Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 6, 2018

Instructions

This exam has four questions and is worth 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly.

Use WORDS to explain your reasoning.

Good luck!

1. (25 pts) Consider a binary relation $\succsim$ on $\mathbb{R}^L_+$.

   (a) (4 pts) Define what it means for $\succsim$ to be representable by a function $u : \mathbb{R}^L_+ \to \mathbb{R}$.

   (b) (4 pts) Suppose $\succsim$ is representable. What are all the conditions $\succsim$ must therefore satisfy? (Define these conditions, don't just name them.) Prove your answer.

   (c) (4 pts) What general conditions on $\succsim$ are sufficient for it to be representable? (Again, define these conditions, don't just name them.)

   (d) (13 pts) Sketch a proof of your answer to (c), under the extra assumption that $\succsim$ is monotone (in the MWG sense that $x \succ y$ if $x \gg y$).

2. (25 pts) Consider a competitive one-output firm with a production function $f : \mathbb{R}^{L-1}_+ \to \mathbb{R}$ that is C^2 and has a strictly positive gradient and a negative definite Hessian matrix at each $z \in \mathbb{R}^{L-1}_+$. Assume that at a given initial price vector (p, w), the firm demands a positive amount of each input. Prove the following statements, where each "increase" and "decrease" is meant in the strict sense.

   (a) (15 pts) An increase in the output price always increases the profit-maximizing level of output.

   (b) (5 pts) An increase in the output price increases the demand for some inputs.

   (c) (5 pts) An increase in the price of an input leads to a decrease in the demand for that input.

3. (25 pts)

   (a) (6 pts) State carefully a fixed point theorem.

   (b) (7 pts) State carefully a general equilibrium theorem that uses this theorem and explain how the theorem is used in the proof.

   (c) (6 pts) For the theorem you identified in (b), the conclusion will not necessarily hold if the economy is such that the assumptions of the fixed point theorem are not satisfied. Give an example of an economy for which the conclusion of the general equilibrium theorem fails, and describe how it fails.

   (d) (6 pts) In (c) you were asked to describe an economy for which the conclusion of the general equilibrium theorem failed. Give an example of an economy for which the conclusion of the general equilibrium theorem in (b) HOLDS, but for which the assumptions of the fixed point theorem do NOT hold.

4. (25 pts) Consider an exchange economy with one physical commodity, two states of the world, $j = 1, 2$, and two agents, $i = 1, 2$. Let $p \in (0, 1)$ be the probability of state 1. The agents are von Neumann-Morgenstern expected utility maximizers, and agent i has a concave differentiable utility function, $u_i : \mathbb{R} \to \mathbb{R}$, satisfying $u_i' > 0$. Denote a state contingent allocation by (x^1, x^2), where $x^i = (x_1^i, x_2^i)$ and x_j^i is agent i's consumption of the commodity in state j. Denote the aggregate endowment of the commodity in state j by w_j.

   (a) (7 pts) Suppose the aggregate quantity of the commodity is the same in each state. What are the competitive equilibrium prices? (Assume the equilibrium allocation is strictly positive.)

   (b) (6 pts) Suppose the aggregate quantity of the commodity is greater in the first state: $w_1 > w_2$. Show that if u_2 is strictly concave, then any strictly positive Pareto efficient allocation (x^1, x^2) satisfies $x_1^1 > x_2^1$.

   (c) (6 pts) Suppose the agents' endowments are $e^1 = (2, 0)$ and $e^2 = (0, 1)$, that $u_1(x) = x$, and that u_2 is strictly concave. Explain concisely which of the agents will benefit more from trading to a competitive equilibrium allocation, and why.

   (d) (6 pts) Suppose again that the aggregate quantity of the commodity is the same in each state, but that the agents disagree about the probability of state 1. Agent i believes the probability is p_i, with $p_1 > p_2$. What can you say about the the quantities agent 1 consumes in the two states, x_1^1 and x_2^1, at a strictly positive Pareto efficient allocation (x^1, x^2)?