Stuff related to Economics 704, Victor Rios Rull

In the following there are 11 questions for 100 points. Be as BRIEF as you can and good luck.

Brewers, Workers, and Entrepreneurs in a Growth Model

Consider an economy with a large measure of infinitely lived agents. They have one unit of time and depending on what they do their utility takes three levels indexed $i \in \{b, 0, e\}$. Their preferences are given by $\log c - \chi \, u^i$, with $\chi > 0$, and $u^b < 0 < u^0 < u^e$. They discount the future at rate $\beta \in (0, 1)$. We refer to the activity that is enjoyable $i = b$ as *brewers*, activity 0 as working, and the least enjoyable one as *entrepreneurs*.

Households have a stochastic shock to their ability in each activity s^i that follows a Markov chain with transition Γ^s. The ability to be a brewer is constant, that of being a worker oscillates between high and low $\{\ell, h\}$, while that of being an entrepreneur has the highest variance with many levels s^{e_j}.

Activity $i = b$ or brewing requires fixed amount of capital $\overline{k}^b$ and yields output f^b. It does not use outside labor.

Entrepreneurs output have a decreasing production function $s^{e_i} \, f(k, n)$ where n is the amount of efficiency units of labor that they hire.

All capital depreciates at rate δ.

Workers supply their unit of time in a labor market and they get paid their ability times the price of an efficient units of labor.

Assume first that the only asset is capital that can be rented out to entrepreneurs and that the economy is in steady state.

1. (5 points) What is the state of a household?

2. (20 points) Assume that the household chooses its savings conditional of what activity it is undertaking. Write down the problem of a household, explicitly distinguishing the choices of savings, activities and hiring if any.

3. (15 points) Define a steady state.

4. (5 points) Define a set of natural borrowing constraint for entrepreneurs if the shocks are iid. Do they depend on their wealth? What about on their entrepreneurial ability?

5. (10 points) Characterize as much as possible the steady state if all shocks are i.i.d. In particular, is the average worker richer or poorer than the average brewer? What about the average high ability worker?

   Imagine now that there is also a corporate sector that operates a constant returns to scale production $F(K, N)$

6. (10 points) How does the steady state differ from the previous one?

7. (5 points) In the absence of any borrowing possibility and with persistence of the shocks is it possible that two workers with the same working ability and wealth save different amounts? Explain why.

Lucas Trees with two groups of households, search, and work as sellers

Consider a Lucas-tree type economy with a measure one of trees. The amount of fruit in each tree, z, is random and Markovian with transition $\Gamma_{z,z'}$. There are two groups of households that discount the future at rate β, and have preferences each period given by $u(c) - \theta^i \ell$, θ^i is a group specific iid shock to preferences with common probability distribution γ, c is fruit. The household has one unit of time that can be allocated either to find trees to buy fruit and we denote it d, or to sell fruit in a frictionless market and we denote n (it takes 1 unit of time to sell B units of fruit) or to enjoy leisure ℓ. There is a matching function $M(T, D)$ that determines the number of matches, given the trees T and search effort D. Consequently, fruit can be bought either directly from the trees at price p^T (in terms of shares of the tree) or in the frictionless market at price p^M.

Assume that there are complete markets in what follows.

8. (5 points) Write the problem of the agent and define state variables.

9. (10 points) Define recursive equilibrium.

10. (5 points) Characterize as much as possible how time is allocated.

11. (10 points) Define recursive equilibrium in the absence of state contingent markets.