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# Relative Price Dispersion: Evidence and Theory

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# Relative Price Dispersion: Evidence and Theory*

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**Abstract**

We use a large dataset on retail pricing to document that a sizeable portion of the cross-sectional variation in the price at which the same good trades in the same period and in the same market is due to the fact that stores that are, on average, equally expensive set persistently different prices for the same good. We refer to this phenomenon as relative price dispersion. We argue that relative price dispersion stems from sellers' attempts to discriminate between high-valuation buyers who need to make all of their purchases in the same store, and low-valuation buyers who are willing to purchase different items from different stores. We calibrate our theory and show that it is not only consistent with the extent and sources of dispersion in the price that different sellers charge for the same good, but also with the extent and sources of dispersion in the prices that different households pay for the same basket of goods, as well as with the relationship between prices paid and the number of stores visited by different households.

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# 1 Introduction

Using a large dataset on retail pricing, we document that a significant fraction of the cross-sectional variation in the price at which the same good is sold in the same period of time and in the same market is due to the fact that retailers that are, on average, equally expensive set persistently different prices for that particular good. We refer to this phenomenon as *relative price dispersion*. We propose a theory of relative price dispersion in an equilibrium model of the retail market, where buyers and sellers, respectively, demand and supply multiple goods. We argue that relative price dispersion stems from the sellers' incentive to discriminate between high-valuation buyers who need to make all of their purchases in the same store, and low-valuation buyers who are willing to purchase different items from different stores. We calibrate our theory of relative price dispersion and we show that it captures well the differences between the extent and cause of variation in the price at which the same good is sold by different sellers, and the extent and cause of variation in the prices that different households pay for the same basket of goods.

In the first part of the paper, we measure the extent and sources of dispersion in the price at which the same good is sold in the same week and in the same geographical area by different retailers. We carry out the analysis using the Kilts-Nielsen Retail Scanner Dataset (KNRS), which provides weekly price and quantity information for 1.5 million goods (defined by their Unique Product Code) at around 40,000 stores in over 2,500 counties across 205 Designated Market Areas (which are geographical areas of roughly the same size as Metropolitan Statistical Areas).

Using the KNRS, we compute the average price of a particular good in a given week and in a given geographical area. We normalize the price of the good at each different store by expressing it as a percentage deviation from the average price in the relevant week and market. We break down the normalized price of the good at each different store into a *store component*—defined as the average of the normalized price of all the goods sold by the store in the week—and a *store-good component*—defined residually as the difference between the price of the good at the store and the average price of the store. We then compute the variance of the price of the good at different stores, and we break down this variance into a store component and a store-good component. We find that, on average, the standard deviation of prices for the same good in the same week and market is 15.3%, and the variance is 2.34%. We also find that only 15% of the variance is due to the variance in the store component—i.e. due to the fact that the same good is sold at stores that have a different average price—while 85% is due to the variance of the store-good component—

i.e. due to the fact that the same good is sold at different prices at stores that are equally expensive on average.

We then use the time dimension in KNRS to identify the *persistent component* and the *transitory component* of the price of a particular good at a particular store. To this aim, we follow an approach that is commonly used in the literature on labor economics to analyze wage inequality (see, e.g, Gottschalk and Moffitt 1994 and Blundell and Preston 1998) but that had never been applied before to study price dispersion. Specifically, we estimate a statistical model for prices, in which both the store and the store-good component of prices are given by the sum of a fixed effect, an AR process and an MA process. When estimated, the statistical model fits the auto-covariances of prices very well. The estimated model implies that almost all of the cross-sectional variance of the store component of prices stems from persistent differences in average store prices. Also, the estimated model implies that roughly 1/3 of the variance of the cross-sectional variance of the store-good component of prices stems from persistent differences in the price of the good at equally expensive stores, while roughly 2/3 of the variance stems from transitory differences. Overall, a sizable fraction of the variation in the price of the same good across different stores in the same week and in the same market is due to the fact that, equally expensive stores, charge *persistently different* prices for the same good. We refer to the persistent differences in the price of a particular good relative to the average price of the store as relative price dispersion.

The literature offers compelling theories of short-term differences in the price of the same good across equally expensive stores. For instance, according to the theory of intertemporal price discrimination (see, e.g., Conlisk, Gerstner and Sobel 1984, Sobel 1984, Menzio and Trachter 2015a), sellers find it optimal to occasionally lower the price of a particular good in order to discriminate between low-valuation customers who are willing to do their shopping at any time during the month, and high-valuation customers who need to make their purchases on a specific day of the month. As different sellers implement these occasional price reductions at different times, the equilibrium may feature short-term differences in the price of the same good across equally expensive stores. According to the inventory management theory (see, e.g., Aguirregabiria 1999), a seller finds it optimal to increases the price of a good as the inventory of the good falls, and it finds it optimal to lower the price when the inventory of the good is replenished. As different sellers have different inventory cycles, the equilibrium may feature short-term differences in the price of the same good across equally expensive stores. In contrast, the literature does not offer much theoretical guidance in the way of understanding relative price dispersion, i.e. long-term differences in the price of the same good across equally expensive stores.

In the second part of the paper, we develop a theory of relative price dispersion. We consider a retail market in which sellers and buyers respectively supply and demand two goods. Sellers are ex-ante homogeneous, both with respect to their cost of producing the goods and in terms of the type of buyers they meet. Buyers are ex-ante heterogeneous. One type of buyers, which we call *busy*, has a relatively high valuation for goods and needs to make all purchases in the same store. The other type of buyers, which we call *cool*, has a relatively low valuation and can purchase different items at different stores. The retail market is imperfectly competitive. As in Butters (1977) and Burdett and Judd (1983), we assume that buyers do not have access to all the sellers, but only to a subset of them. In particular, a fraction of buyers can access a single seller, and a fraction of buyers can access multiple sellers.

We find that, for some parameter values, the equilibrium of the retail market must display relative price dispersion. This result follows from two properties of the equilibrium: (i) some sellers find it optimal to post different prices for the two goods; (ii) for every seller that posts a lower price for one good, there is another seller posting a lower price for the other good. There is a simple intuition behind the first property of the equilibrium. Consider a seller that sets the same price for the two goods and suppose that this common price lies in between the valuation of the cool buyers and the valuation of the busy buyers. Given this common price, the seller trades with some of the busy buyers, but never trades with the cool buyers. Now, suppose that the seller lowers the price of the first good and increases the price of the second good, so as to keep the average price constant. Since the busy buyers must purchase both goods in the same location, they are indifferent to the change in prices. Hence, the seller trades the same quantity of goods to this type of buyers. However, as the seller keeps lowering the price of the first good, at some point if reaches the valuation of the cool buyers. Hence, at some point, the seller trades the good to this type of buyers. Overall, the seller is strictly better off setting different prices for the two goods, rather than setting a common price. The second property of the equilibrium follows from the fact that, if there were more seller charging a lower price for one good than for the other, then there would be some unexploited profit opportunities. When taken together, the two properties of the equilibrium imply that there is dispersion in the price of the same good among sellers that are, on average, equally expensive.

According to our theory, relative price dispersion is the general equilibrium consequence of a new type of price discrimination. The difference in valuation between the busy buyers and the cool buyers gives sellers a reason to try and price discriminate. The difference in the ability of the busy and the cool buyers to make their purchases at different locations gives the seller a way to price discriminate. Specifically, price discrimination is achieved by setting asymmetric prices for the two goods, so as to charge a high price for the basket of goods to the high-valuation buyers who need to purchase the basket of goods in the same retailer, and to charge a low price (on one item) to the low-valuation buyers who have the ability to purchase different items from different retailers.

In the last part of the paper, we calibrate and validate our theory of relative price dispersion. We calibrate the theory so as to match the extent of dispersion in the persistent component of prices at which different stores sell the same good, the contribution of the (persistent) store component and of the (persistent) store-good component to the dispersion of prices, and the elasticity of the price paid by a household for a given basket of goods with respect to the number of stores from which the household shops. We find that our theory can match these features of the data very well. Dispersion in the (persistent) store component of prices obtains for the same reason as in the single product retail market models of Butters (1977) and Burdett and Judd (1983). Dispersion in the (persistent) store-good component of prices is a general equilibrium consequence of price discrimination between low-valuation buyers who can shop at multiple locations, and high-valuation buyers who must shop at a single location. The negative elasticity of the price paid by a household for a given basket of goods with respect to the number of stores from which it shops obtains because, when there is relative price dispersion, buyers can achieve lower prices by purchasing different items at different stores.

We validate the theory using the Kilts-Nielsen Consumer Panel (KNCP), which tracks the shopping behavior of 50,000 households drawn from 54 different markets. We compute the extent and sources of dispersion in the prices that different households pay for the same basket of goods, in the same quarter and in the same market. After removing the dispersion due to transitory variation in the store-good component of prices, we find that the standard deviation of prices paid by different households for the same basket of goods is 7.8%, and the variance is 0.61%. We also find that 55% of the variance is due to persistent differences in the store component of prices, while 45% is due to persistent differences in the store-good component of prices (and to the covariance term). We then compute the extent and the sources of dispersion in the prices paid by different households that are implied by our theory. We find that the predictions of the theory line up very well with the data. The theory correctly predicts that the dispersion of prices paid by different households for the same basket of goods is lower than the dispersion of prices posted by different stores for the same good. Also, the theory correctly predicts that variation in the store component of prices contributes to a larger share of the dispersion of prices paid by households than to the dispersion of prices posted by stores. Intuitively, the predictions of the theory are correct because the variation of the store-good component of prices of individual goods washes out in the basket price of those households who need to purchase all goods from the same store.

Overall, our model of the retail market offers a parsimonious explanation for the extent and sources of dispersion in the price that different sellers charge for the same good, and for the extent and sources of dispersion in the price that different households pay for the same basket of goods.

The paper contributes to the large empirical literature documenting price dispersion. Ours is the first paper to use a large-scale dataset that covers multiple products, each sold at multiple stores and each observed over a long period of time. The data allows us to use techniques borrowed from labor economics to estimate the overall variance of prices, the variance due to temporary and persistent differences in the store component of prices, and the variance due to temporary and persistent differences in the store-good component of prices. The variance decomposition identifies a novel feature of retail prices: relative price dispersion. Part of the previous empirical literature has documented the extent of price dispersion for particular products: cars and anthracite coal in Stigler (1961), 39 products in Pratt, Wise, and Zeckhauser (1979), several books and CDs in Brynjolfsson and Smith (2000), 4 academic textbooks in Hong and Shum (2006), illegal drugs in Galenianos, Pacula, and Persico (2012), mortgage brokerage services in Woodward and Hall (2012). There are also studies that have used small-scale data that covers multiple products, each sold at multiple stores and each observed over time: car insurance policies in Alberta in Dahlby and West (1986), prescription drugs in upstate New York in Sorensen (2000), 4 products in Israel in Lach (2002). However, none of these papers has attempted to decompose price dispersion as in our paper and, hence, identified relative price dispersion.

