704 Part II
Summer 2018

In the following there are 2 questions for 100 points. Be as BRIEF as you can and good luck.

1. **Industry Equilibria**

   Imagine an industry, say the duck decoys industry, with demand given by $p = a - b\,Y$, where Y is the total level of output, and a and b are positive parameters. Imagine that there are many firms in that industry, each indexed by its technology level s, that, how else, follows a Markov chain with transition matrix Γ. New firms draw their s from cdf F. Imagine that capital can be rented at rate r, there is full depreciation, and that labor can be hired at rate w. Output of a firm, y is given by $\min\{s\ f(k,n), \alpha y\}$, where k is capital, n is labor f is a decreasing returns to scale production function and α is a technical coefficient that determines the amount of a special material need to attract ducks in each decoy. Consequently, $c = \alpha y$ is the amount of such special material needed by each firm. The price of wood q is given by $q = \gamma_1 + \gamma_2 C$ where C is the total amount of cork used by the bobbers industry (the only users of cork since this is a wineless world).

   (a) (10 points) Define a stationary equilibrium with free entry.

   (b) (5 points) Give a formula for the value of a firm with shock $\bar{s}$.

   (c) (10 points) What are the long run effects of cork tax at a rate of .25.

   (d) (15 points) Define equilibrium when the industry is in a stationary situation without taxes but the tax is imposed by surprise. THis is define the perfect foresight equilibrium that involves a transition to a new steady state with taxes.

2. **Lucas Trees**

   Assume there is a representative agent economy. There are two trees in the economy one produces avocados and the other blueberries. Consumers in this society like avocados in odd years and blueberries and avocados in even years (anthropologists are working hard trying to understand why) in the following way. We write those preferences as

$$E\left\{\sum_{t=1}^{\infty}\left[\beta^{2t-1}\,\alpha\,\frac{a_t^{1-\sigma}}{1-\sigma}\ +\beta^{2t}\,\frac{\left(\alpha\ a_t^{\rho}\ +(1-\alpha)\ b_t^{1-\rho}\right)^{1-\sigma}}{1-\sigma}\right]\right\}$$

   where a_t and b_t stand for avocados and blueberries respectively. Avocado production follows a Markov process with transition Γ and the blueberries tree produces a unit each period.

   (a) (20 points) Define equilibria recursively. Make sure that you define the state space and the equilibrium conditions.

   (b) (10 points) Write a formula for the price in an odd period of an option to buy one share of the blueberry tree at price p the period after and again the period after that.

   Imagine now that the utility function has an additional separable term that is decreasing in search effort d. Imagine also that acquiring avocados requires not only to purchase them but to find them using search effort. The amount of avocados found depends on a matching function between avocados and aggregate search effort D, $M(A,D)$.

(c) (10 points) Define Competitive Recursive Equilibria when the search protocol is competitive search and markets are indexed by price and market tightness.

(d) (20 points) Write down the first order conditions that are satisfied in equilibrium when $\alpha = \rho = 1$.