

Rehypothecation and Intermediary Leverage

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This paper provides a theory of endogenous leverage through rehypothecation in collateralized intermediation. Overcollateralization with an rehypothecation option arises as an optimal contract form between broker dealers and their clients to mitigate adverse selection on collateral quality. Such contract prevents the broker dealers from taking advantage of private information on collateral quality, thus enabling them to repledge the collateral with lower margins or resell it with lower discounts, and obtain "money for nothing". This type of unsecured funding increases the broker dealers' risk-shifting incentive. The clients expecting this will increase the compensation requirement for granting rehypothecation rights, which affects the broker dealers' optimal choice of leverage through rehypothecation. As collateral becomes riskier, this leverage first increases due to the margin spread, and then decreases due to the adverse selection problem. When the broker dealers and their clients are trading in over-the-counter market, their leverage through rehypothecation is inefficiently too high when the quality of collateral starts to become questionable, and too low when it is too questionable.

I. Introduction

Motivation.— The recent financial crisis features sizable collateral liquidity dry up, in which 40% is due to the reduction of collateral velocity¹. Collateral circulates in the form of re-use or rehypothecation², in which banks or other broker-dealers use the collateral neither initially owned by themselves nor outrightly purchased from other sellers.

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¹According to Singh (2012), the overall amount of collateral declined by one half, and the collateral velocity declined by 20% during 2007-2010.

²These two terms both refer to the practice of using the collateral not initially owned by the user or outrightly purchased from the seller, and are sometimes used interchangeably in the U.S. Outside U.S., the term re-use applies if collateral is posted on the basis of title transfer, while rehypothecation applies if the collateral is pledged. The right of re-use is an part of property right from title transfer, while the right of rehypothecation should be granted by the pledgor. This paper will focus on rehypothecation.

However, following the collapse of Lehman Brother, hedge funds in particular started to insist on contracts that limit rehypothecation, so as to reduce counterparty risk. With this striking fact, two questions arise naturally. First, if rehypothecation involves counterparty risk, then why do market participants voluntarily expose themselves to such risk in the first place. Second, is such exposure efficient or not? The purpose of this article is to provide a first theoretical framework to address these questions.

The answer is not obvious. First, the asset taker side counterparty risk does not exist if the asset is outrightly sold, thus we need microfoundation for collateral contract. Second, such risk is not problematic if the asset taker does not have incentive to increase it, thus we need to model why the taker's receivable cash cannot fully rule out this incentive. Third, in the highly interconnected market where one asset taker is simultaneously trading with multiple asset givers, the contractual term in each bilateral transaction cannot affect the risk-taking incentive of the taker as a whole, which means there must be another reason for credit rationing in rehypothecation.

The existing literature does not provide a satisfactory solution. The most classic approach relies on liquidation discount, as in Hart and Moore (1994)³. The asset giver does not sell the collateral simply because it is worth less to the taker. In this approach, the loan repayment in the optimal bilateral contract will not be lower than the liquidation value of collateral, which means the loan repayment has sufficiently secured the asset giver from losing the collateral, and granting rehypothecation right does not impose any additional cost on the giver. The most recent approach is based on the asset taker side adverse selection problem, as in Gorton and Ordoñez (2014). The asset giver promises to buy back the collateral just to reduce the taker's incentive of information acquisition. When collateral contract is used, no information on the collateral value is revealed, and credit rationing in rehypothecation cannot happen in the interconnected market. Other approaches are even less relevant. For example, the "skin in the game" approach formalized by Holmstrom and Tirole (1997) does not apply in the situation when the collateral has already been created as in the repo and derivative market, and the search approach in Monnet (2012) can hardly incorporate two-sided risk which is the crux in rehypothecation.

My paper borrows an insight from Demarzo and Duffie (1999) that adverse selection can be alleviated by risk retention, and builds a model with optimal contract preventing information leakage along the collateral chain, information rent tied to the right of rehy-

³In Hart and Moore (1994), entrepreneurs do not sell their project because value of human capital is lost after the ownership transfer. Lorenzoni (2008) endogenizes the liquidation discount in a general equilibrium framework. Lacker (2001) further proves that the liquidation discount is necessary for a collateral contract to exist if the only friction is in ex post state verification. Sirto (2012) analyzes the effect of this friction in a dynamic equilibrium framework. In Fostel and Geanakoplos (2008) and Simsek (2013), belief disagreement has a similar flavor, since now the liquidation discount is due to the difference in subjective valuation.

pothecation, and a funding spread between the initial use and re-use of the same piece of collateral. This model is flexible enough to allow for insufficient rehypothecation due to information rent in re-use, and potentially too much leverage due to the collateral taker side risk-shifting. These two mechanisms feed back to each other in an equilibrium of over-the-counter (OTC hereafter) market, not allowing for quick conclusion on efficiency issues.

A Stripped-Down Version.— A risk-neutral firm is trading with a risk-neutral bank. The firm has access to a riskless investment project with variable size and net return rate $r^F > 0$. The bank has a similar project except that it is risky with expected net return rate $r^e \in (0, r^F)$. Both projects are non-pledgeable and require cash input. Since the firm's project is superior, the first best allocation is to invest all cash into the firm's project. Assume that the firm cannot commit⁴ to pay back the bank after the project matures, then, cash transfer in terms of unsecured loan is not implementable. Now suppose the firm holds a bond with expected value 1, and can sell the bond to the bank, but it involves adverse selection if the firm knows the true value of the bond v_θ with $\theta \in \{H, L\}$, and the bank only knows the distribution of v_θ . The probability of (v_H, v_L) is $(p, 1 - p)$. Assume the bank makes a take-it-or-leave-it (TIOLI) offer. If only the firm with v_L can sell the bond due to adverse selection, then the expected firm investment is $\frac{1-p}{1+r^F} v_L$.

Collateral contract can help alleviate the adverse selection. Without secondary market, there are two equivalent ways to improve the allocation. The firm can either sell a tranche of the bond, or directly pledge the bond. In both cases, the contract is pooling, the bank does not learn v_θ , and both firms can invest $\frac{1}{1+r^F} v_L$. Otherwise, if the bank also has commitment problem in the secondary market, the whole bond can provide additional 1 unit of liquidity to the bank while the tranche can only provide v_L . In another word, rehypothecation provides more liquidity than securitization.

Compared with tranching, rehypothecation has two problems. First, adverse selection leads to insufficient re-use. Since the bank's investment project is risky, it is possible that the bank cannot return the bond. It incurs some cost to the economy not because the bond is misallocated, but because it reintroduces adverse selection. More specifically, suppose there is a probability $1 - q$ of bank failure, and denote $\Delta v = v_H - v_L$, then in order to have both firms willing to trade, it is necessary to provide some compensation to the firm with v_H . The firm investment size becomes $\frac{v_L + (1-q)\Delta v}{1+r^F}$, which is larger if the bank risk and bond return spread is larger. In this sense, collateral re-use improves the

⁴This is not surprising in high frequently trading where the enforcement cost is much higher than the transaction returns. We can also assume that the firm's project has a negligible risk of failure which is not verifiable to the bank.

allocation by reducing the monopolistic rent of the bank. However, the bank has to offer the same term and thus providing information rent $\frac{1-q}{1+r^F} \Delta v$ to the firm with v_L to prevent mimicking. Apparently, the surplus created by the additional firm investment $r^F \frac{1-q}{1+r^F} \Delta v$ is very likely to be smaller than the expected information rent $(1-p) \frac{1-q}{1+r^F} \Delta v$. In this case, adverse selection becomes a private cost to the bank, and the bank might not be willing to request rehypothecation right even if it generates positive surplus.

Second, moral hazard leads to too much rehypothecation. When the bank re-uses the bond, there is no adverse selection in the secondary market. Assume the bank still makes the offer, then the expected liquidity provided by the bond is 1, which is larger than the expected cash payment v_L from the firm. Such a spread resembles a unsecured funding source and introduces risk-shifting incentive to the bank.

The full-fledged model makes extensions along three dimensions. First, the contract space between the firm and bank is fully flexible, and all tradeoff in the optimal contract can be shown analytically. Second, Δv has a distribution, so that we can discuss the rise and fall of safe asset, as well as collateral contingent macro prudential policies. Third, in the equilibrium, both adverse selection and moral hazard are endogenous, thus the feed back effect between them can also be discussed.

Main Insights.— The main insights are summarized in the following.

(i) Collateral contract instead of outright selling contract is used because the former enables partial transactions on the collateral with v_h , which is not possible in outright selling when the adverse selection problem is severe.

(ii) The bank can obtain a funding spread through collateral intermediation because the optimal collateral contract between the bank and the pledging firms are pooling, which does not allow information leakage. As a result, the bank does not have adverse selection problem when re-using the collateral in the secondary market.

(iii) Even though collateral contract enables all collateral to be fully pledged, the pledgors who have private information still extract information rent when rehypothecation right is granted, because the firm with v_L collateral can now mimic the other type and requires more compensation for the occasionally collateral lost.

(iv) Exposing to counterparty risk is just a way to reduce the information friction in asset tradings, and increases the liquidity of the asset.

(v) The extent of private information on the collateral value creates two problems in the collateral chain. First, when rehypothecation is possible, information rent drives the private gains from requesting rehypothecation right below the private surplus. Second, when rehypothecation right is granted in an OTC market, the private cost of granting

rehypothecation is lower than the social cost, leading to uninternalized risk-shifting. The private adverse selection and the collective moral hazard interacts in the equilibrium

(vi) When Δv has a distribution, the margin of information rent depends on the marginal Δv , while the margin of risk-shifting depends on the accumulated Δv up to the marginal Δv . As a result, this tradeoff can be summarized in a few sufficient statistics.