The paper also contributes to the search-theoretic literature on price dispersion. In particular, we develop a version of the search-theoretic model of equilibrium price dispersion of Burdett and Judd (1983) in which buyers and sellers trade multiple goods and buyers are heterogeneous with respect to both their valuation and their ability to purchase different items at different stores. Our version of the model leads to relative price dispersion and fits well both data on the dispersion of prices across stores, and data on the dispersion of prices across households. There are many search-theoretic models of equilibrium price dispersion in the retail market for a single good, such as Butters (1977), Varian (1980), Burdett and Judd (1983), Stahl (1989) where price dispersion emerges because some buyers are in contact with a single seller and others are in contact with multiple sellers, Reinganum (1979) and

Albrecht and Axell (1984) where price dispersion emerges because of heterogeneity among buyers or sellers, Rob (1985), Stiglitz (1987), and Menzio and Trachter (2015b) where price dispersion obtains because sellers are large and have an impact on reservation prices. There are much fewer search-theoretic models of equilibrium price dispersion in the retail market for multiple goods as these models are famously difficult to analyze. The main exceptions are McAfee (1995), Zhou (2014), Rhodes (2015), Baughman and Burdett (2015) and Rhodes and Zhou (2015). Yet, none of these papers considers the type of buyers' heterogeneity on which our theory of relative price dispersion is built.

Outside of search theory, there are multiproduct models of the retail market in which a retailer finds it optimal to charge different prices for goods that are equally valued by buyers and equally costly to sellers (see, e.g., Lal and Matutes 1994). Asymmetric pricing is optimal because it is assumed that buyers are aware of the price of only a subset of goods when deciding where to shop. However, these theories typically imply that different retailers should all charge lower prices for the same subset of goods. Hence, these theories do not generate relative price dispersion. There are also some multiproduct models of the retail market in which, like in our model, buyers differ with respect to their ability to purchase different items in different locations (see, e.g., Lal and Matutes 1989, and Chen and Rey 2012). These models, even though different from ours in many dimensions, can also generate relative price dispersion. Unlike ours, these models have not been confronted with the data.

# 2 Relative Price Dispersion: Evidence

In this section, we jointly analyze the dispersion and dynamics of the prices of identical goods in the same geographical area and over the same period of time. We use a detailed dataset on prices that includes the time series of the price of a large number of goods at each of a large number of stores. We use these data to estimate a rich stochastic process for the average price level of a store, and for the price of a good at a store relative to the average price level of the store. We then use the estimated stochastic process to decompose the variance of the price of the same good in the same period of time and same geographic area. The new finding that we emphasize is that a substantial fraction of the cross-sectional variance of prices is due to the fact that stores that are, on average, equally expensive set persistently different prices for the same good. We refer to this phenomenon as relative price dispersion.

## 2.1 Framework and estimation strategy

Let $p_{jst}$ denote the quantity-weighted average price of good $j$ at store $s$ in time period $t$. In our application a time period is one week and a good is defined by its UPC (barcode). We first decompose the log of each price $p_{jst}$ into three additively separable components: a component that reflects the average price of the good in period $t$; $\mu_{jt}$; a component that reflects the expensiveness of the store selling the good, $y_{st}$; and a component that reflects factors that are unique to the combination of store and good, $z_{jst}$.[1] Formally, we decompose the log of $p_{jst}$ as

$$\log p_{jst} = \mu_{jt} + y_{st} + z_{jst}. \tag{1}$$

We model both the store component of the price, $y_{st}$, and the store-good component of the price, $z_{jst}$, as the sum of a fixed effect, a persistent part and a transitory part. This statistical model is motivated by the empirical shape of the auto-correlation functions of $y_{st}$ and $z_{jst}$, which are illustrated in Figure 1. The auto-correlation functions of $y_{st}$ and $z_{jst}$ display a sharp drop at short lags, followed by a smoothly declining profile that remains strictly positive even at very long lags. The initial drop in the auto-correlation suggests the presence of a transitory component in both $y_{st}$ and $z_{jst}$. We model the transitory components as an MA(q) process, rather than an IID process, to allow for the possibility that the transitory component may reflect temporary sales. Indeed, since sales may last longer than one week and since the timing of sales may not correspond to the weekly reporting periods, they are better captured by a process with some limited persistence than with a weekly IID process.[2] The smoothly declining portion of the auto-correlation function is consistent with the presence of an AR(1) component. Finally, the fact that the auto-correlation function remains positive even after 100 weeks suggests the presence of a fixed effect.

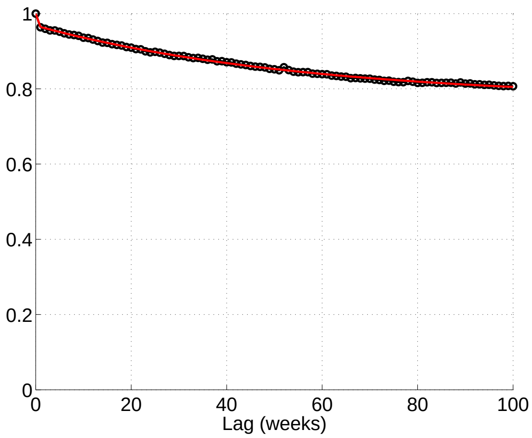
Formally, the statistical model for $y_{st}$ and $z_{jst}$ is given by

$$\begin{aligned}
y_{st} &= y_s^F + y_{st}^P + y_{st}^T, & z_{jst} &= z_{js}^F + z_{jst}^P + z_{jst}^T, \\
y_{st}^P &= \rho_y y_{s,t-1}^P + \eta_{s,t}^y, & z_{jst}^P &= \rho_z z_{js,t-1}^P + \eta_{js,t}^z, \\
y_{st}^T &= \varepsilon_{s,t}^y + \sum_{i=1}^{q} \theta_{y,i} \varepsilon_{s,t-i}^y, & z_{jst}^T &= \varepsilon_{js,t}^z + \sum_{i=1}^{q} \theta_{z,i} \varepsilon_{js,t-i}^z, \\
y_s^F &= \alpha_s^y, & z_{js}^F &= \alpha_{js}^z,
\end{aligned} \tag{2}$$

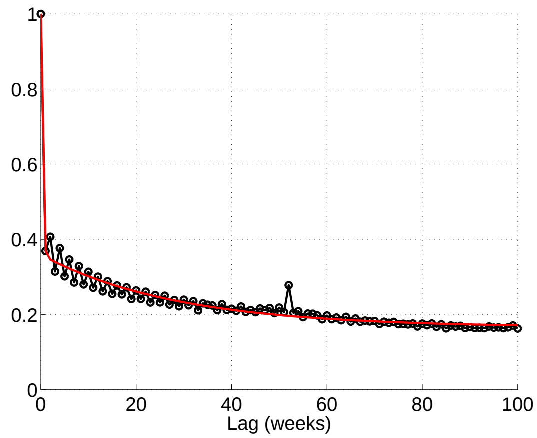
[1] We work with the natural logarithm of quantity-weighted average prices. This reflects an assumption that innovations to prices enter multiplicatively, which is convenient when jointly analyzing prices of many different goods.

[2] Later in this section we show that our findings are robust to alternative specifications for the transitory component.

Figure 1: Auto-correlation function of prices




(a) Store component




(b) Store-good component

*Notes:* The figure plots the empirical autocorrelation functions of the store and store-good components, $\hat{y}_{st}$ and $\hat{z}_{jst}$, together with their counterparts from the fitted statistical model.

where $y_s^F$ and $z_{js}^F$ denote the fixed-effects of the store and of the store-good components, $y_{st}^P$ and $z_{jst}^P$ denote the persistent parts of the store and of the store-good components, and $y_{st}^T$ and $z_{jst}^T$ denote the transitory parts of the store and of the store-good components. The parameters $\alpha_s^y$ and $\alpha_{js}^z$ are random variables with mean zero and variance $\sigma^2_{\alpha^y}$ and $\sigma^2_{\alpha^z}$. The parameters $\rho_y$ and $\rho_z$ are the auto-regressive parameters of the AR(1) part of the store and store-good components, while $\eta_{s,t}^y$ and $\eta_{js,t}^z$ are the innovations to the AR(1) part and are assumed to be random variables with mean zero and variance $\sigma^2_{\eta^y}$ and $\sigma^2_{\eta^z}$. Finally, the parameters $\theta_{y,i}$ and $\theta_{z,i}$ are the coefficients of the MA(q) part of the store and store-good components, while $\varepsilon_{s,t}^y$ and $\varepsilon_{js,t}^z$ are the innovations to the MA(q) part and are assumed to be normal random variables with mean zero and variance $\sigma^2_{\varepsilon^y}$ and $\sigma^2_{\varepsilon^z}$. All random variables are independent across goods, stores and times. In our baseline model we set $q = 1$.

We estimate the parameters of the statistical model in (2) using data on quantity-weighted average prices, $p_{jst}$, for a large number of goods $j = 1 \ldots J$, at a large number of stores $s = 1 \ldots S$ in a single geographic market $m$ at a weekly frequency $t = 1 \ldots T$. Given the large number of goods, stores and time periods, and the presence of unobserved components in prices, estimating this model via Maximum Likelihood (or with Panel Data Instrumental Variables regressions) is not feasible. Instead, we estimate the model using a multi-stage Generalized Method of Moments approach that is analogous to techniques that are commonly used when estimating models of labor earnings dynamics (see, e.g., Gottschalk and Moffitt

1994, Blundell and Preston 1998, Kaplan 2012).

The estimation procedure involves four steps.

**Step 1**. We estimate the good-time mean, $\mu_{jt}$, as the average of the log price, $\log p_{jst}$, across all stores $s$ in the market of interest, i.e.

$$\hat{\mu}_{jt} = \frac{1}{S}\sum_{s=1}^{S} \log p_{jst}. \tag{3}$$

We construct normalized prices as

$$\tilde{p}_{jst} = \log p_{jst} - \hat{\mu}_{jt}. \tag{4}$$

**Step 2**. We estimate the store component $\hat{y}_{st}$ by taking sample means of the normalized prices across all goods in store $s$, i.e.

$$\hat{y}_{st} = \frac{1}{n_{jst}}\sum_{j=1}^{n_{jst}} \tilde{p}_{jst}, \tag{5}$$

where $n_{jst}$ is the number of goods for which we have data for store $s$ in period $t$. In some instances $n_{jst} < J$ because not every store-good combination will meet our sample selection requirements in every week. We estimate the store-good component $z_{jst}$ as

$$\hat{z}_{jst} = \tilde{p}_{jst} - \hat{y}_{st}. \tag{6}$$

The above process leads to a $S \times T$ panel of store components $\{\hat{y}_{st}\}$, and a $(J \times S) \times T$ panel of store-good components $\{\hat{z}_{jst}\}$ (where there may be missing data for some combinations of $(j, s, t)$).

**Step 3**. We construct the auto-covariance matrix of each of these panels up to $L$ lags.

**Step 4**. We minimize the distance between the theoretical auto-covariance matrices implied by the model and the empirical auto-covariance function from step three. We use a diagonal weighting matrix that weights each moment by $n_{jst}^{0.5}$. However, the main results are not sensitive to using an identity weighting matrix instead.

## 2.2 Kilts-Nielsen Retail Scanner Dataset

We estimate the statistical model in (2) using the Kilts-Nielsen Retail Scanner Dataset (KNRS). The KNRS contains store-level weekly sales and unit average price data at the

UPC level. The dataset covers the period 2006 to 2012. The full dataset contains weekly price and quantity information for over 1.5 million UPCs at around 40,000 stores in over 2,500 counties across 205 Designated Market Areas (DMA). A DMA is a geographic area defined by Nielsen that is roughly the same size as a Metropolitan Statistical Area (MSA). Since our estimation procedure requires computing a full auto-covariance matrix at the store-good-week level, it is not feasible to estimate the model using anywhere near the full set of UPCs. For example, in the Minneapolis-St Paul DMA alone, the full data set would consist of over 200 million observations of $p_{jst}$ per year. Thus in order to keep the size of the analysis manageable, we restrict attention to a subset of the data.

