Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 3, 2019

Instructions

You have 2.5 hours to answer all questions.

There are 4 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (20 pts) A price-taking firm has a differentiable, strictly increasing, and strictly concave production function $f(z_1, z_2)$ for producing a single output q. Rather than maximizing profit, this firm chooses (z_1, z_2) to maximize revenue. Furthermore, it is cash constrained: the amount it spends on inputs cannot exceed some constant C. Your econometrician friend tells you that at any prices $(p, w_1, w_2) \gg 0$ and constant $C > 0$, the firm's maximal revenue takes the form
$$R(p, w_1, w_2, C) = p\left[\gamma + \ln C - \alpha \ln w_1 - (1-\alpha)\ln w_2\right].$$
Find the firm's optimal input bundle in terms of the variables (p, w_1, w_2, C).

2. (30 pts) A consumer has a C^2 Bernoulli utility function $u : \mathbb{R} \to \mathbb{R}$ satisfying $u' > 0$. Let $A := -u''/u'$ denote her Arrow-Pratt measure of absolute risk aversion.

   (a) (5 pts) Show that if this consumer is risk neutral, then her preferences over gambles do not depend on her non-random wealth w.

   (b) (5 pts) Show the same is true if the consumer is not risk neutral but does exhibit CARA: $A(\cdot) \equiv \lambda$ for some $\lambda \neq 0$. (In this case we can assume $u(x) = -\lambda e^{-\lambda x}$.)

   (c) (10 pts) Show that if another consumer has a C^2 Bernoulli utility function $\hat{u} : \mathbb{R} \to \mathbb{R}$ satisfying $\hat{u}' > 0$, and the two consumers have identical preferences over gambles, then $\hat{A} = A$, where $\hat{A} := -\hat{u}''/\hat{u}'$.

   (d) (10 pts) Show that if the preferences over gambles represented by $\mathbb{E}u(\tilde{x} + w)$ do not depend on w, then u exhibits CARA. (Hint – the result of part (c) may be useful.)

3. (25 pts) Consider an exchange economy with three agents and three commodities. The initial endowments for the three agents are $\omega_1 = (1, 1, 0)$, $\omega_2 = (1, 0, 1)$, and $\omega_3 = (0, 1, 1)$. The agents have the same utility function, $U(x, y, z) = xyz$.

   (a) (6 pts) Show that any interior core allocation, $w = (w^1, w^2, w^3)$, of this economy must have the form $w^1 = (a, a, a)$, $w^2 = (b, b, b)$, and $w^3 = (c, c, c)$. That is, each agent's consumption bundle in a core allocation must have equal quantities of the three commodities.

   (b) (6 pts) Show that there is no core allocation in which some agent receives nothing. That is, $w^i = (0, 0, 0)$ is not part of a core allocation. (**Hint:** the answer to (a) is useful here.)

   (c) (6 pts) Part (b) only shows that no agent can get 0 of each commodity in a core allocation. Are there boundary core allocations in which an agent gets a positive quantity of some commodity and 0 of another? Explain.

   (d) (7 pts) What is the maximum quantity agent 3 can get of the commodities in any core allocation in which agents 1 and 2 get the same bundle, (a, a, a)?

4. (25 pts) Consider a standard two-period economy, dated $t = 0$ and $t = 1$. Agents consume in both periods. There are three states of nature in the second period. There is a single consumption good, and it is used as a numeraire; hence, the spot price of a unit of consumption at either date is 1. At date 0 agents can trade in two primary securities. Security 1 has the second-period payoff vector $r_1 = (1, 0, 0)$, and security 2 has the second-period payoff vector $r_2 = (1, 2, 3)$. The prices of these securities at date 0 are $q_1 = 0.5$ and $q_2 = 1.3$.

   In addition, there are two *derivative* securities, denoted 3 and 4. Security 3 is a call option on security 2 with a strike price of 1, and a price $q_3 = 0.5$. Security 4 is a call option on security 2 with a strike price of 2, and a price $q_4 = 0.2$.

   All quantities are defined in units of the consumption good.

   (a) (10 pts) Show that this system is arbitrage free.

   (b) Assume these prices arise in an incomplete markets equilibrium with the specified four securities.

      i. (5 pts) What is the date 0 price of a contingent claim to deliver one unit of consumption at date 1 in state 3?

      ii. (5 pts) What would be the market price of a put option on asset 4 with a strike price of 1?

      iii. (5 pts) What would be the risk-free interest rate on a loan taken at date $t = 0$?