1 The Stochastic Neoclassical Growth Model with Stochastic Government Spending

Consider the stochastic neoclassical growth model with stochastic government spending. The government spending shock G_t follows an exogenous N-state Markov chain. Let $\mathcal{G} = \{G_1, G_2, \ldots G_N\}$ be the state space of the Markov chain, and let $\pi(G_{t+1}|G_t)$ denote the Markov transition matrix of the chain. Assume that

$$0 < G_1 < G_2 < \ldots < G_N$$

The timing is such that all choices in the current period are taken after the current shock G_t is realized. Both private agents as well as the social planner take the government spending process as exogenously given and beyond their control. The social planner chooses stochastic consumption, labor and capital allocations $\{c_t, l_t, k_{t+1}\}$ to solve the following maximization problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

s.t.

$$c_t + k_{t+1} + G_t = (k_t)^{\alpha} (l_t)^{1-\alpha} (G_t)^{\gamma}$$

with $\beta \in (0,1)$, $\alpha \in (0,1)$ and $\gamma \geq 0$ being parameters. The initial endowment of capital k_0 and the initial exogenous state G_0 is given.

1. Let $N = 3$ for the rest of the question, and suppose the Markov transition function is of the form

$$\pi = \begin{pmatrix} 0.5 - \frac{\nu}{2} & 0.5 - \frac{\nu}{2} & \nu \\ 0.5 - \frac{\nu}{2} & 0.5 - \frac{\nu}{2} & \nu \\ 0 & 0 & 1 \end{pmatrix}$$

   For all $\nu \in [0, 1]$ determine the set of stationary distributions.

2. Formulate the problem of the social planner recursively. State clearly what the state variables and control variables are.

3. Use the first order conditions and the envelope conditions to derive the Euler equation and the intratemporal optimality condition (in recursive form).

4. Suppose for the rest of the question that the utility function is of the form

$$U(c_t, l_t) = \log\left(c_t - \frac{(l_t)^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}}\right)$$

where $\chi > 0$ is a parameter. For what values of (γ, χ) does a *current* negative government spending shock (a low G_t) cause a recession (a decline in *current* output)? You have to give a formal argument (i.e. use equations) to justify your answer.

5. Now consider the competitive equilibrium. Denote by S the aggregate state of the economy. Assume that government spending is financed through taxation such that the government budget constraint reads as

$$G = \theta T(S) + (1-\theta)\tau(S)w(S)L(S)$$

where T, τ is a lump sum tax and a proportional labor income tax, respectively, and $w(S), L(S)$ is the wage and aggregate labor supply. Here $\theta \in \{0, 1\}$ is the share of tax revenue levied through the lump-sum tax. Also, denote by $r(S)$ the return on capital. Define a recursive competitive equilibrium. First, clearly state what variables constitute the aggregate state of the economy S.

6. For a given aggregate state S, is equilibrium labor higher under lump-sum taxation ($\theta = 1$) or under proportional taxation ($\theta = 0$), and how does it compare to labor chosen by the social planner? How does the answer depend on the parameters (γ, χ). Give some intuition for your answer.