Treasury auctions during a crisis*

Seher Gupta† Rohit Lamba‡

November 2017

Abstract

Treasuries all over the world sell sovereign bonds through an auction which is typically conducted by the central bank. When volatility in financial markets is high, auctions may fail to elicit the true price of the bond. To study the impact of increased uncertainty on bidder behavior in treasury auctions, we introduce (a) risk averse preferences and (b) common uncertainty in the valuation of the underlying security. Using detailed bid-level data on the Indian Treasury Bill market around the (in)famous episode called taper tantrum, we estimate bidders' valuations in a model of multi-unit discriminatory price auction. We find that average bid shading increases substantially during this period leading to a big loss to the exchequer. A large part of the increase in bid shade is explained by the rise in uncertainty as measured by activity in the secondary market. We also uncover systematic heterogeneity across bidders. While some bidders bid at low prices because their valuations are low, others bid less as a strategic response to the increased uncertainty. We evaluate two alternative selling mechanisms – uniform price auction and a fixed price tender. We find that switching the pricing rule to uniform does not reduce bidder surplus. A fixed price mechanism, on the other hand, can help stabilize the market without affecting revenue much.

*The authors are indebted to Alessandro Lizzeri, Christopher Conlon, Konrad Menzel, and N Prabhala for their advice and encouragement. We would also like to thank Shoumitro Chatterjee, Boyan Jovanovic, Rishabh Kirpalani, Vijay Krishna, Isabelle Perrigne, Karl Schurter, Laura Veldkamp and Venky Venkateswaran for their comments. We are extremely grateful to CAFRAL and the Reserve Bank of India for their wonderful hospitality and for sharing data and institutional details with us.

†New York University, seher.gupta@nyu.edu

‡Penn State University, rlamba@psu.edu

1 Introduction

Markets do their job well when prices aggregate all possible information associated with trade. A widely used instrument for *price discovery* in large markets is auctions. The designer (or auctioneer) could be interested in finding out the right price from the perspective of direct revenue or efficiency of the downstream market. A necessary condition for price discovery through auctions is limited *bid shading*, defined as the difference between the buyer's value and bid for the object being sold. In a crisis situation, characterized informally as increased uncertainty in the price of underlying security, standard auctions may fail to fulfill the said role. How can we measure the quantity and the quality of this failure? Should the auctioneer then try something different?

In the summer of 2013, the Federal Reserve Board of the United States of America (henceforth the Fed) signaled an intent to tighten monetary policy by tempering the so-called quantitative easing and raising the base interest rate. This, it is largely believed, sent financial markets in emerging economies into a tizzy.[1] Exchange rates appreciated against the dollar and the domestic bond yields rose rapidly. It is important to note the Fed eventually did not follow through with this intent, hence the episode is often refereed to as the "taper tantrum". Understanding what actually happened at the micro level that led to a significant macro shock is widely regarded as an important question.[2] This paper takes a step in that direction by documenting the tumultuous episode through bidding behavior in the primary auctions for government bonds in India. In the process, it extends the tool kit of the empirical multi-unit auctions literature to incorporate uncertainty and addresses the general question of auction design for sovereign debt during times of crisis.

The empirical facts are as follows. Once the announcement of the Fed was internalized by monetary policy in India, yields on short-term and long-term paper started rising, that is the price bidders were willing to pay started to fall. Figure 1 plots the market clearing prices in the primary auction for the three month treasury bond around the taper tantrum period. These auctions are conducted by the central bank- Reserve Bank of India.[3] Forty two weekly held auctions are depicted from April 2013 to January 2014. The sequence of auctions between the vertical markers will be referred to as "during",

[1] See Basu, Eichengreen, and Gupta [2015] for an overview of the impact of the Fed's announcement on financial markets in India.

[2] Commenting on the taper tantrum episode, the chairman of the Fed at the time later wrote (Bernanke [2016]), "This line of research is interesting and important. Given the sometimes severe consequences of financial instability, we have to take these issues very seriously. Unfortunately, we don't understand these phenomena as well as we would like."

[3] This is a zero coupon bond. The central bank promises to pay the holder of one bond Rs 100 at the end of three months. The auction asks them to bid how much they would be willing to pay and how many pieces of paper they would buy.

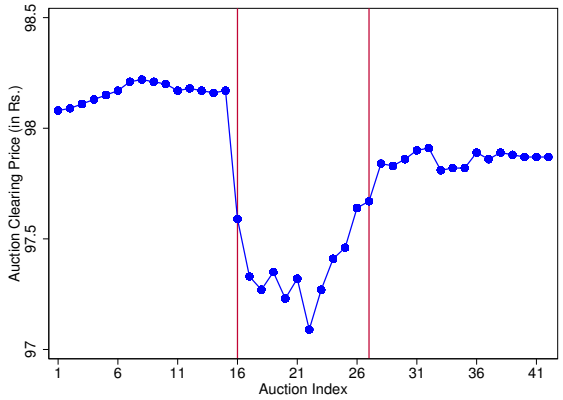
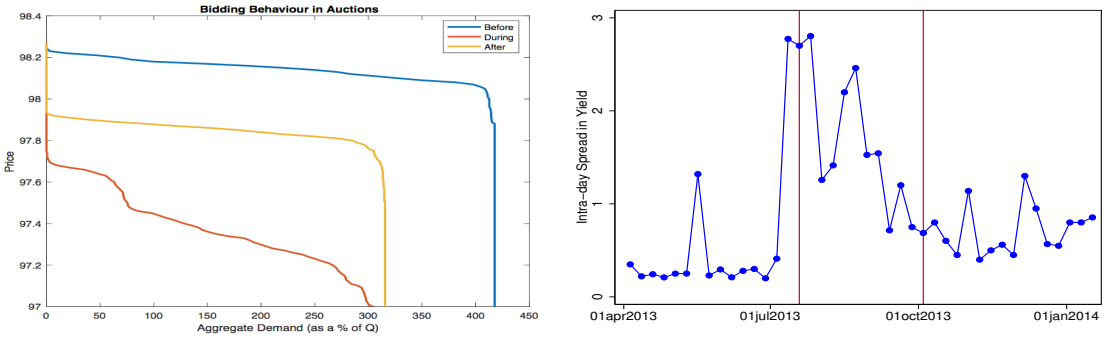


Figure 1: Auction clearing prices for the 3-month T-Bill, April 2013-Jan 2014



(a) aggregate demand

(b) intra day price differential

Figure 2: Aggregate demand in the primary auction became steep and secondary market exhibited greater variance in the price of trades

ones to the left as "before" and the ones to the right as "after".

The bids in the auction were widely dispersed. Figure 2a plots the aggregate sum of demands at every price point; the slope during the taper tantrum period is excessively steep relative to the norm. Therefore, the bidders were either speculative and/or their valuations for the bond were highly varied. After the primary auction these bonds can be traded on the secondary market. In fact many bidders come to the primary auction solely as an intermediary to resell their wins. In this paper we use the secondary market data to quantify uncertainty. The volatility in the secondary market spiked. Figure 2b reports the intra day difference in transaction prices; these were unusually high during the episode. The total activity too increased with a large increase in volume of trade.

Why did the prices fall so much? Was it purely a due a shift in fundamentals, that is, the valuation of the sovereign bond by the market? Why did the price rise up again so quickly? Did the quality of India's debt temporarily change by a substantial amount? There are three plausible reasons for the fall in the market clearing price – (i) decrease in the *valuation* of the bond, (ii) increase in *uncertainty* leading to precautionary bid-

ding, and (iii) a *purely strategic response* by some market participants to earn a quick buck culminating in greater bid shading. We shall refer to them as the valuation effect, uncertainty effect and strategic effect respectively. Documenting the existence and extent of these effects requires a structural model, which we write down and estimate in this paper.

While the aforementioned uncertainty effect is supported by the data on secondary market activity, to make the case for the strategic effect, we appeal to the number of participants and winners in the primary auction both of which increased significantly during the taper tantrum period (numbers provided in section 2.2). It is perhaps surprising that a steep decline in price was concomitant with a sharp increase in the number of bidders and the number of winners. It points to the fact that strategic considerations on part of the bidders should definitely be explored as a potential reason for the decline in price.

In order to capture the effects of both uncertainty and strategic bidding, we draw from the tools developed by the literatures on empirical auctions and financial economics (see Kastl [2016] for an elucidation of the research agenda). We build on a great body of work on the empirical analysis of treasury auctions (see Hortaçsu [2002], Kang and Puller [2008], Hortaçsu and McAdams [2010], Kastl [2011], Hortaçsu and Kastl [2012] and Cassola, Hortaçsu, and Kastl [2013]).[4] In particular we augment the canonical empirical multi-units auction framework with risk averse bidders and introduce aggregate or common uncertainty into the setup.

We model the bidders as being risk averse for three reasons. First, it ensures that uncertainty is not just "integrated out" and actually affects bidders' behavior at the optimum. Second, it helps bridge the gap between the literatures on empirical auctions and financial economics, most leading models in the latter assume market participants to be risk averse (see for example Kyle [1989], Biais, Glosten, and Spatt [2005] and Vives [2011]).[5] Third, it qualitatively squares with the empirical fact of reduced price and increased entry during the taper tantrum period. A risk-neutral model would not predict increased entry unless accompanied by a substantial but temporary change in idiosyncratic values for the sovereign bond, whereas the risk averse model provides a direct reason for entry with increased uncertainty. We expound upon the argument in greater detail in section 4.4.

[4]See Paarsch and Hong [2006] for a textbook treatment and Athey and Haile [2007] for an exhaustive survey on the general state of the art in empirical auctions. Hortaçsu and McAdams [2016] provide a recent synthesis of the research on empirical analysis of multi-unit auctions.

[5]On modeling financial institutions as being risk averse Biais, Glosten, and Spatt [2005] eloquently write "To speak to this issue it could be fruitful to analyze theoretically the internal organization of these financial institutions. For example, suppose the dealers need to exert costly but unobservable effort to be efficient and take profitable inventory positions. To incentivize them to exert effort, it is necessary to compensate them based on the profits they make. In this context, even if diversifiable risk does not enter the objective function of the financial institution, it plays a role in the objective function of an individual dealer quoting bid and ask prices."

The valuation of bidders in our model is driven by both a private idiosyncratic component and commonly unknown part, that is $v_i = z_i + A$, where each z_i is independently drawn and A is independently distributed and commonly unknown at the time of bidding.[6] This too departs from the standard set up in the estimation of multi-unit treasury auctions which assumes pure independent private values, equivalent to A being deterministic in our setup. We want the reader to think of A as the fundamental value of the sovereign bond and z_i as the private component driven by idiosyncratic demand to either hold the bond for regulatory requirements or to buy and sell as an intermediary.[7] We measure the distribution of A through the set of prices at which trade occurs in the secondary market, providing a first attempt at incorporating the larger market structure into the empirical analysis of treasury auctions.

Using this augmented set up, we derive a novel first-order necessary condition that optimal bids must satisfy in this multi-unit auction framework. The condition pins down the tradeoff between the marginal benefit and marginal cost of placing a particular bid step. The tradeoff can be broken down into two components. First is the probability of winning (marginal benefit) and the payment conditional on winning (marginal cost), which captures the standard intuition from single unit first price auctions. Second, risk aversion and common uncertainty introduce another tradeoff - the bidder faces the endogenous risk of the probability of winning (marginal benefit) that makes her bid more aggressively, but she also faces the pure exogenous risk of the value of A being low ex post (marginal cost) that makes her bid cautiously. We find that this latter tradeoff is resolved in favor of excessive precaution.[8]

Our main empirical result is that bid shading during the taper tantrum period goes up substantially. The average bid shade across bidders goes from 0.04 price points before taper tantrum to 0.44 during to 0.12 after it.[9] For a typical supply of 7×10^8 pieces of papers worth Rs 100 each, an extra bid shading of 0.4 price points means a loss of Rs 2.8×10^8 (or US$ 4 million) per auction.[10] This episode cost the Indian government a large sum of money, and the primary cause was not a change in the fundamental quality of debt but, as we will argue below, it was increased uncertainty in the market for sovereign

[6]The valuation is a downward sloping function of quantities (the true idiosyncratic demand function), therefore A sets the intercept of the demand and z_i its slope: $v_i(q) = z_i(q) + A$.

[7]Broadly speaking there are two types of bidders in these auctions. The buy and hold types are primarily banks who need to maintain a certain ratio of their assets in government bonds to satisfy a regulatory requirement known as the statuary liquidity ratio. The buy and sell types are intermediaries who act as market makers in the secondary market for sovereign bonds.

[8]The single unit version of this latter tradeoff is studied in Esö and White [2004] and Gupta, Lamba, and Muratov [2017].

[9]A bid shading of 0.04 price points means that a bidder that values the bond at Rs 98 would bid Rs 97.96 and a bid shading of 0.44 means that the bidder would bid at 97.56.

[10]Typically three to five auctions of varying maturities are conducted by the Reserve Bank of India every week.

bond and strategic behavior on part of the bidders.

Next, we set out to explain the decrease in bids and simultaneous increase in bid shading, seeking to add texture to the aggregate numbers. We take the following three pronged approach. First, we compare the distribution of valuations across auctions. We find that the distribution during the taper tantrum period is first-order stochastically dominated by that before and after. This points to the valuation effect– valuations did indeed go down, which in turn reduced the level of bids. However, digging a bit further into the change in valuations, we find that almost all of the change is driven by a shift in the distribution of the common component, there is almost no change in the private margin to own the bond. Formally, change in the distribution of $v_i = z_i + A$ is almost entirely driven by the change in distribution of A, while the distribution of z_i remains almost the same.