We focus our analysis on a single DMA: Minneapolis-St Paul. However, there is nothing particularly special about Minneapolis St-Paul - in Section 2.4 we show that our findings are robust to extending the analysis to cover a broad set of geographically dispersed markets. For the Minneapolis-St Paul DMA, we focus on the 1000 UPCs with the largest quantities of sales in the state of Minnesota in the first quarter of 2010. These 1,000 products span 50 product groups, shown in Table 7 in Appendix A.[3] Table 8 shows how these 1000 sample UPCs are distributed across goods departments. To give a sense of how frequently these products are purchased, in the Minneapolis market areas in 2010:Q1, the product with the largest quantity of units sold (2.9 million units) was in the Fresh Eggs product module. The least frequently sold of these 1000 products (with just under 50,000 units sold) was in the Liquid Cocktail Mixes product module. Even after restricting attention to these 1000 products, the dataset is extremely large. Over the 7 year period from 2006 to 2012, we have over 40 million observations of prices $p_{jst}$. To ensure that our findings are not specific to this particular bundle of goods, in Section 2.4 we re-estimate the model using a number of alternative sets of UPCs, chosen in various ways.

We always estimate the model separately for each geographic area that we consider. For a given set of UPCs and a given geographic area, we select stores, goods and weeks that satisfy two criteria:

1. For each store/week combination, we have quantity and price data for at least $N_1$ of the UPCs in in the given set. In our baseline estimation we set $N_1 = 250$, and we report results for $N_1 \in \{50, 500\}$.

[3]Nielsen divides the full set of UPCs in the product database into 10 "departments", which are subdivided into around 125 "product groups", which are further subdivided into around 1,075 "product modules". For example, different sized bottles of Heinz tomato ketchup are distinct UPCs in the "Catsup" product module, which is one of 34 product modules in the "Condiments, Gravies and Sauces" product group, which is one of 38 product groups in the "Dry Grocery" department.

2. For each good/week combination, we have quantity and price data for at least $N_2$ stores. In our baseline estimation we set $N_2 = 50$,and we report results for $N_2 \in \{25, 100\}$.

These selection criteria ensure that we focus only on store/goods/weeks where we have sufficient data to reliably estimate the good-time means and store-time means in the first and second stages of the estimation procedure. In addition, to avoid the influence of large outliers when computing the empirical auto-covariance function, we drop observations of the store components and store-good components whose absolute value is greater than one.

## 2.3 Baseline decomposition

We first present results for the baseline set of 1000 goods in the Minneapolis-St Paul designated market area. We then consider the robustness of these results to a range of alternative specifications, including whether they vary significantly across markets in the US.

Figure 1 displays the fit of the auto-correlation function for the store component (Panel A) and the store-good component of prices (Panel B) out to 100 lags. The parameter estimates that correspond to this model are reported in Table 9 in Appendix A. For both components, the statistical model provides an excellent fit to the shape of the auto-correlation function. Several features of the auto-correlation functions are worthy of mention. First, the auto-correlation of the store component is very high even at long lags, foreshadowing our finding that almost all of the store component is persistent in nature. The auto-correlation of the store component asymptotes to around 0.8, equivalent to an auto-covariance of 0.0029 which is what identifies the variance of the fixed effect (whose standard deviation is estimated to be 5.3%). Second, the sharp drop in the auto-correlation of the store-good component after one lag suggests the presence of a large transitory component in prices. Third, the slow exponential decay and then flattening out of the store-good component suggest the presence also of a substantial persistent part of the store-good component. The auto-correlation of the store-good component asymptotes to around 0.17, equivalent to an auto-covariance of 0.0033 which is what identifies the variance of the fixed effect (whose standard deviation is estimated to be 5.7%). Fourth, the spike at 52 weeks reflects the fact that some products display annual regularities in their prices. Finally, the zig-zag pattern of the auto-correlation of the store-good component is due to regularities in the patterns of sales that are not captured by our statistical model.

Overall, the estimated model fits the data very well and, for this reason, we are comfortable using it to decompose the cross-sectional variance of the price at which the same good

Table 1: Dispersion in prices: persistent and transitory

| | Variance | | Percent | | Std. Dev. |
|---|---|---|---|---|---|
| Store component | | | | | |
| Transitory | 0.000 | | 3.2 | | 0.011 |
| Fixed plus Pers. | 0.004 | | 96.8 | | 0.059 |
| Total Store | | 0.004 | 100.0 | 15.5 | 0.060 |
| Store-good component | | | | | |
| Transitory | 0.013 | | 64.1 | | 0.113 |
| Fixed plus Pers. | 0.007 | | 35.9 | | 0.084 |
| Total Store-good | | 0.020 | 100.0 | 84.5 | 0.141 |
| Total | | 0.023 | | 100.0 | 0.153 |

*Notes:* The left panel presents the cross-sectional variances of UPC prices, as well as the store and store-good components separately. The middle panel presents the decomposition of this variance into persistent and transitory components. The right panel presents the cross-sectional standard deviations.

is sold in the same week and in the same market. The variance decomposition is reported in Table 1. The overall variance of prices is 0.023, equivalent to a standard deviation of 15.3%. The variance of the store component accounts for 15% of the overall variance of the price, while the variance of the store-good component accounts for the remaining 85%. That is, most of the variation in the price at which a good is sold is not due to the fact that the good is sold at stores that are, on average, more or less expensive. Rather, most of the variation in the price at which a good is sold is due to the fact that the good is sold at different prices at stores that are, on average, equally expensive.

The variation in prices associated with the store and store-good components could be due to either the transitory or persistent components. The statistical model (2) is designed to distinguish between these two sources of variation. Since the estimated persistence of the AR(1) component of prices is extremely close to unity for both the store and store-good components (the estimates of $\rho_z$ and $\rho_y$ for the baseline model are 0.965 and 0.983, respectively), we group the fixed effect and AR(1) components together and refer to these as the persistent part of the price, and we refer to the MA components as the transitory part of the price.[4]

The decomposition in Table 1 reveals that nearly all of the price variance that is due to variation in the store component comes from persistent differences in the average price of different stores. In contrast, 64% of the price variance that is due to the variation in the

[4]The estimation results remain very similar if we adopt an identity weighting matrix for the GMM objective, instead of a diagonal weighting matrix based on sample sizes.

store-good component comes from transitory differences across stores in the price of the good relative to the average price of the store. Yet, over one-third of the price variation that is due to the variation in the store-good component comes from persistent differences across stores in the relative price of the good. This is what we call relative price dispersion. Relative price dispersion is a feature of the data that had not been documented before and, at first blush, it seems hard to rationalize. Why do stores that are, on average, equally expensive choose to systematically charge different prices for the very same good?

Finally, notice that while variance decompositions are a convenient tool for breaking down dispersion into orthogonal elements, the fact that variances are measured in squared prices makes the comparison of the various elements somewhat hard to interpret. For this reason, the last column of Table 1 reports the standard deviation of each of the orthogonal components of prices implied by the model. The overall standard deviation of prices is 15% and the standard deviation due to persistent differences in relative prices is 8%. These figures further emphasize the point that persistent differences in relative prices are an important feature of the retail market.

## 2.4 Robustness

The estimates in Table 1 highlight two important features of price dispersion, both of which turn out to be extremely robust. First, the vast majority of price dispersion is due to variation in the store-good component of prices, rather than the store component of prices. Second, of the variation in the store-good component, at least one-third is due to highly persistent differences across stores in the price of the good relative to the price of the store.[5] In this section, we present results from a series of estimates under alternative assumptions about the sample selection criteria, the statistical model for prices, for different types of products, and different geographic areas.

### 2.4.1 Sample Selection

Our baseline selection criteria required that a minimum of $N_1 = 250$ of the 1000 goods in our sample be sold at a given store in a given week for that store/week to be included in the estimation sample. Table 2 reports variance decompositions for $N_1 \in \{50, 500\}$ and shows that the results are not sensitive to this particular threshold. Our baseline selection criteria

[5] An immediate implication of these two features of the data is that the price ranking of stores for a particular good differs a lot from one good to another, and that the correlation over time of these rankings is non-negligible.

Table 2: Robustness to sample criteria

| | $N_1 = 50$ | | $N_1 = 500$ | | $N_2 = 25$ | | $N_2 = 100$ | |
|---|---|---|---|---|---|---|---|---|
| | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% |
| Store | | | | | | | | |
| Transitory | 0.017 | 5.0 | 0.011 | 3.6 | 0.011 | 3.2 | 0.011 | 3.3 |
| Fixed plus Pers. | 0.075 | 95.0 | 0.055 | 96.4 | 0.058 | 96.8 | 0.063 | 96.7 |
| Total Store | 0.077 | 19.0 | 0.056 | 13.6 | 0.059 | 15.3 | 0.064 | 16.8 |
| Store-good | | | | | | | | |
| Transitory | 0.126 | 63.3 | 0.113 | 65.4 | 0.111 | 63.8 | 0.114 | 65.0 |
| Fixed plus Pers. | 0.096 | 36.7 | 0.082 | 34.6 | 0.084 | 36.2 | 0.084 | 35.0 |
| Total Store-good | 0.158 | 81.0 | 0.140 | 86.4 | 0.139 | 84.7 | 0.141 | 83.2 |
| Total | 0.176 | 100.0 | 0.151 | 100.0 | 0.151 | 100.0 | 0.155 | 100.0 |

*Notes:* This table presents a robustness exercise comparing our baseline results to results obtained using alternative cutoffs for required numbers of observations.

also required that a minimum of $N_2 = 50$ stores have positive sales for a given good in a given week for that good/week to be included in the sample. Table 2 also reports variance decompositions for $N_2 \in \{25, 100\}$ and shows that the results are also not sensitive to this threshold. Thus, it is unlikely that relative price dispersion is a statistical artifact of small samples and/or insufficient overlap of goods across stores. This is important because for some of the sets of UPCs considered below, we are require to set $N_1 = 50$ and $N_2 = 25$ in order to have sufficient overlap for reliable estimation.

### 2.4.2 Statistical Model

In our baseline specification, we modeled the transitory part of the store-good component as an MA(1), which implicitly assigns all price changes with a duration greater that one week to the persistent component. Since the transitory component is intended to capture the effects of temporary sales, the reader may be concerned that, if some sales last more than one week, our baseline specification may be interpreting some sales-induced price variation as relative price dispersion. To show that this is not the case, Table 3 reports the variance decomposition when we model the temporary component as an MA(5) or MA(10), thus allowing the transitory component to capture sales that potentially last up to 10 weeks. The decomposition is barely affected - the persistent parts of the store-good component still account for at least one-third of the variance of the store-good component.

