Microeconomic Theory II
Preliminary Examination Solutions

1. **(45 points)** Consider the following normal form game played by Bruce and Sheila:

		Sheila	
		L	R
	T	$-1, 0$	$3, 3$
Bruce	M	$1, x$	$0, 0$
	B	$0, 0$	$4, 1$

(a) Suppose $x = -1$. What are all the pure and mixed strategy Nash equilibria of this game when $x = -1$? When $x = 1$? [5 points]

Solution: Since T is strictly dominated by B, Bruce does not play T in any Nash equilibrium, irrespective of the value of x.

Suppose $x = -1$. R strictly dominates L, and so the game has a unique Nash equilibrium: BR.

Suppose $x = 1$. The game has two pure strategy Nash equilibria, ML and BR, and one mixed strategy equilibrium, $(\frac{1}{2} \circ M + \frac{1}{2} \circ B, \frac{4}{5} \circ L + \frac{1}{5} \circ R)$. ■

Consider now the **once** repeated game, with players observing first period play before making second period choices. Payoffs in the repeated game are the sum of payoffs in the two periods.

(b) Describe the information sets and pure strategies of the extensive form of the repeated game. [5 points]

Solution: Bruce has 7 information sets, the initial information set (or null history) $\varnothing$, and the information sets reached by the six possible first period action profiles $a^0 \in \{T, M, B\} \times \{L, R\}$. Sheila has a similar collection of information sets. Denoting Bruce and Sheila's information sets by the histories that reach them, $H = \{\varnothing, \{T, M, B\} \times \{L, R\}\}$ denotes the collection of information sets for both Bruce and Sheila.

A pure strategy for Bruce is a mapping $\sigma_B : H \to \{T, M, B\}$, and a pure strategy for Sheila is a mapping $\sigma_S : H \to \{L, R\}$. ■

(c) Suppose $x = -1$. Describe a pure strategy Nash equilibrium of the once repeated game in which Bruce plays T in the first period. Is it subgame perfect? Explain why or why not. [5 points]

Solution: A simple profile with the needed properties specifies TR in the first period, BR in the second period after T is played in the first, and ML if Bruce deviates in the first period. Sheila's deviations are ignored. More formally, the profile is (σ_B, σ_S), where

$$\sigma_B(h) = \begin{cases} T, & \text{if } h = \varnothing, \\ B, & \text{if } h = Ta_2^0, \text{ and} \\ M, & \text{otherwise.} \end{cases}$$

and

$$\sigma_S(h) = \begin{cases} R, & \text{if } h = \varnothing, \\ R, & \text{if } h = Ta_2^0, \text{ and} \\ L, & \text{otherwise.} \end{cases}$$

This profile is Nash: Bruce is optimizing in the second period, and in the first period, following σ_B gives a payoff of $3 + 4 = 7$, while deviating to B in the first period yields at most $4 + 1 = 5 < 7$.

Sheila's deviations are ignored and she is myopically optimizing.

Hence both players are playing a best response.

The profile is not subgame perfect: the second period behavior after BR is ML, which is not a Nash equilibrium of the stage game when $x = -1$. ■

(d) Suppose $x = +1$. Describe a pure strategy subgame perfect equilibria of the once repeated game in which Bruce plays T in the first period. [5 points]

Solution: We use the same profile (σ_B, σ_S) as in part 1(c). The only change in analysis is that since now $x = +1$, the specification of second period behavior after BR of ML is a Nash equilibrium of the stage game. Hence, the specification of second period behavior after every history is a Nash equilibrium of the stage game.

■

Consider now the **infinitely** repeated game, with players observing past play before making choices (so that the game has perfect monitoring). Payoffs in the infinitely repeated game are the average discounted sum of payoffs, with discount factor $\delta \in (0, 1)$.

