Econ 703

August Prelim

Instructions:

- This is a *closed* book examination.
- The total points are 120.
- You have 2.5 hours to answer questions.
- Please give a complete and intelligible solution.
- All the best.

1. (40 points) Consider the following method of dissolving a partnership. Two partners each own one share of a firm. They have a dispute and each partner wishes to either buy-out, or be bought-out by the other partner. Partner i values both his own and her partner's share at θ_i per share. These values θ_1 and θ_2 are independently and uniformly distributed on $[0,1]$. Each partner i knows her own θ_i but not θ_{-i}.

(1) (10 points) Suppose that each partner i chooses a bid $b_i \geq 0$ for the other's share. The highest bidder wins and obtains her partner's share for the amount of the opponent's (losing) bid. So if i is the winner, i's payoff is $2\theta_i - b_j$ and the loser's payoff is b_j, where $j \neq i$. In the event of a tie, each partner wins with equal probability.

Compute the interim expected payoff of bidder i with type θ_i and bid b_i, when the opponent uses a linear symmetric strategy $b_j(\theta_j) = \alpha + \beta\theta_j$ with $\beta > 0$.

(2) (10 points) Find a linear symmetric equilibrium.

(3) (10 points) Suppose now that the winner pays the amount of her winning bid. Find a linear symmetric equilibrium. (Hint: In this case, FOC is NOT enough to find an equilibrium.)

(4) (10 points) Compare the equilibrium payoffs in parts (2) and (3) for each type θ_i. Discuss.

2. (40 points) There are two players, and infinitely many periods $t = 1, 2, \cdots$. Player 1 moves in odd periods, and chooses "Stop" or "Go." Similarly, player 2 moves in even periods, and chooses "Stop" or "Go." Once someone chooses "Stop," the game ends. If the game ends in some odd period t, player 1's payoff is $\delta^{t-1}a$ and player 2's payoff is $\delta^{t-1}b$. If the game ends in some even period t, player 1's payoff is $\delta^{t-1}b$ and player 2's payoff is $\delta^{t-1}a$. If no one chooses "Stop," then the payoff is $(0,0)$. Assume that $\delta \in (0,1)$.

Find all *pure-strategy* subgame-perfect equilibrium for the following regions of the parameter space. To have a full credit, prove that you indeed find *all* equilibria.

(1) (10 points) $a > 0$ and $a > \delta b$.

(2) (10 points) $a < 0$ and $a < \delta b$.

(3) (10 points) $a > 0$ and $a < \delta b$.

(4) (10 points) $a < 0$ and $a > \delta b$.

3. (40 points) Consider the following signaling game. Player 1 is an entrepreneur, who owns all of the stock in her company. She wants to start a new project, but to do so she must have an investment of $I = 1$ from player 2, a venture capitalist. The only way player 1 can do that is by selling player 2 an equity stake in the company.

The profitability π of the company is either 0 or 1, which is private information for player 1. Player 2 believes that the probability of $\pi = 1$ is $\frac{1}{2}$. Player 1 offers player 2 a stake $s \in [0, 1]$ of the company. Player 2 observes s but not π, and then either accepts or rejects the offer. If player 2 rejects, player 1's payoff is π while player 2's payoff is zero. If player 2 accepts, player 1's payoff is $(1 - s)(\pi + V)$ while player 2's payoff is $s(\pi + V) - I$, where $V = 2$ is the value of the new project.

(1) (5 points) Show that player 2 accepts any offer $s > \frac{1}{2}$, regardless of her posterior belief.

(2) (10 points) Is there any pure-strategy PBE in which both types of player 1 make the same offer $s = 0$ and player 2 rejects?

(3) (15 points) Is there any pure-strategy PBE in which both types of player 1 offer the same share $s > 0$ and player 2 accepts?

(4) (10 points) Is there a pure-strategy PBE in which two types of player 1 make different offers and at least one of them is accepted?