The reader may also be concerned that modeling temporary sales as an MA process of any order may fail to capture salient features of sales dynamics, and that this may lead

Table 3: Robustness to statistical model

| | MA(5) | | MA(10) | | Skewed MA(1) | | Sales | |
|---|---|---|---|---|---|---|---|---|
| | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% |
| Store-good | | | | | | | | |
| Transitory | 0.114 | 65.6 | 0.114 | 66.1 | 0.113 | 64.1 | 0.098 | 57.7 |
| Fixed plus Pers. | 0.082 | 34.4 | 0.082 | 33.9 | 0.084 | 35.9 | 0.084 | 42.3 |
| Total Store-good | 0.141 | 100.0 | 0.141 | 100.0 | 0.141 | 100.0 | 0.141 | 100.0 |

*Notes:* This table shows variance decompositions for the store-good component after replacing the MA(1) process with: (i) an MA(5); (ii) an MA(10); (iii) an MA(1) that allows skewness in the disturbances; and (iv) an explicit model of sales, as described in the main text.

us to understate the importance of sales. To show that this is not the case, we consider two alternative approaches to modeling sales. First, we recognize that the cross-sectional distribution of prices induced by periodic sales is likely to be negatively skewed, which may help in identifying the component of price dispersion that is due to sales. To account for this possibility we allow that the transitory innovation to the store-good component, $\varepsilon^{z}_{js,t}$ is drawn from a skewed distribution, whose skewness we estimate alongside the other parameters of the model.[6] As expected, the estimated skewness is mildly negatively skewed (coefficient of skewness $= -0.6$), consistent with sales. However, the decomposition of the variance of prices, shown in Table 3 is not affected.

Second, we depart from the assumption of an MA process for the transitory store-good component, and replace it with a process that more explicitly resembles temporary sales. In this model, the transitory store-good component is modeled as 2-point process. In a given week, there is a probability $\phi$ that each good is on sale, and sales are independently distributed across goods and over time. If a good is on sale, then it is discounted from its regular price by a fraction $\delta$. We assume that all sales last exactly one week, and the day that a good goes on sale is uniformly distributed within the week. This means that each sale will affect the observed price of the good in two adjacent weeks, so the auto-covariance of prices is impacted at both the 0 and 1st lag (as with an MA(1) process). Since we work with normalized prices, there is an additional restriction that the mean value of the transitory component, after accounting for sales, is zero. The estimated weekly sales probability is $\phi = 4.65\%$, and the corresponding average discount is $\delta = 52\%$.[7] The associated variance

[6]Note that our estimation procedure does not require distributional assumptions on the innovations. Our baseline specification and procedure uses only second moments of the price data and only estimates the distribution of price innovations up to their second moments. In order to achieve identification of third moments, we include joint third moments of prices.

[7]As with the skewed MA process, the sales process requires joint third moments of prices to be included in the GMM objective in order to achieve identification. We have also explored richer specifications in which

decomposition, show in Table 3, reveals that the persistent components of the store-good variance are even larger than in the baseline. We conclude that relative price dispersion is not driven by temporary sales, and is a distinct feature of price distributions.

### 2.4.3 Products

Our baseline set of goods is the 1000 most-commonly purchased products in Minnesota in the first quarter of 2010. We now show that relative price dispersion is not specific to this set of goods, but rather is a robust phenomenon that is present among samples of products chosen in a broad variety of ways. The analysis also serves the purpose of rejecting some simple explanations for relative price dispersion, such as managerial inattention, store-good cost differentials, and different styles of shelf management.

**Frequency of purchase** Our baseline procedure weights each good equally when constructing the good-time means and the store components. In Table 4, we report the variance decomposition when we use quantity weights to construct the good-time means and store components. The decomposition is barely affected by this change.

Our baseline sample comprises only goods that are purchased very frequently. We examine whether relative price dispersion is a feature of the data for less frequently purchased goods. To do this we select a sample of the 1000 goods ranked 9001 to 10,000 in terms of their frequency of purchase in Minnesota in the first quarter of 2010. This choice is motivated by our desire to select substantially less-commonly purchased goods than in our baseline sample, while still satisfying the requirement that the goods are sufficiently commonly purchased so that there is enough overlap across stores and enough continuity in weekly sales to meet our two inclusion criteria. The types of goods in this alternative sample, shown in Table 10 in Appendix A, are quite different from those in the baseline sample. However, the variance decomposition for this set of goods, shown in Table 4 (labelled "UPC-alt"), is extremely similar to the baseline.

Lastly, we selected a different sample of goods based on frequently-purchased goods *nationwide* in the first quarter of 2010, rather than frequently-purchased goods in Minnesota. Selecting a sample in this way is useful for when we extend our analysis to other parts of the country below. To construct this sample we created two lists of the most commonly

the sales discount $\delta$ is itself a (possibly negatively skewed) random variable, and none of our main findings are affected.

Table 4: Robustness

| | Baseline | | Weighted | | UPC-alt | | UPC-national | |
|---|---|---|---|---|---|---|---|---|
| | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% |
| Store | | | | | | | | |
| Transitory | 0.011 | 3.2 | 0.019 | 21.0 | 0.006 | 1.4 | 0.011 | 2.2 |
| Fixed plus Pers. | 0.059 | 96.8 | 0.037 | 79.0 | 0.054 | 98.6 | 0.072 | 97.8 |
| Total Store | 0.060 | 15.5 | 0.041 | 6.5 | 0.055 | 18.7 | 0.073 | 18.9 |
| Store-good | | | | | | | | |
| Transitory | 0.113 | 64.1 | 0.124 | 62.6 | 0.082 | 51.1 | 0.119 | 61.0 |
| Fixed plus Pers. | 0.084 | 35.9 | 0.096 | 37.4 | 0.080 | 48.9 | 0.095 | 39.0 |
| Total Store-good | 0.141 | 84.5 | 0.156 | 93.5 | 0.114 | 81.3 | 0.152 | 81.1 |

| | Low price | | High price | | Low durability | | High durability | |
|---|---|---|---|---|---|---|---|---|
| | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% |
| Store | | | | | | | | |
| Transitory | 0.024 | 8.7 | 0.025 | 15.6 | 0.013 | 4.0 | 0.027 | 27.9 |
| Fixed plus Pers. | 0.078 | 91.3 | 0.059 | 84.4 | 0.062 | 96.0 | 0.043 | 72.1 |
| Total Store | 0.082 | 20.6 | 0.065 | 15.9 | 0.063 | 19.3 | 0.051 | 19.4 |
| Store-good | | | | | | | | |
| Transitory | 0.122 | 57.4 | 0.130 | 77.0 | 0.103 | 64.0 | 0.077 | 55.8 |
| Fixed plus Pers. | 0.105 | 42.6 | 0.071 | 23.0 | 0.077 | 36.0 | 0.069 | 44.2 |
| Total Store-good | 0.161 | 79.4 | 0.148 | 84.1 | 0.129 | 80.7 | 0.103 | 80.6 |

| | Unilever | | Coca-cola | | State: MN | | County: Hennepin | |
|---|---|---|---|---|---|---|---|---|
| | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% | Sd | Dec/% |
| Store | | | | | | | | |
| Transitory | 0.035 | 27.4 | 0.030 | 15.5 | 0.011 | 2.5 | 0.015 | 6.2 |
| Fixed plus Pers. | 0.058 | 72.6 | 0.070 | 84.5 | 0.070 | 97.5 | 0.058 | 93.8 |
| Total Store | 0.068 | 21.3 | 0.076 | 26.2 | 0.071 | 17.6 | 0.060 | 12.5 |
| Store-good | | | | | | | | |
| Transitory | 0.101 | 60.9 | 0.106 | 68.9 | 0.120 | 60.9 | 0.128 | 64.4 |
| Fixed plus Pers. | 0.081 | 39.1 | 0.071 | 31.1 | 0.096 | 39.1 | 0.095 | 35.6 |
| Total Store-good | 0.130 | 78.7 | 0.127 | 73.8 | 0.154 | 82.4 | 0.159 | 87.5 |

*Notes:* This table presents a robustness exercise comparing our baseline results to results obtained using alternative specifications: quantity weighting in constructing the store and store-good components, alternative samples of UPCs (UPC-alternative and UPC-national), the low and high price samples, the low and high durability samples, the Unilever and Coca-cola samples, alternative definitions of a market (state of Minnesota and Hennepin county). Tables 7, and 10-17 in Appendix A illustrate the diversity in product groups across the alternative samples.

purchased UPCs, one based on quantity and one based on revenue. We then selected the 1,463 goods which appear in either list. The decomposition for this set of 1,463 goods is also shown in Table 4 (labelled "UPC-national"). For this set of goods, the store component accounts for slightly more of the overall price variation, but the persistent components of prices account for even more of the variance of the store-good component. Hence relative price dispersion is larger in this set of goods than in the baseline.

**High-price and low-price goods** A simple explanation for relative price dispersion is managerial inattention (see, e.g., Ellison, Snyder, and Zhang 2015). Equally expensive stores may set persistently different prices for the same good because managers choose to not pay much attention to the price of low-ticket items.[8] This potential explanation for relative price dispersion motivates us to decompose price dispersion for low and high-price goods separately. We divide our baseline sample of 1000 UPCs according to their average unit price. The low-price subsample of 430 UPCs has a median unit average price of 99 cents, a 5th percentile of 39 cents and a 95th percentile of $1.79; the high-price subsample of 315 UPCs has a median unit average price of $3.59 cents, a 5th percentile of $2.39 and a 95th percentile of $6.99. The variance decompositions for these two subsamples are shown in Table 4. The low-price subsample features more relative price dispersion than the full sample – the store-good component accounts for 79% of the overall variance of prices, of which the persistent components account for 43%. The high-price subsample features less relative price dispersion than the full sample, but relative price dispersion is still a substantial fraction of overall price dispersion. Hence, relative price dispersion is not only a feature of low-price, low-revenue goods, and thus is unlikely to be entirely due to managerial inattention.

**Products from a single distributor** Another possible explanation for relative price dispersion is that equally expensive stores set persistently different prices for the same good because they have better or worse relationships (and, hence, are charged lower or higher prices) with the wholesaler.[9] This potential explanation motivates us to decompose price dispersion for a subset of products produced and distributed by a single wholesaler. Indeed, if relative price dispersion is caused by different retailer-wholesaler relationships, relative price dispersion should be absorbed by the store component when we restrict attention to products from a single wholesaler.

[8]We thank Stephan Seiler for suggesting this hypothesis.

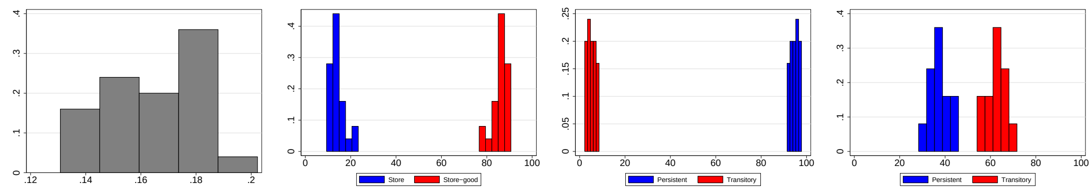
[9]We thank Matthew Gentzkow for suggesting this hypothesis.

We consider two subsamples of goods. In the first subsample, there are only products from Coca-Cola. In the second subsample, there are only products from Unilever. The 3,608 UPCs in our Coca-Cola subsample are primarily various types of beverages. The 10,866 UPCs in our Unilever subsample come from a variety of product groups – "Hair Care" is the product group with the largest fraction of UPCs (32%), followed by "Personal Soap and Bath Additives" (13%), "Deodorant" (12%) and "Skin Care Preparations" (10%).