Second, we separately evaluate bid shading in the standard risk neutral model. This extricates our aggregate numbers from the effect of common uncertainty. If the bidders were only best responding to a decrease in valuation of other market participants this exercise would capture the interaction of the valuation and strategic effects. We find then that the standard model would explain only twenty five percent of the average bid shade. The rest therefore is driven by the uncertainty effect and its interaction with strategic effects.[11]

Third we separately calculate average shading of each bidder that participates in primary auction in the taper tantrum period. We find significant heterogeneity in these numbers. It suggests that the ex ante symmetric assumption on bidders may not be a good one. We follow a creative iterative procedure used by Cassola, Hortaçsu, and Kastl [2013] and club the bidders into groups on the basis on their bid shades and redo the estimation. Here we are somewhat limited by the data – a full fledged heterogenous evaluation of all or even a fine partition of bidders is elusive.[12] But, by partitioning the set of bidders into two or three groups in multiple ways we find the heterogeneity to be robust. The intermediaries, affected perhaps the most by uncertainty and expected to gain from it too, bid shade the highest. Moreover, bidders who shade the least are the ones who enter only during the taper tantrum period, revealing themselves to be fringe bidders who participate simply because of the significant drop in equilibrium price. This adds further force to the strategic effect, that some bidders are strategically responding to

[11] By interaction of valuation and uncertainty effects with strategic effect we mean the change in bidding behavior of the participants due to a change in the distribution from which their opponents' types and common uncertainty respectively are drawn. For example, a pure increase in uncertainty would lead to greater bid shading in equilibrium.

[12] The data considered by Cassola, Hortaçsu, and Kastl [2013] has about 300 bidders per auction, so they are able to create bidder groups without loosing much power, whereas the number of bidders per auction in our data never exceeds 55.

valuation and uncertainty effects in trying to make hay during the crisis.

A natural question to ask on absorbing the above analysis is this – can the central bank tweak the auction design in a way that decreases bid shading, improves price discovery and increases revenue? The commonly held view is that uniform price auctions perform better during times of financial stress.[13] In fact the Reserve Bank of India changed the format of the primary auction for long-term government securities from discriminatory to uniform, so in a given week the bidders were participating in a uniform price auction for long-term bonds and a discriminatory auction for short-term T-bills. Using bid level data in uniform price auctions, we show that the switch did not help. In fact the switch itself seems to have spooked the bidders. Bidder surplus increased quite substantially after the switch, suggesting that the tweak in auction design may need to be along a different dimension.

Inspired from a nascent but growing literature on quantity based (as opposed to price based) market mechanisms (see, for example, Vives [2010] and Duffie and Zhu [2017]), we propose such a tweak that can be evaluated using counterfactual calculations. We consider fixed price and flexible quantity tenders, as opposed to the prevalent fixed quantity and flexible price ones. Vives [2010] suggests that "an optimal demand schedule for the central bank should be more elastic when the information problem is more severe." We calculate the counterfactual (ex post) revenue for fixed prices under the marginal value estimates during the taper period. We find that in most auctions the central bank would have exhausted the supply set in the original auction and done reasonably well in terms of revenue by choosing a fixed price equal to the average of secondary market prices during the previous week plus a markup.

2 The background and data

2.1 The summer of taper tantrum

In May 2013, the then chairman of the Fed spoke about the possibility of tapering off the purchases of long-term bonds, effectually signaling a withdrawal or at least a tempering of quantitative easing. The Economist later wrote "the announcement that it [Fed] would start tapering the pace of its quantitative-easing programme caused money to stampede for safety."[14] It is important to note that foreign institutional investors in India are limited in the amount of treasury bonds they can buy. This is done precisely to keep the hot

[13] We learnt this fact through personal conversations with the auctioneers at the Reserve Bank of India. The underlying reasoning is intuitively plausible – a unique price can help to reduce uncertainty during crisis episodes. Friedman [1960] offered a similar reasoning to argue for uniform price over discriminatory price for the primary auctions conducted by the US Fed.

[14] The Economist [2015]

money phenomenon in check. However, through large movements in the exchange rate which translated into a change in the interest rate and a general rise in uncertainty in the market, the bidder behavior of the domestic actors was affected.

A full recap of the episode is beyond the scope here, we refer the readers to Sahay et al. [2014] for a detailed analysis of the lessons on emerging market volatility and to Basu, Eichengreen, and Gupta [2015] for the specific impact on India. Our endeavor is to use this as an example to explore the general question of auction design by central banks during times of crisis and in the process provide a revealed preference foundation to the macro question of volatility in financial securities in emerging economies.

2.2 The primary auction

Sovereign bonds in India are broadly classified into two categories: treasury bills (or T-Bills) and government securities (or G-Secs); the former are short-term classified into three distinct maturity baskets- 91, 182 and 364 days and the latter are long-term with maturities like 5, 10, even 20 years. Given the data we have, and owing to their simpler structure, we primarily analyze the three months T-Bills, with the exception of Section 6 where also look at the ten years long-term security. The Reserve Bank of India (like most other central banks over the world) issues new bonds into the market through a *primary auction*. The primary auction for the three months T-Bill is held weekly and the auction calendar is announced well in advance.

T-Bills are zero coupon securities, issued at a discount and redeemed at face value. For one piece of paper at a face value of Rs. 100, we will typically see a market clearing price in the set $[96, 99]$ for the three 3-months T-Bill. For example, for the auctions conducted between April 2013 - Januray 2014, the market clearing prices ranged from 97.09 to 98.22, as can be seen in Figure 1. An auction of Rs. 7×10^{10} bond will involve selling of 7×10^8 pieces of "paper" (denoted by $Q = 7 \times 10^8$) each worth Rs. 100. The bidders can submit multiple bids in the form of price-quantity pairs. A typical bid is of the form $\{(p_1, y_1), (p_2, y_2)\}$ with $100 \geq p_1 \geq p_2$; it means that the bidder is willing to buy y_1 pieces of paper at a total price $p_1 y_1$, and $y_1 + y_2$ pieces of paper at price $p_1 y_1 + p_2 y_2$, and so on. If both bids win, the bidder makes a transfer of $p_1 y_1 + p_2 y_2$ to the central bank, and is paid $100(y_1 + y_2)$ at the time of maturity.

The main features of the T-Bill auctions are as follows. First, treasury auctions world over are share auctions – multi unit divisible goods auctions. For a total notified amount Q, each player can bid for any fraction of Q. Multiple bidders can "win" the auction. Second, the auction is characterized by a market clearing price. After all the bids are in they are arranged in descending order of prices. As we go down the list, the price at which the cumulative quantity demanded exceeds Q is christened the market clearing

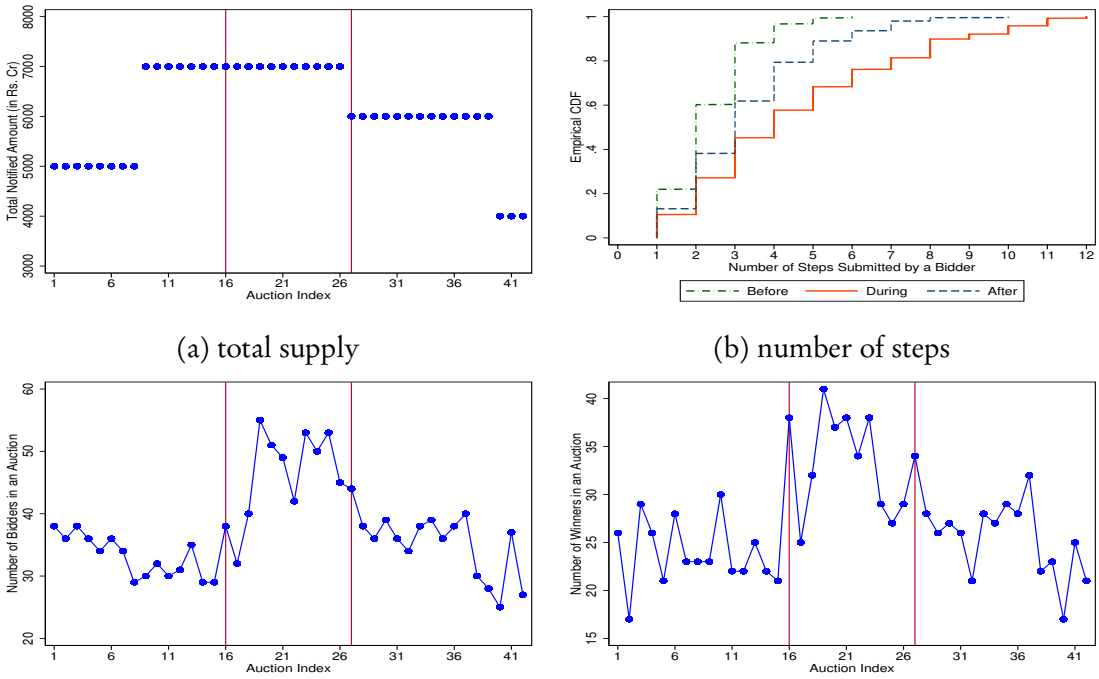
price. Third is the payment rule. For the quantities won, do the bidders pay the price they bid or the market clearing price? In the example above, if $p^m \leq p_2 \leq p_1$ is the market clearing price, does the bidder pay $p_1y_1 + p_2y_2$ or $p^m(y_1 + y_2)$? The former is called discriminatory price auction and latter uniform price auction. The T-Bill auction in India is discriminatory.

We use bid level data from April 2013 - January 2014 for the 3-month T-Bill auction. The auction is held every week on a Wednesday. On the Friday of the week before, the RBI posts an announcement on its website informing the bidders about the date and time of the auction, and the total amount on sale. After the auction ends, the market clearing price, number of bids received and the total amount sold are published. For all the 42 auctions conducted between April 3rd 2013 – January 15th 2014, our data set consists of all the bids submitted (including both prices and quantities), and the quantity won by each bidder. We do not observe the exact identity of the bidder, but can track it across auctions through a unique identity number. Moreover, each bidder is assigned a particular category by the central bank which we observe- for example is the bidder a national bank, foreign bank, pension fund or mutual fund, etc.

Table 1 presents the summary statistics. The total quantity supplied varied during the forty two auctions, but it remained almost constant during the taper tantrum period (see Figure 3a), which helps control for an important auction covariant. The market clearing price as we have already discussed saw a big dip (Figure 1). In lieu of a complete demand function, bidders submit steps in these multi unit auctions. Figure 3b plots the distribution of steps submitted by all bidders, the one during the episode first-order stochastically dominates those before and after. The larger number of steps during taper tantrum potentially exhibits increased uncertainty in the bidders' evaluation of the market clearing price. The number of bidders increased significantly as the price was falling. Figures 3c and 3d plot respectively the time series of the number of bidders and the number of winners across auctions, latter indicates that the market was becoming more fragmented. Finally, the bid-cover ratio (aggregate quantity demanded as a percentage of total supply) remained comfortably above one, so aggregate demand was strong.

	Mean	**Median**	**Min**	**Max**
Total supply (in Rs 10^{10})	6.26	7	4	7
Auction clearing price	97.72	97.83	97.09	98.22
# steps	2.44	2	1	19
# bidders	40.19	38	25	55
% Q demanded	3.55	3.43	1.63	7.16

Table 1: Summary statistics of forty two auction between April 2013 to January 2014



(a) total supply

(b) number of steps

(c) number of bidders

(d) number of winners

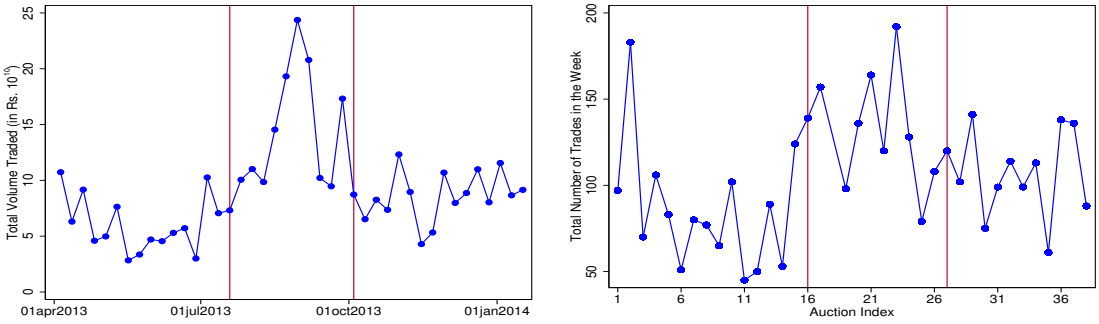
Figure 3: Total supply, CDF of number of steps, and total number of bidders and winner

2.3 The secondary market

There is an active secondary market for Treasury Bills in India, organized through an anonymous order matching module called Negotiated Dealing System - Order Matching (NDS-OM). The Reserve Bank of India website describes NDS-OM as "an order driven electronic system, where participants can trade anonymously by placing orders on the system or accepting the orders already placed by other participants". After being issued on a Wednesday, most of the trade of the new T-bills takes pace on Wednesday, Thursday and Friday of the same week. In addition to the data on primary auctions, we have trade-level data on the secondary market. We see the price at which trade occurs, the quantity traded, the date and time of the trade, and the maturity date of the traded security.

Under the broad rubric of empirical market microstructure, a large literature in financial economics has furthered our understanding of financial securities through their trade in secondary markets (see for example Hasbrouck [2007]). However, the secondary market has not been used much in the structural estimation of treasury auctions. Secondary markets act as a fair indicator of the market clearing price in the primary auction and vice-versa.[15] We shall use the price of trades in the secondary market as a measure of the fundamental value of the bond and the common component of the bidders' demand

[15]Outside the taper tantrum episode, the maximal difference between the average secondary market price and the auction clearing price that week for the financial year 2013-14 is only 0.02.



(a) total volume traded (b) total number of trades

Figure 4: Secondary market activity spiked

functions.