(e) Suppose $x = +1$. For some values of δ, there is a simple pure strategy equilibrium of the infinitely repeated game with outcome path $(TR)^\infty$. Describe it, and the bounds on δ for which the profile is a subgame perfect equilibrium. [10 points]

Solution: The desired profile is the trigger profile in which TR is played on the equilibrium path and any deviation triggers Nash reversion: play ML thereafter. The automaton representation is $\{\mathcal{W}, w^0, f, \tau\}$, where

- the set of states is $\mathcal{W} = \{w_{TR}, w_{ML}\}$;
- the starting state is $w^0 = w_{TR}$;
- the output function is $f(w_a) = a$;
- the transition function is

$$\tau(w_a, a') = \begin{cases} w_{TR} & \text{if } a' = TR \text{ and } w_a = w_{TR}, \\ w_{ML} & \text{otherwise.} \end{cases}$$

The automaton is illustrated in Figure 1.

It is immediate that $V_B(w_{TR}) = V_S(w_{TR}) = 3$, and $V_B(w_{ML}) = V_S(w_{ML}) = 1$.

The profile is subgame perfect if no player has an incentive to deviate in any state.

Since w_{ML} is absorbing, and ML is a Nash equilibrium of the stage, no player has an incentive to deviate in that state.

In state w_{TR}, Sheila has no incentive to deviate (for all δ), since she is myopically optimizing *and* $V_S(w_{TR} \geq V_S(w_{ML})$.

Bruce has no incentive to deviate if

$$V_B(w_{TR}) = 3 \geq 4(1 - \delta) + \delta V_B(w_{ML}) = 4 - 3\delta$$

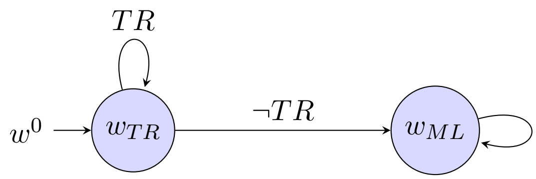

Figure 1: The automaton from Question 1(e).

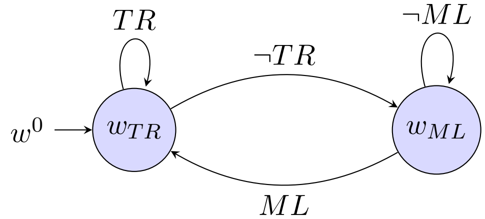

Figure 2: The first automaton from Question 1(f).

$$\iff \delta \geq \frac{1}{3}.$$

Thus the profile is subgame perfect if and only if $\delta \geq \frac{1}{3}$. ■

(f) Suppose $x = -1$. For some values of δ, there is a pure strategy equilibrium of the infinitely repeated game with outcome path $(TR)^\infty$. This profile is necessarily more complicated than the profile from part 1(e). Why? Describe it, and the bounds on δ for which the profile is a subgame perfect equilibrium. [15 points]

Solution: The profile in part 1(e) is a trigger profile using Nash reversion as the punishment. But when $x = -1$, Nash reversion is not available as a punishment: the stage game has a unique Nash equilibrium, and in this equilibrium, Bruce has a higher payoff than at TR.

One possible profile uses ML as a punishment, but since Sheila has an incentive not to play L, the profile must provide incentives for Sheila to play L when specified. These incentives are provided by having play return to TR only after Sheila has played L.

The desired profile has the automaton representation $\{\mathcal{W}, w^0, f, \tau\}$, where

- the set of states is $\mathcal{W} = \{w_{TR}, w_{ML}\}$;
- the starting state is $w^0 = w_{TR}$;
- the output function is $f(w_a) = a$;
- the transition function is

$$\tau(w_a, a') = \begin{cases} w_{TR} & \text{if } a' = TR \text{ and } w_a = w_{TR}, \text{ or } a' = ML \text{ and } w_a = w_{ML}, \\ w_{ML}, & \text{otherwise.} \end{cases}$$

The automaton is illustrated in Figure 2.

