Microeconomic Theory II
Preliminary Examination
June 4, 2018

The exam is worth 120 points in total.

There are **3** questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **[35 points]** Consider the following game between players I (the row player) and II (the column player):

	L	C	R
U	$5,5$	$3,9$	$0,-1$
D	$3,3$	$5,1$	$0,-1$

(a) What is the unique Nash equilibrium of this game? **[10 points]**

(b) Now suppose that player I has the option of either publicly choosing his action of U or D before II chooses, or of making his choice in secret (so that II does not know I's choice when choosing). The cost to I of publicly choosing before II is 1 util.

i) Describe an extensive form fitting this description in which player I publicly choosing U and player II responding with C is a subgame perfect equilibrium outcome. **[10 points]**

ii) Describe an extensive form with the same reduced normal form as the game in part i), but in which player I publicly choosing U and player II responding with C is *not* a subgame perfect equilibrium outcome. **[15 points]**

[Question 2 is on the next page.]

2. [**45 points**] Consider the following version of a reputation game: There are two periods and two players: a "long-lived" row player (he chooses in both periods and total payoffs are the sum of payoffs from each period) and a column player choosing in the second period. The row player is either an aggressive type (denoted t_A), in which case payoffs are as described in the first pair of payoff matrices, or a passive type (t_P), in which case payoffs are as described in the second pair of payoff matrices (only the row player's payoffs differ by type). The column player receives a payoff of 0 in the first period, irrespective of the choice of the row player.

T_1	4, 0
B_1	3, 0

Period 1

	L	R
T_2	4, 1	2, 0
B_2	3, 1	1, 2

Period 2

row player is t_A

T_1	3, 0
B_1	4, 0

Period 1

	L	R
T_2	3, 1	1, 0
B_2	4, 1	2, 2

Period 2

row player is t_P

The prior probability that the column player assigns to the row player being aggressive is p. Assume $p \in (\frac{1}{2}, 1)$. In the second period, the column player chooses her action simultaneously with the row player, not knowing his type, but having observed the choice of the row player in period 1.

(a) What restrictions on second-period behavior of the row player are implied by sequential rationality (**Hint:** These restrictions are also implied by perfect Bayes and sequential equilibrium)? Describe the signaling game thus induced. [**10 points**]

(b) Prove that there is no separating perfect Bayesian equilibrium of the induced signaling game. [**10 points**]

(c) Describe the two pooling perfect Bayesian equilibria of the induced signaling game in which both types of row player choose the same action in period 1. [**15 points**]

(d) One of the pooling equilibria of the signaling game is more plausible than the other. Which one and why? [**10 points**]

[**Question 3 is on the next page.**]

3. **[40 points]** A seller will run an auction to sell a one-of-a-kind replica of the Sydney Harbour Bridge. There are two potential buyers, with buyer i's value for the replica, θ_i, distributed on $[0, \bar{\theta}_i]$, independently of buyer $j \neq i$. As usual, outcomes are a function of type profiles denoted by

$$(\rho, t) : [0, \bar{\theta}_1] \times [0, \bar{\theta}_2] \to \Delta(\{0, 1, 2\}) \times \mathbb{R}^2,$$

where $\rho(\theta)$ is the probability distribution over who obtains the object (with 0 meaning the seller retains the good), and $t_i(\theta)$ is the transfer from buyer i to the seller. Define (where $j \neq i \in \{1, 2\}$)

$$p_i(\theta_i) := \int_{\theta_j} \rho_i(\theta_i, \theta_j) d F_j(\theta_j) \text{ and } T_i(\theta_i) := \int_{\theta_j} t_i(\theta_i, \theta_j)\, d F_j(\theta_j),$$

where F_j is the distribution function of θ_j. Each F_j has a strictly positive density f_j on its support $[0, \bar{\theta}_i]$.

(a) By the Bayesian revelation principle, the seller can restrict attention to incentive-compatible individually-rational direct mechanisms. Carefully define the terms "direct mechanism," "incentive compatibility," and "individual rationality." **[10 points]**

(b) Suppose (ρ, t) is an incentive-compatible individually-rational direct mechanism. Then p_i is nondecreasing and

$$T_i(\theta_i) = -k_i + p_i(\theta_i)\theta_i - \int_0^{\theta_i} p_i(\tilde{\theta}_i) d\tilde{\theta}_i,$$

for some $k_i \geq 0$. Using this fact, prove that buyer i's expected payment in the direct mechanism is given by

$$\int_0^{\bar{\theta}_i} \left[\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)}\right] p_i(\theta_i)\, d F_i(\theta_i) - k_i,$$

where f_i is the density of F_i. **[10 points]**

(c) Suppose $\bar{\theta}_1 = 1$, $\bar{\theta}_2 = 2$, and both buyers' valuations are uniformly distributed. Using part (b), derive the allocation rule in the revenue-maximizing incentive-compatible individually-rational direct mechanism. Is the allocation efficient? **[10 points]**

(d) Suppose the seller values the replica at $v_s \in (0, 2)$, so that the seller's ex post payoff from the direct mechanism (ρ, t) is

$$\begin{aligned} &\rho_0(\theta_1, \theta_2) v_s + t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2) \\ &\qquad = (1 - \rho_1(\theta_1, \theta_2) - \rho_2(\theta_1, \theta_2)) v_s + t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2). \end{aligned}$$

What is the seller's optimal incentive-compatible individually-rational direct mechanism? **[10 points]**