Large variance in the price of financial securities is often accompanied by an increase in the aggregate activity in the market- this correlation typically operates through the mechanism of a rise in speculative trade. Figures 4a reports the total volume traded in the secondary market and Figure 4b reports the total number of transactions. These along with Figure 2b clearly point towards a spike in volatility and activity in the secondary market during the taper tantrum period. It seems natural that this effect be incorporated in the analysis of the primary auction.

3 Model

In this section we present a model of multi-unit discriminatory auctions and derive a necessary condition for optimal bids. The necessary condition gives us a mapping from the data to the fundamentals of the model, which forms the basis for our estimation. We build on the model in Kastl [2012] with two critical differences: (i) bidders in our model are risk averse, and (ii) there is both a common component and a private component in their demand for securities.

3.1 Primitives

Let Q be the total amount of T-bills up for sale, and $\mathcal{N} = \{1, 2, ..., N\}$ be the set of bidders. Each bidder receives a private signal $s_i \in [0, 1]$ that parameterizes her valuation function, $v^i : [0, 1] \times [0, 1] \to \mathbb{R}_+$. Here $v^i(q, s)$ is the marginal value of bidder i for a share q (or total quantity qQ) of T-bills when she receives a private signal s.

Assumption M1. $S = (s_i)_{i=1}^N$ is distributed on $[0, 1]^N$ according to F that admits a continuous density f, and the signals across bidders are drawn independently.

Assumption M2. For each i, v^i is continuous and weakly decreasing in q, and strictly increasing in s.

Assumption M3. For each i, $v^i(q,s) = z^i(q,s) + A$, where A is an exogenous constant distributed according to μ on a discrete grid $\{a_1, ..., a_M\}$.

Assumptions M1 and M2, define the independent private values (IPV) part of the demand function, and ensure that it is downward sloping. Assumption M3 makes it clear that the marginal valuation of each bidder is additively composed of a pure private component and a pure common component. The private (and independent) signal s_i perfectly informs agent i about the marginal value function $z^i(., s_i)$. However, all the agents are symmetrically (un)informed about A, we assume that it is exogenously distributed. We call this the parameter of *common uncertainty*. Note that ours is neither a classical IPV nor a common values set up.

There are typically two types of bidders in these auctions- the buy and hold types and the buy and sell and types, the former participate for portfolio reasons or regulatory requirements whereas the latter are intermediaries who sell to other clients. For each of them z^i represents the idiosyncratic demand. The common component A refers to fundamental value of the sovereign bond to which z^i is appended as the private margin. The variance of A measures uncertainty in the bond market. We will rely on the secondary market prices and trades to estimate A and μ.

Further, bidder i submits a bid $b^i : [0,1] \times [0,1] \to \mathbb{R}_+$, where $b^i(q,s)$ is the price bidder i with private signal s is willing to pay for qQ units of T-bills. It is restricted to be weakly decreasing in q. The total payment for share q can be written as:

$$B^i(q,s) = \int_0^q b^i(x,s)dx$$

Writing $V^i(q,s) = \int_0^q v^i(x,s)dx$ to be the total value function, bidder i's utility from being allocated q fraction of T-bills when her signal is s is given by

$$U^i(q,s) = u(V^i - B^i \circ (q,s))$$

Assumption M4. $u : \mathbb{R}_+ \to \mathbb{R}$ is a von-Neumann-Morgenstern utility function that satisfies $u(0) = 0$, $u' > 0$ and $u'' < 0$.

That is, the bidders evaluate their net surplus (total valuation of the amount won minus total payment to the seller) according to a strictly concave function. This assumption

departs from the standard linear utility model used in the literature. Next, we assume, as is empirically relevant, that bids are steps functions.

Assumption M5. A typical bid is of the form $\left(b_k^i, q_k^i\right)_{k=1}^{K_i}$, where $K_i \in \{1, 2, ..., K\}$, $b_1 > b_2 > ... > b_{K_i}$, and $q_1 < q_2 < ... < q_{K_i} \leq 1$.

The share of issue amount demanded at any price p can be succinctly expressed as: $y^i(p|s_i) = q_k^i \mathbb{1}_{\left(b_{k+1}^i < p \leq b_k^i\right)}$. The price at which the market clears then has a simple definition: $p^c = \max\{p \mid D(p) = 1\}$ where $D(p) = \sum_{i=1}^{N} y^i(p|s_i)$ and the max operator breaks ties in favor of the auctioneer.[16]

Assumption M6. The total quantity is randomly distributed on $\left[\underline{Q}, \overline{Q}\right]$ with strictly positive density conditional on $s_i \; \forall \; i$.

This assumption is a technical requirement for precluding mass points at any quantity in the bidder's demand at the optimum. Economically, it is a tool of equilibrium selection. Intuitively, one can think of this as the uncertainty faced by the bidder on where to place her last step. We want the reader to think of the distribution as being tightly centered around Q, say with support $[Q - \epsilon, Q + \epsilon]$ and a small variance.

3.2 Bidder behavior

The multi-unit auction model is hard to precisely pin down analytically. General theoretical results are elusive (see Ausubel et al. [2014] for the "solvable" cases). The approach has therefore been to push the theory to provide a set of necessary conditions that optimizing bidders must satisfy and invoke those to put structure on bid-level data to back out valuations and its distribution. An added layer of challenge in our framework is the non-linearity of bidders' utility function.

Our bidder faces two tradeoffs in placing a particular bid step (b_k, q_k). First is the standard intuition from single unit first price auctions. A higher bid price increases the probability of winning at that step, but conditional on winning it increases the payments (see Krishna [2010], Chapter 2). A second tradeoff is introduced by risk aversion and common uncertainty. The bidder is faced with two "lotteries"- whether she wins at that step and the ex post value of the bond on winning. Again theoretical results from single unit auctions tell us that the first will make the bidder bid more aggressively while the

[16] Note that since aggregate demand is also a step function, it is very likely (in fact with probability one in equilibrium) that the vertical total supply line intersects the demand function in the horizontal part of the step rather than at the edge. In such a situation the quantities are rationed. We apply, as is standard in the literature, the pro-rata rule of rationing, that is, quantities on the last step are allocated according to the intensity of demand at the step. This is also the methodology used by the Reserve Bank of India.

second will make her bid conservatively (see Esö and White [2004] and Gupta, Lamba, and Muratov [2017]).

Both aforementioned tradeoffs are prevalent for each step of the bidder, and since we are dealing with a multi-unit auction, these are then linked across steps in a bidder's strategy. This makes a direct generalization of the theoretical results that generate these intuitions to multi-unit auctions non-trivial, in fact mostly intractable. However, the spirit of the intuition carries through to the empirical analysis we do around the first-order necessary condition that we derive next.

3.3 First-order necessary condition

Using a perturbation approach we can derive a necessary condition that the quantity demanded at any step of a pure strategy Bayesian Nash equilibrium must satisfy. This condition will be used to extract the fundamentals (marginal values) from the observable (bids). In what follows, let v_m^i be the marginal value V_m^i be the area under the value curve when the common component is $A = a_m$.

Proposition 1. *Under Assumptions M1-M6, in any K-step equilibrium of a discriminatory price auction, for almost all s_i, every step $k < K_i$ in the equilibrium bid function $y_i(\cdot|s_i)$ has to satisfy:*

$$\sum_{m=1}^{M} \mu_m \left\{ \mathbb{P}\left(b_k > p^c > b_{k+1} | s_i\right) \left(v_m^i(q_k, s_i) - b_k\right) \left[u'\left(V_m^i - B^i \circ (q_k, s)\right) \middle| b_k > p^c > b_{k+1}, s_i \right] \right\}$$
$$= \sum_{m=1}^{M} \mu_m \left\{ \mathbb{P}\left(b_{k+1} \geq p^c | s_i\right) \left(b_k - b_{k+1}\right) \mathbb{E}\left[u'\left(V_m^i - B^i \circ (q_k, s)\right) \middle| b_{k+1} \geq p^c, s_i \right] \right\} \quad (1)$$

and at the last step K_i it has to satisfy $v(\bar{q}, s_i) = b_{K_i}$ where $\bar{q}$ is the largest quantity allocated to bidder i of type s_i is equilibrium.

Proof. See Appendix. □

Equation (1) is derived by perturbing the expected payoff of the bidder i around the quantity she bids at the k^{th} step, keeping the price component of the bid fixed. Note that $\mathbb{P}\left(b_k > p^c > b_{k+1} | s_i\right)$ refers to the probability of the bidder winning her k^{th} step but not the $(k+1)^{th}$ step, while $\mathbb{P}\left(b_{k+1} \geq p^c | s_i\right)$ refers to the probability that she wins her $(k+1)^{th}$ step and maybe more. To fix ideas first consider the thought experiment that the bidder is risk neutral, then all the u' terms vanish. The left hand side of equation captures the expected marginal benefit of $v(q_k, s_i) - b_k$ from demanding quantity slightly in excess of q_k at step k. The right hand side measures the expected marginal cost of paying an

extra $b_k - b_{k+1}$ for the increase in quantity at step k. Then, solving for $v(q_k, s_i) - b_k$ gives us the bid shade of bidder i at step k.[17]

Now, add to the picture the fact that the bidder is risk-averse. Then, both the marginal benefit and the marginal cost are scaled by the marginal utility of the bidder at the monetary value generated by the area under the value curve minus bidding curve. The right hand side has an expectation over marginal utility because perturbation of the k^{th} step can change the expected allocation at the optimum- when the market clearing price is weakly less than the $(k+1)^{th}$ bid, the allocation can be different for different values of p^c. Optimality requires equation (1) to hold for all steps individually, and unlike the risk-neutral case, there is inter-dependence between the equation for each step through the marginal utility term which is evaluated at the aggregate monetary value. Finally, both sides are averaged over the common uncertainty term which has a discrete density given by $\{\mu_m\}$.

If the analyst knows or observes all fundamentals other than the bidder's marginal value curve, then with a little more structure, equation (1) allows him to back out valuation of the bidder at each step that she bids at.

4 Estimation

The main question we ask is this- why did prices fall so much during the taper period? Did the value for T-bills decrease or was the fall precipitated by increased uncertainty and strategic bidding? To answer this question, in what follows, we estimate the marginal valuations of bidders in the primary auctions. Equation (1) gives us a relationship between the bids we observe in the data and the valuations that rationalize them.

There are three steps in the estimation procedure: (1) estimating the probability distribution of the market clearing prices, (2) estimating the probability distribution μ of the common component A, and finally (3) estimating the marginal valuation $v(q_k, s_i)$ for every bidder in every auction at the observed quantities. We explain each step in detail below.

4.1 The distribution of market clearing prices: an algorithm

The first step involves estimating the probability expressions in equation (1) : $\mathbb{P}\left(b_k > p^c > b_{k+1} | s_i\right)$ and $\mathbb{P}\left(b_{k+1} \geq p^c | s_i\right)$. To estimate these probabilities we employ a standard "resampling" approach. We partition the set of bidders into three groups, the exact method of the group formation is explained in section 5.2 and then later in the appendix. Since the

[17] For the risk neutral case Proposition 1 would simplify to Proposition 1 in Kastl [2012].

probabilities are specific to a bidder in an auction, for a fixed auction and a fixed bidder i with $N = N_1 + N_2 + N_3$ bidders we use the following algorithm.

1. Fix an auction and a bidder with the bid $(b_k^i, q_k^i)_{k=1}^K$. If bidder i belongs to group 1, draw $N_1 - 1$ bids from all the bids of group 1 bidders and N_2 and N_3 bids from all the bids of groups 2 and 3 respectively. Symmetrically if bidder i is in group 2, draw N_1, $N_2 - 1$ and N_3 bids from the three groups.[18]

2. The resampled vector of rival bids represents one simulation of the state of the world from the point of view bidder i. Intersect this vector with the fixed bid to get one possible realization of the market clearing price.

3. Repeat the first two steps 5000 times to get an empirical distribution of market clearing price conditional on the fixed bid of bidder i.

We use this simulated distribution of market clearing prices to get empirical estimates for the probabilities in equation (1). The asymptotic properties of the resulting estimators established in Hortaçsu and Kastl [2012] extend to our setting. A more detailed explanation is provided in the appendix. The following assumption underlies the resampling procedure.

Assumption E1. The private information of bidders is identically distributed within their respective groups and independent across bidders and auctions.

4.2 The distribution of common uncertainty

To estimate the support and probability distribution of A, we use data from the secondary market. The support of A is taken to be set of prices at which trade takes places all week preceding the primary auction, and probability of each element is the frequency with which trades take place at that price. Since the data is publicly available, our estimate satisfies the assumption the bidders know the distribution of common uncertainty when they bid in the auction. We could also measure the support and distribution of A using the secondary market price for working days after the auction date and before the next auction, thereby invoking a rational expectations assumption. Our empirical results are robust to this alternate specification. Suppose the list of prices at which trades take place in a given week is given by R and it constitutes m distinct values: $\{a_1, a_2, ..., a_M\}$. Then, we assume the following.

[18] When the number of bidders in one auction is not large enough, following Hortaçsu and Kastl [2012], we pool the bids from two nearby auctions. We cannot, however, pool data from more auctions because there is a trend in the auction clearing prices.

Assumption E2. $A \in \{a_1, a_2, ..., a_M\}$, and $\mu_m := \mathbb{P}(a_m) = \frac{\#a_m \in R}{\#R}$, where $\#a_m \in R$ is the number of times a_M appears in the set of trades and $\#R$ is the total number of trades.

4.3 Adding structure to the valuation function

Recall that the total valuation term is defined as $V^i(q,s) = \int_0^q v^i(x,s)dx$. Therefore, we need to add more structure to V^i to be able to back out the valuation curve from equation (1). We assume that the akin to the bids, the valuations are also decreasing step functions. make the following functional form assumption:

Assumption E3. There exists a set of values $\{v_1(s), v_2(s), ..., v_K(s)\}_{s\in[0,1]}$ such that $v(q,s) = v(q_k,s) = v_k(s) \quad \forall \quad q \in (q_{k-1}, q_k]$, where $q_0 = 0$.

