

$$D_H(q)^{new} \equiv D_m(q) + D_s(q)$$

$$\text{where } D_s(q) = \frac{\phi_s(q) X_s}{\int_Q \phi_s(q') P(q')^{1-\sigma} dq'}$$

11. Calculate the input price index:

$$c(q)^{new} \equiv \left[\frac{\theta_m(q)}{V(q)} \int_Q \phi_y(q,q')\phi_v(q,q') \left[P(q')^{1-\sigma}\right] dq'\right]^{\frac{1}{1-\sigma}}$$

12. Update and iterate until inner loop converges:

$$D_H(q)^{(n+1)} = D_H(q)^{(n)} + 0.2\left[D_H(q)^{new} - D_H(q)^{(n)}\right]$$

$$c(q)^{(n+1)} = c(q)^{(n)} + 0.2\left[c(q)^{new} - c(q)^{(n)}\right]$$

H Model with No Complementarity

We report the parameter estimates and the fit of moments for a special case of the model, where we shut down the two sources of complementarity in matching ($\nu_v \to \infty$) and in production ($\nu_y = 0$). We match the exact same set of moments conditional on average firm wage quintiles, except for the wage sorting patterns which the special case cannot match by assumption. In particular, since all firms distribute their spending equally across supliers' qualities in the special case, the predicted sorting moments are all zero.

Table A10 reports the parameter estimates, and Table A11 reports the data and model moments. Except for the excluded sorting moments, the fit of this special case is very similar to the general model. Due to the lack of sorting, we need slightly larger standard deviation of the quality capability σ_{ω_1} to account for the overall concentration of network

Table A10: Parameter Estimates for Special Case with No Complementarity

	Parameter	Estimate	Standard error
Matching friction	κ	0.00095	(0.00176)
Directed search	$\nu_v \to \infty$	-	-
Complementarity	$\nu_y = 0$	-	-
Sd of quality capability	σ_{ω_1}	0.134	(0.002)
Sd of efficiency capability	σ_{ω_0}	0.128	(0.000)
Correlation	ρ	0.136	(0.006)
Efficiency cost of quality	$\overline{\omega}_2$	-0.105	(0.003)
Mean of log export cost	μ_E	-4.05	(0.03)
Sd of log export cost	σ_E	1.67	(0.05)
Foreign demand shifter	b_1	70.26	(62.87)
Foreign demand curvature	b_2	0.41	(0.01)

sales. Since the firm capability is more dispersed, the model requires a flatter export demand schedule b_2 to explain rising export intensity across firms of different wages.

Table A11: Model Fit – Targeted Moments (Special Case with No Complementarity)

	Quintiles of average wage per worker				
	1	2	3	4	5 (largest)
Mean number of suppliers					
Data	5.8	6.7	5.8	11.4	25.8
Model	6.7	5.2	6.1	8.5	28.4
Mean number of customers					
Data	5.6	7.0	6.7	11.7	25.1
Model	8.4	7.0	7.8	9.9	22.2
Standard deviation of log sales					
Data	1.37	1.34	1.37	1.52	1.79
Model	1.45	1.36	1.38	1.41	1.75
Share of total network sales					
Data	0.03	0.04	0.04	0.10	0.78
Model	0.07	0.04	0.05	0.09	0.76
Fraction of exporters					
Data	0.08	0.18	0.16	0.34	0.57
Model	0.17	0.16	0.19	0.28	0.56
Export intensity of exporters					
Data	0.24	0.21	0.23	0.23	0.26
Model	0.17	0.21	0.23	0.24	0.29
Shift-share IV coefficient (5% export shock)					
Data	0.21%				
Model	0.21%				

I Model with Endogenous Targeting

I.1 Theory with Endogenous Targeting

We modify the model to allow firms to endogenously choose the direction of their search. In the main text, the ads posted to find customers are distributed according to a normal density $\phi_v(q', q)$ with mean equal to the firm's own quality level q. Here, the firm chooses the mean. We also add an iceberg-type cost for firms to post ads far from their own quality. For each v, the mass of ads directed at quality q' posted by a firm of quality q centered around τ is

$$\phi_v(q, \tau, q') = \tilde{\phi}_v(q, \tau) \exp[-\nu_c(\tau - q')^2]$$

where $\tilde{\phi}_v(q, \tau)$ is the density of a normal distribution with mean τ and variance parameter ν_v as before, and $\exp[-\nu_c(\tau - q')^2]$ is an added iceberg cost that the firm incurs if it posts ads far from its own quality, where ν_c is a parameter.

Using the same derivation of (25), the sales to other manufacturing firms of a firm with price p, quality q, posting v ads to find customers centered around τ is:

$$p^{1-\sigma} v \tilde{D}_m(q,\tau)$$
$$\text{where} \quad \tilde{D}_m(q,\tau) = \alpha_m \frac{\sigma-1}{\sigma} \int_Q \frac{\theta_v(q')}{M(q')} \phi_y(q',q)\phi_v(q',\tau)c(q')^{\sigma-1}X(q')dq'$$