Relationship to the Literature: As far as I know, this is the first paper to discuss systemic risk in rehypothecation. Eren (2014) models rehypothecation as a way of funding for the dealer bank. Infante (2014) explains the funding spread of in rehypothecation, and the corresponding pledgor side bank run. Bottazzi, Luque and Páscoa (2012) and Maurin (2014) explicitly incorporate rehypothecation in a general equilibrium framework. Gottardi and Kubler (2015) models the possibility of cross-netting in a dynamic derivative market, and discuss the efficiency issue. Andolfatto, Martin and Zhang (2014) focuses on the efficiency of collateral circulation, but does not distinguish between outright selling and repo contract. All of these papers have two limitations. First, they do not carefully explain why collateral contract with rehypothecation right is used in asset trading, even if it involves higher counterparty risk. Second, they do not model the endogenous granted rehypothecation right, hence are not relevant to macro prudential policies which target on systemic risk.

This paper also provides an alternative model for the shadow banking system. Similar to the originate-to-distribute (OTD) model (see Vanasco 2013), my model also has interaction between adverse selection and moral hazard, and the inefficiency also arises from the no commitment problem⁵ in adverse selection. The difference is that the OTD model has moral hazard in the primary market and adverse selection in the secondary market, while in my model, it is reversed. Figure 1 demonstrates the comparison.

Another important contribution is to discuss the interaction between contracting externality and information frictions in affecting the systemic risk. I incorporate moral hazard and adverse selection into private contract as in McAfee and Schwartz (1994), Segal (1999) and Acharya and Bisin (2014), and naturally apply it to the problem of rehypothecation. Different from the standard private contract models, my model can discuss how information asymmetry affects the externality in private contract models. Also, this paper combines the insight from the literature of privately optimal but socially inefficient financial contract, such as Lorenzoni (2008), and the network externalities, such as Far-

⁵OTC refers to over-the-counter market in which transactions are unobservable to the public. The bank thus cannot commit to the contract menus, leading to OTC externality.

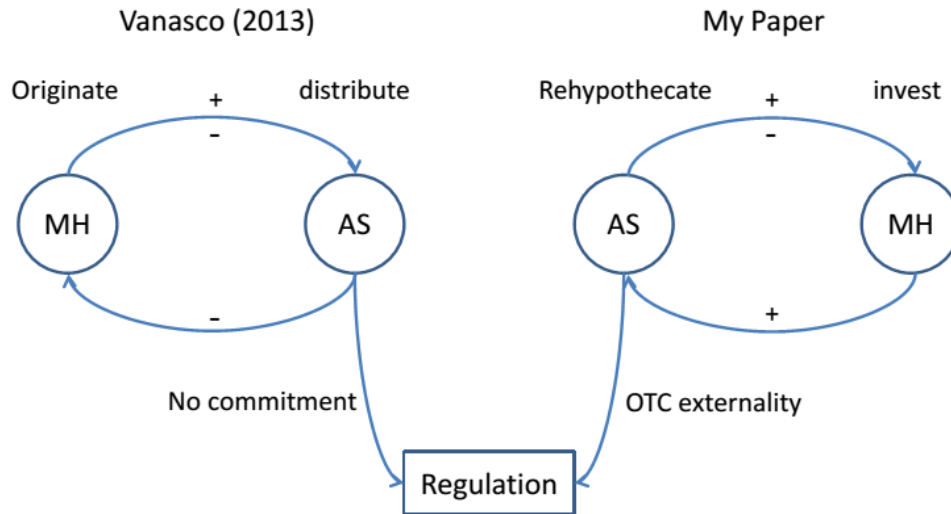


FIGURE 1. THE COMPARISON BETWEEN ORIGINATE-TO-DISTRIBUTE AND COLLATERAL REUSE

boodi (2014). Hence, this framework can be used to discuss how financial frictions affect the interconnectness of the financial market. In addition, I also sheds light on the social value of safe asset, as in Hanson and Sunderam (2013). But differently, creating safe asset through credit transformation affects the efficiency not because of the information externality, but because of the tradeoff between private adverse selection and collective moral hazard.

II. Rehypothecation Contract

In this section I model rehypothecation as optimal contract facilitating security trading. I first lay out the basic model with frictions that prevents efficient transactions. Then I build a flexible framework allowing for continuous forms of contracts and show when overcollateralization with an rehypothecation option can be optimal.

A. Setting

Trading Motive. Consider a model in which a large bank, "the principal", is trading with a continuum $[0, 1]$ of ex ante identical small firms, "the agents". For convenience, they are denoted as B and F respectively hereafter. Both the bank and the firms are risk neutral and maximizing their payoff π^τ defined later on, where $\tau \in \{B, F\}$. Each firm has superior investment technology requiring cash input, but is only endowed with bonds normalized to 1 unit. The bank has superior technology to liquidate the bond, but is only

endowed with cash of value $C > 0$. The first best allocation can be implemented by having the firms selling their bonds to the bank, and the bank giving loans to the firms as much as possible.

Frictions. Two frictions impede the first best allocation. First, suppose the value of each unit of bond v has a distribution with a two point support, or

$$v = \begin{cases} v_H = 1 + (1 - p) \Delta v & \text{with probability } p > 0 \\ v_L = 1 - p \Delta v & \text{with probability } 1 - p > 0 \end{cases} .$$

Each firm has either $v = v_H$ or $v = v_L$, and it is ex ante private information. Denote $\theta \in \{H, L\}$ as the type of bond, and $\Delta v = v_H - v_L > 0$ as the spread of values. If Δv is large enough, then outright selling is subject to adverse selection, and only the bond with v_L can be sold. Second, suppose bankruptcy procedure is too costly in high frequency trading, and without bankruptcy, only the bond can be seized. Then, the repayment to unsecured loans cannot be enforced.

Technology. Investments are constant return-to-scale. The firms have homogenous risk free net return rate $r^F > 0$, while the bank only has risky r^B with

$$r^B = \begin{cases} r^B(1) = \frac{r^e}{q} + \frac{1-q}{q} & \text{with probability } q > 0 \\ r^B(0) = -1 & \text{with probability } 1 - q \geq 0 \end{cases} .$$

Denote $s \in \{0, 1\}$ of the state of bank return, and $r^e = \mathbb{E}^q [r^B(s)]$ as the expected bank return. Assume $r^e \in [0, r^F)$.

ASSUMPTION 1 (Secret Keeper): *The bank can sell bond at its expected value in the secondary market, and buy it back in the same price at any time.*

B. Financial Contract

Contract. The bank makes a take it or leave it (TIOLI) menu offer $X = \{x_H, x_L\}$ to all firms, in which $x_\theta = (l_\theta, \lambda_\theta, (b_\theta(s), d_\theta(s))_{s \in \{0,1\}})$. This contract has two stages. In the contracting stage, each firm transfers all⁶ their bond to the bank and the bank transfers cash with value $l > 0$ to the firm. In the settlement stage, $b(s) > 0$ units of bonds and $d(s) > 0$ value of cash are returned at state s if the firm does not breach the contract. Otherwise, nothing is returned. See Figure 2 for illustration. In addition, the contract also specifies the restriction of reuse and requires the bank to have a segregated account

⁶Partial collateral transfer is not optimal, and not mentioned here for simplicity and clarity. However, the reason is non-trivial before we do not know which contract terms will be used as the screening device. See the appendix for details.

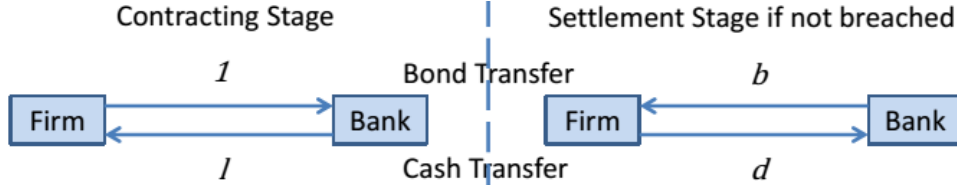


FIGURE 2. TWO STAGES OF THE CONTRACT

for part of the collateral. Since $r^B(0) = -1$, $b(0)$ cannot exceed the size of segregated account which cannot exceed $b(1)$. $\lambda b(1)$ is the size with $\lambda \in [0, 1]$, and $b(0) \leq \lambda b(1)$. The feasibility constraints can be summarized as

$$(l_\theta, \lambda_\theta, b_\theta(1), b_\theta(0), d_\theta(1), d_\theta(0)) \in \mathbb{R}_+ \times [0, 1] \times [0, 1] \times [0, \lambda_\theta b_\theta(1)] \times \mathbb{R}_+ \times \mathbb{R}_+.$$

Denote $\mathcal{X} \subseteq \mathbb{R}_+^{6 \times 2}$ as the set of all feasible contract menus.

Timing. The timing of this paper can be summarized in the following

- 1) Each firm learns θ .
- 2) The bank offers $X \in \mathcal{X}$. Each firm accepts $x_{\bar{\theta}}$, and invests $l_{\bar{\theta}}$.
- 3) The bank reuses the bond not in the segregated account.
- 4) The bank invests $I^B(X) = C + \mathbb{E}^P[-l_\theta + (1 - \lambda_\theta b_\theta(1))v_\theta]$.
- 5) s realizes, and $(r^F, r^B(s))$ pays off.
- 6) $(b_\theta(s), d_\theta(s))$ is implemented if the contract is not breached.
- 7) v pays off.

Payoffs. The payoffs of firms and the bank are the expected values of their positions.

$$\begin{aligned} \pi^F(x_{\bar{\theta}}, v_\theta | q) &= r^F l_{\bar{\theta}} + (l_{\bar{\theta}} - v_\theta) + \mathbb{E}^q[\max\{b_{\bar{\theta}}(s)v_\theta - d_{\bar{\theta}}(s), 0\}], \\ \pi^B(X, q) &= r^e I^B(X) + \mathbb{E}^P[r^F l_\theta - \pi^F(x_\theta, v_\theta | q)]. \end{aligned}$$

Optimal Contract. The optimal contract is a feasible menu maximizing $\pi^B(X, q)$

$$\begin{aligned} \text{s.t. } (IC_{\theta\bar{\theta}}) \pi^F(x_\theta, v_\theta) &\geq \pi^F(x_{\bar{\theta}}, v_\theta), \\ (IR_\theta) \pi^F(x_\theta, v_\theta) &\geq 0. \end{aligned}$$

C. Using and Reusing Collateral

In this subsection, I will solve the optimal contract problem, and analyze the trade-off in using and reusing collateral. The following assumption on parameters is needed.