That is, we assume that the marginal valuation function is a step function which is constant between the quantities that are observed in a bid, and the constants are given by the marginal values at exactly the observed quantities. Hence, for a bid $(b_k^i, q_k^i)_{k=1}^K$ in the data, if we know the vector of marginal values at the observed steps $\{v_1, v_2, ..., v_K\}$ then we know the entire marginal value function that generates that bid.[19]

Note that Assumption E3 actually violates the continuity clause of Assumption M2. There are two ways of justifying our choice. First, we can consider a downward sloping continuous valuation function very close to the step function- exactly equal except in a very small neighborhood of values where the steps change. Informally speaking, this makes our estimates of marginal values almost equal to those for the continuous function. Second, for a fixed bid, valuations as step functions give the lowest possible values of bid shading in the class of all possible downward sloping valuation functions. Since we are interested in showing an increase bid shading during the taper tantrum period, this choice gives a lower bound.[20]

4.4 Risk aversion

The final unknown object in the estimation equation is the utility function of the bidders. Here we assume that the bidders evaluate their net surplus using the following CRRA utility function.

Assumption E4. $u(V^i - B^i) = \frac{(V^i - B^i)^{1-\sigma}}{1-\sigma}$

[19] Since $v^i = z^i + A$, technically the Assumption E3 is imposed on the function $z^i(q,s)$.

[20] Through interviews with bidders we also reached the conclusion that it is a good model of how bidders actually think of their "true demand functions".

Figure 5: Correlation between uncertainty and number of bidders that participate in the auction

This is, as we mentioned in the introduction, a departure from the literature on empirical analysis of treasury auctions, but very much in line with the literature on financial economics. The literature on empirical analysis of auctions with risk averse bidders is sparse, arguably because of the technical difficulty it presents both the theorist and the econometrician.[21] A suggestive test for risk aversion of our bidders in the empirical fact of increased participation by bidders in the auctions during the taper tantrum period. Figure 5 presents a scatter plot of the variance in secondary market prices, our proxy for the level of uncertainty, and the number of bidders that participated in the auction, the raw correlation coefficient is 0.5.[22]

For the risk neutral model to predict increased entry, it must be that the valuation of the frequent bidders goes down substantially whereas the value of the "fringe" bidders does not change much, which would then lead the latter to enter the auction. However, the model with risk averse bidders and common uncertainty predicts an increased entry simply through a rise in common uncertainty. As the variance of the common component goes up, the equilibrium price declines, which then encourages the fringe bidders to enter the auction.

There at least two papers that estimate Bayesian models of auctions with risk averse bidders. In a lab experiment, Bajari and Hortaçsu [2005] find that the risk averse model predicts bidder behavior much better than the risk neutral one in standard first price auction. They use a CRRA utility function and estimate the coefficient of relative risk aversion to be $\sigma = 0.77$. Lu and Perrigne [2008] too invoke the CRRA model and find $\sigma = 0.35$ for bidders in the auctions for timber in California. We estimate our model for

[21] Standard tools of mechanism design are not easily applicable with risk averse agents, see Maskin and Riley [1984] for a leading exception to the rule. Moreover, a striking result by Guerre, Perrigne, and Vuong [2009] shows that simultaneous estimation of risk aversion and distribution of valuations is impossible in many standard settings.

[22] Note that there are various other reasons that we do not observe which may influence the marginal or fringe bidders' decision on whether to participate in the auction.

all values of $\sigma \in [0, 1]$ and report the results for $\sigma = 0.3$ and $\sigma = 0.8$ in section 5.

4.5 Putting it together

Using the CRRA formulation, the first-order condition (equation 1) for bidder i's k^{th} step in the observed bid $(b_k^i, q_k^i)_{k=1}^K$, can be re-written as follows:

$$\begin{aligned}
&\sum_{m=1}^{M} \mu_m \left\{ \mathbb{P}\left(b_k > p^c > b_{k+1} | s_i\right) \left(v_k - b_k\right) \left[\sum_{l=1}^{K} (z_l + a_m - b_l)(q_l - q_{l-1}) \right]^{-\sigma} \right\} \\
&= \sum_{m=1}^{M} \mu_m \cdot \mathbb{P}\left(b_{k+1} \geq p^c | s_i\right) \left(b_k - b_{k+1}\right) \left[\sum_{j=k+1}^{K} \frac{\mathbb{P}\left(b_j > p^c > b_{j+1}\right)}{\mathbb{P}\left(b_{k+1} \geq p^c\right)} \left(\sum_{l=1}^{j} \left(z_l + a_m - b_l\right) \left(q_l - q_{l-1}\right) \right)^{-\sigma} \right]
\end{aligned} \tag{2}$$

If we plug-in our estimated distributions of market clearing prices and common uncertainty, and a numerical value for the risk aversion parameter σ, then for a K-step bid, we get a set of K simultaneous non-linear equations involving K unknowns $\{z_1, z_2, ..., z_K\}$. Under the assumption of a Bayes optimizing agent, we can numerically solve the system of equations to back out the vector of marginal valuations for every bidder in every auction.

5 Results

We present the results piecemeal, starting with the aggregate numbers across bidders, followed by the decomposition of the aggregates, and heterogeneity across bidders.

5.1 Averages

Our main empirical result is depicted in Figure 6. It plots the market clearing price and average marginal value under the standard risk-neutral model and our model with risk averse bidders and common uncertainty. While three curves follow each other fairly closely before the dip, they diverge quite a bit during the taper tantrum period. This wide gulf represents the increase in average bid shading, the associated failure of price discovery and the consequent loss to the exchequer.

Table 2 presents the average bid shading across bidders before, during and after taper tantrum period for three different values of σ. Looking at each column individually it is quite clear that average shading went up significantly during the crisis episode. While we use $\sigma = 0.3$ for our leading estimates, we enlist the value for the risk neutral model and for higher risk aversion to argue that whether bidders are risk averse or not is more

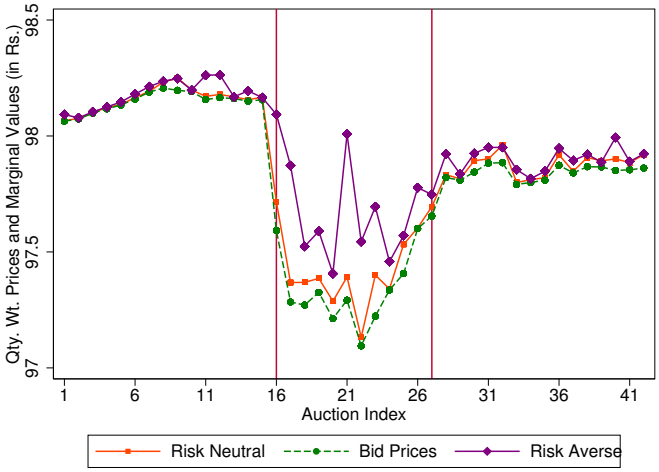


Figure 6: Bid shading increases significantly during the taper tantrum period

important up to a first-order for the estimates than how risk averse they are.[23]

	$\sigma = 0$	$\sigma = 0.3$	$\sigma = 0.8$
Before	0.0109 [0.0074,0.0164]	0.0474 [0.0302,0.772]	0.0537 [0.0244,0.1217]
During	0.1018 [0.0603,0.1889]	0.4434 [0.3630,0.5770]	0.4444 [0.3367,0.6523]
After	0.0393 [0.0249,0.0860]	0.1091 [0.0419,0.2338]	0.1174 [0.0509,0.2874]

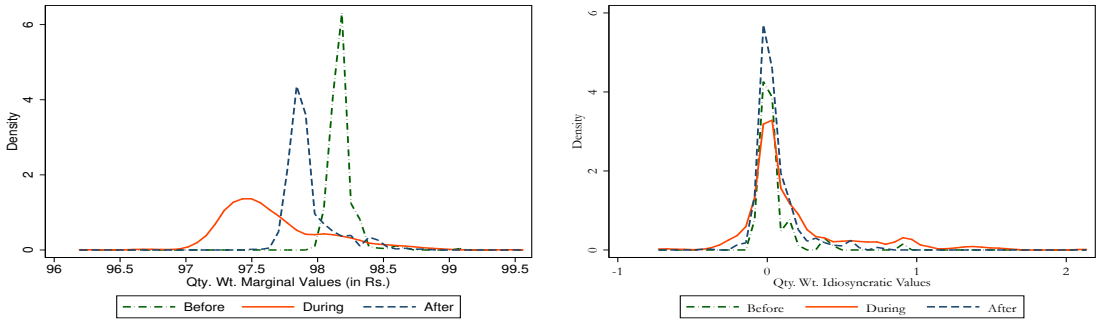
Table 2: Average bid shading before, during and after the taper tantrum period

As we can see from Figure 6, the risk neutral model, since it integrates out all of the uncertainty, would predict that most of the drop in price is due to a change in valuations. There is still significant bid shading (it increases by 900 percent in the first column of Table 2), but the sudden change in price is majorly attributable to a drop in actual valuations or "change in fundamentals". As we show in what follows, our model, because it allows us to incorporate uncertainty in a nuanced fashion, concludes that the driving force behind the steep fall and then rise in price is the spike in common uncertainty, and the strategic effect associated with it.

5.2 Decomposition and heterogeneity

There are broadly three reasons for the increase in bid shading. The distribution of the bidders' valuations stochastically declined. There was an increase in common uncertainty, that is variance of the component component. And, the bidders strategically responded to the change in the two distributions. We call these three the valuation effect,

[23] Note that for $\sigma > 0$, the marginal value of a bidder is calculated as $v_i = z_i + \mathbb{E}[A]$.



(a) average marginal values (b) idiosyncratic component

Figure 7: Density of marginal values before, during and after taper tantrum

uncertainty effect and strategic effect, respectively. While the valuation effect would exogenously lead to a decline in price, it is the interaction of the valuation and uncertainty effects with the strategic effect that drives the endogenous decline.

We start the decomposition by evaluating the change in marginal valuations. Figure 7a plots the density of the average marginal values across the three time periods.[24] The values during the taper tantrum period constitute an almost complete leftward shift in comparison to those before and after. This makes it clear that the aggregate valuation effect is strong- the value of the bond in the market did go down. Moreover, the bell becomes fatter, which implies that values also become more dispersed.

However, recollect that $v_i = z_i + A$, and Figure 7a makes a statement about the average value, that is $z_i + \mathbb{E}[A]$. What about the distribution of the idiosyncratic component, the private margin for the bidders? Figure 7b plots the distribution of the private idiosyncratic component, z_i, across bidders for the three time periods. These are small numbers (in magnitude) because the private margin most bidders hope to earn in these auctions are mostly in two decimal points. The change in distribution is minimal at best, while the mean remains more or less the same, there is a slight increase in variance. The striking observation then is that yes there was a change in valuations, but it was driven only by the change in fundamentals through a shift in the distribution of the common component. From the perspective of private values, the bond did not change much in value for the bidders.

Next, we want to understand how much of the average bid shade is explained by the valuation effect. As a proxy, we appeal to estimates of our model for $\sigma = 0$. Here the uncertainty effect is completely shut down. From the numbers in Table 2 we can conclude that about 25 percent of the total bid shade can be explained by the valuation effect and

[24] Figure 7a plots the distribution of quantity weighted marginal values. Suppose $K = 2$ and the quantity demanded at the two steps is given by y_1 and y_2. Then, the quantity weighted marginal value is given by $\frac{v_1 y_1 + v_2 y_2}{y_1 + y_2}$. This is right notion of average marginal value for a bidder. Similarly in Figure 7b we define quantity weighted private component as $\frac{z_1 y_1 + z_2 y_2}{y_1 + y_2}$.

its interaction with the strategic effect of bidders reacting to a change in the distribution of fellow bidders' valuations. Therefore a large component of the bid shading, about three-fourths, during the taper tantrum period is explained by the uncertainty effect and its interaction with the strategic effect.

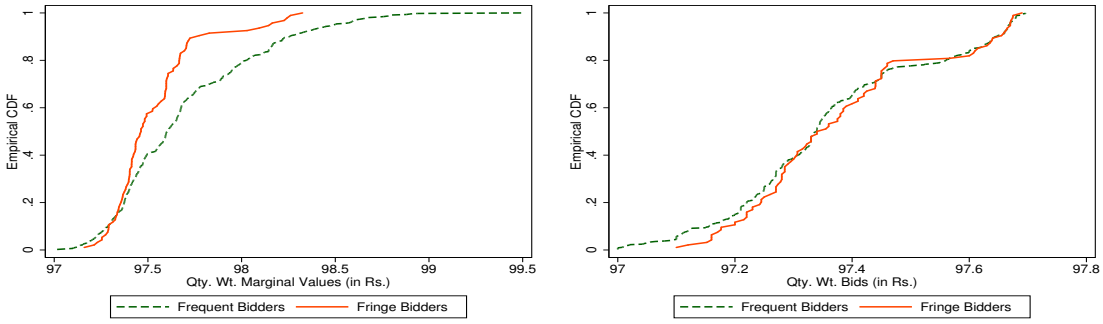
Finally, we want to understand the extent of heterogeneity across bidders. Here we use the methodology of Cassola, Hortaçsu, and Kastl [2013] of dividing the bidder into groups. The procedure has to be tailored to our model and data, and as is discussed is the appendix. We see the bidders split themselves broadly into three categories. A significant fraction of bidders come to almost every auction, we call them group 1. Another group of bidders comes reasonably frequently, importantly these bidders appear in all three time periods – before, during and after taper tantrum, we refer to them as group 2. However, there is a third groups of bidders that appears only during the taper tantrum period, we call them group 3.

