Economics 702, Dirk Krueger

1 A Stochastic Pure Exchange Economy and Asset Pricing

Consider a stochastic pure exchange economy where the current state of the economy is described by $s_t \in S = \{1, 2\}$. Event histories are denoted by s^t. Probabilities of event histories are given by $\pi_t(s^t)$. There are 2 different types of households with equal mass each normalized to 1. Households potentially differ in their endowment stream $\{e_t^i(s^t)\}$, their initial asset position a_0^i, their time discount factors $\beta_i \in (0, 1)$ and their utility functions U_i. Preferences for each household over consumption allocations $c^i = \{c_t^i(s^t)\}$ are given by

$$u^i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} (\beta_i)^t \, \pi_t(s^t) U_i(c_t^i(s^t)).$$

where each $U_i(.)$ is strictly increasing, strictly concave and twice differentiable.

1. Suppose that $\pi_t(s^t)$ is Markov with transition matrix

$$\pi(s'|s) = \begin{pmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{pmatrix}$$

   where $\rho \in [0, 1]$ is a parameter. For which parameter value is there an associated invariant distribution $\Pi = (0.3, 0.7)$?

2. Fix an initial state s_0. Households can trade a full set of Arrow securities. Define a sequential markets equilibrium.

3. Define a recursive competitive equilibrium.

4. Give the price of an Arrow security traded at s^t that pays one unit of consumption in state $\hat{s}_{t+1}$ and the price of a risk-free bond traded at s^t, as functions of equilibrium consumption allocations. You **cannot** make any assumptions beyond those stated in the question thus far (and in particular, cannot assume that the endowment process is Markov or that the utility function is of CRRA form).

5. For the rest of this question, now suppose that $s_0 = 1$, and that the aggregate endowment is given by

$$e_t(s^t) = 2(1+g)^t$$

for all t, s^t. Individual endowments are such that $e_t^i(s^t) = e_t(s^t)$ whenever $s_t = i$. That is, when the current state is $s_t = i$, household i owns the entire endowment, and the other household has no endowment in that state. Furthermore, assume that both households have the same time discount factor β, identical utility function $U(c) = \log(c)$, and that the stochastic process for s_t is Markov with transition matrix as in question 1, and with $\rho = 1$. Finally, assume that the initial asset positions are given by $a_0^2 = W$ and $a_0^1 = -W$, where $W \in [0, \frac{1}{1-\beta}]$ is a parameter. As function of the parameters of the model g, β, W, characterize the sequential markets equilibrium consumption allocation and Arrow securities prices.

6. For what values of W is the consumption allocation associated with a sequential markets equilibrium Pareto-efficient? You do not have to prove anything, but you do have to justify your answer.

7. How would you go about characterizing the whole set of Pareto efficient consumption allocations without solving the social planner problem? You again do not have to prove anything, but you do have to justify your answer.