ASSUMPTION 2 (Parameters): $(r^F, p, \Delta v)$ satisfies

$$(1 - p - r^F) \frac{\Delta v}{v_H} > pr^F.$$

The optimal contract problem here differs from the standard screening problem by introducing limited enforcement. Due to hidden types, we cannot simply impose enforcement constraints, and need further justification. The following lemma shows how.

LEMMA 1 (Enforcement): *The set of contracts in which $b_\theta(s) v_L - d_\theta(s) \geq 0$ is weakly payoff dominant, when Assumption 2 is imposed.*

Consider a situation where the bank is offering a pooling contract to all firms, and the enforcement constraints for L firm is binding. Since the enforcement constraint for H firm is still slack, a potential way for the bank to increase payoff is to raise l_H by ε and d_H by $(1 + r^F)\varepsilon$. However, the L firm may have incentive to mimic the H firm and then breaches the contract in the settlement stage. A potential way to prevent this from happening is to also raise l_L but keep d_L unchanged. However, such an alternative contract does not yield higher payoff for the bank if r^F is sufficiently small. A proof for the general case is shown in the appendix.

Given Lemma 1, the standard property of the screening problem that (IC_{LH}) and (IR_H) bind while (IC_{HL}) and (IR_L) are redundant also holds here. Then, l_θ can be expressed as a function of $(b_\theta(s), d_\theta(s))$

$$\begin{aligned} l_H &= \frac{v_H - \mathbb{E}^q [b_H(s) v_H - d_H(s)]}{1 + r^F}, \\ l_L &= \frac{v_L - \mathbb{E}^q [b_L(s) v_L - d_L(s)]}{1 + r^F} + \frac{\mathbb{E}^q [1 - b_H(s)] \Delta v}{1 + r^F}. \end{aligned}$$

Substituting them into the bank's payoff function, we get

$$\begin{aligned}
\pi^B(X, q) &= r^e I^B(X) - \mathbb{E}^P [\pi^F(x_\theta, v_\theta | q) - r^F l_\theta] \\
&= \underbrace{(r^F - r^e) \mathbb{E}^P [l_\theta]}_{\text{Benefit from loans}} + \underbrace{r^e \{C + \mathbb{E}^P [(1 - \lambda_\theta b_\theta(1)) v_\theta]\}}_{\text{Benefit from bank investment}} \\
&\quad - \underbrace{(1 - p) \mathbb{E}^q [1 - b_H(s)] \Delta v}_{\text{Cost of information rent}} \\
&= \frac{r^F - r^e}{1 + r^F} \{1 - \mathbb{E}^P [\mathbb{E}^q [b_\theta(s) v_\theta - d_\theta(s)]]\} \\
&\quad + r^e \{C + \mathbb{E}^P [(1 - \lambda_\theta b_\theta(1)) v_\theta]\} \\
&\quad - (1 - p) \left(1 - \frac{r^F - r^e}{1 + r^F}\right) \mathbb{E}^q [1 - b_H(s)] \Delta v.
\end{aligned}$$

From this expression, it is straight forward to have the following lemma.

LEMMA 2 (Corner Solution): *The set of contracts in which $b_\theta(s) v_L - d_\theta(s) = 0$ and $\lambda_L b_L(1) = 0$ is weakly payoff dominant, when Assumption 2 is imposed.*

$b_\theta(s) v_L - d_\theta(s) = 0$ is true because $d_\theta(s)$ is a free variable that does not involve any trade-off. Increasing $d_\theta(s)$ and $l_\theta(s)$ while keeping (IC_{LH}) and (IR_H) binding is essentially making more loans to firms and collecting all surplus generated by it. $\lambda_L b_L(1) = 0$ is true as in the standard screening problem, in which no distortion is associated with the L type trading. substituting them into π^B yields

$$\begin{aligned}
\pi^B(X, q) &= \frac{r^F - r^e}{1 + r^F} + r^e \{C + 1\} - (1 - p) \left(1 - \frac{r^F - r^e}{1 + r^F}\right) \Delta v \\
&\quad + \underbrace{(1 - p) \Delta v \mathbb{E}^q [b_H(s)]}_{\text{information rent}} - \underbrace{\frac{r^F - r^e}{1 + r^F} \Delta v \mathbb{E}^q [b_H(s)]}_{\text{haircut}} - \underbrace{pr^e \lambda_H v_H b_H(1)}_{\text{segregated account}}.
\end{aligned}$$

This expression explicitly demonstrates the trade-off in using collateral. First, collateral saves information rent. Second, using collateral reduces loans to firms through haircuts. More specifically, $(1 - p) \frac{r^F - r^e}{1 + r^F} \Delta v$ is the direct effect from haircut, and $p \frac{r^F - r^e}{1 + r^F} \Delta v$ is the indirectly one. Since $b_\theta(s) v_L - d_\theta(s) = 0$ binds, less information rent implies less loans to firms, thus higher haircuts, which strengthens the directly effect. Third, given λ , the more collateral the bank promises to return, the more collateral the bank has to keep in the segregated account, which reduces the bank investment. Given Assumption 2, the net effect is always positive, and we have the following lemma.

LEMMA 3 (Collateral): *The set of contracts in which $b_H(1) = 1$ and $b_H(0) = \lambda_H$ is weakly payoff dominant, when Assumption 2 is imposed.*

Substituting them into π^B yields

$$\begin{aligned} \pi^B(X, q) = & \frac{r^F - r^e}{1 + r^F} (1 - p \Delta v) + r^e \{C + 1\} - pr^e v_H + \underbrace{pr^e v_H (1 - \lambda_H)}_{\text{segregated account}} \\ & - \underbrace{(1 - q)(1 - p) \Delta v (1 - \lambda_H)}_{\text{information rent}} + \underbrace{(1 - q) \frac{r^F - r^e}{1 + r^F} \Delta v (1 - \lambda_H)}_{\text{haircut}}. \end{aligned}$$

Since λ_H determines how much collateral can be promised to return, reusing collateral has the opposite trade-off as in using collateral. One difference is that both the information rent and haircut effects are multiplied by $1 - q$, thus whether collateral reuse is profitable still depends on how risky the bank is. The following lemma is a summary.

PROPOSITION 1 (Optimal Contract): *Given Assumption 2, any contract satisfying the following term achieves the maximal bank payoff:*

- (1) $b_H(1) = 1$ and $b_H(0) = \lambda_H$,
- (2) $d_\theta(s) = b_\theta(s) v_L$,
- (3) $l_\theta = \frac{v_H - [1 - (1 - q)(1 - \lambda_H)] \Delta v}{1 + r^F}$,
- (4) $\lambda_L b_L(1) = 0$,
- (5) $1 - \lambda_H = 1$ if $\left(1 - p - \frac{r^F - r^e}{1 + r^F}\right) (1 - q) \frac{\Delta v}{v_H} - pr^e \leq 0$.

It is worthwhile to mention that the optimal contract can be separating even if the model is linear. This separation is different from that in the standard screening contract. In the screening contract, separation partially restores trading with the H type. However, in my model, separation happens when it is too expensive to have $1 - \lambda_H > 0$, resembling the trade collapse in the screening contract. Since reuse collapse does not imply the entire trade collapse, the contract is still separating in my model in this case. In addition, the pooling contract happens not because trade collapses as in the screening contract, but because the information rent becomes sufficiently low after multiplied by $1 - q$, such that the bank prefers to restore the trade with H firm even if it involves rent.

III. Intermediary Leverage

This section applies the results from the optimal contract to demonstrate how intermediary leverage can be obtained from rehypothecation, further explores the microfoundation of it, and takes comparative statics analysis.

A. Margin Spread

The bank can obtain additional cash from rehypothecation because the collateral margin is smaller in it. According to Proposition 1, the additional cash obtained from rehypothecation by the bank is

$$\begin{aligned}
 I^B - C &= \mathbb{E}^P [-l_\theta + (1 - \lambda_\theta b_\theta(1)) v_\theta] \\
 &= -\frac{v_H - [1 - (1 - q)(1 - \lambda_H)] \Delta v}{1 + r^F} + (1 - p\lambda_H) v_H - (1 - p) \Delta v \\
 &= \frac{(1 + r^F)(1 - p\lambda_H) - 1}{1 + r^F} v_H + \frac{[1 - (1 - q)(1 - \lambda_H)] - (1 + r^F)(1 - p)}{1 + r^F} \Delta v
 \end{aligned}$$

In Assumption 1, the bank has been assumed to reuse the collateral with no cash discount. Here, I will show that it is indeed consistent with the optimal contract, and provide microfoundation for the margin spread and the corresponding contract forms. From Proposition 1, it is straight forward to have the following corollary.

COROLLARY 1 (Observations): *Given Assumption 2, any contract satisfying the following terms is weakly payoff dominant:*

- (1) *Homogenous margins:* $1 - l_\theta = \frac{r^F + [p - (1 - q)(1 - \lambda_H)] \Delta v}{1 + r^F}$.
- (2) *Homogenous returns:* $\frac{d_\theta(1)}{l_\theta} - 1 = \frac{1 - \frac{\Delta v}{v_H}}{1 - [1 - (1 - q)(1 - \lambda_H)] \frac{\Delta v}{v_H}} (1 + r^F) - 1$.

This result looks surprising because the margins and returns to both H firm and L firm are homogenous regardless of whether $\lambda_H = \lambda_L$ or not. The reason is that reusing L bond does not need compensation, while reusing H bond needs to raise the compensation to both H firm and L firm to the same amount, such that L firm does not want to mimic. As a result,

Margin Spread.

Pecking Order Rights.

Non-monotonic Leverage.