$\sigma = 0.3$	**Group 1**	**Group 2**	**Group 3**
Before	0.05 [0.02,0.12]	0.04 [0.01,0.08]	
During	**0.55** [0.37,0.84]	**0.35** [0.25,0.51]	**0.15** [0.11,0.30]
After	0.13 [0.04,0.28]	0.07 [0.03,0.16]	

Table 3: Average bid shading across the three different bidder groups

Table 3 reports bid shading across groups. It is clear that on average group 1 bidders shade the most, group 2 intermediate, and group 3 shade the least. The level of bid shading is inversely proportional to frequency of participation in the auction. Group of 1 consists mostly of the primary dealers that are required, due to a contractual agreement with the Reserve Bank of India, to appear in every auction. Majority of them act as pure intermediaries and sell their wins in the secondary market. Group 2 consists mostly of large non-primary dealers such as big public and private sector banks. Majority of them are the buy and hold types. And, group 3 mostly consists of small banks and financial institutions.

The intermediaries are the ones most exposed to the uncertainty, and mostly likely to benefit from the uncertainty- they bid shade a lot. The large banks can afford to bid conservatively while the taper tantrum lasts, making sure they bid with the market sentiment. The "fringe" bidders, those in group 3, not only shade the least, they also have the lowest marginal values. Figure 8a plots distributions of the fringe and frequent (groups 1 and 2) bidders during the taper tantrum period. We would miss this stochastic dominance that we see in marginal values if we just plotted the bids of the fringe and



(a) values (b) bids

Figure 8: Distributions of bids and marginal values of fringe vs frequent bidders during the taper tantrum period

frequent bidders, as in Figure 8b. Therefore, as the price of the bond is declining, in large part due a rise in uncertainty, it becomes attractive for those at the margin in terms of actual valuations to enter the auction and buy some of the bonds.

6 The switch that did not help: uniform price

It is folk wisdom, at least since Friedman [1960] and Friedman [1991], that the uniform price auction may do better than the discriminatory auction in terms price discovery and revenue. A majority of sovereigns though still use the discriminatory price auction, the US being an exception. The theoretical literature does not point towards a clear winner and the empirical evidence is at best mixed.[25]

Creating counterfactual estimates for a switched multi unit auction format has so far been elusive, the state of the art in the literature is to construct the market clearing price and the associated revenue under the assumption that bidders bid truthfully. The revenue estimate for the hypothetical auction then gives an upper bound on the revenue that can be collected in any equilibrium of a uniform price auction.[26] In our setup with risk averse bidders, these estimates are a looser upper bound than in the risk neutral model, since this exercise does not take into account the risk faced by the bidders. Table 4 enlists the counterfactual exercise for uniform price auctions.

We find that if the bidders did bid close to being truthful, the uniform price auction

[25]In a recent survey paper, Hortaçsu and McAdams [2016] conclude that "when bidders have private information the ranking of the pay-as-bid and uniform-price auction is ambiguous, both in terms of revenue and efficiency."

[26]This is the approach used in Hortaçsu and McAdams [2010]. Note however that Kastl [2011] shows the bidders may actually bid higher than their marginal values in a uniform price auction, so there is no clear mathematical result that establishes revenue from truthful bidding as an upper bound. But, even in his sample few bidders actually bid above their marginal value. Based on that, the analysis done in Hortaçsu and McAdams [2010] and our own conversations with bidders we think this is still a worthwhile number to analyze.

Auction	Actual auction clearing price	Uniform auction price	Revenue as a fraction of actual revenue
16	97.59	97.98	1.0032
17	97.33	97.70	1.1037
18	97.27	97.39	1.0008
19	97.35	97.53	1.0038
20	97.23	97.31	1.0004
21	97.32	97.94	1.0062
22	97.09	97.40	1.0026
23	97.27	98.00	1.0079
24	97.41	97.48	1.0006
25	97.46	97.50	1.0003
26	97.64	97.67	1.0002
27	97.67	97.68	1.0000

Table 4: The uniform price estimates for the 3-month T-Bills under truthful bidding

would have led to better "price discovery" (by definition), and would have done almost as well in terms of revenue. Of course, we have no systematic way of knowing how the bidders would actually bid in the auction. But the exercise does provide some credence to the aforementioned folk wisdom. Luckily, due to a critical decision made by the Reserve Bank of India during the summer of 2013, we can say a bit more.

At the onset of the taper tantrum crisis, the Reserve Bank of India actually switched the format of the primary auction for long term securities from discriminatory to uniform. Anticipating the impact of announcements by the Fed, the Reserve Bank preemptively tried to lessen the turmoil by changing the format.[27] Figure 9 plots the market clearing prices for the 10 year sovereign bond during the same time period we looked at before. The auction for the these maturities happen less frequently. The vertical lines enclose the same calendar weeks as in Figure 1, however, the switch actually happens at the first dot.

A casual glance at the numbers reveals that the switch perhaps did not work. The price declined substantially, much more than what the change in interest rate at the time would suggest. We can again ask the same question we asked for the 3-month T-Bill, did the valuation for the sovereign bond decline substantially or was the decline driven in large part by uncertainty and strategic considerations? In order to answer this question, we rebuild the theoretical and empirical machinery we developed earlier, now for the uniform price auction. The details are completely analogous and are hence relegated to the appendix. The fundamental difference to note is that the payment function is now

[27] In personal conversations with the auctioneers, it was revealed to us that the switch was done in anticipation of the effect of the Fed announcement. The central bank prefers to have a unique price during times of stress while in times of normalcy the discriminatory auction potentially earns higher revenue.

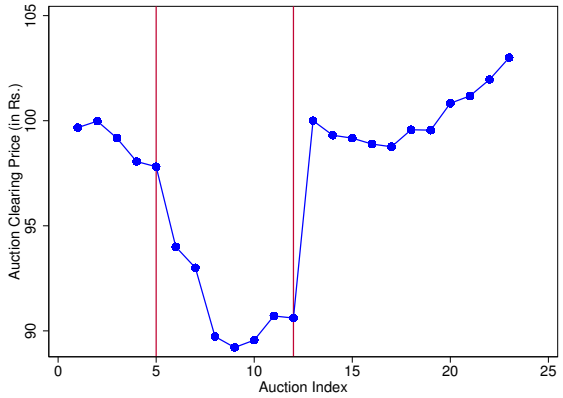


Figure 9: Auction clearing prices for the 10 year bond fresh issues, April 2013-Jan 2014

simpler. The total payment for a share q of the total quantity is given by

$$B^i(q,s) = qQp^c$$

where p^c is the ex post market clearing price. We prove an analogous first-order necessary condition for uniform price auctions with risk averse bidders and common uncertainty.

Proposition 2. *Under Assumptions M1-M5 in any K-step equilibrium of a uniform price auction, for almost all s_i, every step k in the K-step bid function $y_i(\cdot|s_i)$ has to satisfy:*

$$\sum_{m=1}^{M} \mu_m \mathbb{P}(b_k > p^c > b_{k+1}) \cdot \mathbb{E}\left[u'\left(V_m^i - q_k p^c\right)\{v_m^i(q_k, s_i) - p^c\} \middle| b_k > p^c > b_{k+1}\right] + \frac{\partial}{\partial q}\mathbb{E}\left[u(V_m^i - q_k p^c(q)); b_k \geq p^c \geq b_{k+1}\right] = 0 \quad (3)$$

The tradeoff faced by the bidder in the uniform price auction is different than that faced in the discriminatory auction. Here the bidder is only concerned with his expected position in the winners' demand schedule relative to the residual supply function she faces. The risk-neutral version of the equation is analogous to third-degree price discrimination. Our model adjusts for the standard trade-off with risk averse bidders.

The average bidder surpluses for the period we defined as "before", "during" and "after" are 0.6897, 0.5562 and 0.2837 respectively. This shows that the bidders were anything but truthful and were "shading" a lot even in the uniform price auction format. Average bid shading in the discriminatory auction just before the switch was 0.078, which leads us to conclude that switch just before an impeding crisis could have spooked the bidders even more increasing the uncertainty effect. This is not to say that uniform price auction can't do better in times of crisis, just that we cannot disentangle at the margin how much the switch actually helped given the general rise in uncertainty in the market,

the timing of the switch itself and limitations of counterfactual estimates. However, given the pure decline in price and our estimated numbers for bidder surplus, it seems to fair to conclude that switch on the ground did not work.

7 Tweaking the design: fixed price tenders

A thought experiment worth exploring is this – what if the central bank simply posted prices and let the market decide how much to buy? There are at least three reasons why this simple procedure may be attractive in times of crisis. First, if the central bank knows "more than the market", it might be in a better position to set prices. The auction might actually tie its hand in a crisis episode. Second, it could arrest uncertainty by plugging the speculation channel at least in the primary auction, which may have desired downstream effects on the secondary market. Third, it is not unusual for the central bank to take an out of the box policy lever in times of distress; arguably the Fed did the same in the aftermath of the collapse of Lehman Brothers in 2008-09.

Since the estimation procedure arms us with the marginal value curve of individual bidders and hence the aggregate market, we can precisely determine how much of the quantity will be sold and revenue raised at each price point. The exact exercise is as follows. Fix a posted price. Given this price, we can compute the quantity that maximizes the expected utility of each bidder. Assume that all bidders who participated in an auction will be willing to buy their optimum quantity at the posted price. Then, sum up the individual quantities to compute the counterfactual aggregate demand for the fixed price.

A natural question to ask then is what should this posted price be? How does the central bank determine this price? We perform the counterfactual calculation using secondary market prices as our marker. The posted price equals the average of all secondary market trades in the days leading up to the auction plus a small markup. In Table 5 below, we report the aggregate quantity that the central bank can potentially sell when the markup is 0.02 price points.[28] Only in 3 out of the 12 auctions does aggregate demand fall significantly below the actual quantity sold in the auction, and in three other auctions much more quantity is sold than the actual supply set by the central bank. In fact, a simple back of the envelope calculation show that the total quantity sold and revenue raised in the 12 fixed price sales would be higher than that in the actual auctions.

To come up with a fixed price, this counterfactual calculation takes the secondary market price as being fixed. But, in reality the secondary market price itself may react to the fixed price chosen by the central bank and the quantity sold at that price. It is worthwhile therefore to perform an exercise equivalent to that in section 6 and come

[28] We use this markup because in our data (outside of the taper period) the maximum difference between the secondary market price before the auction and market clearing price in the auction is 0.02 price points.

Auction	Actual Q Sold (10^8)	Fixed Price	Q Sold at Fixed Price (10^8)
16	7	97.77	9.154
17	7	97.92	2.759
18	7	97.46	2.216
19	7	97.50	6.604
20	7	97.33	3.952
21	7	97.21	15.207
22	7	97.33	8.775
23	7	97.31	17.126
24	7	97.47	9.998
25	7	97.49	8.103
26	7	97.65	10.462
27	6	97.69	6.309

Table 5: The fixed price counterfactual

up with the fixed price that would exactly exhaust the total supply originally set by the central bank, assuming the bidders optimally choose their quantity. The numbers turn out to be close to those under truthful bidding - the price is always slightly lower than the third column of Table 4 and the fraction of revenue is slightly lower too, though still always above one. The exercise is presented in Table 6.[29]

Auction	Actual auction clearing price	Price which exhaust Q	Revenue as a fraction of actual revenue
16	97.59	97.98	1.0032
17	97.33	97.69	1.1036
18	97.27	97.33	1.0001
19	97.35	97.49	1.0034
20	97.23	97.29	1.0002
21	97.32	97.92	1.0060
22	97.09	97.35	1.0020
23	97.27	98.00	1.0079
24	97.41	97.48	1.0006
25	97.46	97.50	1.0003
26	97.64	97.65	1.0000
27	97.67	97.68	1.0031

Table 6: The fixed price estimates for the 3-month T-Bills for optimizing bidders

The analysis for uniform price auction and fixed price auction makes us conclude that

[29]The main difference between Table 6 and Table 4 is that in Table 6 we are assuming that bidders optimally respond to the price set by the central bank in choosing their quantity, whereas in Table 4 we assumed that they buy at the their expected marginal value curve, the latter does not take into account the risk borne by the bidders.

suitably chosen fixed price tenders could have potentially had the desired effect that the switch to uniform price could not. By monitoring prices in the secondary market and quantity sold in the primary sale every week, the Reserve Bank of India, by setting a suitable fixed price, could have ensured steady supply of debt and revenue to the exchequer, while tempering down uncertainty around the value of the bond. Of course, a deeper analysis into the question would require a richer model and experimental data which is a good agenda for future research.

8 Final thoughts

Taper tantrum led to what may be regarded as the "spillovers debate"- what effect monetary policy in one country can have on another and what are the channels through which it propagates? While we take the first as given in starting out from the decline in bond prices, we provide some explanation to the second question by appealing to the rise of uncertainty and speculative behavior associated with it. More formally, through bid level data in the primary auctions for sovereign debt in India, we record a significant rise in average bid shading and hence bidder surplus. The steep decline in price was not so much a sign of changing fundamentals but rather a function of the rise of uncertainty about the true value of the bond and a strategic response of the key financial players in the domestic market.

We make a technical contribution towards the empirical analysis of multi-unit auctions by introducing risk averse preferences for bidders and common uncertainty in valuations. We take both to be natural assumptions for treasury bills. The financial economics literature has often invoked risk averse agents to analyze such markets. Risk aversion also squares better with the empirical facts around the taper tantrum period. And, given that value of a government bond is linked to quality of the sovereign debt, we include a common component for all bidders whose variance can spike in times of uncertainty. We hope these additions to the burgeoning literature on empirical multi-unit auctions allows us to model crisis episodes well.

