Microeconomic Theory II
Preliminary Examination
August 6, 2018

The exam is worth 120 points in total.

There are 3 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. [**40 points.**] Bruce and Sheila have to share a pie, which has a cherry on top. To keep notation straight, denote Bruce as player 1 and Sheila as player 2. Both Bruce and Sheila would like more of the pie, as well as the cherry, with agent $i \in \{1,2\}$ valuing the cherry at $\theta_i \in [0,1]$. A division of the pie is denoted by $x \in [0,1]$, where x is the size of the slice with the cherry, with the other slice having size $1-x$ (it is not possible to divide the cherry). Assume the cherry has zero width, so that $x = 0$ is a feasible choice (effectively, one person gets the cherry and the other gets the pie). An allocation is (x,i) where $i \in \{1,2\}$ is the recipient of the slice with the cherry. Payoffs are

$$v_j(x,i) = \begin{cases} \theta_j + x, & \text{if } j = i, \\ 1-x, & \text{if } j \neq i, \end{cases}$$

for $i, j \in \{1,2\}$. Suppose Bruce and Sheila agree to Bruce dividing the pie into two slices, and Sheila choosing which slice to take (with Bruce taking the remaining slice).

   (a) Suppose the values of θ_1 and θ_2 are public information. Describe the extensive form *and* the normal form of the game in which Bruce divides the pie. What is the backward induction solution? Why are there multiple solutions when $\theta_1 = \theta_2$? [**15 points**]

   (b) Suppose agent i's value of the cherry is private information, known only by agent i. It is common knowledge that the values are uniformly and independently distributed on $[0,1]$. Describe the resulting game of incomplete information. [**10 points**]

   (c) Describe the restrictions implied by weak perfect Bayesian equilibrium. What is the weak perfect Bayesian equilibrium? [**15 points**]

[**Question 2 is on the next page.**]

2. [**40 points.**] A seller can either exert effort (H) or not (L) in the production of a good. A buyer must decide, not knowing the effort choice of the seller, whether to buy (B) the good or not (D). The resulting game is (with the seller being the row player and the buyer the column player):

	B	D
H	$3,4$	$-1,0$
L	$4,2$	$0,0$

(a) Describe the set of feasible payoffs and the subset of individually rational payoffs. What is the largest lower bound on the set of subgame perfect equilibrium seller payoffs in the infinitely repeated game (and why)? [**10 points**]

(b) Suppose the game is infinitely repeated, and both buyer and seller have discount factor $\delta \in (0,1)$. For some values of δ, there is a simple pure strategy equilibrium of the infinitely repeated game with outcome path $(HB)^\infty$. Describe it, and the bounds on δ for which the profile is a subgame perfect equilibrium. [**20 points**]

(c) How would your answer to part (b) change if the buyer makes his purchase decision after observing the seller's effort choice? [**10 points**]

[**Question 3 is on the next page.**]

3. **[40 points.]** A community of size n must decide on whether to provide a public good. The *per capita* cost of the public good is $c > 0$, and if the public good is provided, all agents contribute c. Agent valuations for the good are private (known only to the agent). Suppose each agent i's valuation for the public good net of per capita cost, denoted v_i, is distributed according to the distribution F with density f and support $[\underline{v}, \bar{v}]$, independently of the other agents. Assume $\underline{v} < 0 < \bar{v}$ (so some types value the public good less than its per capita cost). An allocation is $(p, \tau_1, \ldots, \tau_n) = (p, \tau)$, where p is the probability of of provision of the public good and τ_i is the expected adjustment to the contribution of c from agent i towards the public good. The payoff to agent i is $pv_i - \tau_i$. Budget balance requires the adjustments to contributions sum to zero, i.e.,

$$\sum_{i=1}^{n} \tau_i = 0.$$

(a) Carefully describe an incentive-compatible direct mechanism. Why can the community restrict attention to incentive compatible direct mechanisms when maximizing expected revenue? **[5 points]**

(b) Describe the class of Groves mechanisms that efficiently provide the public good. What attractive property do Groves mechanisms have in terms of strategic behavior? **[10 points]**

For the remainder of this question, use the following notation: If $(p, \tau_1, \ldots, \tau_n)$ is an incentive-compatible direct mechanism, denote the expected payoff if an agent with type v_i from reporting $\hat{v}_i$ by

$$U_i(\hat{v}_i, v_i) := \int_{\underline{v}}^{\bar{v}} \cdots \int_{\underline{v}}^{\bar{v}} [p(\hat{v}_i, v_{-i})v_i - \tau_i(\hat{v}_i, v_{-i})] \prod_{j \neq i} F(dv_j) =: p_i(\hat{v}_i)v_i - t_i(\hat{v}_i)$$

and the expected payoff from reporting $\hat{v}_i = v_i$ by

$$U_i^*(v_i) := U_i(v_i, v_i).$$

(c) Suppose $n = 2$ and $\underline{v} + \bar{v} < 0$. Prove that no Groves mechanism can be budget balanced and satisfy voluntary participation (assume the payoff from nonparticipation is 0). **[15 points]**

(d) Outline the proof that for general incentive-compatible direct mechanisms

$$U_i^*(v_i) = U_i^*(\underline{v}) + \int_{\underline{v}}^{v_i} p_i(\tilde{v}_i) d\tilde{v}_i.$$

[10 points]