Explaining the Puzzles. All results are driven by adverse selection and limited enforcement. First, repo prevails. Without adverse selection, outright selling is no worse than repo, and without limited enforcement, unsecured loan is no worse than repo. Since Assumption 2 implies $(1 - p) \frac{\Delta v}{v_H} - pr^F > 0$, trade collapses with H firm in outright selling. The optimality of repo helps partially restore the trade with H firm. However, repo helps the bank even if the trade does not collapse with H firm. A sufficient condition is $\left(1 - p - \frac{r^F - r^e}{1 + r^F}\right) \frac{\Delta v}{v_H} - pr^e > 0$. In the likely situation in which trade does not

collapse but the bank still finds it profitable to use repo contract, repo leads to higher bank payoff while outright selling leads to higher social surplus.

Second, the bank obtains excessive money through intermediation. By assumption, the bank can reuse the collateral as if there is no asymmetric information. This assumption can be justified by the optimal contract. Suppose no asymmetric information in the secondary market is weakly payoff dominant for the bank, then given the optimal contract, if the bank chooses $(\lambda_H, \lambda_L) = (0, 0)$, then no information leaks and if the bank chooses $(\lambda_H, \lambda_L) = (1, 0)$, then only L bond can be circulated in the secondary market, and there is still no adverse selection problem in the secondary market. While the presence of adverse selection in the secondary market leads to liquidation discount in OTD models, the presence of adverse selection in the primary market leads to liquidation premium in my model. As long as the bank reuses sufficient collateral, excessive money is obtained.

Third, information rent in the primary market is the cost of collateral reuse. In my model, the bank does not liquidate the collateral in some cases not because the liquidating collateral in the secondary market involves discount, but because the right of collateral reuse may be too expensive to purchase.

Testable Implications. Before describing the testable implications, it is helpful to first derive some observable aspects of the optimal contract.

Based on this, there are a few testable implications that distinguish my theory from other theories of collateralized loans. First, repo contracts are heterogeneous across different categories of collateral, but highly homogeneous within each category, regardless of whether the right of reuse is granted or not and how risky the cash borrower is. This is different from Lacker (2001) and Sirtto (2012), in which the investment risk affects the incentive the lender needs to provide to the borrower through margin requirement. This is also different from Gorton and Ordoñez (2014), which emphasizes the role of collateral as insurance against the borrower side investment risk.

Second, if $p + q > 1$, which means collateral risk and bank risk are not extremely high at the same time, then we should observe negative correlation between haircut and repo rate in collateral with certain values, such as treasury bills, but no clear correlation in low rated asset, such as BBB corporate bonds. Theories emphasizing the borrow side risk can easily generate the negative correlation, as in Ewerhart and Tapping (2009) and Eren (2014), but are silent on which set of categories has this pattern.

Third, any category of collateral satisfying the following condition

$$\frac{\Delta v}{v_H} \in \left(\max \left\{ \underbrace{\frac{r^F}{r^F + (1-q)}}_{\text{negative repo rate}}, \underbrace{\frac{pr^F}{1-p-r^F}}_{\text{Assumption 2}} \right\}, \underbrace{\frac{pr^e}{\left(1-p-\frac{r^F-r^e}{1+r^F}\right)(1-q)}}_{\text{reuse all collateral}} \right)$$

can have negative repo rate. In another word, negative repo rate is more likely to happen in repo contract with a haircut neither too high nor too low. Eren (2014) also has negative repo rate, but the result is not collateral contingent. Duffie (1996) emphasizing the short of demand on some "special" collateral, but predicts that more liquid instrument is more special in repo, thus more likely to have lower repo rate. In contrast, my model predicts that it is non-monotonic. A necessary condition is characterized in the appendix.

IV. Endogenous Bank Risk

In this section, I introduce endogenous bank default by assuming that the bank can endogenously choose q and potentially has incentive of risk-shifting. In addition, I consider a realistic bilateral repo market, in which there is a continuum of bond types, and all transactions are over-the-counter. First, the optimal risk-shifting is characterized. Then, I define a OTC equilibrium, and show that under moderate assumption, all results in the optimal contract can be directly applied to the equilibrium. Based on this, I also discuss how adverse selection and moral hazard affect the source of inefficiency.

A. Optimal Risk-Shifting

Moral Hazard. Moral hazard is modeled as the bank side risk-shifting problem. Assume the bank can choose an investment project from a list (r^e, q) , and for each specific $q \in [\underline{q}, 1]$, there is a unique corresponding $r^e(q) \in [0, r^F]$.

ASSUMPTION 3 (Risk-Shifting): $r^e(\cdot)$ is twice continuously differentiable on the interval $[\underline{q}, 1] \subsetneq [0, 1]$, with $r^e(\underline{q}) = 0$, $r^e(1) = \bar{r}^e \in (0, r^F)$, $r^{e'}(q_0^+) = +\infty$, $r^{e'}(1^-) = 0$, and $r^{e''}(q) < 0$ on $[\underline{q}, 1]$.

Given the financial contract, when deciding how much risk to take, the bank is trading off two margins. On the one hand, increasing q results in higher $r^e(q) I^B(X)$. This is the return margin. On the other hand, increasing q leads to higher probability of $s = 1$, in which the bank's net transfer to the firms is no smaller than that in $s = 0$. This is the

default margin. Denote $q^*(X)$ as the optimal risk-shifting rule as a function of X , then the optimality condition can be expressed in the following proposition:

PROPOSITION 2 (Optimal Risk-Shifting): *When X is optimal, $q^*(X)$ satisfies*

$$r^e(q^*(X)) I^B(X) = p(1 - \lambda_H) \Delta v.$$

PROOF:

Given contract X , the first order condition yields

$$\underbrace{r^e(q^*(X)) I^B(X)}_{\text{return margin}} = \max \left\{ \underbrace{\mathbb{E}^P [b_\theta(1) v_\theta - d_\theta(1)] - \mathbb{E}^P [b_\theta(0) v_\theta - d_\theta(0)]}_{\text{default margin}}, 0 \right\}.$$

Substituting the optimal contract described in Proposition 1 yields this expression.

Proposition 1 and 2 highlight the interaction between adverse selection and moral hazard in this paper. First, larger Δv results in higher incentive of risk-shifting through the haircut effect, but larger Δv also implies that $1 - \lambda_H$ is more likely to be 0, which reduces the incentive of risk-shifting. In another word, adverse selection may have either positive or negative effect on moral hazard, depending on how large Δv is. Second, as the bank has more incentive to increase $1 - q$, the cost of collateral reuse (information rent) also increases, and $1 - \lambda_H$ is more likely to be 0, thus moral hazard strengthens the adverse selection problem. Furthermore, this strengthening effect reduces the incentive of risk-shifting, leading to smaller $1 - q$.

B. OTC Equilibrium

Multiple Bonds. Now think about a situation where Δv is no more a constant, but has a cumulative distribution function $\Omega(\cdot)$ with a well defined density function $\omega(\cdot)$, and support $[\underline{\Delta v}, \overline{\Delta v}]$. Assumption 2 holds for $\underline{\Delta v}$. Denote $\mathbf{X} = \cup_{\Delta v \in [\underline{\Delta v}, \overline{\Delta v}]} X_{\Delta v}$, then the optimal risk-shifting rule given the optimal contract can be expressed as

$$r^e(q^*(\mathbf{X})) I^B(\mathbf{X}) = p \int_{\underline{\Delta v}}^{\overline{\Delta v}} (1 - \lambda_{H, \Delta v}) \Delta v d\Omega(\Delta v).$$

The bilateral repo market is an over-the-counter market. Following Acharya and Bisin (2014), it is modeled as private contract market, in which firms cannot observe the contracts received by other firms. In order to characterize the equilibrium, I need to specify firms' belief on the default probability of bank $1 - q$, off the equilibrium path, as in

McAfee and Schwartz (1994), but in a more general way.

ASSUMPTION 4 (Non-aggressive Belief): *Given the optimal risk-shifting rule $q^*(\cdot)$ and the contract menus \mathbf{X}^* in the equilibrium, any firm with bond Δv receiving contract menu $X_{\Delta v}$ believes that the bank's success probability is*

$$\mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*) \begin{cases} = q^*(\mathbf{X}^*) & \text{if } X_{\Delta v} = X_{\Delta v}^* \\ \in \mathcal{B}(\mathbf{X}, \mathbf{X}^*) & \text{if } X_{\Delta v} \neq X_{\Delta v}^* \end{cases}$$

where $\mathcal{B}(\mathbf{X}, \mathbf{X}^*)$ satisfies $\lim_{\Delta m \rightarrow 0} \mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*) = 0$, and Δm denotes the measure of firms observing the deviating contract offer made by the bank.

This assumption is moderate. One example is "passive belief", which assumes that the firm observing bank deviation treats it as a trembling, and believes that the bank is still offering the on-the-equilibrium contract menu to other firms, as is highlighted by McAfee and Schwartz (1994) and Segal (1999), and justified by Segal (2003) in a different context. Another one is "correct belief", which assumes that the firm observing bank deviation takes some negligible effort to figure out what the bank is actually offering to other firms. As long as the firm which the bank is privately and bilaterally deviating to does not estimate the probability of bank failure with higher order bias, the problem of deviation in infinite dimensions can be greatly simplified. These two special cases can be expressed as

$$\begin{aligned} \text{Passive belief:} \quad & \mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*) = q^*(\mathbf{X}^*) \text{ if } X_{\Delta v} \neq X_{\Delta v}^*, \\ \text{Correct belief:} \quad & \mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*) = q^*(\mathbf{X}) \text{ if } X_{\Delta v} \neq X_{\Delta v}^*. \end{aligned}$$

DEFINITION 1 (OTC Equilibrium): *The OTC equilibrium is a contract menu $\mathbf{X}^* = \cup_{\Delta v \in [\underline{\Delta v}, \overline{\Delta v}]} X_{\Delta v}^*$, and belief $\mu(\cdot)$, such that*

$$\begin{aligned} \mathbf{X}^* & \in \operatorname{argmax}_{\mathbf{X} \in \mathcal{X}[\underline{\Delta v}, \overline{\Delta v}]} \mathbb{E}^\Omega [\pi^B(X_{\Delta v}, q^*(\mathbf{X}))] \\ \text{s.t. } 0 & \leq \pi^F(x_{\theta, \Delta v}, v_{\theta, \Delta v} | \mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*)) - \pi^F(x_{\hat{\theta}, \Delta v}, v_{\theta, \Delta v} | \mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*)), \\ 0 & \leq \pi^F(x_{\theta, \Delta v}, v_{\theta, \Delta v} | \mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*)), \\ & \text{and Assumption 4 holds.} \end{aligned}$$

The equilibrium is difficult to solve because the deviation from \mathbf{X}^* has infinite dimension, and the bank's problem has to internalize $q^*(\mathbf{X})$. Here I simplify the analysis by solving a relaxed problem. In the relaxed problem, the bank is taking its own success

probability as given, resembling a "price taker", and doing pairwise optimization.