With a final look at Figure 6, we emphasize that our model predicts a *failure of the auction* only during taper tantrum period, both before and after periods see the marginal value line follow the market clearing price relatively closely. It is useful for central banks to perhaps have a flexible schedule of quantity at such times and even take the initiative in setting prices in the market. Of course, the success of the approach depends critically on the credibility of the treasury and the central bank, for which it is important that such a clause of temporarily abandoning the auction and using posted prices not be abused.

9 Appendix

9.1 Proof of Proposition 1

The result is established through a perturbation argument. For the ease of notation, we suppress the common uncertainty term in much of the proof, and show how the proof statement and proof generalize as a last step. We follow the proof style in Kastl [2012] closely- adjusting at each step for the fact that bidders are risk averse bidders.

For a step $k < K$, let us perturb the quantity demanded from q_k to $q' = q_k - \epsilon$. Given the bid price b_k of this bidder at the k^{th} step, depending on the realization of the random variables (Q, S) and hence market clearing price, the states of the world can be partitioned into five subsets:

$\theta_1(q_k)$: When the market clearing price is above the bid price at k^{th} step, the bidder does not win anything at this step and hence the total share won in equilibrium (Q^c) is less than the cumulated share demanded at k^{th} step, i.e.

$$b_k < p^c \quad and \quad Q^c < q_k$$

$\theta_2(q_k)$: When the market clearing price is exactly the same as the price bid at k^{th} step, the bidder wins a proportion of the quantity demanded at k^{th} step and hence the total share won in equilibrium is *still* less than the cumulated share demanded at this step, i.e.

$$b_k = p^c \quad and \quad Q^c < q_k$$

$\theta_3(q_k)$: Market clearing price is between the k^{th} and the $(k+1)^{th}$ bid price, so the bidder wins all the share demanded at k^{th} step. Thus:

$$b_k \geq p^c > b_{k+1} \quad and \quad Q^c = q_k$$

$\theta_4(q_k)$: When the market clearing price is weakly lower the $(k+1)^{th}$ bid price and there is *no tie* at b_{k+1}, so the total share won in equilibrium is more than the cumulated share demanded at k^{th} step. That is:

$$b_{k+1} \geq p^c \quad and \quad Q^c > q_k$$

$\theta_5(q_k)$: Now the market clearing price is exactly the $(k+1)^{th}$ bid price, and so the total share won in equilibrium is more than the cumulated share demanded at k^{th} step. That is:

$$b_{k+1} = p^c \quad and \quad q_{k+1} > Q^c > q_k$$

The difference between θ_4 and θ_5 is that in the states in θ_5 the bidder could be *rationed* at the $(k+1)^{th}$ step, and so the total share won could potentially be less than q_{k+1}.

Now, when we slightly reduce the quantity demanded at the k^{th} step, the market clearing price will either remain the same or decrease. As a result, the probability weights on the thetas defined above changes. In particular, the states in $\theta_2(q_k)$ in which price decreases due to the perturbation now move to $\theta_3(q')$. Let us denote these states as $\omega_2(q')$. Similarly, we can define the new partition after perturbation as follows:

$$\begin{aligned}
\theta_1(q') &= \theta_1(q_k) \\
\theta_2(q') &= \theta_2(q_k) - \omega_2(q') - \omega_4(q') \\
\theta_3(q') &= \theta_3(q_k) + \omega_2(q') - \omega_3(q') \\
\theta_4(q') \cup \theta_5(q') &= \theta_4(q_k) \cup \theta_5(q_k) + \omega_3(q') + \omega_4(q')
\end{aligned} \tag{4}$$

where $\omega_3(q')$ and $\omega_4(q')$ are the states that move from $\theta_3(q_k)$ and $\theta_2(q_k)$ to $\theta_4(q_k) \cup \theta_5(q_k)$.

As noted above, the main difference between the states θ_4 and θ_5 is in the allocation. Since, there is *no tie* at b_{k+1} in θ_4, the bidder wins all the quantity demanded at $k+1^{th}$ step. Therefore, when we perturb to q', the bidder surely wins the ϵ as well. However, in state θ_5 and in the event of a tie, the bidder might not win all of the ϵ share.

Expected Utility under q_k:

We denote the expected utility in each of the different sets of states of the world $\theta_j(q_k)$ (where $j \in \{1,2,3,4,5\}$) as:

$$\mathbb{E}\left[u\left(V_j - B_j\right)\middle|\theta_j(q_k)\right]$$

and we define V_j and B_j for each j below.

Let us start with $\theta_1(q_k)$. In this set, the bidder does not win anything on the k^{th} step. So, the expected utility does not change with the perturbation. Thus we can ignore these states of the world.

When the state is in $\theta_2(q_k)$, the bidder wins all the quantity demanded at $(k-1)^{th}$ and also some rationed amount of $(q_k - q_{k-1})$, let us call it $c_2 \in (q_{k-1}, q_k]$. Then, the total value and payment can be written as follows:

$$V_2 = \int_0^{q_{k-1}+c_2} v^i(x, s_i)dx$$
$$B_2 = \sum_{m=1}^{k-1} (q_m - q_{m-1})\, b_m + (c_2 - q_{k-1})b_k$$

where $q_0 = b_0 = 0$. For example: if $k = 2$, $B_2 = q_1 b_1 + (c_2 - q_1)b_2$, and if $k = 3$, then $B_2 = q_1 b_1 + (q_2 - q_1)b_2 + (c_2 - q_2)b_3$.

Similarly, when state is in $\theta_3(q_k)$, the bidders wins exactly q_k. So the corresponding total value and payment expressions are:

$$V_3 = \int_0^{q_k} v^i(x, s_i)dx$$
$$B_3 = \sum_{m=1}^{k} (q_m - q_{m-1})\, b_m$$

Now, when the state is in $\theta_4(q_k)$, the market clearing price is weakly lower than the price at this bidder's $(k+1)^{th}$ step and there is no tie at the $(k+1)^{th}$ step. Therefore, the bidder wins at least q_{k+1}. If the bidder is rationed at the l^{th} step where $l > (k+1)$, then we have:

$$V_4 = \int_0^{q_{l-1}+c_4} v^i(x, s_i)dx$$
$$B_4 = \sum_{m=1}^{l-1} (q_m - q_{m-1})\, b_m + (c_4 - q_{l-1})b_l$$

where $c_4 \in (q_{l-1}, q_l]$.

For the last case, when state is in $\theta_5(q_k)$, the bidder wins all the quantity demanded at $(k)^{th}$ step and also some rationed amount of $(q_k - q_{k+1})$. Thus:

$$V_5 = \int_0^{q_k+c_5} v^i(x, s_i)dx$$
$$B_5 = \sum_{m=1}^{k} (q_m - q_{m-1})\, b_m + (c_5 - q_k)b_{k+1}$$

where $c_5 \in (q_k, q_{k+1}]$.

Expected Utility under q':

With the perturbation some quantity is reduced from k^{th} step and increased at the $(k+1)^{th}$ step. All other demands remain the same. This will affect the allocations (both rationed and otherwise) and payments in the different state of the world. Again, let us denote the expected utility as below:

$$\mathbb{E}\left[u\left(V_j - B_j\right)\middle|\theta_j(q')\right]$$

where $q' = q_k - \epsilon$ and $j \in \{1,2,3,4,5\}$.

For states in $\theta_2(q')$ – These are the states in which the bidder is rationed at the k^{th} step. The allocation after rationing will reduce because the total amount demanded at the k^{th} step is less due to the perturbation, and so:

$$V_2' = \int_0^{q_{k-1}+c_2'} v^i(x, s_i)dx$$
$$B_2' = \sum_{m=1}^{k-1}(q_m - q_{m-1})\, b_m + (c_2' - q_{k-1})b_k$$

where $c_2' < c_2$ and $c_2' \in (q_{k-1}, q_k]$.

For states in $\theta_3(q')$ – There is no rationing in this set of states. The bidders wins everything till the k^{th} step. But since we are perturbing the demand on the k^{th} step, the bidders wins $q' < q_k$. Hence:

$$V_3' = \int_0^{q'} v^i(x, s_i)dx$$
$$B_3' = \sum_{m=1}^{k}\left(q_m' - q_{m-1}'\right) b_m$$

where $q_m' = q_m \; \forall m < k$ and $q_k' = q_k - \epsilon$.

Now, in $\theta_4(q')$ there is no tie at b_{k+1}, so allocations do not change at the k^{th} and the $k+1^{th}$ step, but the payment on ϵ will change. Again, if the bidder is rationed at the l^{th} step when $l > (k+1)$, then:

$$V_4' = \int_0^{q'_{l-1}+c'_4} v^i(x, s_i)dx$$
$$B_4' = \sum_{m=1}^{l-1} \left(q'_m - q'_{m-1}\right) b_m + (c'_4 - q'_{l-1})b_l$$

$$\text{where :} \quad \begin{cases} c'_4 = c_4 \quad and \quad c'_4 \in (q'_{l-1}, q'_l], \\ q'_m = q_m \quad \forall \quad k+1 < m < k, \\ q'_k = q_k - \epsilon, \quad and \\ q'_{k+1} = q_{k+1} + \epsilon. \end{cases}$$

In states $\theta_5(q')$, the bidder is rationed at the $(k+1)^{th}$ step. Thus, the rationed quantity changes because the demand on the $(k+1)^{th}$ step is now $q'_{k+1} = q_{k+1} + \epsilon$. It is important to note here that the bidder might not win all of ϵ. The total value and payments, thus, are:

$$V_5' = \int_0^{q'_k+c'_5} v^i(x, s_i)dx$$
$$B_5' = \sum_{m=1}^{k} \left(q'_m - q'_{m-1}\right) b_m + (c'_5 - q'_k)b_{k+1}$$

$$\text{where :} \quad \begin{cases} c'_5 \in (q'_k, q'_{k+1}], \\ q'_m = q_m \quad \forall \quad m < k, \\ q'_k = q_k - \epsilon, \quad and \\ q'_{k+1} = q_{k+1} + \epsilon. \end{cases}$$

Note that when we write the expected utility for $\theta_4(q_k) \cup \theta_5(q_k)$, we will write the expression using V_4 and B_4 using $l \geq (k+1)$. Now, we have take the difference between the expected utilities before and after the perturbation.

Difference between the two Expected Utilities:

$$EV(s_i|q_k) = \Pr(\theta_1) \cdot \mathbb{E}\left[u\left(V_1 - B_1\right)\middle|\theta_1(q_k)\right] + \Pr(\theta_2) \cdot \mathbb{E}\left[u\left(V_2 - B_2\right)\middle|\theta_2(q_k)\right] + \Pr(\theta_3) \cdot \left[u\left(V_3 - B_3\right)\right] + \Pr(\theta_4 \cup \theta_5) \cdot \mathbb{E}\left[u\left(V_4 - B_4\right)\middle|\theta_4 \cup \theta_5(q_k)\right] \quad (5)$$

and

$$EV(s_i|q') = \Pr(\theta_1) \cdot \mathbb{E}\left[u\left(V_1' - B_1'\right)\middle|\theta_1(q')\right] + \Pr(\theta_2) \cdot \mathbb{E}\left[u\left(V_2' - B_2'\right)\middle|\theta_2(q')\right]$$
$$+ \Pr(\theta_3) \cdot \mathbb{E}\left[u\left(V_3' - B_3'\right)\middle|\theta_3(q')\right] + \Pr(\theta_4 \cup \theta_5) \cdot \mathbb{E}\left[u\left(V_4' - B_4'\right)\middle|\theta_4 \cup \theta_5(q')\right]$$

Using the definitions of the partition for q', we get:

$$\begin{aligned}
&= \Pr(\theta_1) \cdot \mathbb{E}\left[u\left(V_1' - B_1'\right)\middle|\theta_1(q_k)\right] + \Pr(\theta_2) \cdot \mathbb{E}\left[u\left(V_2' - B_2'\right)\middle|\theta_2(q_k)\right] \\
&\quad - \Pr(\omega_2) \cdot \mathbb{E}\left[u\left(V_2' - B_2'\right)\middle|\theta_2, \omega_2\right] - \Pr(\omega_4) \cdot \mathbb{E}\left[u\left(V_2' - B_2'\right)\middle|\theta_2, \omega_4\right] \\
&+ \Pr(\theta_3) \cdot \left[u\left(V_3' - B_3'\right)\right] + \Pr(\omega_2) \cdot \left[u\left(V_3' - B_3'\right)\middle|\theta_3, \omega_2\right] - \Pr(\omega_3) \cdot \left[u\left(V_3' - B_3'\right)\middle|\theta_3, \omega_3\right] \\
&\quad + \Pr(\theta_4 \cup \theta_5) \cdot \mathbb{E}\left[u\left(V_4' - B_4'\right)\middle|\theta_4 \cup \theta_5(q_k)\right] + \Pr(\omega_3) \cdot \mathbb{E}\left[u\left(V_4' - B_4'\right)\middle|\theta_4 \cup \theta_5, \omega_3\right] \\
&\quad + \Pr(\omega_4) \cdot \mathbb{E}\left[u\left(V_4' - B_4'\right)\middle|\theta_4 \cup \theta_5, \omega_4\right]
\end{aligned} \tag{6}$$

Taking the difference:

$$\begin{aligned}
EV(s_i|q_k) - EV(s_i|q') &= \Pr(\theta_2) \cdot \mathbb{E}\left[u\left(V_2 - B_2\right) - u\left(V_2' - B_2'\right)\middle|\theta_2\right] \\
&\quad + \Pr(\theta_3) \cdot \left[u\left(V_3 - B_3\right) - u\left(V_3' - B_3'\right)\right] \\
&\quad + \Pr(\theta_4 \cup \theta_5) \cdot \mathbb{E}\left[u\left(V_4 - B_4\right) - u\left(V_4' - B_4'\right)\middle|\theta_4 \cup \theta_5(q_k)\right] \\
&\quad + \Pr(\omega_2) \cdot \left\{\mathbb{E}\left[u\left(V_2' - B_2'\right)\middle|\theta_2, \omega_2\right] - u\left(V_3' - B_3'\right)\right\} \\
&\quad + \Pr(\omega_3) \cdot \left\{u\left(V_3' - B_3'\right) - \mathbb{E}\left[u\left(V_4' - B_4'\right)\middle|\theta_4 \cup \theta_5, \omega_3\right]\right\} \\
&\quad + \Pr(\omega_4) \cdot \left\{\mathbb{E}\left[u\left(V_2' - B_2'\right)\middle|\theta_2, \omega_4\right] - \mathbb{E}\left[u\left(V_4' - B_4'\right)\middle|\theta_4 \cup \theta_5, \omega_4\right]\right\}
\end{aligned} \tag{7}$$

Taking the Limit:

The next step is to divide the difference $EV(s_i|q_k) - EV(s_i|q')$ by $q_k - q'$ and take the limit $q' \to q_k$. Let us look at each term in equation (5) separately.

