Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 6, 2018

Instructions

This exam has four questions and is worth 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly.

Use WORDS to explain your reasoning.

Good luck!

1. (25 pts) Joe lives in a two-good world. His income is $m = 18$. He has a continuous monotonic utility function U on $\mathbb{R}^2_+$, and the expenditure function

$$e(p, u) := (p_1 + \tfrac{1}{2}p_2)u.$$

   (a) (10 pts) Find U, carefully explaining the steps in your reasoning.

   (b) (15 pts) Find Joe's compensating variation for the price change from $p^0 = (4, 1)$ to $p^1 = (1, 1)$. (Use MWG's convention, so this CV should be positive.)

2. (25 pts) A consumer has wealth w that she must consume over two periods. In period 2 she has a random income shock, $\theta\tilde{y}$, where $\theta > 0$, $\mathbb{E}\tilde{y} = 0$, and $\mathbb{E}\tilde{y}^2 > 0$. Her expected utility when she chooses to save an amount x is

$$u(w - x) + \mathbb{E}v(x + \theta\tilde{y}).$$

   She can save any amount, i.e., x can be any real number. The functions u and v are C^3, with strictly positive first derivatives and strictly negative second derivatives. Let $x^*(w, \theta)$ denote her optimal savings function.

   (a) (10 pts) Suppose $u = v$. Assuming $u''' > 0$, What can you infer about the value of x^*/w?

   (b) (15 pts) Assuming $v''' > 0$, show that this consumer is "prudent" in the sense that an increase in the size of the shock increases the amount she will save, i.e., show that $x^*_\theta > 0$ everywhere.

3. (25 pts) (*Discrete Goods*) Consider an economy in which there are two goods, knives and forks. Each is indivisible, that is, the only feasible bundles are (k, f) where k and f are nonnegative integers. There are n agents and agent i has initial endowment (k_i, f_i). Knives and forks are useful only in pairs: all agents have the same utility function $u(k, f) = \min\{k, f\}$.

   (a) (10 pts) Find a Walrasian equilibrium for this economy.

   (b) (5 pts) How would the answer in part (a) change if agent i has the utility function $u(m_i k, m_i f)$, where each $m_i > 0$ and u is still $u(k, f) = \min\{k, f\}$?

   (c) (10 pts) Suppose now that there are three agents. Two agents each have one fork and no knives and the third agent has one knife and no forks. In class we noted that when defining the core of an economy, we distinguished between blocking when *all* members of a blocking coalition had to be strictly better off, and blocking when all members of a blocking coalition had to be at least as well off as in the proposed allocation and one member strictly better off. In the case of divisible goods we argued that the set of unblocked allocations (the core) was the same whichever definition we used. Is that the case in this example?

4. (25 pts) (*Public Goods*) Consider a two-person, two-period economy with one private good and one public good. Each agent is endowed with 3 units of the private good at the beginning of period one and no endowment in period two. In period one each agent chooses an amount of her private good to consume, and uses the remainder to produce public good to be consumed in period two. Each unit of private good used for public good production results in one unit of public good. There are two equally likely states of the world in period two. If agent $i = 1, 2$ consumes an amount x_i of private good in period one and y is the amount of public good in period two, his utility in state $t = 1, 2$ is $x_i + v_t(y)$. The agents are exepected utility maximizers. Let

$$v_1(y) = \ln y, \quad v_2(y) = 3 \ln y.$$

   (a) (8 pts) If both agents consume the same amount of the private good in the first period, what is the efficient level of consumption of private good?

   (b) (8 pts) Suppose each agent's initial endowment was one unit of private good in period 1 instead of 3. What would then be the efficient level of private good consumption in period 1 (still assuming that both agents consume the same amount of private good in period 1)?

   (c) (9 pts) Returning to the case in which each agent's endowment of private good in the first period is 3, what is the minimum expected utility one agent can get at an efficient outcome? (Note that agents do not have to consume the same amount of private good here.)