DEFINITION 2 (Relaxed Problem): *The relaxed problem of the OTC equilibrium is a contract menu $\mathbf{X}^R = \cup_{\Delta v \in [\underline{\Delta v}, \overline{\Delta v}]} X_{\Delta v}^R$, such that*

$$\begin{aligned} X_{\Delta v}^R &\in \operatorname{argmax}_{X \in \mathcal{X}} \pi^B (X_{\Delta v}, q^*(\mathbf{X}^R)) \\ \text{s.t. } 0 &\leq \pi^F (x_{\theta, \Delta v}, v_{\theta, \Delta v} | q^*(\mathbf{X}^R)) - \pi^F (x_{\bar{\theta}, \Delta v}, v_{\theta, \Delta v} | q^*(\mathbf{X}^R)), \\ 0 &\leq \pi^F (x_{\theta, \Delta v}, v_{\theta, \Delta v} | q^*(\mathbf{X}^R)). \end{aligned}$$

Since the belief is fixed at $q^(\mathbf{X}^R)$, Assumption 4 is no more needed here.*

PROPOSITION 3 (Price Taker): *Any solution to the OTC equilibrium is also a solution to the relaxed problem.*

The idea of proof is similar to "Envelop Theorem". The only exception is that we need to deal with the IC and IR constraints. For any local deviation in which the bank only deviate to one firm, the private contract assumption incorporated in Assumption 4 implies that the IC and IR constraints of other firms do not change. For the specific firm observing the deviation, Assumption 4 also guarantees that the corresponding belief is close enough to the equilibrium belief, thus considering deviation within \mathcal{X} under the equilibrium belief does not violate the actual IC and IR conditions. With this local property, "Envelop Theorem" can be applied to the bank's problem as in the unconstrained optimization problems, and the bank would not internalize its own risk in the local deviation. In the local deviation, all changes are negligible. Assumption 4 guarantees that the change in bank's payoff is small in higher orders compared to the change in contract menus. A complete proof is shown in the appendix.

C. Social Planner

DEFINITION 3 (Social Planner): *Denote $\Gamma \subseteq \mathcal{X}^{[\underline{\Delta v}, \overline{\Delta v}]}$, and $\mathbf{X}^*(\Gamma)$ as the OTC equilibrium offer when \mathcal{X} is replaced with Γ . Then, the social planner's problem is*

$$\Gamma^{SP} \in \operatorname{argmax}_{\Gamma \subseteq \mathcal{X}^{[\underline{\Delta v}, \overline{\Delta v}]}} \left\{ \begin{array}{l} \mathbb{E}^\Omega [\pi^B (X_{\Delta v}^*(\Gamma_{\Delta v}), q^*(\mathbf{X}^*(\Gamma)))] \\ + \mathbb{E}^{\Omega, P} [\pi^F (x_{\theta, \Delta v}, v_{\theta, \Delta v} | q^*(\mathbf{X}^*(\Gamma)))] \end{array} \right\}$$

ASSUMPTION 5 (Regulation Instruments): *The only available regulation instruments Γ is a Δv contingent cap on $1 - \lambda_{\theta, \Delta v}$.*

In principle, the regulator could regulate the intermediation service operated by the bank directly, redesign the market structure, and impose richer constraints on the legal form of repo contracts. For two reasons, I am not considering them in the paper. First, it is very difficult to regulate the shadow banking system. The shadow banking activity is difficult to understand for an outsider, and also changing its forms rapidly. The OTC market exists due to some other reasons not modeled in this paper. And the form of contract is difficult to regulate if the transaction itself is difficult to monitor. Second, I will not try to characterize the constrained optimal repo contract, but focus on the source of inefficiency in it given the unchangeable trading environment in the short run.

ASSUMPTION 6 (Monotonicity): *The parameters and functional forms satisfy*

- (1) $C \geq p$;
- (2) $p - (1 - \underline{q}) \geq 0$;
- (3) $1 + p \frac{r^{e''}(q)}{(r^{e'}(q))^2} \leq 0$ for $\forall q \in [\underline{q}, 1]$ satisfying $r^{e'}(q) \leq 1$.

In this assumption, (1) guarantees that the bank's initial cash endowment is sufficient, and we do not need to consider the cash when bank's investment is possibly negative. We also want a q^* monotonic in collateral reuse. This is not in general true because firms' belief should be consistent with the bank's behavior, which is a fixed point problem and involves feed back effect. (2) imposes constraints on the feed back effect. (3) is a restriction of the curvature of $r^e(q)$, such that the incentive effect is stronger than the feed back effect. See the appendix for the use of this assumption.

PROPOSITION 4 (Pecking Order): *In $\mathbf{X}^*(\Gamma^{SP})$, there exists a $\Delta v^{SP} \in [\underline{\Delta v}, \overline{\Delta v}]$,*

$$\text{such that } \lambda_{\theta, \Delta v}^*(\Gamma^{SP}) = \begin{cases} 0 & \text{if } \Delta v < \Delta v^{SP} \\ 1 & \text{if } \Delta v > \Delta v^{SP} \end{cases} .$$

The intuition is straight forward. First, It is privately optimal to reuse the collateral with smaller Δv first because it is cheaper. From Proposition 3, we know that the relaxed problem with pairwise optimization, taking the equilibrium belief as given, is a necessary condition for the OTC equilibrium, thus all results in the optimal contract problem can be directly applied in the OTC equilibrium. Hence, Proposition 1 implies that the bank will reuse Δv first. Second, it is also socially optimal because more loans can be made to the firms without increasing the bank's risk-shifting incentive (non-trivial) if collateral with smaller Δv is reused first. A complete proof can be found in the appendix.

PROPOSITION 5 (Uniqueness): *The solution to the relaxed problem is unique.*

This is true due to monotonicity. The privately optimal reuse cutoff Δv^{cut} is non-increasing in the equilibrium belief of $1 - q^*$, while the optimal $1 - q^*$ consistent with the belief is non-decreasing in Δv^{cut} . See the appendix for the proof. Since the solution to the original problem is a subset to the relaxed problem, the equilibrium may be unique or not exist. The following analysis does not rely on the existence of the equilibrium.

Represent the whole contract menu by a simple cutoff Δv^{cut} , and with slight abuse of notation, the social planner's problem can be simplified to

$$\Delta v^{SP} \in \underset{\Delta v^{cut} \in [\underline{\Delta v}, \overline{\Delta v}]}{\operatorname{argmax}} r^F I^F (\Delta v^{cut}) + r^e (q^* (\Delta v^{cut})) I^B (\Delta v^{cut}).$$

Since the right-hand side is differentiable in Δv^{cut} , a necessary condition is

$$0 \geq r^F I^{F'} (\Delta v^{SP}) + r^e (q^* (\Delta v^{SP})) I^{B'} (\Delta v^{SP}) \\ + r^{e'} (q^* (\Delta v^{SP})) q^{*'} (\Delta v^{SP}) I^{B'} (\Delta v^{SP}).$$

The equality holds if $\Delta v^{cut} < \overline{\Delta v}$. A convenient way to check whether Δv^{SP} increases surplus is to check the wedge at Δv^{SP} , or whether the bank has private incentive to have Δv^{cut} larger than Δv^{SP} , when $\Delta v^{SP} < \overline{\Delta v}$. The regulatory cap on collateral reuse is binding if the following condition holds:

$$r^F I^{F'} (\Delta v^{SP}) + r^e (q^* (\Delta v^{SP})) I^{B'} (\Delta v^{SP}) - \operatorname{rent}_{\Delta v^{cut}} (\Delta v^{SP}, q^* (\Delta v^{SP})) > 0,$$

where $\operatorname{rent} (\Delta v^{cut}, q^* (\Delta v^{SP})) = \mathbb{E}^{\Omega, p} [\pi^F (x_{\theta, \Delta v} (\Delta v^{cut}), v_{\theta, \Delta v} | q^* (\Delta v^{SP}))]$. Equivalently, this can be expressed as a condition of the wedge when $\Delta v^{SP} < \overline{\Delta v}$.

$$-r^{e'} (q^* (\Delta v^{SP})) q^{*'} (\Delta v^{SP}) I^{B'} (\Delta v^{SP}) > \operatorname{rent}_{\Delta v^{cut}} (\Delta v^{SP}, q^* (\Delta v^{SP})).$$

This wedge implies two ways of self-regulation. There is potentially too much collateral reuse when the externality in the OTC market is larger than the information rent in repo contract with collateral reuse. The externality is the uninternalized moral hazard, while the information rent comes from the adverse selection problem in collateral reuse, which offsets the externality. In addition, $\mathbb{E}^{\Omega} [\Delta v]$ imposes upper bound on the cash that can be obtained from collateral reuse. The strength of self-regulation depends on the quantity and the distribution of collateral in the economy.

Replace the collateral capacity of firms (normalized to 1 previously) by Q , then increasing C and Q in the same proportion only scales up the economy. The following two propositions characterize how the wedge is affected by Q/C and Ω .