The first term on the RHS involves the probability of θ_2. This set contains the states in which the bidders is rationed at the k^{th} step. So, we can invoke Lemma 1 here and argue that in equilibrium: $\Pr(\theta_2) = 0$. Thus, this term vanishes.

The second term can be re-written as:

$$\Pr(\theta_3) \cdot \lim_{q' \to q_k} \left[\frac{u\left(V_3 - B_3\right) - u\left(V_3' - B_3'\right)}{q_k - q'} \right]$$

$$= \Pr(\theta_3) \cdot \frac{d}{dq_k} \left\{ u\left(V_3 - B_3\right) \right\}$$

$$= \Pr(\theta_3) \cdot u'\left(V_3 - B_3\right) \cdot \frac{d}{dq_k} \left(V_3 - B_3\right)$$

Using the definitions of V_3 and B_3, we get:

$$= \Pr(\theta_3) \cdot u'\left(V_3 - B_3\right) \cdot \left(v^i(q_k, s_i) - b_k\right)$$

$$= \Pr\left(b_k > p^c > b_{k+1} | s_i\right) \left[v(q_k, s_i) - b_k\right] \left[u'\left(V^i - B^i\right) \middle| b_k > p^c > b_{k+1}, s_i \right] \tag{8}$$

For the third term, following the same steps as above and we get:

$$\Pr(\theta_4 \cup \theta_5) \cdot \mathbb{E}\left[u'\left(V_4 - B_4\right) | \theta_4 \cup \theta_5(q_k)\right] \frac{d}{dq_k} \left(V_4 - B_4\right)$$

Since the perturbation does not affect the allocation in these states, the derivative of V with respect to q_k is zero. Thus, we are left with:

$$\Pr(\theta_4 \cup \theta_5) \cdot \mathbb{E}\left[u'\left(V_4 - B_4\right) | \theta_4 \cup \theta_5(q_k)\right] \frac{d}{dq_k} \left(-B_4\right)$$

$$\Rightarrow \Pr\left(b_{k+1} \geq p^c | s_i\right) \cdot \mathbb{E}\left[u'\left(V^i - B^i\right) \middle| b_{k+1} \geq p^c, s_i \right] \left[-\left(b_k - b_{k+1}\right)\right] \tag{9}$$

Now, we are left with the ω-terms. Define $\nu_j := \lim_{q' \to q_k} \omega_j$, where $\in \{2, 3, 4\}$. Note that to generate the movement in states due to the perturbation, it must be that the states in ν_2 are such that the residual supply function is vertical at q_k and it goes through b_k but not b_{k+1}. [See Figure 1(a) for an illustration.] Thus, the allocation in state ν_2 is q'.

Similarly, for states in ν_3 residual supply is vertical at q_k, and goes through b_{k+1} but not b_k, as shown in Fig 1(b). And finally for states in ν_4, residual supply is vertical at b_k and goes through *both* b_k and b_{k+1}, as shown in Fig 1(c). As a result, the allocation in both ν_3 and ν_4 is q_k.

Given these allocations, the utility terms that are subtracted in the ω-terms in equation (5) are equivalent. Hence, all ω-terms will be 0 in the limit.

The difference between the expected values must be 0 in the limit, which gives us the necessary condition:

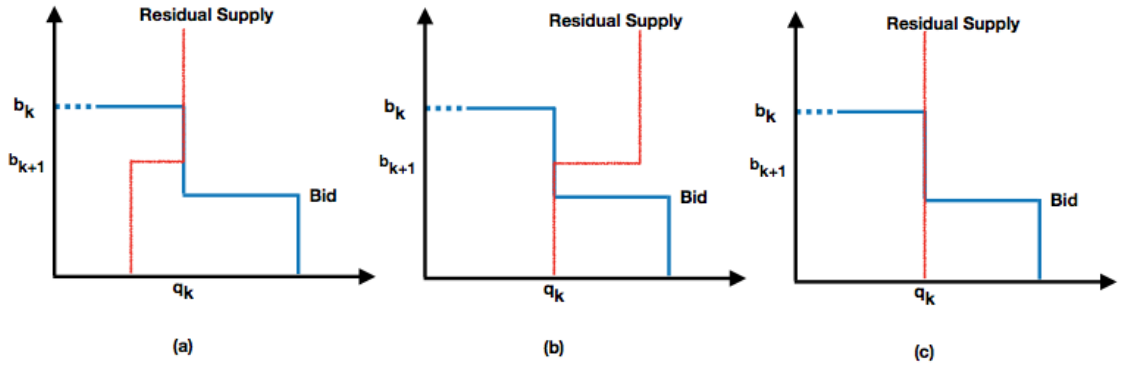


Figure 10: Illustration of the states in v_2, v_3 and v_4

$$\Pr\left(b_k > p^c > b_{k+1}|s_i\right)\left[v(q_k, s_i) - b_k\right]\left[u'\left(V^i - B^i\right)\middle|b_k > p^c > b_{k+1}, s_i\right]$$
$$= \Pr\left(b_{k+1} \geq p^c|s_i\right) \cdot \mathbb{E}\left[u'\left(V^i - B^i\right)\middle|b_{k+1} \geq p^c, s_i\right](b_k - b_{k+1})$$

This completes the proof for Proposition 1. □

9.2 On the consistency of the estimation procedure

In this section, we show the consistency of the resampling estimator for the distribution of market clearing prices. To do that we first define a V-statistic below. Recall that given a K-step bid $(b_k^i, q_k^i)_{k=1}^K$ for bidder i, the share of issue amount demanded by her at any price p is denoted as $y^i(p|s_i)$. Now, fix an auction with total supply Q and the number of bidders $N = N_1 + N_2 + N_3$. Suppose that bidder i belongs to Group 1. Then, conditional on the bid of bidder i, we define an indicator of excess supply at a price p as:

$$\Phi(y^1, ..., y^{N_1-1}, ..., y^{N_1+N_2-1}, ..., y^{N-1}) = \mathbb{1}\left(Q - \sum_{j=1}^{N-1} y^j(p|s_{-i}) \geq y^i(p|s_i)\right)$$

where s_{-i} is the set of signals of all bidders other than i.

Now consider the following V-statistic:

$$\xi(\hat{\Gamma};p) = \frac{1}{(N_1T)^{N_1-1}} \cdot \frac{1}{(N_2T)^{N_2}} \cdot \frac{1}{(N_3T)^{N_3}} \times \sum_{\alpha_1=(1,1)}^{(T,N_1)} \dots \sum_{\alpha_{N_1-1}=(1,1)}^{(T,N_1)} \times \sum_{\alpha_{N_1+1}=(1,N_1+1)}^{(T,N_1+N_2)} \dots \sum_{\alpha_{N_1+N_2}=(1,N_1+1)}^{(T,N_1+N_2)} \times \sum_{\alpha_{N_1+N_2+1}=(1,N_1+N_2+1)}^{(T,N)} \dots \sum_{\alpha_{N-1}=(1,N_1+N_2+1)}^{(T,N)} \Phi(y^{\alpha_1}, \dots y^{\alpha_{N-1}};p)$$

where $\alpha_i = \{(1,1), \dots (T,N)\}$ is the index of the bid in the subsample and $\hat{\Gamma}$ is the empirical distribution of bids if T is the total number of auctions in our sample. This statistic represents the probability with which the market clearing price is weakly lower than p in the data if we draw all possible subsamples (with replacement) of size $(N_1 - 1) + N_2 + N_3$ from the full sample of $N \times T$ data points.

Let us denote the resampling estimator of $\mathbb{P}(p^c \leq p|s_i)$ as $\hat{H}(p)$. Note that $\hat{H}^B(p)$ is a simulator of the statistic $\xi(\hat{\Gamma};p)$ in which only B subsamples are randomly drawn instead of all possible subsamples.

Lemma 1. *Suppose that the data is i.i.d across all T auctions and bidders, all bidders are ex-ante symmetric and N is fixed. Then, as $T \to \infty$ and $\frac{B}{T} \to \infty$, $\hat{H}^B(p) \to \xi(\hat{\Gamma};p)$.*

Proof. Analogous to proof of Lemma 2 in Hortaçsu and Kastl [2012] □

This lemma assumes that the bidders are ex-ante symmetric. In our resampling procedure bidders are symmetric only within the groups but the signals are independent across groups as well. To prove consistency for within group symmetric bidders, we use Theorem 8.1 in Hoeffding (1948) which proves the result in Lemma 1 for the case when the signal for each bidder is drawn from a different distribution.

Proposition 3. *Suppose the data is independent across all auctions and all groups, bidders are symmetric within their groups, and N is fixed. Then, as $T \to \infty$ and $\frac{B}{T} \to \infty$, $\hat{H}^B(p) \to \xi(\hat{\Gamma};p)$.*

Proof. Since $\Phi(\cdot)$ is an indicator function, it is uniformly bounded. Therefore, it satisfies all the conditions for Theorem 8.1 in Hoeffding (1948). Hence, our estimator is consistent. □

The consistency of the resampling estimator of $\mathbb{P}(b_k > p^c > b_{k+1}|s_i)$ can be established in an analogous analysis where the indicator function $\Phi(\cdot)$ is defined with the appropriate strict inequality.

9.3 On the iterative procedure to create bidder groups

In the model, bidders are ex-ante symmetric. In the data, however, there are two clear categories of bidders – those who bid in all the auctions in our sample period (Group 1), and those who enter the auctions intermittently (Group 2). The second group can be divided further into those who only enter during the "dip" period (Group 3). To account for this asymmetry, we differentiate between these three categories in the resampling procedure. We split the total number of bidders in an auction ($N = N_1 + N_2 + N_3$) into three groups, and draw a fixed number of bids N_j from each group j.

Ideally, we would define the groups only on the basis of how many auctions they participate in. However, the number of *purely new entrants* in the 12 auctions during the dip period is not very high. Hence, we use a second criterion to identify the bidders in Group 3 – the average bid shade across the 12 auctions in the dip period. Group 3 consists of the bidders that form the (lowest) 20th percentile of the average bid shade during the taper tantrum period.

To ensure that our definition of Group 3 is robust, we adopt the following iterative algorithm:

Step 1. We start the algorithm by defining only two groups of bidders: those who appear in almost every auction form Group 1 and those who appear less frequently form Group 2. We estimate the model using these groups and obtain the marginal values of each bidder in every auction.

Step 2. We use the estimates to compute the amount by which each bidder shades her bid below her true valuation. We find significant heterogeneity among bidders in Group 2 during the taper tantrum period. This suggests a further division of Group 2 – we define the set of bidders that belong to the lowest 20th percentile of average shade in the crisis period as Group 3.

Step 3. The model is re-estimated using three groups.

Step 4. We identify the set of bidders in the lowest 20th percentile of average shading during the crisis period. If this set coincides with the current definition of Group 3, the algorithm is terminated. Otherwise, we repeat Step 3 with this new set as Group 3.

In practice, there are always some bidders who switch between Groups 2 and 3. However, a fixed subset of the new entrants are *always* among the bidders we identify as Group 3. Note that the composition of Group 1 stays fixed throughout the exercise. The iteration only affects whether a bidder belongs to Group 2 or 3.

9.4 Proof of Proposition 3

Given a K-step bid vector $\left(b_k^i, q_k^i\right)_{k=1}^K$ for bidder i, we want to derive the condition that ensures that the bidder does not have a profitable local deviation. Consider a perturbation of q_k to $q' = q_k - \epsilon$ such that the rest of the quantity-bid vector remains the same, that is, $q'_m = q_m \forall m \neq k$.

Fix the bid and the quantity demanded at the k-th step as (b_k, q_k). Now, the states of world – the market clearing price and the allocation to this bidder – can be partitioned into the following different sets:

$\theta_1(q_k)$: market clearing price is above b_k and the bidder does not win anything on the k-th step. Hence, the perturbation has no effect on these states.

$\theta_2(q_k)$: here the market clearing price is exactly equal to b_k and the bidder is rationed at the k-th step. So, perturbing q_k affects her allocation.

$\theta_3(q_k)$: This set includes all states of the world such that the market clearing price is in interval (b_{k+1}, b_k) and the bidders wins her full k-step demand.

$\theta_4(q_k)$: Here we have all the states in which the market clearing price is exactly equal to b_{k+1}, and the bidder is rationed. Again, the perturbation affects her allocation.

$\theta_5(q_k)$: Finally, when the market clearing price is weakly less than b_{k+1} and the bidder is not rationed on the $k+1$-th step, perturbing q_k does not affect the bidder's payoff. This is because the aggregate share demanded on the $k+1$-th step is the same.