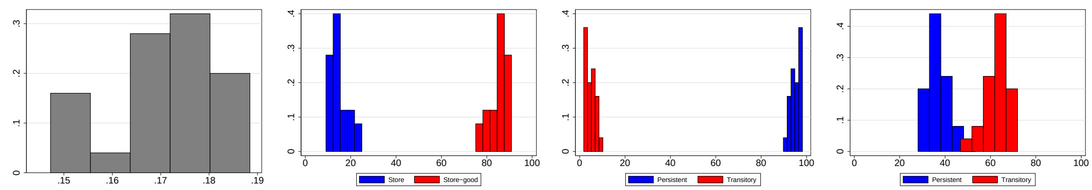
The variance decompositions for each these two subsamples of goods is shown in the bottom row of Table 4. For both samples, the overall degree of price dispersion is very similar to the degree of price dispersion in our baseline sample. However, the fraction of variation that is due to the store component is somewhat larger – 21% for Unilever and 26% for Coca-Cola, compared with 16% for the baseline. This is consistent with the hypothesis that some part of price dispersion is due to different relationships between particular stores and particular distributors. However, for both of these distributors, the vast majority of price dispersion is due to the store-good component, and of this, the persistent parts account for 39% (Unilever) and 31% (Coca-Cola). Thus relative price dispersion exists even when only considering goods from the same distributor, and so is not only driven by heterogeneity in distributional relationships.

**High-durability and low-durability products** Another natural explanation for relative price dispersion is shelf management. Some stores may keep perishable goods on their shelves for longer and, for this reason, sell them at systematically lower prices, while other stores may remove perishable goods sooner and, for this reason, sell them at systematically higher prices. This observation motivates us to decompose price dispersion separately for two subsamples of goods: low-durability goods (i.e., perishable goods) and high-durability goods.[10] The variance decompositions for these two subsamples are shown in Table 4. Even though the two subsamples contain very different sets of products, the overall decomposition of price dispersion is quite similar. For both subsamples, the store component accounts for approximately 20% and the store-good component for 80% of the cross-sectional variance of prices. For both subsamples, the transitory part accounts for roughly 2/3 and the persistent part for roughly 1/3 of the cross-sectional variance of the store-good component of prices.

[10]We thank Boyan Jovanovic for suggesting us to measure relative price dispersion for low and high-durability products. We also thank George Alessandria for sharing the durability indexes constructed in Alessandria, Kaboski, and Midrigan (2010). We merge this index to the Nielsen database at the product module level by comparing descriptions of products. We define low durability goods as those with a durability index of less than 2 months, and high durability goods as those with a durability index of more than 140 months.




(a) Sd of prices (b) Store vs store-good (c) Store (d) Store-good




(e) Sd of prices (f) Store vs store-good (g) Store (h) Store-good

*Notes:* These histograms present a robustness exercise looking at how our results for price dispersion and the variance decompositions vary across geographic regions across the country. The top row considers designated market areas across the US, and the bottom counties across the US.

Figure 2: Price dispersion and variance decompositions across geographic areas

These findings suggest that relative price dispersion is unlikely to be a phenomenon caused by different styles of shelf management for perishable goods. Indeed, relative price dispersion turns out to be slightly more important in the subsample of goods that are less perishable.

#### 2.4.4 Markets

So far our analysis has focused on a single geographic region - the Minneapolis-St Paul designated market area. Here, we show that none of our results are specific to this level of geographic aggregation or this part of the country. First, we consider alternative levels of geographic aggregation for the definition of a market. In Table 4 we report the variance decomposition when we use a broader definition of market (the state of Minnesota) and a narrower definition of a market (Hennepin county, which is contained in the Minneapolis-St Paul DMA). All our findings are robust to switching to either of these alternative levels of aggregation.

Second, we extend our analysis to the whole of the United States in order to verify that our findings are not specific to Minneapolis-St Paul. We present results both at the level of a DMA and the county level. For each level of geographic aggregation, we selected the 25 largest areas by revenue and repeated the estimation for each market, using the same set of 1,463 UPCs. As described above, this set of UPCs was chosen to reflect UPCs that are commonly purchased nationwide.

Figure 2(a) displays a histogram of the standard deviation of prices in each of the 25 DMAs. The corresponding variance decomposition between the store and store-good components is shown in Figure 2(b), and the fraction of the variance of each component that is due to transitory versus persistent factors is shown in Figures 2(c) and 2(d), respectively. The analogous statistics are displayed for the 25 counties in the bottom row of Figure 2. These figures show very clearly that our findings are not unique to any one particular region but instead are a general feature of price dynamics and distributions. For all geographic areas, virtually all of the variance of prices occurs is due to the store-good component, rather than the store component, and a substantial part of the variance of the store-good component (between one-third and one-half) is very persistent in nature.

# 3 Relative Price Dispersion: Theory

In this section, we develop a theory of relative price dispersion. According to our theory, relative price dispersion emerges as a strategy that retailers use in order to price discriminate between high-valuation buyers who need to make all of their purchases in the same location, and low-valuation buyers who have the time to purchase different items in different places. In subsection 3.1, we describe the model, which is an extension to multiple goods and multiple types of buyers of the canonical theory of price dispersion of Burdett and Judd (1983). In subsection 3.2, we establish some general properties of equilibrium. In subsection 3.3, we characterize the equilibrium when competition between sellers is weak. In this equilibrium, sellers post prices that are attractive only to the high-valuation buyers. Since high-valuation buyers purchase all goods in the same location, the model behaves like the one-good model of Burdett and Judd (1983). In subsection 3.4, we characterize the equilibrium when competition between sellers is stronger. In this equilibrium, some sellers post some prices that are attractive to low-valuation buyers. Since low-valuation buyers can purchase different goods from different locations, the model does not behave like Burdett and Judd (1983) and, indeed, the model features relative price dispersion. In subsection 3.5, we briefly describe the equilibrium when competition between sellers is strongest.

## 3.1 Environment

We consider a retail market populated by homogeneous sellers and heterogeneous buyers who trade two goods (i.e., good 1 and good 2). Specifically, the market is populated by a measure $s > 0$ of identical sellers.[11] Every seller is able to produce each of the two goods at the same, constant marginal cost, which we normalize to zero. Every seller chooses a price for good 1, $p_1$, and a price for good 2, $p_2$, so as to maximize its profits taking as given the distribution $H(p_1, p_2)$ of the vector of prices across sellers. We find it useful to denote as $F_i(p)$ the fraction of sellers whose price for good $i \in \{1, 2\}$ is smaller than $p$, and with $\lambda_i(p)$ the fraction of sellers whose price for good $i \in \{1, 2\}$ is equal to $p$. We refer to $F_i(p)$ as the distribution of prices for good $i \in \{1, 2\}$. Similarly, we find it useful to denote as $G(q)$ the fraction of sellers whose prices $p_1$ and $p_2$ sum up to less than $q$, and with $\nu(q)$ the fraction of sellers whose prices sum up to $q$. We refer to $G(q)$ as the distribution of basket prices.

On the other side of the retail market there is a measure 1 of buyers. A fraction $\mu_b \in (0, 1)$ of buyers are of type $b$ and a fraction $\mu_c = 1 - \mu_b$ of buyers are of type $c$, where $b$ stands for *busy* and $c$ stands for *cool*. A buyer of type $b$ demands one unit of each good, for which he has valuation $u_b > 0$. A buyer of type $c$ demands one unit of each good, for which he has valuation $u_c$, with $u_b > u_c > 0$. More specifically, if a buyer of type $i \in \{b, c\}$ purchases both goods at the prices $p_1$ and $p_2$, he attains a utility of $2u_i - p_1 - p_2$. If a buyer of type $i \in \{b, c\}$ purchases one of the two goods at the price $p$, he attains a utility of $u_i - p$. If a buyer of type $i \in \{b, c\}$ does not purchase any of the goods, he attains a utility of zero.

In the retail market, trade is frictional. We assume that a buyer cannot purchase from just any seller in the market, as each buyer has only access to a small network of sellers. In particular, a buyer of type $b$ can access only one seller with probability $\alpha_b$, and he can access multiple (namely, two) sellers with probability $1 - \alpha_b$, with $\alpha_b \in (0, 1)$. Similarly, a buyer of type $c$ can access only one seller with probability $\alpha_c$, and two sellers with probability $1 - \alpha_c$, with $\alpha_c \in (0, 1)$. For the sake of simplicity, we let $\alpha_b = \alpha_c = \alpha$.[12] We refer to a buyer who can only access one seller as a *captive buyer*, and to a buyer who can access multiple sellers as a *non-captive buyer*. We interpret the above restrictions on the buyers' access to sellers as physical constraints (i.e., sellers that the buyer can easily reach) rather than

[11] We assume that sellers are ex-ante identical. If sellers had different costs or faced different populations of buyers, it would be easy to generate relative price dispersion. However, such explanations of relative price dispersion would be basically unfalsifiable, as data on wholesale prices and demand curves faced by different retailers is generally unavailable. Indeed as noted by Stigler (1961): "*It would be metaphysical, and fruitless, to assert that all dispersion is due to heterogeneity.*" To further strengthen this point, let us draw a parallel with the literature on temporary sales. Clearly, one could explain temporary sales with temporary declines in wholesale prices or with temporary increases in the elasticity of demand faced by retailers. Yet, because such explanations are hard to falsify and seem empirically implausible, the literature has developed theories in which sellers choose to have temporary sales in a stationary environment (see, e.g., Conlisk, Gerstner and Sobel 1984, Sobel 1984 or Albrecht, Postel-Vinay and Vroman 2013).

[12] It is tedious but straightforward to generalize our theory of relative price dispersion to the case of $\alpha_c > \alpha_b$.

as informational constraints [13] (i.e. sellers of which the buyer is aware). We also assume that a buyer of type $b$ must always make all of his purchases from just one of the sellers in his network. In contrast, a buyer of type $c$ can purchase different goods from different sellers in his network. We interpret this assumption as heterogeneity in the buyers' ability or willingness to visit multiple stores when shopping.

A few comments about the environment are in order. We consider a version of the canonical model of price dispersion of Butters (1977) and Burdett and Judd (1983). In this model, the co-existence of buyers who have access to only one seller and of buyers who have access to multiple sellers induces identical sellers to post different prices for the same good. We depart from the canonical model by considering a market in which buyers and sellers trade two goods. Obviously, to develop a theory of relative price dispersion, it is necessary to consider multiproduct retailing. Two products is the simplest case of multiproduct retailing. We also depart from the canonical model by considering a market in which buyers are heterogeneous. In particular, we assume that buyers of type $b$ are willing to pay higher prices for the goods than buyers of type $c$, and they are less willing to purchase different goods from different retailers than buyers of type $c$. It is natural to think about type-$b$ buyers as buyers whose time has high value in the labor market, and about type-c buyers as buyers whose time has low value in the labor market. Hence, type-$b$ buyers are willing to pay higher prices but they are hesitant to spend time shopping at different retailers.[14] The negative correlation in the buyers' willingness to pay and ability to shop from multiple retailers is the key to our theory of relative price dispersion.

Our model of the retail market is static, as Butters (1977) and Burdett and Judd (1983). We interpret the equilibrium price distribution of our static model as a long-term outcome. Indeed, in a repeated version of the model, it is immediate to see that sellers would have nothing to gain from changing their prices over time. Moreover, in the presence of any type of adjustment costs, sellers would face a loss from changing their prices over time. Thus, in a repeated version of the model, sellers would keep their prices constant. Given our interpretation of the model, we compare the equilibrium price distribution to the distribution of the persistent component of seller's prices.[15]

[13] The reader may want to think of our model as a version of Hotelling (1929) in which buyers have a zero transportation cost to access some sellers, and an infinite transportation cost to access some other sellers.