PROPOSITION 6 (Collateral Scacity): Denote $\Delta v^{OTC|SP}$ as the bank side privately optimal reuse cutoff given belief q^* (Δv^{SP}), then for $\forall \Omega$ with well defined ω , in which the wholes support satisfying Assumption 2, $\exists \underline{Q} > 0$, s.t. for $\forall Q/C \in [0, \underline{Q}]$, $\Delta v^{OTC|SP} = \Delta v^{SP} = \overline{\Delta v}$,

This proposition characterizes a special cases in which collateral reuse is not too much. As $C/Q \rightarrow +\infty$, even if all categories of collateral are reused, the risk-shifting incentive is still small, hence it is both privately and socially desirable to reuse all the collateral. This proposition is straight forward, and the proof is omitted. There is a similar case as $C/Q \rightarrow 0$. However, this violates Assumption 6. Still, the mechanism is similar, and drives Δv^{SP} back to $\overline{\Delta v}$ when Q is large enough, which implies no excessive collateral reuse. The dependence on collateral distribution does not have a closed form solution, and will be shown in the numerical example.

D. Numerical Illustration

Now I illustrate the efficiency analysis with a numerical example. All parameters are summerized in Table 1. Impose the following functional form for $r^e(\cdot)$

$$r^e(q) = \bar{r} \sqrt{1 - \left(\frac{1-q}{1-\underline{q}} \right)^2},$$

and $\omega(\cdot)$ is assumed to be linear. Figure 3 demonstrates the comparison between Δv^{SP} and $\Delta v^{OTC|SP}$. $\Delta v^{SP} < \Delta v^{OTC|SP}$ means too much reuse in the equilibrium. From this figure, we know that collateral reuse is too much only when Q is neither too large nor too small, and this interval is shrinking in $\mathbb{E}^\Omega[\Delta v]$.

There are a few policy implications. First, in contrast to Andolfatto, Martin and Zhang (2014), collateral scarcity does not increases the likelihood of too much collateral reuse, but decreases it because it restricts the leverage of the bank. Second, when the bank is heavily involved in the collateral reuse activity, which means A/Q is small, regulation on collateral reuse cannot improve allocation efficiency, because the bank's risk-shifting incentive is not very sensitive to it. Third, when there a sudden increase in the information dispersion on collateral value in the recession, regulation on collateral reuse does not help because now the bank actually have much less private incentive to reuse collateral.

C	r^F	\bar{r}	p	\underline{q}	$\underline{\Delta v}$	$\overline{\Delta v}$
1	0.5	0.4	0.6	0.8	0.1	1.5

TABLE 1—PARAMETERS FOR THE NUMERICAL EXAMPLE

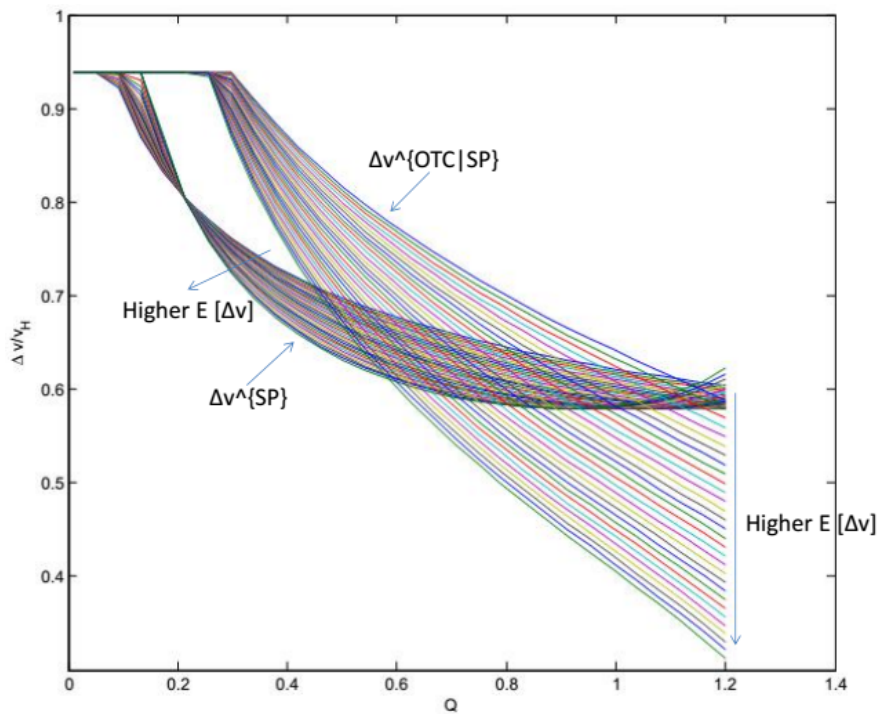


FIGURE 3. COMPARISON BETWEEN THE SOCIALLY AND PRIVATELY OPTIMAL CUTOFF

V. Concluding Remarks

This paper builds a theoretical framework for collateral reuse. This framework is consistent with three stylized facts in repo market: (1) repo is prevailing even if outright selling is available; (2) the bank engaged in collateral reuse can possibly obtain excessive cash; (3) the collateral is more likely to be reused if the asymmetric information problem is not too large. In addition, this framework predicts that given the collateral category, haircut and repo rate do not depend on whether the collateral is reused, and negative repo rate arises more likely in the collateral neither too liquid nor too illiquid. Collateral reuse is too much in the OTC market only when Q/A is neither too large nor too small, and less likely when $\mathbb{E}^\Omega[\Delta v]$ is larger.

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VI. Appendix

A. Proof of Lemma 1

Suppose $b_L(s)v_L - d_L(s) < 0$ for some $s \in \{0, 1\}$, replace X with X' such that $(b'_L(s), d'_L(s)) = (0, 0)$. By construction,

$$\begin{aligned}\pi^F(x'_H, v_H|q) &= \pi^F(x_H, v_H|q), \\ \pi^F(x'_L, v_L|q) &= \pi^F(x_L, v_L|q),\end{aligned}$$

thus (IR_H) and (IR_L) trivially hold. In addition,

$$\begin{aligned}\pi^F(x'_L, v_L|q) &= \pi^F(x_L, v_L|q) \geq \pi^F(x_H, v_L|q) = \pi^F(x'_H, v_L|q), \\ \pi^F(x'_H, v_H|q) &= \pi^F(x_H, v_H|q) \geq \pi^F(x_L, v_H|q) \geq \pi^F(x'_L, v_H|q),\end{aligned}$$

hence (IC_{LH}) and (IC_{HL}) also holds. Since $\pi^B(X', q) = \pi^B(X, q)$, excluding the case of $b_L(s)v_L - d_L(s) < 0$ does not lower the maximal π^B .

Suppose $b_H(s)v_L - d_H(s) < 0$ for some $s \in \{0, 1\}$, replace X with X' such that

$$\begin{aligned}b_H(s)v_L - d'_H(s) &= 0, \\ (1+r^F)l'_\theta - \mathbb{E}^q[d'_H(s)] &= (1+r^F)l_\theta - \mathbb{E}^q[d_H(s)],\end{aligned}$$

By construction, (IR_H) and (IC_{HL}) hold because

$$\pi^F(x'_H, v_H|q) = \pi^F(x_H, v_H|q) \geq \pi^F(x_L, v_H|q) > \pi^F(x'_L, v_H|q).$$

(IC_{LH}) holds because

$$\begin{aligned}\pi^F(x_L, v_L|q) &\geq \pi^F(x_H, v_L|q), \\ \pi^F(x'_L, v_L|q) - \pi^F(x_L, v_L|q) &= \pi^F(x'_H, v_L|q) - \pi^F(x_H, v_L|q).\end{aligned}$$

(IR_L) holds because

$$\pi^F(x'_L, v_L|q) \geq \pi^F(x'_H, v_L|q) \geq \pi^F(x'_H, v_H|q) = \pi^F(x_H, v_H|q).$$

$l_\theta \geq 0$ because $\pi^F(x_\theta, v_\theta|q) \geq 0$ implies

$$(1+r^F)l_\theta \geq v_\theta - \mathbb{E}^q[\max\{b_\theta(s)v_\theta - d_\theta(s), 0\}] \geq 0.$$

$\pi^B(X', q) \geq \pi^B(X, q)$ because

$$\begin{aligned}
& \pi^B(X', q) - \pi^B(X, q) \\
&= r^F \mathbb{E}^P [l'_\theta - l_\theta] - \mathbb{E}^P [\pi^F(x'_\theta, v_\theta | q) - \pi^F(x_\theta, v_\theta | q)] \\
&= -\mathbb{E}^P [l'_\theta - l_\theta] - \mathbb{E}^P [\mathbb{E}^q [b_\theta(s) v_\theta - d'_\theta(s)] - \mathbb{E}^q [b_\theta(s) v_\theta - d_\theta(s)]] \\
&= p [\mathbb{E}^q [d'_H(s) - d_H(s)]] - \mathbb{E}^P [l'_\theta - l_\theta] = -(1 - p - pr^F) \mathbb{E}^P [l'_\theta - l_\theta],
\end{aligned}$$

in which $\mathbb{E}^P [l'_\theta - l_\theta] < 0$, and Assumption 2 implies that $1 - p - pr^F \geq 0$.