All firms with the same quality choose the same mean so that the demand shifter is

$$D_m(q) = \max_\tau \{\tilde{D}_m(q,\tau)\}$$

I.2 Estimation and Counterfactual with Endogenous Targeting

We find it hard to separately identify the variance parameter ν_v and the iceberg cost parameter ν_c. The model simulations are unstable if $\nu_c \approx 0$ because all firms want to target their ads to more productive firms, and more productive firms locate where the ads are concentrated. But for a wide range of positive cost parameter ν_c there is a corresponding variance parameter ν_v that allows the model to match the data moments almost equally well. The intuition is that while an increase in iceberg cost makes it more costly to target qualities further away, it can partly be offset by an increase in the variance parameter of directed search. To see this, we report moments two calibrated models in Table A12, one with $\nu_c = 1$, $\nu_v = 3.04$ (Endogenous target 1), and the other with $\nu_c = 0.20$, $\nu_v = 2.87$ (Endogenous target 2), while the remaining parameters are fixed at the baseline estimated value. The cost parameter can be interpreted as the following: when targeting one standard deviation away from its own quality, a seller loses 68% of its ads if $\nu_c = 1$, and it loses only 20% of its ads if $\nu_c = 0.20$. Clearly in Table A12, despite the difference in ν_c and ν_v, the two endogenous targeting models generate very similar moments. These model moments are also close to the moments implied by the exogenous targeting model and the data in Table A12. Given the lack of identification, we restrict our baseline model and estimation to the simpler exogenous-targeting case.

We further investigate the robustness of our baseline counterfactual results. After a 5% increase in export demand, the average wage of all firms increases by 1.26% and 1.39% in the endogenous targeting models, which is very close to the 1.22% increase in the exogenous targeting case. The average changes for exporters and non-exporters in Figure A4 also confirm our counterfactual results are robust to endogenous targeting.

Table A12: Model Fit

	Quintiles of average wage per worker				
	1	2	3	4	5 (largest)
Mean number of suppliers					
Data	5.8	6.7	5.8	11.4	25.8
Exogenous Target ($\nu_c \to \infty$)	4.7	4.7	6.0	9.1	29.4
Endogenous Target 1 ($\nu_c = 1.00$)	4.7	4.7	5.9	9.1	29.4
Endogenous Target 2 ($\nu_c = 0.20$)	4.8	4.7	6.0	9.2	29.6
Mean number of customers					
Data	5.6	7.0	6.7	11.7	25.1
Exogenous Target ($\nu_c \to \infty$)	5.4	5.9	7.6	10.9	23.8
Endogenous Target 1 ($\nu_c = 1.00$)	5.4	6.0	7.6	10.9	23.8
Endogenous Target 2 ($\nu_c = 0.20$)	5.6	6.1	7.7	10.9	23.7
Standard deviation of log sales					
Data	1.37	1.34	1.37	1.52	1.79
Exogenous Target ($\nu_c \to \infty$)	1.20	1.18	1.20	1.24	1.55
Endogenous Target 1 ($\nu_c = 1.00$)	1.20	1.18	1.20	1.24	1.55
Endogenous Target 2 ($\nu_c = 0.20$)	1.20	1.18	1.20	1.24	1.55
Share of total network sales					
Data	0.03	0.04	0.04	0.10	0.78
Exogenous Target ($\nu_c \to \infty$)	0.04	0.03	0.05	0.11	0.78
Endogenous Target 1 ($\nu_c = 1.00$)	0.04	0.03	0.05	0.11	0.78
Endogenous Target 2 ($\nu_c = 0.20$)	0.04	0.03	0.05	0.11	0.77
Fraction of exporters					
Data	0.08	0.18	0.16	0.34	0.57
Exogenous Target ($\nu_c \to \infty$)	0.11	0.13	0.18	0.29	0.60
Endogenous Target 1 ($\nu_c = 1.00$)	0.11	0.13	0.18	0.29	0.60
Endogenous Target 2 ($\nu_c = 0.20$)	0.11	0.13	0.18	0.29	0.60
Export intensity of exporters					
Data	0.24	0.21	0.23	0.23	0.26
Exogenous Target ($\nu_c \to \infty$)	0.18	0.21	0.22	0.23	0.25
Endogenous Target 1 ($\nu_c = 1.00$)	0.18	0.21	0.22	0.23	0.25
Endogenous Target 2 ($\nu_c = 0.20$)	0.18	0.21	0.22	0.23	0.25
Unweighted average log wage of suppliers					
Data	-	0.01	0.01	0.04	0.14
Exogenous Target ($\nu_c \to \infty$)	-	0.02	0.04	0.07	0.12
Endogenous Target 1 ($\nu_c = 1.00$)	-	0.02	0.04	0.07	0.12
Endogenous Target 2 ($\nu_c = 0.20$)	-	0.02	0.04	0.07	0.12
Weighted average log wage of suppliers					
Data	-	0.02	0.02	0.07	0.23
Exogenous Target ($\nu_c \to \infty$)	-	0.04	0.07	0.11	0.17
Endogenous Target 1 ($\nu_c = 1.00$)	-	0.04	0.07	0.11	0.17
Endogenous Target 2 ($\nu_c = 0.20$)	-	0.04	0.07	0.11	0.18
Shift-share IV coefficient (5% export shock)					
Data	0.21%				
Exogenous Target ($\nu_c \to \infty$)	0.21%				
Endogenous Target 1 ($\nu_c = 1.00$)	0.22%				
Endogenous Target 2 ($\nu_c = 0.20$)	0.21%				

Notes: This table compares the targeted data moments with the simulated moments from our baseline model with exogenous targeting and two models with endogenous targeting. Firms are ranked according to their average wage per worker. We target the following moments by quintile of firm wage: the mean number of suppliers (5 moments), the mean number of customers (5 moments), the share in total network sales (5 moments), the standard deviation of sales (5 moments), the fraction of exporters (5 moments), the export intensity of exporters (5 moments), the unweighted average log wage of suppliers (4 moments), and the average log wage of suppliers weighted by spending share (4 moments), where the latter two are normalized with respect to the first quintile. Besides, we also target the shift-share IV coefficient (1 moment).

Figure A4: Baseline Counterfactual Wage Response