It is immediate that $V_B(w_{TR}) = V_S(w_{TR}) = 3$. Moreover,

$$\begin{aligned} & V_B(w_{ML}) = (1-\delta) + \delta V_B(w_{TR}) = 1 + 2\delta, \\ \text{and} \qquad & V_S(w_{ML}) = -(1-\delta) + \delta V_S(w_{TR}) = -1 + 4\delta. \end{aligned}$$

As before, subgame perfection requires no player have an incentive to deviate in any state. Since both players receive a higher payoff in state w_{TR} than in w_{ML}, in states where a player

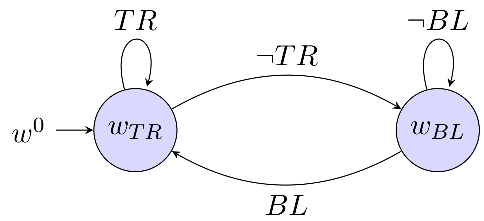

Figure 3: The second automaton from Question 1(f).

is myopically optimizing, that player has no incentive to deviate. Thus, we need only check Bruce in w_{TR} and Sheila in w_{ML}.

Bruce is optimizing in w_{TR} if

$$\begin{aligned} V_B(w_{TR}) = 3 \geq 4(1-\delta) + \ \delta V_B(w_{ML}) &= 4 - 3\delta + 2\delta^2 \\ \iff 0 \geq 1 - 3\delta + 2\delta^2 &= (1-\delta)(1-2\delta), \end{aligned}$$

which requires $\delta \geq \frac{1}{2}$.

Sheila is optimizing in w_{ML} if

$$\begin{aligned} V_S(w_{ML}) &\geq 0 \times (1-\delta) + \delta V_S(w_{ML}) \\ \iff V_S(w_{ML}) &\geq 0 \iff \delta \geq \frac{1}{4}. \end{aligned}$$

Thus the profile is subgame perfect for $\delta \geq \frac{1}{2}$.

Another possible profile uses BL as a punishment. Since (again) Sheila has an incentive not to play L, this profile must also provide incentives for Sheila to play L when specified. These incentives are provided by having play return to TR only after Sheila has played L.

The desired profile has the automaton representation $\{\mathcal{W}, w^0, f, \tau\}$, where

- the set of states is $\mathcal{W} = \{w_{TR}, w_{BL}\}$;
- the starting state is $w^0 = w_{TR}$;
- the output function is $f(w_a) = a$;
- the transition function is

$$\tau(w_a, a') = \begin{cases} w_{TR} & \text{if } a' = TR \text{ and } w_a = w_{TR}, \text{ or } a' = BL \text{ and } w_a = w_{BL}, \\ w_{BL}, & \text{otherwise.} \end{cases}$$

The automaton is illustrated in Figure 3.

It is immediate that $V_B(w_{TR}) = V_S(w_{TR}) = 3$. Moreover,

$$\begin{aligned} & V_B(w_{BL}) = 0 \times (1-\delta) + \delta V_B(w_{TR}) = 3\delta, \\ \text{and} \qquad & V_S(w_{BL}) = 0 \times (1-\delta) + \delta V_S(w_{TR}) = 3\delta. \end{aligned}$$

As before, subgame perfection requires no player have an incentive to deviate in any state. Since both players receive a higher payoff in state w_{TR} than in w_{BL}, in states where a player is myopically optimizing, that player has no incentive to deviate. Thus, we need only check Bruce in w_{TR} and Sheila in w_{BL}.

Bruce is optimizing in w_{TR} if

$$V_B(w_{TR}) = 3 \geq 4(1-\delta) + \ \delta V_B(w_{BL}) = 4 - 4\delta + 3\delta^2$$

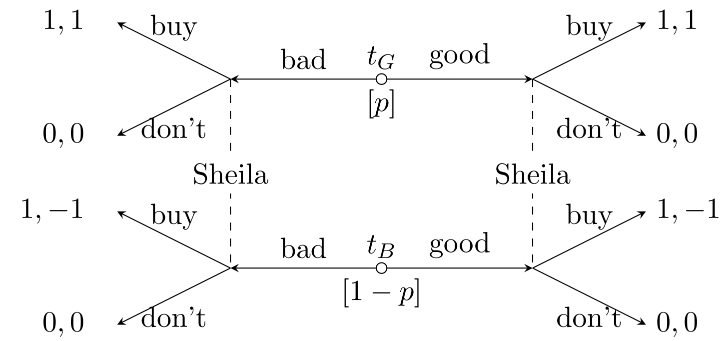

Figure 4: The extensive form for Question 2(a), where p is the probability that Sheila assigns to Bruce's car being of good quality (Bruce having type t_G).

$$\iff 0 \geq 1 - 4\delta + 3\delta^2 = (1-\delta)(1-3\delta),$$

which requires $\delta \geq \frac{1}{3}$.