B. Proof of Full Pledge More Generally

The optimal contracting problem can be generalized to allow for partial pledge. Denote $\kappa_\theta \in [0, 1]$ as the quantity of bond transfer in the contracting stage, and normalize all contract terms by κ_θ . Then the firms and the bank's payoff functions become

$$\begin{aligned}
\pi^F(x_{\bar{\theta}}, v_\theta | q) &= r^F l_{\bar{\theta}} \kappa_{\bar{\theta}} + (l_{\bar{\theta}} - v_\theta) \kappa_{\bar{\theta}} + \mathbb{E}^q [\max \{b_{\bar{\theta}}(s) v_\theta - d_{\bar{\theta}}(s), 0\} \kappa_{\bar{\theta}}], \\
\pi^B(X, q) &= r^e I^B(X) + \mathbb{E}^P [r^F l_\theta - \pi^F(x_\theta, v_\theta | q)].
\end{aligned}$$

In the Proof of Lemma 1, $b_L(s) v_L - d_L(s) \geq 0$ regardless of κ_θ . By the same logic, we can prove that $b_H(s) v_H - d_H(s) \geq 0$. Given this, for $\forall \kappa_\theta < 1$, replace X with X' such that $\kappa'_\theta = 1$ and

$$\begin{aligned}
l'_\theta \kappa'_\theta = l_\theta \kappa_\theta &\implies l'_\theta = l_\theta \kappa_\theta \\
(b'_\theta(s) - 1) \kappa'_\theta = (b_\theta(s) - 1) \kappa_\theta &\implies b'_\theta(s) = 1 - (1 - b_\theta(s)) \kappa_\theta \\
d'_\theta(s) \kappa'_\theta = d_\theta(s) \kappa_\theta &\implies d'_\theta(s) = d_\theta(s) \kappa_\theta \\
(1 - \lambda'_\theta b'_\theta(1)) \kappa'_\theta = (1 - \lambda_\theta b_\theta(1)) \kappa_\theta &\implies \lambda'_\theta = \frac{1 - (1 - \lambda_\theta b_\theta(1)) \kappa_\theta}{1 - (1 - b_\theta(1)) \kappa_\theta}
\end{aligned}$$

By construction, X' keeps $\pi^F(x_\theta, v_\theta | q)$ and $\pi^B(X, q)$ unchanged. Since

$$\begin{aligned}
& l'_\theta \in [0, l_\theta] \subseteq \mathbb{R}_+ \\
& \lambda'_\theta \in [\lambda_\theta, 1] \subseteq [0, 1] \\
& b'_\theta(s) \in [b_\theta(s), 1] \subseteq [0, 1] \\
& d'_\theta(s) \in [0, d_\theta(s)] \subseteq \mathbb{R}_+ \\
& b'_\theta(0) = 1 - (1 - b_\theta(0)) \kappa_\theta \leq 1 - (1 - \lambda_\theta b_\theta(1)) \kappa_\theta = \lambda'_\theta b'_\theta(1)
\end{aligned}$$

the feasibility constraint still holds in the following

$$(l'_\theta, \lambda'_\theta, b'_\theta(1), b'_\theta(0), d'_\theta(1), d'_\theta(0)) \in \mathbb{R}_+ \times [0, 1] \times [0, 1] \times [0, \lambda'_\theta b'_\theta(1)] \times \mathbb{R}_+ \times \mathbb{R}_+.$$

$(IC_{\theta\tilde{\theta}})$ still holds because by construction

$$\begin{aligned} & \mathbb{E}^q \left[\max \left\{ \left(b'_{\tilde{\theta}}(s) - 1 \right) v_{\theta} - d'_{\tilde{\theta}}(s), -v_{\theta} \right\} \right] \\ & \leq \mathbb{E}^q \left[\max \left\{ \left(b_{\tilde{\theta}}(s) - 1 \right) v_{\theta} - d_{\tilde{\theta}}(s), -v_{\theta} \right\} \kappa_{\tilde{\theta}} \right], \end{aligned}$$

which is equivalent to $\pi^F(x'_{\tilde{\theta}}, v_{\theta}|q) \leq \pi^F(x_{\tilde{\theta}}, v_{\theta}|q)$.

C. A Necessary Condition for Negative Repo Rate

Negative repo rate requires that $1 - \lambda_H = 1$, which requires

$$\frac{\Delta v}{v_H} \leq \frac{pr^e}{\left(1 - p - \frac{r^F - r^e}{1+r^F}\right)(1-q)}.$$

Negative repo rate requires that

$$\frac{\Delta v}{v_H} > \frac{r^F}{(1-q)(1-\lambda_H) + r^F} = \frac{r^F}{1-q+r^F}.$$

Assumption 1 requires that

$$\frac{\Delta v}{v_H} > \frac{pr^F}{1-p-r^F}.$$

Hence a necessary condition for the existence of negative repo rate is

$$\frac{pr^e}{\left(1 - p - \frac{r^F - r^e}{1+r^F}\right)(1-q)} > \max \left\{ \frac{r^F}{1-q+r^F}, \frac{pr^F}{1-p-r^F} \right\}.$$

A necessary condition for this is

$$\frac{p}{(1-p)(1-q)} > \max \left\{ \frac{1}{1-q+r^F}, \frac{p}{1-p-r^F} \right\},$$

which is equivalent to

$$\underbrace{(1-2p)(1-q)}_{\text{negative repo rate}} < \underbrace{pr^F}_{\text{Assumption 2}} < p(1-p)q.$$

One straight forward way to have a non-empty set of parameters such that the repo rate is possibly negative is to have p larger than $\frac{1}{2}$ but not too close to 1, q not close to 0, r^F close to 0 and r^e close to r^F .

D. Proof of Proposition 3

Due to the linearity of the problem, there exists a uniformed $M > 0$ such that

$$\mathbf{X}^* \in \operatorname{argmax}_{\mathbf{X} \in \mathcal{X}[\underline{\Delta v}, \bar{\Delta v}]} \left\{ \mathbb{E}^\Omega \left[\pi^B (X_{\Delta v}, q^*(\mathbf{X})) \right] + \mathbb{E}^\Omega \left[\Xi_{\Delta v} (X_{\Delta v} | \mu_{\Delta v}(\mathbf{X} | \mathbf{X}^*)) \right] \right\}.$$

where the punishment function $\Xi(\mathbf{X})$ is

$$\Xi_{\Delta v} (X_{\Delta v} | \mu_{\Delta v}(\mathbf{X} | \mathbf{X}^*)) = M \mathbb{E}^P \left[\begin{array}{l} \min \left\{ \pi^F (x_{\theta, \Delta v}, v_{\theta, \Delta v} | \mu_{\Delta v}(\mathbf{X} | \mathbf{X}^*)), 0 \right\} \\ + \min \left\{ \begin{array}{l} \pi^F (x_{\theta, \Delta v}, v_{\theta, \Delta v} | \mu_{\Delta v}(\mathbf{X} | \mathbf{X}^*)) \\ -\pi^F (x_{\bar{\theta}, \Delta v}, v_{\theta, \Delta v} | \mu_{\Delta v}(\mathbf{X} | \mathbf{X}^*)) \end{array}, 0 \right\} \end{array} \right].$$

Consider a specific form of deviation $\mathbf{X}^* + \Delta \mathbf{X}^D(\Delta v, \Delta m)$ in which

$$\Delta X_{\Delta v}^D(\Delta v, \Delta m) = \begin{cases} D_{\Delta v}^{\sim}(\Delta m) & \text{if } \widetilde{\Delta v} \in \mathcal{I}(\Delta v, \Delta m) \\ 0 & \text{if } \widetilde{\Delta v} \notin \mathcal{I}(\Delta v, \Delta m) \end{cases},$$

where $\mathcal{I}(\Delta v, \Delta m)$ is a set contains Δv with measure $\Delta m > 0$, $X^* + D_{\Delta v}^{\sim}(\Delta m)$ satisfies all constraints, and $D_{\Delta v}^{\sim}(\Delta m)$ is continuous in both $\widetilde{\Delta v}$ and Δm . Consider the net benefit from deviating to $\mathbf{X}^* + \Delta \mathbf{X}^D(\Delta v, \Delta m)$ (or $\mathbf{X}^* + \Delta \mathbf{X}^D$ for short),

$$\begin{aligned} & \frac{1}{\Delta m} \mathbb{E}^\Omega \left[\pi^B (X_{\Delta v}^* + \Delta X_{\Delta v}^D(\Delta v, \Delta m), q^*(\mathbf{X}^* + \Delta \mathbf{X}^D(\Delta v, \Delta m))) \right] \\ & - \frac{1}{\Delta m} \mathbb{E}^\Omega \left[\pi^B (X_{\Delta v}^* + \Delta X_{\Delta v}^D(\Delta v, \Delta m), q^*(\mathbf{X}^*)) \right] \\ & + \frac{1}{\Delta m} \mathbb{E}^\Omega \left[\pi^B (X_{\Delta v}^* + \Delta X_{\Delta v}^D(\Delta v, \Delta m), q^*(\mathbf{X}^*)) \right] \\ & - \frac{1}{\Delta m} \mathbb{E}^\Omega \left[\pi^B (X_{\Delta v}^*, q^*(\mathbf{X}^*)) \right] \\ & + \frac{1}{\Delta m} \mathbb{E}^\Omega \left[\Xi (X_{\Delta v}^* + \Delta X_{\Delta v}^D | \mu_{\Delta v}(\mathbf{X} + \Delta \mathbf{X}^D | \mathbf{X}^*)) - \Xi (X_{\Delta v}^* | q^*(\mathbf{X}^*)) \right] \\ = & \mathbb{E}^\Omega \left[\frac{\partial}{\partial q} \pi^B (X_{\Delta v}^* + \Delta X_{\Delta v}^D(\Delta v, \Delta m), q^*(\mathbf{X}^* + \alpha \Delta \mathbf{X}^D(\Delta v, \Delta m))) \right] \\ & \cdot \frac{q^*(\mathbf{X}^* + \Delta \mathbf{X}^D(\Delta v, \Delta m)) - q^*(\mathbf{X}^*)}{\Delta m} \\ & + \frac{\mathbb{E}^\Omega \left[\pi^B (X_{\Delta v}^* + \Delta X_{\Delta v}^D(\Delta v, \Delta m), q^*(\mathbf{X}^*)) - \pi^B (X_{\Delta v}^*, q^*(\mathbf{X}^*)) \right]}{\Delta m} \\ & + \frac{\mathbb{E}^\Omega \left[\Xi (X_{\Delta v}^* + \Delta X_{\Delta v}^D | \mu_{\Delta v}(\mathbf{X} + \Delta \mathbf{X}^D | \mathbf{X}^*)) - \Xi (X_{\Delta v}^* | q^*(\mathbf{X}^*)) \right]}{\Delta m}. \end{aligned}$$

where $\exists \alpha \in [0, 1]$. Consider the limit when $\Delta m \rightarrow 0$,

$$\begin{aligned} \lim_{\Delta m \rightarrow 0} q^* (\mathbf{X}^* + \alpha \Delta \mathbf{X}_{\Delta v}^D (\Delta v, \Delta m)) &= q^* (\mathbf{X}^*), \\ \limsup_{\Delta m \rightarrow 0} \left| \frac{q^* (\mathbf{X}^* + \Delta \mathbf{X}_{\Delta v}^D (\Delta v, \Delta m)) - q^* (\mathbf{X}^*)}{\Delta m} \right| &< +\infty. \end{aligned}$$