Let us denote market clearing price as p^c and the share that bidder i wins in equilibrium as Q^c. Then, the different states can be summarized as follows:

$$
\begin{aligned}
\theta_1(q_k) \rightarrow \quad & p^c > b_k \quad and \quad Q^c < q_k \\
\theta_2(q_k) \rightarrow \quad & p^c = b_k \quad and \quad Q^c \leq q_k \\
\theta_3(q_k) \rightarrow \quad & b_{k+1} < p^c < b_k \quad and \quad Q^c = q_k \\
\theta_4(q_k) \rightarrow \quad & p^c = b_{k+1} \quad and \quad q_k < Q^c < q_{k+1} \\
\theta_5(q_k) \rightarrow \quad & p^c \leq b_{k+1} \quad and \quad Q^c \geq q_{k+1}
\end{aligned}
$$

Now, due to the perturbation, the market clearing price might decrease which will cause some downward movement in the states. We define $\omega_k(q')$ as follows:

$$\omega_2(q') := \theta_2(q_k) \cap \theta_3(q')$$
$$\omega_3(q') := \theta_3(q_k) \cap \theta_4(q')$$
$$\omega_4(q') := \theta_2(q_k) \cap \theta_4(q')$$

The set $\omega_2(q')$ includes the states in which bidder i was rationed at price b_k and after perturbing q_k to q' she gets her full demand. $\omega_3(q')$ are the states which move from $\theta_3(q_k)$ to $\theta_4(q')$, that is, the bidder originally received her full demand at k-th step but now she will be rationed at the $k+1$-step and will win a higher quantity. Finally, the set $\omega_4(q')$ is made up of the states in which the bidder was rationed at k-th step, and will now be rationed at $k+1$-th step. Hence, we can express the probabilities of the sets $\theta_j(q')$ using the $\omega_j(q')$ as follows:

$$\mathbb{P}(\theta_2(q')) = \mathbb{P}(\theta_2(q_k)) - \mathbb{P}(\omega_2(q')) - \mathbb{P}(\omega_4(q'))$$
$$\mathbb{P}(\theta_3(q')) = \mathbb{P}(\theta_3(q_k)) + \mathbb{P}(\omega_2(q')) - \mathbb{P}(\omega_3(q'))$$
$$\mathbb{P}(\theta_4(q')) = \mathbb{P}(\theta_4(q_k)) + \mathbb{P}(\omega_3(q')) - \mathbb{P}(\omega_4(q'))$$

To derive our local optimality condition, we want to compute the following limit:

$$\lim_{q' \to q_k} \frac{\mathbb{E}_{s_{-i}} u(s_i|q_k) - \mathbb{E}_{s_{-i}} u(s_i|q')}{q_k - q'} \tag{10}$$

As discussed above, the perturbation affects the payoff in states $\theta_2, \theta_3 \; and \; \theta_4$. Hence, we can write the above equation(10) as:

$$\lim_{q' \to q_k} \frac{\sum_{j=2}^{4} \left\{ \mathbb{E}_{s_{-i}} \left[u(V_j^i - B_j^i); \theta_j(q_k) \right] - \mathbb{E}_{s_{-i}} \left[u(V_j^i - B_j^i); \theta_j(q') \right] \right\}}{q_k - q'} \tag{11}$$

Let us evaluate the different components under the summation of equation (11) one by one.

State θ_3.

Define the gross utilities and payments as follows:

$$V_3 = \int_0^{q_k} v^i(x, s_i)dx \quad ; \quad V_3' = \int_0^{q'} v^i(x, s_i)dx$$
$$B_3 = q_k p^c(q_k), \quad B_3' = q' p^c(q') \quad ; \quad \bar{B}_3 = q' p^c(q_k)$$

Then:

$$\begin{aligned}&\mathbb{E}\left[u(V_3 - B_3) - u(V_3' - B_3'); \theta_3\right] \\ &= \mathbb{E}\left[u(V_3 - B_3) - u(V_3' - \bar{B}_3) + u(V_3' - \bar{B}_3) - u(V_3' - B_3'); \theta_3\right]\end{aligned}$$

Taking the limit $q' \to q_k$:

$$\left(\frac{1}{q_k - q'}\right)\left\{\lim_{q' \to q_k} \mathbb{E}\left[u(V_3 - B_3) - u(V_3' - \bar{B}_3); \theta_3\right] + \lim_{q' \to q_k} \mathbb{E}\left[u(V_3' - \bar{B}_3) - u(V_3' - B_3'); \theta_3\right]\right\}$$
$$= \mathbb{E}\left[u'(V_3 - B_3) \cdot \left\{v^i(q_k, s_i) - p^c(q_k)\right\}; \theta_3\right] + \frac{\partial}{\partial q}\mathbb{E}\left[u\left(V_3 - q_k p^c(q)\right); \theta_3\right]$$

therefore, we have:

$$\begin{aligned}= \mathbb{P}(b_k > p^c > b_{k+1}) \cdot \mathbb{E}\left[u'\left(V^i - q_k p^c\right) \cdot \left\{v^i(q_k, s_i) - p^c\right\} \middle| b_k > p^c > b_{k+1}\right] \\ + \frac{\partial}{\partial q}\mathbb{E}\left[u\left(V^i - q_k p^c(q)\right); b_k > p^c > b_{k+1}\right] \quad (12)\end{aligned}$$

State θ_2.

The set $\theta_2(q_k)$ consists of all the states of world in which the market clearing price is exactly equal to bidder i's k^{th} step b_k. Hence, the bidder will win all her demand at the $(k-1)^{th}$ step but could be rationed at the k^{th} step. Let us denote the amount she wins as $q_{k-1} + c_2$ where $c_2 \in (q_{k-1}, q_k]$. Then, her gross utility and payment can be written as:

$$V_2 = \int_0^{q_{k-1}+c_2} v^i(x, s_i)dx \quad ; \quad V_2' = \int_0^{q_{k-1}+c_2'} v^i(x, s_i)dx$$
$$B_2 = (q_{k-1} + c_2)p^c(q_k), \quad B_2' = (q_{k-1} + c_2')p^c(q') \quad ; \quad \bar{B}_2 = (q_{k-1} + c_2')p^c(q_k)$$

Then:

$$\begin{aligned}&\mathbb{E}\left[u(V_2 - B_2) - u(V_2' - B_2'); \theta_2\right] \\ &= \mathbb{E}\left[u(V_2 - B_2) - u(V_2' - \bar{B}_2) + u(V_2' - \bar{B}_2) - u(V_2' - B_2'); \theta_2\right]\end{aligned}$$

Taking the limit $q' \to q_k$:

$$\left(\frac{1}{q_k - q'}\right)\left\{\lim_{q' \to q_k} \mathbb{E}\left[u(V_2 - B_2) - u(V_2' - \bar{B}_2); \theta_2\right] + \lim_{q' \to q_k} \mathbb{E}\left[u(V_2' - \bar{B}_2) - u(V_2' - B_2'); \theta_2\right]\right\} \quad (13)$$

Note that the expression $\left[u(V_2 - B_2) - u(V_2' - \bar{B}_2)\right]$ in the first term of the equation above differs only in the amount that is allocated to bidder i in the states that belong to

the set θ_2. It follows from Lemma A1 in Kastl [2012] that whenever a bidder is rationed it is because she is the marginal bidder. So, the perturbation does not affect the allocation in state θ_2. Thus, $\left[u(V_2 - B_2) - u(V_2' - \bar{B}_2)\right] = 0$, and equation (13) simplifies to:

$$\frac{\partial}{\partial q}\mathbb{E}\left[u\left(V^i - q_k p^c(q)\right); b_k = p^c\right] \tag{14}$$

State θ_4.

The analysis for states in θ_4 is analogous to the one for θ_2 with the difference that now the market clearing price is exactly equal to b_{k+1} and so the bidder is rationed at her $(k+1)^{th}$ step. Following the steps exactly as for state θ_2 we get:

$$\lim_{q' \to q_k}\left(\frac{1}{q_k - q'}\right)\mathbb{E}\left[u(V_4 - B_4) - u(V_4' - B_4'); \theta_4\right]$$

$$= \frac{\partial}{\partial q}\mathbb{E}\left[u\left(V^i - q_k p^c(q)\right); b_{k+1} = p^c\right] \tag{15}$$

This completes the analysis for all the different cases.[30] Finally, combining equations (12), (14) and (15), we conclude that:

$$\lim_{q' \to q_k} \frac{\mathbb{E}_{s_{-i}} u(s_i|q_k) - \mathbb{E}_{s_{-i}} u(s_i|q')}{q_k - q'} = 0$$

$$\Rightarrow \mathbb{P}(b_k > p^c > b_{k+1}) \cdot \mathbb{E}\left[u'\left(V^i - q_k p^c\right)\{v^i(q_k, s_i) - p^c\}\middle| b_k > p^c > b_{k+1}\right]$$
$$+ \frac{\partial}{\partial q}\mathbb{E}\left[u(V^i - q_k p^c(q)); b_k \geq p^c \geq b_{k+1}\right] = 0$$

Hence, proved. □

References

S. Athey and P. A. Haile. Nonparametric approaches to auctions. In J. J. Heckman and E. E. Leamer, editors, *Handbook of Econometrics, Volume 6A*, chapter 60, pages 3847–3965. Elsevier, 2007.

L. M. Ausubel, P. Cramton, M. Pycia, M. Rostek, and M. Weretka. Demand reduction and inefficiency in multi-unit auctions. *The Review of Economic Studies*, 81(4):1366–1400, 2014.

[30] We invoke Lemma A1 from Kastl [2012] to claim that $\lim_{q' \to q_k} \omega_{jk}(q') = 0$.

P. Bajari and A. Hortaçsu. Are structural estimates of auction models reasonable? evidence from experimental data. *Journal of Political Economy*, 113(4):703–741, 2005.

K. Basu, B. Eichengreen, and P. Gupta. From tapering to tightening: The impact of the Fed's exit on India. In S. Shah, A. Panagariya, and S. Gokarn, editors, *India Policy Forum 2014-15, Volume 11*, chapter 1, pages 1–66. Sage Publications India Pvt Ltd, 2015.

B. S. Bernanke. Tantrums and hot money: How does Fed policy affect global financial stability? *Brookings Blog*, January 6, 2016.

B. Biais, L. Glosten, and C. Spatt. Market microstructure: A surveyof microfoundations, empirical results, and policy implications. *Journal of Financial Markets*, 8(2):217–264, 2005.

N. Cassola, A. Hortaçsu, and J. Kastl. The 2007 subprime market crisis through the lens of European Central Bank auctions for short-term funds. *Econometrica*, 81(4): 1309–1345, 2013.

D. Duffie and H. Zhu. Size discovery. *The Review of Financial Studies*, 30(4):1095–1150, 2017.

P. Esö and L. White. Precautionary bidding in auctions. *Econometrica*, 72(1):77–92, 2004.

M. Friedman. *A Program For Monetary Stability*. Fordham University Press, 1960.

M. Friedman. How to sell government securities? *Wall Street Jounral*, August 28, 1991.

E. Guerre, I. Perrigne, and Q. Vuong. Nonparametric identification of risk aversion in first-price auctions under exclusion restrictions. *Econometrica*, 77(4):1193–1227, 2009.

S. Gupta, R. Lamba, and O. Muratov. First price auctions with risk averse bidders and common uncertainty. New York University and Penn State University, 2017.

J. Hasbrouck. *Empirical Market Microstructure: The Institutions, Economics, and Econometrics of Securities Trading*. Oxford University Press, 2007.

A. Hortaçsu. Bidding behavior in divisible good auctions: Theory and evidence from the Turkish treasury auction market. University of Chicago, 2002.

A. Hortaçsu and J. Kastl. Valuing dealers' informational advantage: A study of Canadian treasury auctions. *Econometrica*, 80(6):2511–42, 2012.

A. Hortaçsu and D. McAdams. Mechanism choice and strategic bidding in divisible good auctions: An empirical analysis of the turkish treasury auction market. *Journal of Political Economy*, 118(5):833–865, 2010.

A. Hortaçsu and D. McAdams. Empirical work on auctions of multiple objects. University of Chicago and Duke University, 2016.

B.-S. Kang and S. L. Puller. The effect of auction format on efficiency and revenue in divisible goods auctions: A test using Korean treasury auctions. *Journal of Industrial Economics*, 56(2):290–332, 2008.

J. Kastl. Discrete bids and empirical inference in divisible good auctions. *The Review of Economic Studies*, 78(3):974–1014, 2011.

J. Kastl. On the properties of equilibria in private value divisible good auctions with constrained bidding. *Jounral of Mathematical Economics*, 48(6):339–352, 2012.

J. Kastl. Recent advances in empirical analysis of financial markets: industrial organization meets finance. Princeton University, 2016.

V. Krishna. *Auction theory*. Elsevier, 2010.

A. Kyle. Informed speculation with imperfect competition. *The Review of Economic Studies*, 56(3):317–355, 1989.

J. Lu and I. Perrigne. Estimating risk aversion from ascending and sealed-bid auctions: the case of timber auction data. *Journal of Applied Econometrics*, 23(7):871–896, 2008.

E. Maskin and J. Riley. Optimal auctions with risk averse buyers. *Econometrica*, 52(6): 1473–1518, 1984.

H. J. Paarsch and H. Hong. *An introduction to the structural econometrics of auction data*. MIT Press, 2006.

R. Sahay, V. Arora, T. Arvanitis, H. Faruqee, P. N'Diaye, and T. Mancini-Griffoli. Emerging market volatility: Lessons from the taper tantrum. *IMF Satff Discussion Note*, 2014.

The Economist. Mismatch point: The rise of the dollar will punish borrowers in emerging markets, 2015. *March 2015 Print Edition*.

X. Vives. Asset auctions information and liquidity. *Journal of European Economic Association*, 8(2-3):467–477, 2010.

X. Vives. Strategic supply function competition with private information. *Econometrica*, 79(6):1919–1966, 2011.