Sheila is optimizing in w_{BL} if

$$V_S(w_{BL}) \geq 1 \times (1-\delta) + \delta V_S(w_{BL})$$
$$\iff V_S(w_{BL}) \geq 1 \iff \delta \geq \frac{1}{3}.$$

Thus the profile is subgame perfect for $\delta \geq \frac{1}{3}$.

■

2. **(15 points)** Bruce owns a car he would like to sell to Sheila. Only Bruce knows the quality of the car: it is either good or bad (a lemon). Sheila wants to buy the car only if it is good. Bruce can try to convince Sheila about the car's quality before Sheila makes her purchase decision. To keep things simple, suppose that Bruce receives a payoff of 1 if he sells the car, and 0 otherwise, and that Sheila receives a payoff of +1 from buying a good car, a payoff of −1 from buying a bad car, and 0 otherwise.

   (a) Describe the associated extensive form game, where we interpret "Bruce can try to convince Sheila about the car's quality" as Bruce can announce "good" or "bad" before Sheila makes her decision, and this announcement has not direct payoff implications. Be precise in any additional assumptions you must make to obtain a fully specified extensive form game. [3 points]

   Solution: Bruce has two types, t_G and t_B, reflecting the car's quality. As a function of his type, Bruce makes an announcement about the car's quality. Sheila then, as a function of Bruce's announcement, makes her purchase decision. The extensive form is in Figure 4 for the case of two announcements.

   A critical part of the specification is the belief that Sheila assigns to Bruce having a good quality car, and Bruce's beliefs about that belief. In the extensive form, nature determines the type of Bruce, and this distribution is both Sheila's beliefs and known by Bruce.

   ■

   (b) Is there an equilibrium of this game in which Sheila buys the car if and only if it is good? Why or why not? [5 points]

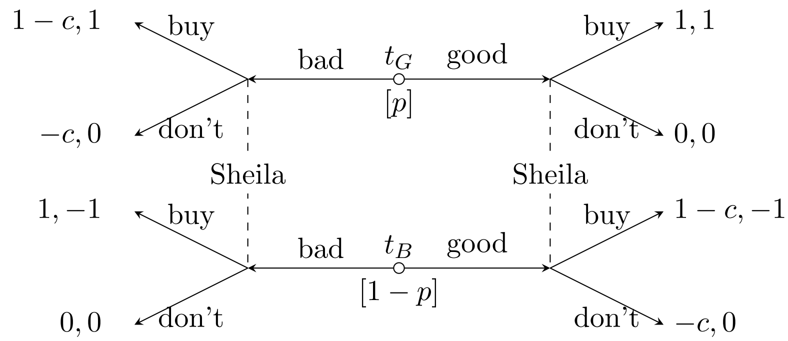

Figure 5: The extensive form for Question 2(c), where p is the probability that Sheila assigns to Bruce's car being of good quality (Bruce having type t_G).

Solution: There is no such equilibrium: Proof is by contradiction. If there were such an equilibrium, then one of Bruce's announcements of the car's quality must lead to Sheila buying, and another to not buying. Since Bruce always wants to sell his car, irrespective of the car's quality, Bruce will make an announcement that induces Sheila to buy. Thus, Sheila makes her purchase decision ignoring Bruce's announcement, and only on the basis of her prior beliefs. ■

(c) Suppose that in addition to wishing to sell his car, Bruce does not like lying. How would you alter Bruce's payoffs to reflect this? How does this change your answer to part 2(b) (you may restrict attention to pure strategies)? [7 points]

Solution: Bruce's payoffs now depend upon both his type and announcement, as well as whether Sheila purchases his car. Let c denote the Bruce's disutility from lying. The extensive form is in Figure 5 for the case of two announcements.