[14] As suggested by Kaplan and Menzio (2014), the reader may want to think of type-$b$ buyers as employed ones, and of type-$c$ buyers as the unemployed and the retirees.

[15] It would be easy to extend our model to capture the high-frequency fluctuations in sellers' prices. Menzio and Trachter (2015a) use a version of Butters (1977) and Burdett and Judd (1983) to show that, if the buyers who are more willing to intertemporally substitute their purchases are also in contact with more sellers (or have a lower willingness to pay for goods), then each seller will post different prices at different times of the

## 3.2 General properties of equilibrium

We start the analysis of the model by establishing some general properties of equilibrium. First, we argue that there are no sellers whose prices $(p_1, p_2)$ sum up to more than $2u_b$. Second, we consider sellers whose prices $(p_1, p_2)$ sum up to strictly more than $u_b + u_c$. These are sellers who cannot price an individual good below the willingness to pay of buyers of type $c$, without raising the price of the other good above the willingness to pay of buyers of type $b$. We show that, in any equilibrium, the prices of these sellers must lie in a particular region in $\mathbb{R}^2_+$. Third, we consider sellers whose prices sum up to less than $u_b + u_c$ and strictly more than $2u_c$. These are sellers who can price one of the goods below the willingness to pay of buyers of type $c$, while keeping the price of the other good below the willingness to pay of buyers of type $b$. We show that, in any equilibrium, the prices of these sellers must lie in one of two particular regions in $\mathbb{R}^2_+$.

Lemma 1 below contains two results. First, the lemma shows that a seller never finds it optimal to post a price greater than the willingness to pay of type-$b$ buyers. This result is intuitive, as neither buyers of type-$b$ nor buyers of type-$c$ are willing to purchase goods at a price greater than $u_b$. Second, the lemma establishes that, if the sum $q$ of the prices posted by a seller is strictly greater than $u_b + u_c$, then the seller's prices $p_1$ and $p_2$ are both strictly greater than the willingness to pay of type-$c$ buyers and smaller than the willingness to pay of type-$b$ buyers. That is, if the sum $q$ of the prices posted by a seller is strictly greater than $u_b + u_c$, the seller's prices $p_1$ and $p_2$ must fall in the $R_1$ region in Figure 3. This result is also intuitive. If a seller has a basket price strictly greater than $u_b + u_c$, he must post $p_1 > u_c$ and $p_2 > u_c$ in order to make sure that both of his prices are smaller than $u_b$.

**Lemma 1**: (i) In any equilibrium, no seller posts prices $(p_1, p_2)$, such that $p_1 > u_b$ and/or $p_2 > u_b$. (ii) In any equilibrium, a seller with a basket price $q > u_b + u_c$ posts prices $(p_1, p_2) \in R_1$, where $R_1$ is defined as

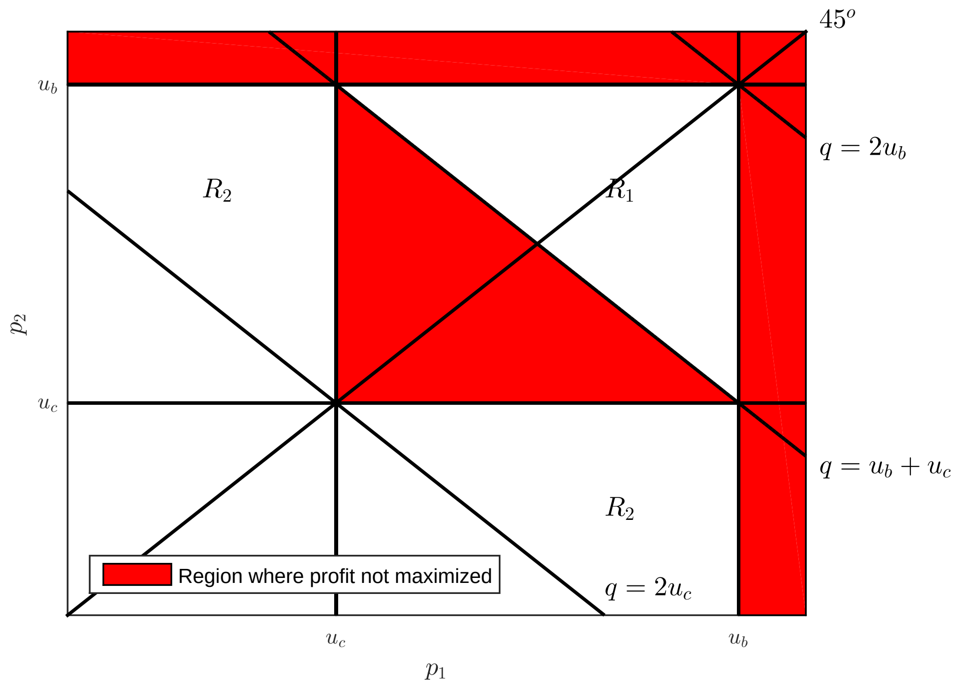
$$R_1 = \{(p_1, p_2) : p_1 \in (u_c, u_b], p_2 \in (u_c, u_b], p_1 + p_2 > u_c + u_b\}. \tag{7}$$

*Proof*: Appendix B. ■

Lemma 2 states that, if the sum $q$ of the prices posted by a seller is smaller than $u_b + u_c$ (but strictly greater than $2u_c$), then the seller's prices $p_1$ and $p_2$ are such that one of them is smaller than the willingness to pay of type-$c$ buyers, and the other one is greater than the

week in order to discriminate between different types of buyers.

Figure 3: Pricing decision of sellers




*Notes:* This figure considers the pricing decision of sellers discussed in the text, illustrating which regions of the $(p_1, p_2)$ space will not be profit-maximizing. Conditional on a basket price in the interval $u_b + u_c \leq q \leq 2u_b$, sellers will not price outside $R_1$. Conditional on a basket price in the interval $2u_c \leq q \leq u_b + u_c$, sellers will not price outside $R_2$.

willingness to pay of type-$c$ buyers. That is, if the sum q of the prices posted by a seller is smaller than ub+uc and strictly greater than $2u_c$, the seller's prices $p_1$ and $p_2$ must fall in one of the two region marked $R_2$ in Figure 3.

Let us provide some intuition for Lemma 2. Consider a seller with a basket price of $q \in (2u_c, u_b + u_c]$ who posts the same price $p_1 = p_2 = q/2$ for both goods. The seller will trade with some type-$b$ buyers. However, the seller will not trade with any type-$c$ buyers, as the price of each good is above their willingness to pay $u_c$. Now suppose that the seller starts lowering the price of one good and, at the same, starts increasing the price of the other good in order to keep the price of the basket constant. The seller will trade with the same number of type-$b$ buyers as before because these buyers, having to make all of their purchases in the same place, only care about the price of the basket. However, when the seller brings the price of the cheaper good below $u_c$, he will start also to trade this good to some type-$c$ buyers. Therefore, a seller with a basket price of $q \in (2u_c, ub + u_c]$ will never find it optimal to post the same price for both goods. For this seller, it is optimal to price the two goods asymmetrically.

**Lemma 2**: In any equilibrium, a seller with a basket price $q \in (2u_c, ub + u_c]$ posts prices $(p_1, p_2) \in R_2$, where $R_2$ is defined as

$$\begin{aligned} R_2 = \quad & \{(p_1,p_2) : p_1 \in [0,u_c], p_2 \in (u_c,u_b], p_1+p_2 \in (2u_c, u_b+u_c]\} \\ & \cup\{(p_1,p_2) : p_2 \in [0,u_c], p_1 \in (u_c,u_b], p_1+p_2 \in (2u_c, u_b+u_c]\}. \end{aligned} \tag{8}$$

*Proof*: Suppose that there is an equilibrium where a seller posts prices $(p_1, p_2)$, with $q \equiv p_1 + p_2 \in (2u_c, ub + u_c]$, $p_1 \in (u_c, u_b)$ and $p_2 \in (u_c, u_b)$. The seller attains a profit of

$$S(p_1, p_2) = \mu_b \left[\alpha + 2(1-\alpha)\left(1 - G(q) + \nu(q)/2\right)\right] q. \tag{9}$$

Let us explain (9) in detail. The seller is in the network of $\mu_b\alpha$ captive buyers of type $b$. A captive buyer of type $b$ will purchase both goods from the seller, as $p_1 \leq u_b$ and $p_2 \leq u_b$. The seller is in the network of $\mu_b 2(1-\alpha)$ non-captive buyers of type $b$. If the second retailer in the network of such a buyer has a basket price $q' > q$, the buyer will purchase both goods from the seller. In fact, if the buyer purchases both goods from the seller, he attains a utility of $2u_b - q$. If the buyer purchases both goods from the second retailer, he attains a utility of $2u_b - q'$ (recall that all sellers post prices below $u_b$). If the buyer purchases only one good, he attains a utility that is strictly smaller than $u_b - u_c$, which is smaller than $2u_b - q$. If the second retailer in the network of the buyer has a basket price $q' = q$, the buyer is indifferent between purchasing both goods from the seller and from the second retailer. We assume that, in this case, the buyer will randomize. If the second retailer in the network of the buyer has a basket price $q' < q$, the buyer will not purchase anything from the seller. Overall, a non-captive buyer of type b will purchase both goods from the seller with probability $1 - G(q) + \nu(q)/2$, and will purchase nothing from the seller with complementary probability. The seller is also in the network of $\mu_c\alpha$ captive buyers of type $c$, and of $\mu_c 2(1-\alpha)$ non-captive buyers of type $c$. However, a buyer of type $c$ does not purchase any good from the seller, as both $p_1$ and $p_2$ are strictly greater than $u_c$.

If the seller deviates from the equilibrium and posts prices $(p_1', p_2')$ with $p_1' = u_c$, $p_2' = q - p_1'$, he attains a profit of

$$\begin{aligned} S(p_1', p_2') \quad &= \mu_b \left[\alpha + 2(1-\alpha) 1 - G(q) + \nu(q)/2\right] q \\ &+ \mu_c \left[\alpha + 2(1-\alpha) 1 - F_1(p_1') + \lambda_1(p_1')/2\right] p_1'. \end{aligned} \tag{10}$$

The expression for $\Pi(p_1', p_2')$ is easy to understand. The seller is in the network of $\mu_b\alpha$ captive buyers of type $b$, and he will sell both goods to each of them. The seller is in the network of $\mu_b 2(1-\alpha)$ non-captive buyers of type $b$, and he will sell both goods to a fraction $1-G(q)+\nu(q)/2$ of them. The seller is in the network of $\mu_c \alpha$ captive buyers of type $c$, and he will sell good 1 to each of them. Finally, the seller is in the network of $\mu_c 2(1-\alpha)$ non-captive buyers of type $c$. If the second retailer in the network of such a buyer has a price for good 1 greater than $p_1'$, the buyer will purchase good 1 from the seller. If the second retailer in the network of such a buyer has a price for good 1 equal to $p_1'$, the buyer will randomize. If the second retailer in the network of such a buyer has a price for good 1 smaller than $p_1'$, the buyer will purchase good 1 from the second retailer. Overall, a non-captive buyer of type $c$ will purchase good 1 from the seller with probability $1 - F_1(p_1') + \lambda_1(p_1')/2$.

