Preliminary Examination

Econ 702-Macroeconomics
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Instructions: There is one question, divided in several subquestions. The number in brackets represents the number of points awarded for answering correctly the each subquestion. Total available points are 100. If the description of the environment seems incomplete to you, explain why, make the assumptions that you deem necessary to proceed and continue. Good Luck!

1. **Political Economy of PAYG Systems.** Consider the following OLG model. The measure of agents born in period $t \geq 1$ is $G_t = G_0 \gamma^t$ with $\gamma > 1$. Every agent lives for two periods (young and old). When young, the agent is endowed with $e_1 > 0$ units of the consumption good. When old, the agent is endowed with $e_2 = 0$ units of the consumption good. Agents have preferences over consumption when young and when old described by the utility function $u(c_1, c_2) = c_1^{1/2} \cdot c_2^{1/2}$. There is also a measure $G_0 = 1$ of old agents in period $t = 1$. They have preferences described by $v(c_2) = A c_2^{1/2}$ with $A > 0$.

    The government considers introducing a Pay As You Go (PAYG) system, according to which the young of generation $t = 1, 2, ...$ contribute a fraction $\tau \in [0, 1]$ of their endowment to the system. The government then pays out the revenues collected from the young of generation t to the old of generation $t - 1$.

    a. (5) Define a Recursive Equilibrium for the economy under the proposed PAYG system. .

    b. (5) Solve for the equilibrium consumption and real interest rate.

    c. (10) Compute the equilibrium welfare of each generation $t = 1, 2, ...$. Compare it with their welfare without a PAYG system. Then compare it with their welfare if the PAYG system is implemented starting in period $t = 2$ rather than in $t = 1$.

    The PAYG proposal is not politically feasible. To see why this is the case, note that the young of generation $t = 1$ (who are the majority of people alive in period $t = 1$) are better off delaying the start of the PAYG system to $t = 2$. However, in $t = 2$, the young of generation $t = 2$ are better off delaying the start of the PAYG system to $t = 3$, etc...

    We now look for a PAYG system that is politically feasible. Specifically, we look for a PAYG system described by a sequence of tax rates $\{\tau_1, \tau_2, \tau_3, ...\}$ with the following properties: (i) The young of generation $t = 1$ would rather implement the system at $t = 1$ than delay its implementation to $t = 2$. Assume that, if the plan is delayed to $t = 2$, the sequence of tax rates will still be τ_1, τ_2, ... That is, the time subscript in τ_t denotes periods since the implementation of the PAYG system rather than calendar time. (ii) Every generation $t = 1, 2, ...$ enjoys the same welfare V. (iii) All the tax rates are such that $\tau_t \in [0, 1]$ for $t = 1, 2, ...$ Condition (i) guarantees that the plan will be implemented right away. Condition (ii) guarantees that, at no point in the future, the majority will want to restart the PAYG plan from scratch.

d. (5) Using condition (ii) find a difference equation for the tax rates, i.e. an expression that gives τ_{t+1} as a function of the tax rate τ_t (given some arbitrary welfare level V).

e. (10) Plot the difference equation in the $\{\tau_t, \tau_{t+1}\}$ space. Is the locus of solutions to the difference equation increasing? concave?

f. (5) Identify the interval of τ_1's such that the PAYG plan is such that $\tau_t \in [0, 1]$ for $t = 1, 2, ...$ (If such an interval exists).

g. (5) Graphically illustrate the effect of increasing the welfare V. Discuss what happens when V is high enough.

h. (10) Solve for the highest possible welfare V^* for which there is a solution to the difference equation with the property that $\tau_t \in [0, 1]$ for $t = 1, 2, ...$

i. (5) Solve for the steady-state tax rate τ^* associated with welfare V^*.

j. (10) Using condition (i) find a restriction on τ_1 and τ_2 such that the young of generation $t = 1$ would rather implement the PAYG system today than delay the implementation to $t = 2$.

k. (10) Illustrate the answer to the previous question in a graph with the 45 degree line, the difference equation for the tax rate associated with V^*.

l. (10) Under what conditions on τ_1, there is a PAYG system such that every generation $t = 1, 2, ...$ attains the same welfare V^*, the sequence of taxes is $\tau_t \in [0, 1]$ for $t = 1, 2, ...$, and such that the young of generation $t = 1$ do not want to delay the implentation of the plan?

m. (10) What is the property of the tax rate in such an equilibrium? Why does the size of the PAYG system (measured by the tax rate) grow over time?