If $c < 1$, then Bruce will still try to induce Sheila to buy, even if he must lie. That is, there is still no pure strategy equilibrium in which Sheila's decision depends on Bruce's announcement. [[If $p < \frac{1}{2}$, there is a mixed strategy equilibrium in which Bruce announces "good" if t_G, randomizes with probability $p/(1-p)$ on "good" if t_B, and Sheila randomizes with probability c on "buy" and $1-c$ on "don't" after "good" and does not buy after "bad".]]

However, if $c \geq 1$, there is an equilibrium in which Bruce truthfully reveals the quality of the car, and Sheila only buys the good car.

■

3. **(35 points)** Suppose that the payoff to a firm from hiring a worker of type θ with education e at wage w is

$$f(e, \theta) - w = 4e\theta^2 - w.$$

The utility of a worker of type θ with education e receiving a wage w is

$$w - c(e, \theta) = w - \frac{e^4}{\theta}.$$

The worker's ability is privately known by the worker. There are at least two firms. The worker (knowing his ability) first chooses an education level $e \in \mathbb{R}_+$; firms then compete for the worker by simultaneously announcing a wage; finally the worker chooses a firm. Treat the wage determination as in class, a function $w : \mathbb{R}_+ \to \mathbb{R}_+$ determining wage as a function of eduction.

Suppose the support of the firms' prior beliefs ρ on θ is $\Theta = \{\theta_L, \theta_H\}$ where $\theta_L = 1$ and $\theta_H = 4$.

(a) What is the full information education level for each type of worker? [5 points]

Solution: The social surplus from a worker of type θ taking education level e is:

$$4e\theta^2 - \frac{e^4}{\theta}.$$

Taking first order conditions,

$$4\theta^2 - \frac{4e^3}{\theta} = 0,$$
$$\implies e = \theta,$$

i.e., the full information education level for worker of type θ is $e = \theta$. ■

(b) Is there a perfect Bayesian equilibrium in which both types of worker choose their full information eduction level? Be sure to verify that all the incentive constraints are satisfied. [5 points]

Solution: The strategy profile $\{e, w\}$ constitutes a perfect Bayesian equilibrium

$$e(\theta) = \theta; w(e) = \begin{cases} 4e & \text{if } e < 4, \\ 64e & \text{if } e \geq 4, \end{cases}$$

with beliefs $\mu(e)$ which place all mass on $\theta = 1$ if $e \neq 4$, and all mass on $\theta = 4$ if $e = 4$.

To verify incentive compatibility observe that on path, a worker of type 1 has a net utility of 3. However, if she deviates to to type 4's education level, she makes a net utility of 0, so this deviation is not profitable.

Similarly, a worker of type 4 on path has a net utility of 192, while by deviating to type 1's education level, she makes a net utility $\frac{15}{4}$. Therefore, this deviation is not profitable.

Next, to verify that this is a PBE, one should carefully check that neither type of worker wishes to deviate to any other education level.

For the low type—deviating to any education level $e' \neq 4$ will still induce a belief that the agent is of low type. By part a), $e = 1$ is education level for an agent of low type when the firms infer she is of low type. So any deviation is not profitable.

For the high type, deviating to any education level $e' \neq 4$ will induce a belief that the agent is of low type. The optimal such deviation solves $\arg\max 4e - e^4/4$. Taking first order conditions, we get $e' = 16^{\frac{1}{3}}$ is the optimal such deviation, plugging back in a simple calculation gets that $4.16^{\frac{1}{3}} - 16^{\frac{4}{3}}/4 < 192$, so no such deviation is profitable.

■

(c) Suppose e_L is the education level undertaken by θ_L, while e_H is the education level taken by θ_H in a separating Perfect Bayesian Equilibrium. What can you conclude about e_H and e_L? Be as precise as possible. [10 points]

Solution: In any separating PBE, the low type of agent takes his full information optimal education level, i.e. $e_L = 1$. Why? Suppose not, suppose there is a separating equilibrium where the low type of agent takes an education level e'_L different from 1. On observing e'_L, firms must believe that the agent is type θ_L for sure, and therefore offer a wage $4e'_L$. In this case deviation to taking an education level of $e_L = 1$ is profitable—we have already argued that

$$4e_L - e_L^4/\theta_L = \max 4e - e^4/\theta_L,$$

and therefore

$$4e_L - e_L^4/\theta_L > 4e'_L - e_L'^4/\theta_L.$$

Next, note that since this is a separating equilibrium, the firms pay $64e_H$ when seeing e_H (and, as we argued before, 4 when seeing $e_L = 1$).