Notice that the first term on the right-hand side of (10) is the same as $S(p_1, p_2)$. This is because buyers of type $b$ base their purchasing decision only on the price of the whole basket of goods, not on the price of individual goods. The second term on the right-hand side of (10) is strictly positive as it is bounded below by $\mu_c \alpha p_1'$. Overall, the right-hand side of (10) is strictly greater than $S(p_1, p_2)$. Therefore, there is no equilibrium in which a seller with a basket price of $q \in (2u_c, u_b + u_c]$ finds it optimal to post $p_1 \in (u_c, u_b)$ and $p_2 \in (u_c, u_b)$. This observation, combined with part (i) of Lemma 1, implies that, if a seller posts prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, u_b + u_c]$, it must be setting one of the two prices below $u_c$, and the other price in between $u_c$ and $u_b$. ■

In Lemma 3, we establish two additional results. First, we show that the distribution of basket prices across sellers, $G(q)$, does not have any mass points. This property of equilibrium obtains for the same reason as it does in Butters (1977), Varian (1980) or Burdett and Judd (1983). Specifically, if there is a mass point at $q_0 > 0$, a seller with a basket price of $q_0$ could lower one of his two prices by an arbitrarily small amount and, instead of selling to half of the non-captive buyers of type $b$ who are in touch with another seller charging $q_0$, he could sell to all of them. Moreover, if there is a mass point at $q_0 = 0$, a seller with a basket price of $q_0$ could raise its prices to $p_1 = p_2 = u_b$ and, instead of attaining a profit of zero, it could attain a strictly positive profit.

Second, we show that the distribution of prices for individual good, $F_i(p)$, does not have any mass points for $p \in (0, u_c]$. The logic behind this result is also similar to the one in Butters (1977), Varian (1980) and Burdett and Judd (1983). However, in this case, we cannot rule out the possibility of a mass point at $p_0 > u_c$ or at $p_0 = 0$. There may be a mass point at $p_0 > u_c$ because, when the price of an individual good is higher than the willingness to pay of type-$c$ buyers, the seller never trades the good in isolation and its price does not play any allocative role. There may be a mass point at $p_0 = 0$ because the fact that a seller trades one good at a price of zero does not imply that the seller's profit is zero.

**Lemma 3**: (i) In any equilibrium, $G(q)$ does not have any mass points. That is, $\nu(q) = 0$ for all $q$. (ii) In any equilibrium, $F_1(p)$ and $F_2(p)$ do not have nay mass points for $p \in (0, u_c]$. That is, $\lambda_1(p) = 0$ and $\lambda_2(p) = 0$ for all $p \in (0, u_c]$.

*Proof*: Appendix B. ■

## 3.3 Bundle Equilibrium

In this subsection, we consider an equilibrium in which every seller in the market sets a basket price $q$ strictly greater than $u_b + u_c$. We have already established that sellers with a basket price $q$ strictly greater than $u_b + u_c$ post prices for the individual goods that are strictly greater than the willingness to pay of type-$c$ buyers, and smaller than the willingness to pay of type-$b$ buyers. Hence, sellers with a basket price $q$ strictly greater than $u_b + u_c$ sell the whole bundle of goods to buyers of type $b$, and do not sell anything to buyers of type $c$. For this reason, we refer to this type of equilibrium as a Bundle Equilibrium. In the first part of this subsection, we characterize the price distribution in a Bundle Equilibrium. In the second part of the subsection, we identify necessary and sufficient conditions for the existence of this type of equilibrium. Even though a Bundle Equilibrium is equivalent to the equilibrium of the one-good model of Burdett and Judd (1983) and does not necessarily feature relative price dispersion, the analysis in this subsection is useful as a stepping stone in the characterization of more complex equilibria.

In a Bundle Equilibrium, all sellers set a basket price $q$ greater than $u_b + u_c$. In Lemma 1, we showed that a seller with a basket price $q$ greater than $u_b + u_c$ posts prices $p_1$ and $p_2$ that are strictly greater than $u_c$ and smaller than $u_b$. That is, he post prices $(p_1, p_2) \in R_1$. In Lemma 4 below we show that a seller who posts a pair of prices $(p_1, p_2) \in R_1$ would attain the same profit by posting any other pair of prices $(p_1', p_2') \in R_1$ as long as $p_1' + p_2' = p_1 + p_2$.

**Lemma 4**: The seller attains the same profit by posting the prices $(p_1, p_2) \in R_1$ and the prices $(p_1', p_2') \in R_1$ as long as $p_1' + p_2' = p_1 + p_2$.

*Proof*: The profit of a seller posting $(p_1, p_2) \in R_1$ is given by

$$S(p_1, p_2) = \mu_b \left[\alpha + 2(1-\alpha)(1 - G(p_1 + p_2))\right](p_1 + p_2). \tag{11}$$

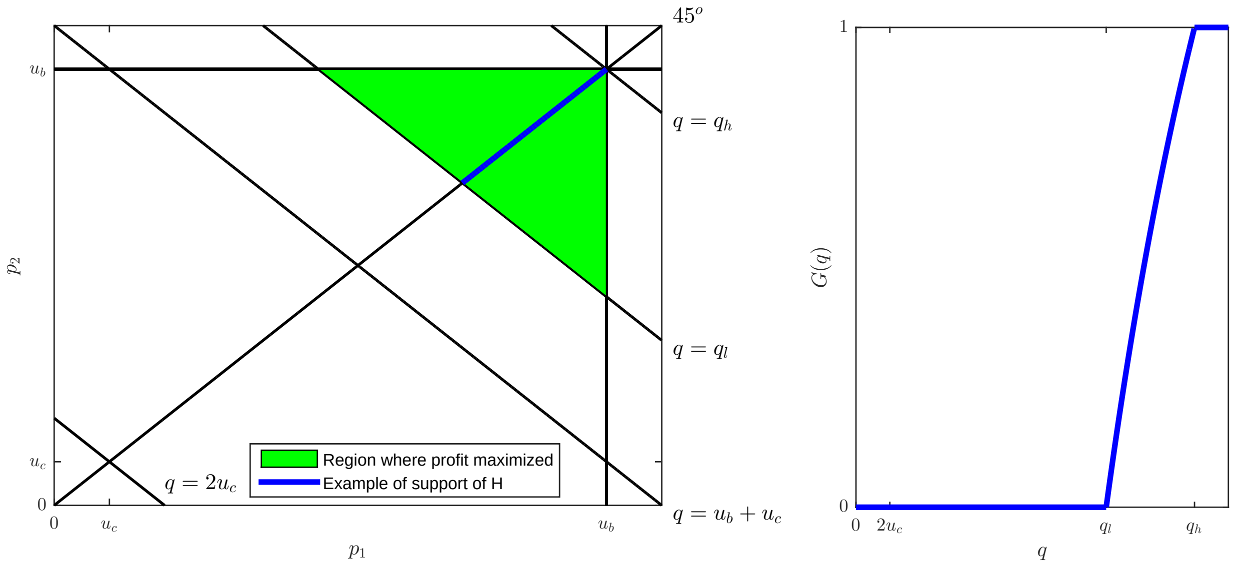
The seller is in the network of $\mu_b\alpha$ captive buyers of type $b$. A captive buyer of type $b$ purchases both goods from the seller with probability 1, since both $p_1$ and $p_2$ are smaller than $u_b$. The seller is also in the network of $\mu_b 2(1-\alpha)$ non-captive buyers of type $b$. A non-captive buyer of type $b$ purchases both goods from the seller with probability $1-G(p_1+p_2)$, which is the probability that the second seller in the buyer's network has a basket price greater than $p_1+p_2$. Finally, the seller is in the network of some buyers of type $c$. However, a buyer of type $c$ never purchases from the seller, since both $p_1$ and $p_2$ are strictly greater than $u_c$. Notice that the seller's profit in (11) only depends on the sum of $p_1$ and $p_2$. Therefore, the seller would attain the same profit by posting any other pair of prices $(p_1', p_2') \in R_1$ such that $p_1' + p_2' = p_1 + p_2$. ■

The above lemma is intuitive. Buyers of type $b$ must make all of their purchases in the same location. Hence, they are indifferent between visiting a seller with prices $(p_1, p_2) \in R_1$ or a seller with prices $(p_1', p_2') \in R_1$ as long as the two sellers charge the same price for the whole basket of goods, i.e. $p_1 + p_2 = p_1' + p_2'$. Buyers of type $c$ never purchase from a seller with prices $(p_1, p_2)$ nor from a seller with prices $(p_1', p_2')$, as both sellers charge prices that are above their willingness to pay. Since both buyers of type $b$ and buyers of type $c$ are indifferent between $(p_1, p_2)$ and $(p_1', p_2')$, so is a seller. In turn, this implies that the profit of a seller with prices $(p_1, p_2) \in R_1$ can be written as a function of $q = p_1 + p_2$, i.e.

$$S_1(q) = \mu_b \left[\alpha + 2(1-\alpha)(1-G(q))\right] q. \tag{12}$$

Now we are in the position to establish two properties of the distribution $G$ of basket prices in a Bundle Equilibrium. First, the highest basket price, $q_h$, on the support of $G$ equals $2u_b$. To see why, suppose that $q_h$ is strictly smaller than $2u_b$. In this case, the profit for a seller with a basket price of $q_h$ is then equal to $\mu_b \alpha q_h$, as this seller is the one with the highest basket price in the economy and, hence, only sells to captive buyers of type $b$. However, is the seller sets a basket price of $2u_b$, he attains a profit of $\mu_b \alpha 2u_b$, as the seller still only sells to captive buyers of type $b$. Since $\mu_b \alpha q_h < \mu_b \alpha 2u_b$, it follows that the seller with a basket price of $q_h$ is not maximizing his profit and, hence, this cannot be an equilibrium. Next, suppose that $q_h$ is strictly greater than $2u_b$. In this case, Lemma 1 is violated. Hence, this cannot be an equilibrium either. Overall, in a Bundle Equilibrium, $q_h = 2u_b$.

Second, the support of $G$ is an interval $[q_\ell, q_h]$. To see why, suppose that the support of $G$ has a gap between the basket price $q_0$ and the basket price $q_1$. In this case, a seller with a basket price of $q_0$ attains a profit of $\mu_b \left[\alpha + 2(1-\alpha)(1-G(q_0))\right] q_0$. A seller with a basket price of $q_1$ attains a profit of $\mu_b \left[\alpha + 2(1-\alpha)(1-G(q_1))\right] q_1$. Since $G$ has a gap between $q_0$ and $q_1$, $G(q_0) = G(q_1)$ and the seller with a basket price of $q_0$ makes the same number of trades as a seller with a basket price of $q_1$, but enjoys a lower profit per trade. Therefore,




*Notes:* This figure shows the possible range of the support of the joint distribution $H(p_1, p_2)$, and the shape of the cumulative distributions $G(q)$, in the Bundle Equilibrium.

Figure 4: Bundle Equilibrium support of $H(p_1, p_2)$ and shape of $G(q)$

the seller with a basket price of $q_0$ does not maximize his profit and, hence, this cannot be an equilibrium.

