Econ 703
June Prelim

Instructions:

- This is a *closed* book examination.
- The total points are 120.
- You have 2.5 hours to answer questions.
- Please give a complete and intelligible solution.
- All the best.

1. (35 points) Consider the following "common-value" auction. A seller has an indivisible object. There are two bidders, and each bidder i has private information θ_i, which follows the uniform distribution on $[0,1]$. The value of the object to each bidder is $v = \theta_1 + \theta_2$; that is, the true value of the object is common for the bidders, and each bidder has only partial information about it.

(a) (7 points) Consider the first-price auction. That is, the bidders are asked to choose their bid simultaneously, and the object is awarded to the highest bidder. The price is the highest bid. Suppose that the opponent is using a linear bidding function $\beta(\theta_j) = k\theta_j$ for some $k > 0$. Compute the expected payoff of bidder i with type θ_i when she bids b_i. (Note that bidder i does not know θ_j, so it should not appear in your answer; you need to consider an appropriate expected value.)

(b) (8 points) Find a symmetric pure-strategy Bayesian Nash equilibrium β.

(c) (10 points) Now suppose that the seller uses the second-price auction. Show that the truthful bidding strategy ($b_i = \theta_i$) is not a Bayesian Nash equilibrium in this common-value environment.

(d) (10 points) Still consider the second-price auction. Show that there are (possibly asymmetric) linear-strategy Bayesian Nash equilibria, in which bidder 1's strategy is $b_1(\theta_1) = k_1\theta_1$ and bidder 2's strategy is $b_2(\theta_2) = k_2\theta_2$ for some $k_1 > 1$ and $k_2 > 1$. Find all values (k_1, k_2) which constitute an equilibrium.

2. (40 points) This question asks you to compare the equilibria of various games with the same payoffs and different information structures. Suppose that there are two firms, and there are two periods. In period one, firm 1 chooses its quantity $q_1 \in [0,7]$. In period two, firm 2 chooses its quantity $q_2 \in [0,7]$. The market price is given by $p = 14 - q$, where $q = q_1 + q_2$ is the total quantity. The marginal cost of production is 2 for each firm, so firm i's payoff is $(p-2)q_i = (14 - q_i - q_{-i} - 2)q_i$.

(1) (10 points) Suppose that firm 2 cannot observe firm 1's quantity q_1. Find a (pure-strategy) subgame-perfect equilibrium.

(2) (10 points) Suppose that firm 2 observes firm 1's quantity q_1 before choosing q_2. Find a (pure-strategy) subgame-perfect equilibrium.

(3) (20 points) Suppose that firm 2 observes a noisy signal about firm 1's quantity. Specifically, after firm 1 chooses q_1, firm 2 observes a signal y which takes a value from $[0,7]$. We assume that with probability $1-\varepsilon$, $y = q_1$. With the remaining probability ε, y follows the uniform distribution $U[0,1]$, regardless of q_1. Note that when $\varepsilon = 1$, the information is perfect, so the game is Stackelberg competition in part (2). On the other hand when $\varepsilon = 0$, the signal is not informative at all, so the game is essentially Cournot competition in part (1).

(a) (5 points) Describe the set of pure strategies of each firm i.

(b) (10 points) For each parameter $\varepsilon \in (0,1)$, find a pure-strategy subgame-perfect equilibrium. You only need to find one equilibrium, for each ε.

(c) (5 points) Interpret your results in part (3-b). How does the commitment power changes for different values of the parameter ε? Explain how firm 2 interprets the signal y in your equilibrium in part (3-b).

3. (45 points) Suppose that a firm (player 1) proposes a wage $w \in [0,1]$ to a worker (player 2), and the worker then accepts or rejects the proposal. If the proposal is accepted, the firm's payoff is $1-w$ (the profit from hiring the worker is 1), and the worker's payoff is w. If the proposal is rejected, each player's payoff is 0.

(1) (5 points) Describe the set of pure strategies for each player, and find all pure-strategy subgame-perfect equilibria. To get a full credit, you have to explain that there are no other equilibria.

(2) (20 points) Now consider Rubinstein's model. In each odd period, the firm makes an offer and the worker accepts or rejects. In each even period, the worker makes an offer and the firm accepts or rejects. There are infinitely many periods, and the game ends once an offer is accepted. Payoffs are discounted by a common discount factor $\delta \in (0,1)$

(a) (10 points) Find a subgame-perfect equilibrium outcome.

(b) (10 points) Consider the following strategy profile:

- In odd periods, the firm always proposes $w=0$, and the worker accepts *any* offer.
- In even periods, the worker always proposes $w=0$, and the firm accepts only if $w=0$.

We know that Rubinstein's model has a unique SPE, so this strategy profile is NOT subgame-perfect. Verify that it is indeed not subgame-perfect. That is, show that there is a subgame in which someone has a profitable deviation. For simplicity, you may assume $\delta = 0.8$.

(3) (20 points) Suppose that the game in part (1) is infinitely repeated. Suppose that players have a common discount factor $\delta = 0.9$ and maximize a discounted sum of stage-game payoffs, $\sum_{t=1}^{\infty} \delta^t u_i^t$, where u_i^t is agent i's utility in period t. Note that the game here is different from Rubinstein's model, in that (i) player 1 always proposes and (ii) even if an offer is accepted in some period, there is still a wage negotiation in the next period.

(a) (10 points) Consider the following strategy profile:

- The play starts with the "cooperative phase" in which the firm proposes $w = 0.4$, and the worker accepts any offer (even $w \neq 0.4$). They stay at the cooperative phase unless the firm deviates. (The worker's deviation is ignored). If the firm deviates in some period (i.e., if the firm proposes $w \neq 0.4$), then the worker accepts this offer, and then they move to the "punishment phase" described below.
- In the punishment phase, the firm offers $w = 0$ and the worker rejects any proposal (even $w > 0$). After that, they return to the cooperative phase immediately, unless the worker deviates by accepting an offer. (The firm's deviation is ignored.) If the worker accepts *any* offer (even $w > 0$), they stay at the punishment phase.

The automaton representation of this strategy profile is given in Figure 1. Show that this strategy profile is NOT a subgame-perfect equilibrium.

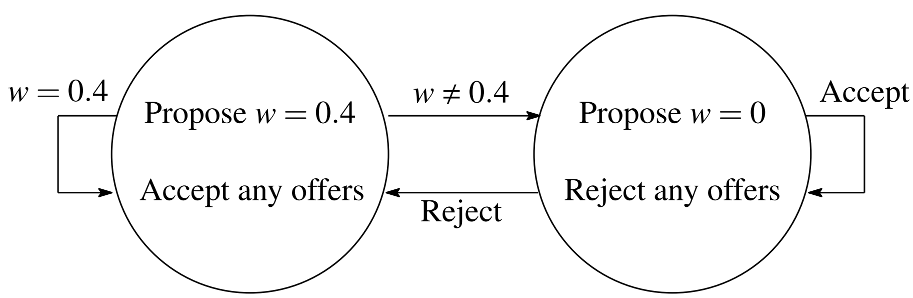

Figure 1: Automaton Representation for (3-a)

(b) (10 points) Find a subgame-perfect equilibrium such that the firm proposes $w = 0.4$ and the worker accepts it each period on the equilibrium path. (Hint:

You may add one more automaton state to the strategy in part (3-a) and modify the transition rule. Probably this is the most challenging question in this exam, so you may try other questions first.)