It is sufficient to consider beliefs in the following form:

$$\mu(e) = \begin{cases} 1 \circ \theta_H & \text{if } e = e_H \\ 1 \circ \theta_L & \text{otherwise} \end{cases}.$$

In this form of proposed equilibria, the low type worker gets 3 and the high type worker gets $64e_H - e_H^4/4$. Given the beliefs specified, the best possible deviation for type $\underline{\theta}$ is e_H; to get the best possible deviation for type $\bar{\theta}$, we solve

$$\max_e U(4, 1, e), \text{ i.e. } \max_e 4e - \frac{e^4}{4}$$

The best possible deviation is to an education level as $4^{\frac{1}{3}}$ and the high type will get $3(4^{\frac{1}{3}})$. The sufficient and necessary conditions such that the strategies of both types are incentive compatible are thus

$$3 \geq 64e_H - e_H^4;$$
$$64e_H - e_H^4/4 \geq 3(4^{\frac{1}{3}}).$$

■

(d) Suppose the firms' prior beliefs ρ are that the worker has type θ_H with probability $\frac{2}{3}$, and type θ_L with probability $\frac{1}{3}$. Is there a pooling Perfect Bayesian equilibrium in this setting? If yes, describe a pooling PBE and argue that it is one. If not, why not? [10 points]

Solution: Yes, a pooling PBE exists.

Consider a putative PBE in which both types of worker take the same education level $e^\star$. Further firms' posterior on observing this education level is the same as their prior, if they observe any other education level their posterior puts all mass on θ_L.

Therefore if the firms observe $e^\star$, they believe the agent is of high type with probability $\frac{2}{3}$ and low type with probability $\frac{1}{3}$. Their expected payoff from hiring such a worker and offering him a wage of w is:

$$\begin{aligned} &\frac{2}{3} \times (4e^\star\theta_H^2) + \frac{1}{3} \times (4e^\star\theta_L^2) - w \\ =&\frac{128}{3}e^\star + \frac{4}{3}e^\star - w \\ =&44e^\star - w. \end{aligned}$$

Since firms are competetive they make 0 profits, i.e. both offer a wage of $44e^\star$. For any other education level $e \neq e^\star$, they offer a wage of $4e$.

Note that a worker of type θ_L who takes the equilibrium education level gets a surplus of $44e^\star - e^{\star 4}$. His optimal education level were he to deviate is $e = 1$ for a net surplus of 3. Therefore for our putative pooling equilibrium to be one, it must be the case that

$$44e^\star - e^{\star 4} \geq 3.$$

Similarly, if a θ_H type deviates, his optimal education level is $2^{\frac{2}{3}}$, for a net surplus of $3 \times 2^{\frac{2}{3}}$ Therefore, it must be the case that

$$44e^\star - \frac{e^{\star 4}}{4} \geq 3 \times 2^{\frac{2}{3}}.$$

Picking $e^\star = 2$ (say) clearly satisfies both inequalities. ■

Now suppose the support of the firms' prior beliefs on θ is $\Theta = \{1, 3, 4\}$.

(e) Is there a perfect bayesian equilibrium in which each type of worker chooses her full information education level? [5 points]

Solution: No. In the putative full information education level equilibrium, a worker of type 3 takes education level 3 and earns a wage of $4 \times 3 \times 3^2 = 108$, for a net utility of 81.