Assumption 3 implies that there is no corner solution for $q^* (\cdot)$, thus

$$\lim_{\Delta m \rightarrow 0} \mathbb{E}^\Omega \left[\frac{\partial}{\partial q} \pi^B (X_{\Delta v}^* + \Delta X_{\Delta v}^D (\Delta v, \Delta m), q^* (\mathbf{X}^* + \alpha \Delta \mathbf{X}_{\Delta v}^D (\Delta v, \Delta m))) \right] = 0.$$

Since $D_{\widetilde{\Delta v}} (\Delta m)$ is continuous in both $\widetilde{\Delta v}$ and Δm , according to Assumption 4,

$$\begin{aligned} &\lim_{\Delta m \rightarrow 0} \frac{\mathbb{E}^\Omega \left[\Xi \left(X_{\Delta v}^* + \Delta X_{\Delta v}^D | \mu_{\widetilde{\Delta v}} (\mathbf{X} + \Delta \mathbf{X}^D | \mathbf{X}^*) \right) - \Xi \left(X_{\Delta v}^* | q^* (\mathbf{X}^*) \right) \right]}{\Delta m} \\ &= \Xi_{\Delta v} (X_{\Delta v}^* + D_{\Delta v} (0) | \mu_{\Delta v} (\mathbf{X} + \Delta \mathbf{X}^D (\Delta v, 0) | \mathbf{X}^*)) - \Xi_{\Delta v} (X_{\Delta v}^* | q^* (\mathbf{X}^*)) \\ &= \Xi_{\Delta v} (X_{\Delta v}^* + D_{\Delta v} (0) | q^* (\mathbf{X}^*)) - \Xi_{\Delta v} (X_{\Delta v}^* | q^* (\mathbf{X}^*)). \end{aligned}$$

Private contract and the continuity of $D_{\widetilde{\Delta v}} (\Delta m)$ implies that

$$\begin{aligned} &\lim_{\Delta m \rightarrow 0} \frac{\mathbb{E}^\Omega \left[\pi^B \left(X_{\Delta v}^* + \Delta X_{\Delta v}^D (\Delta v, \Delta m), q^* (\mathbf{X}^*) \right) - \pi^B \left(X_{\Delta v}^*, q^* (\mathbf{X}^*) \right) \right]}{\Delta m} \\ &= \pi^B (X_{\Delta v} + D_{\Delta v} (\Delta m), q^* (\mathbf{X}^*)) - \pi^B (X_{\Delta v}, q^* (\mathbf{X}^*)) \end{aligned}$$

Hence, this deviation is not profitable for all $\Delta \mathbf{X}^D (\Delta v, \Delta m)$ only if

$$X_{\Delta v}^* \in \operatorname{argmax}_{X \in \mathcal{X}} \left\{ \pi^B (X_{\Delta v}, q^* (\mathbf{X}^*)) + \mathbb{E}^\Omega [\Xi_{\Delta v} (X_{\Delta v} | q^* (\mathbf{X}^*))] \right\}.$$

By construction, this is equivalent to

$$\begin{aligned} X_{\Delta v}^* &\in \operatorname{argmax}_{X \in \mathcal{X}} \pi^B (X_{\Delta v}, q^* (\mathbf{X}^*)) \\ &\text{s.t. all IC and IR conditions hold.} \end{aligned}$$

That is to say, pairwise optimization taken the equilibrium belief $q^* (\mathbf{X}^*)$ as given is a necessary condition for the bank's contracting problem, and solving the relaxed problem yields the set candidates of equilibrium.

E. Proof of Proposition 4

According to Proposition 3, the bank chooses the optimal \mathbf{X} pairwise as if

$$\mu_{\Delta v}(\mathbf{X}|\mathbf{X}^*) = q(\mathbf{X}^*).$$

Hence all results in Proposition 1 can be directly applied here. As a result, given any cut-off regulation Γ requiring $1 - \lambda_H = 0$ for $\forall \Delta v \in [\Delta v^\Gamma, \overline{\Delta v}] \subseteq [\underline{\Delta v}, \overline{\Delta v}]$, $\exists \Delta v^{OTC|\Gamma} \in [\underline{\Delta v}, \Delta v^\Gamma]$ such that the bank maximizes payoff by choosing

$$\lambda_{\theta, \Delta v}^* = \begin{cases} 0 & \text{if } \Delta v < \Delta v^{OTC|\Gamma} \\ 1 & \text{if } \Delta v > \Delta v^{OTC|\Gamma} \end{cases}.$$

Now the question is whether such a regulation is weakly dominant. Suppose not, then $\exists \Delta v_1, \Delta v_2 \in [\underline{\Delta v}, \overline{\Delta v}]$, such that $\Delta v_1 < \Delta v_2$, $1 - \lambda_{H, \Delta v_1}^{SP} < 1$ and $1 - \lambda_{H, \Delta v_2}^{SP} > 0$. Without losing of generality, assume Γ^{SP} is binding. Denote

$$\begin{aligned} \Lambda_0(\Gamma^{SP}) &= \int_{\underline{\Delta v}}^{\overline{\Delta v}} (1 - \lambda_{H, \Delta v}^{SP}) d\Omega(\Delta v), \\ \Lambda_1(\Gamma^{SP}) &= \int_{\underline{\Delta v}}^{\overline{\Delta v}} (1 - \lambda_{H, \Delta v}^{SP}) \Delta v d\Omega(\Delta v), \end{aligned}$$

then, for interior $q \in (0, 1)$, the optimal risk-shifting condition can be written as

$$r^{e'}(q^*) = \frac{p\Lambda_1(\Gamma^{SP})}{C + p\Lambda_0(\Gamma^{SP}) - \left[\frac{1}{1+r^F} - (1-p)\right](1 - p\mathbb{E}[\Delta v]) + \left[p(1-p) - \frac{1-q^*}{1+r^F}\right]\Lambda_1(\Gamma^{SP})}.$$

Moving q^* related terms to the left-hand side yields

$$\begin{aligned} &\frac{p}{r^{e'}(q^*)} + \frac{1-q^*}{1+r^F} - p(1-p) \\ &= \frac{C + p\Lambda_0(\Gamma^{SP}) - \left[\frac{1}{1+r^F} - (1-p)\right](1 - p\mathbb{E}[\Delta v])}{\Lambda_1(\Gamma^{SP})}. \end{aligned}$$

Assumption 6 requires $C > p$, hence

$$C - \left[\frac{1}{1+r^F} - (1-p)\right](1 - p\mathbb{E}[\Delta v]) > 0,$$

and the right-hand side of the last equation is decreasing in $\Lambda_1(\Gamma^{SP})$, and

$$\begin{aligned}
r^{el}(q^*) &\leq \frac{p\mathbb{E}[\Delta v]}{C - \left[\frac{1}{1+r^F} - (1-p)\right](1-p\mathbb{E}[\Delta v]) + \left[p(1-p) - \frac{1-q^*}{1+r^F}\right]\mathbb{E}[\Delta v]} \\
&= \frac{p\mathbb{E}[\Delta v]}{C - \frac{1}{1+r^F} + \frac{1}{1+r^F}p\mathbb{E}[\Delta v] + (1-p) + \frac{p-(1-q^*)}{1+r^F}\mathbb{E}[\Delta v]} \\
&\leq \frac{1}{C+1-p} \leq 1.
\end{aligned}$$

Therefore, $r^{el}(q^*)$ will never be large than 1, and $1 + p \frac{r^{el}(q)}{(r^{el}(q))^2} \leq 0$ in Assumption 6 suffices to guarantee that $\frac{p}{r^{el}(q^*)} + \frac{1-q^*}{1+r^F}$ is increasing in q^* . As a result, if we reduce $\Lambda_1(\Gamma^{SP})$ while keeping $\Lambda_0(\Gamma^{SP})$ unchanged, q^* will be larger.

Now a modification increasing $1 - \lambda_{H,\Delta v_1}^{SP}$ by $\varepsilon > 0$ while decreasing $1 - \lambda_{H,\Delta v_2}^{SP}$ by the same amount only induces larger q^* . Since such a modification also shifts the bank investment to firm investment, which yields higher net return rate, social surplus will be unambiguously larger.

F. Proof of the Proposition 5

Due to Proposition 4, $\frac{\Lambda_0(\Gamma^{SP})}{\Lambda_1(\Gamma^{SP})}$ is non-increasing in reuse cutoff Δv^{cut} , using

$$r^{el}(q^*) = \frac{p}{C - \left[\frac{1}{1+r^F} - (1-p)\right](1-p\mathbb{E}[\Delta v]) + p \frac{\Lambda_0(\Gamma^{SP})}{\Lambda_1(\Gamma^{SP})} + \left[p(1-p) - \frac{1-q^*}{1+r^F}\right]\mathbb{E}[\Delta v]},$$

$1-q^*$ is non-decreasing in Δv^{cut} . Due to Proposition 1, Δv^{cut} is non-increasing in $1-q^*$ in privately optimal collateral reuse. These two conditions pin down the solution of the relaxed problem, and the monotonicity property guarantees the uniqueness.