By deviating and taking education level 4, she earns a wage of $4 \times 4 \times 4^2 = 256$, for a net utility of $\frac{512}{3}$ which is larger than 81. ■

4. **(25 points)** A seller sells to a buyer with type $\theta \in \Theta = [-1, 1]$. If a buyer of type θ receives the good with probability q in return for a payment of p, she has a net utility of:

$$u(q, p|\theta) = e^{(\theta^2)}q - p.$$

The seller uses a direct revelation mechanism. The buyer announces his type θ and receives the good with probability $q(\theta)$ in return for a payment of $p(\theta)$.

(a) From first principles, state and prove necessary and sufficient conditions on $q(\cdot)$ for there to exist a price $p(\cdot)$ such that $(q(\cdot), p(\cdot)$ satisfies IC. Further, derive $p(\cdot)$ as a function of $q(\cdot)$ in this setting. In other words, derive and prove the counterpart of the Fundamental IC Lemma in this setting. [20 points]

Solution: First, clearly, $q(\theta)$ must be (weakly) decreasing from $[-1, 0]$ and then (weakly) increasing from $[0, 1]$.

Next, for $\theta \in [0, 1]$, consider the change of variables $\gamma = e^{(\theta^2)}$, i.e. $\gamma \in \Gamma = [1, e]$. Formally: Define functions $p^\Gamma(\cdot)$ and $q^\Gamma(\cdot)$ as $p^\Gamma(\gamma) = p(\theta), q^\Gamma(\gamma) = p(\theta)$ for $\gamma = e^{(\theta^2)}, \theta \in [0, 1]$. We now have the standard revenue equivalence formula:

$$p^\Gamma(\gamma) = \gamma q^\Gamma(\gamma) - \int_1^\gamma q^\Gamma(g)dg - c,$$

where c is the standard constant/ utility of type $\theta = 0$. Doing the change of variables back, we have

$$p(\theta) = e^{\theta^2}q(\theta) - \int_0^\theta q(e^{t^2})2te^{t^2}dt - c.$$

An analogous argument is used to derive the payment formula for $\theta \in [-1, 0]$.

Finally, one needs to consider the incentive constraint corresponding to θ misreporting as $-\theta$. Observe that the surplus of a buyer of type θ is $\int_0^\theta q(e^{t^2})2te^{t^2}dt - c$, and the surplus from misreporting is $\int_0^{-\theta} q(e^{t^2})2te^{t^2}dt - c$. Therefore it must be the case that for $\theta \in (0, 1]$,

$$\int_0^\theta q(e^{t^2})2te^{t^2}dt = \int_0^{-\theta} q(e^{t^2})2te^{t^2}dt.$$

By observation, it must be that $q(\theta) = q(-\theta)$ for $\theta \in (0, 1]$.

Combining all these results together, we note that type θ and $-\theta$ are treated identically. To verify that our conditions are sufficient, it is therefore enough to argue that no type $\theta \in [0, 1]$ has incentive to deviate to any $\theta' \in [0, 1]$. Note that type θ makes a net surplus of $\int_0^\theta q(e^{t^2})2te^{t^2}dt + c$ if she reports truthfully. If she misreports as θ', she makes a net surplus of

$$(e^{\theta^2} - e^{\theta'^2})q(\theta') + \int_0^{\theta'} q(e^{t^2})2te^{t^2}dt + c$$

Comparing, note that we need that:

$$\int_{\theta'}^{\theta} q(e^{t^2})2te^{t^2}dt \geq (e^{\theta^2} - e^{\theta'^2})q(\theta').$$

This holds for both $\theta' > \theta$ and $\theta' < \theta$ from the monotonicity of q. ■

(b) The seller's type is $\omega \in \Omega = [1, 2]$, and if she sells the the good to the buyer with probability q in return for a payment of p, she has a net utility of:

$$u(q, p|\theta) = p - e^{(\frac{\omega}{2})}q.$$

Suppose the buyer's and seller's types are private to them. Suppose further the buyer's type is drawn uniformly from Θ, while the seller's type is drawn uniformly from Ω, and the two draws are independent. Does there exist an incentive compatible trading mechanism that is individually rational and has no expected subsidy? Why or why not? [5 points]

Solution: No such mechanism. Do the change of variables as above and point out that it clearly satisfies the overlap condition of the Myerson Satterthwaite Impossibility theorem. ■