Next, we can solve for the distribution $G$ of basket prices. In a Bundle Equilibrium, the seller's profit must attain its maximum for any $q$ on the support of $G$. That is, $S_1(q) = S^*$ for all $q \in [q_\ell, q_h]$. Since $S_1(q_h) = S^*$ and $q_h = 2u_b$, the maximized profit $S^*$ is given by $\mu_b \alpha 2u_b$. Since $S_1(q) = S^*$ for all $q \in [q_\ell, q_h]$ and $S^* = \mu_b \alpha 2u_b$, it follows that

$$\mu_b \left[\alpha + 2(1-\alpha)(1 - G(q))\right] q = \mu_b \alpha 2u_b. \tag{13}$$

Solving (13) with respect to $G(q)$, we obtain the equilibrium distribution of basket prices

$$G(q) = 1 - \frac{\alpha}{2(1-\alpha)} \frac{2u_b - q}{q}. \tag{14}$$

Solving $G(q_\ell) = 0$ with respect to $q_\ell$, we obtain the lower bound of the equilibrium distribution of basket prices

$$q_\ell = \frac{\alpha}{2-\alpha} 2u_b. \tag{15}$$

Figure 4 illustrates the equilibrium distribution of basket prices (14). Notice that the distribution of basket prices in (14) is the exactly the same as the equilibrium price distribution of Burdett and Judd (1983). This result is not surprising. In a Bundle Equilibrium, a seller and a buyer either trade the entire basket of goods, or they do not trade at all. Hence, in a

Bundle Equilibrium, our model of multi-product retailing boils down to the single-product model of retailing of Burdett and Judd (1983), where the single product being traded in the market is the entire basket of goods.

In a Bundle Equilibrium, the distribution $G$ of basket prices across stores is uniquely pinned down. The distribution of basket prices is non-degenerate, as there is a positive measure of sellers with any basket price q in the non-empty interval between $q_\ell$ and $q_h$. The fact that the distribution of basket prices is non-degenerate means that the equilibrium features price dispersion across stores, in the sense that some sellers are, on average, expensive and other sellers are, on average, cheap.

In a Bundle Equilibrium, the distribution $H$ of price vectors across stores is not uniquely pinned down. In particular, any distribution of price vectors $H(p_1, p_2)$ that has support inside $R_1$ and that generates the distribution of basket prices $G(q)$ in (14) is such that every price on the support of $H$ maximizes the profit of the seller and, hence, is an equilibrium. For instance, there is an equilibrium in which there are $G'(q)$ sellers with a basket price of $q$ and each one of them posts the price $q/2$ for both good 1 and good 2. This equilibrium does not feature relative price dispersion, as there is no dispersion across sellers in the price of a particular good at a particular seller relative to the average price charged by that seller. However, there always also exists an equilibrium in which there are $G'(q)$ sellers with a basket price of $q$, and each one of them posts prices $(p_1, q - p_1)$ where $p_1$ is randomly drawn from a uniform distribution with support $(u_c, q - u_c]$. In this equilibrium, there is relative price dispersion. Yet, such relative price dispersion is a matter of indifference, as neither buyers nor sellers care about the price of any individual good. In this sense, a Bundle Equilibrium—and, more generally, any model of multiproduct retailing in which buyers always purchase the entire basket of goods in the same location—does not offer a particularly compelling theory of relative price dispersion.

We conclude the analysis by identifying necessary and sufficient conditions for the existence of a Bundle Equilibrium. We find that this type of equilibrium exists if and only if

$$\frac{\mu_c}{\mu_b} \leq \frac{3\alpha - 2}{(2-\alpha)u_c/u_b} - 1. \tag{16}$$

Condition (16) is satisfied if: (i) the market is not too competitive, in the sense that the fraction al of buyers who are in contact with only one seller is smaller than 2/3; (ii) the relative number of type-$c$ buyers, $\mu_c/\mu_b$, and/or the relative willingness to pay of type-$c$ buyers, $u_c/u_b$, is not too large. Intuitively, if the market is too competitive, some sellers would want to set a price $q$ for the basket of goods that is smaller than $u_b + u_c$. Hence, a Bundle Equilibrium would not exist. If the relative number of type-$c$ buyers and/or the relative willingness to pay of type-$c$ buyers are too high, some sellers would want to trade with type-$c$ buyers and set a price $p$ for one or both goods that is smaller than $u_c$. Hence, a Bundle Equilibrium would not exist.

The following proposition summarizes the characterization of a Bundle Equilibrium.

**Proposition 1**: (*Bundle Equilibrium*) (i) In a Bundle Equilibrium, the distribution of basket prices, $G$, is continuous over the support $[q_\ell, q_h]$ and given by (14); the distribution of price vectors across sellers, $H$, is not uniquely pinned down. (ii) A Bundle Equilibrium exists if and only if (16) is satisfied.

*Proof*: Appendix B. ■

## 3.4 Discrimination Equilibrium

In this subsection, we consider an equilibrium in which some sellers have a basket price $q$ greater than $u_b + u_c$, and some sellers have a basket price smaller than $u_b + u_c$ and greater than $2u_c$. We refer to this type of equilibrium as a *Discrimination Equilibrium*, as in this equilibrium some sellers set their prices so as to discriminate between the high-valuation buyers who must purchase all the goods in the same location and the low-valuation buyers who can purchase different goods in different locations. In the first part of the subsection, we characterize the price distribution in a Discrimination Equilibrium and show that it necessarily features relative price dispersion. In the second part of the subsection, we identify necessary and sufficient conditions for the existence of this type of equilibrium.

We start the characterization of a Discrimination Equilibrium by focusing on the sellers with a basket price $q$ strictly greater than $u_b + u_c$. Following the same arguments as in the previous subsection, we can easily show that, among sellers with $q > u_b + u_c$, the distribution of basket prices $G(q)$ has support over the interval $[q^*, q_h]$, with $u_b + u_c < q^* < q_h = 2u_b$. Moreover, using the fact that all of the sellers with $q \in [q^*, q_h]$ must attain the maximized profit $S^*$, we can easily show that the maximized profit $S^*$ is equal to $\mu_b \alpha 2u_b$ and that, for all $q \in [q^*, q_h]$, the distribution of basket prices $G(q)$ is equal to

$$G(q) = 1 - \frac{\alpha}{2(1-\alpha)} \frac{2u_b - q}{q}. \tag{17}$$

Next, we focus on sellers with a basket price $q$ smaller than $u_b + u_c$ and strictly greater than $2u_c$. In Lemma 2, we established that a seller with a basket price $q \in (2u_c, u_b + u_c]$ sets the price of one good below the willingness to pay of type-$c$ buyers, and sets the price of the other good in between the willingness to pay of type-$c$ buyers and the willingness to pay of type-$b$ buyers. That is, a seller with a basket price $q \in (2u_c, u_b+u_c]$, sets prices $(p_1, p_2) \in R_2$. Now, consider a seller who posts prices $(p_1, p_2) \in R_2$. The seller trades the basket of goods to buyers of type $c$ at the price $q = p_1 + p_2$, and it trades the cheaper of the two goods (say good $i$) to buyers of type $b$ at the price $p_i$. Thus, the seller attains a profit of

$$\begin{aligned} S_{2i}(q, p_i) &= \mu_b \left[\alpha + 2(1-\alpha)1 - G(q)\right] q \\ &+\mu_c \left[\alpha + 2(1-\alpha)1 - F_i(p_i)\right] p_i. \end{aligned} \tag{18}$$

Note that (18) makes use of the fact that $G(q)$ does not have any mass points, and $F_i(p)$ does not have any mass points over the interval $(0, u_c]$.

The next lemma shows that, for all $p \in [0, u_c]$, the fraction of sellers charging less than $p$ for good 1 is exactly the same as the fraction of sellers charging less than $p$ for good 2. That is, $F_1(p) = F_2(p) = F(p)$ for all $p \in [0, u_c]$. The lemma implies that the profit of a seller in region $R_2$ is symmetric in the two goods. That is, $S_{21}(q, p) = S_{22}(q, p) = S_2(q, p)$. The lemma is intuitive. If $F_1(p) > F_2(p)$ for $p \in (p_0, p_1)$, with $0 \leq p_0 < p_1 \leq u_c$, then a seller posting the prices $(p, q-p) \in R_2$ would be better off posting the prices $(q-p, p)$ instead. In fact, the seller trades the basket of goods to the same number of type-$b$ buyers and at the same price by posting either $(q-p, p)$ or $(p, q-p)$. However, by posting $(q-p, p)$ rather than $(p, q-p)$, the seller trades the cheaper good to more type-$c$ buyers even though he charges the same price for it. Hence, if $F_1(p) > F_2(p)$ for $p \in (p_0, p_1)$, all sellers posting the prices $(p, q-p) \in R_2$ would be better off switching the price tags of the two goods until $F_1(p) = F_2(p)$.

**Lemma 5**: In a Discrimination Equilibrium, $F_1(p) = F_2(p)$ for all $p \in [0, u_c]$.

*Proof*: Appendix B. ■

The next lemma shows that the profit of a seller in region $R_2$ attains the maximum $S^*$ for all prices of the basket $q$ and prices of the cheaper good $p$ such that $q$ is in the interval $[q_\ell, u_b+u_c]$, and $p$ is in the interval $[p_\ell, u_c]$, where $q_\ell$ denotes the lower bound on the support of the price distribution of baskets and $p_\ell$ denotes the lower bound on the support of the price distribution of an individual good. That is, $S_2(q, p) = S^*$ for all $(q, p)$ such that $q \in [q_\ell, u_b + u_c]$ and $p \in [p_\ell, u_c]$. The proof of the lemma follows the same strategy as Proposition 3 in Menzio and Trachter (2015b). The gist of the proof is to show that—if profits are not constant for all $(q, p)$ such that $q \in [q_\ell, u_b + u_c]$ and $p \in [p_\ell, u_c]$—there are either gaps on the support of the distribution of $G$ over the interval $[q_\ell, u_b + u_c]$, or gaps on the support of the distribution $F$ over the interval $[p_\ell, u_c]$. In turn, if there are gaps on the support of one of the two distributions, there are some sellers that could increase their profit by either the price of the basket, or by increasing the price of one of the cheaper good.

**Lemma 6**: In a Discrimination Equilibrium, $S_2(q,p) = S^*$ for all $(q,p)$ such that $q \in [q_\ell, u_b + u_c]$ and $p \in [p_\ell, u_c]$.

*Proof*: Appendix B. ■

We are now in the position to solve for the lowest basket price $q^*$ posted by sellers in region $R_1$, for the marginal distribution $G(q)$ of basket prices among sellers in region $R_2$, and for the marginal distribution $F(p)$ of prices among sellers in region $R_2$. Lemma 6 implies that a seller posting prices $(u_c, u_b)$ attains the maximized profit $S^*$, i.e.

$$\mu_b \left[\alpha + 2(1-\alpha)\left(1 - G(u_b + u_c)\right)\right](u_b + u_c) + \mu_c \left[\alpha + 2(1-\alpha)\left(1 - F(u_c)\right)\right] u_c = S^*. \tag{19}$$

Similarly, a seller posting prices $(p_1, p_2)$ such that $p_1 \in (u_c, u_b]$, $p_2 \in (u_c, u_b]$ and $p_1 + p_2 = q^*$ attains the maximized profit $S^*$, i.e.

$$\mu_b \left[\alpha + 2(1-\alpha)\left(1 - G(q^*)\right)\right] q^* = S^*. \tag{20}$$

Notice that the fraction of sellers with a basket price smaller than $q^*$ is the same as the fraction of sellers with a basket price smaller than $u_b + u_c$, i.e. $G(q^*) = G(u_b + u_b)$. Also, notice that the fraction of sellers that charge less than $u_c$ for good 1 is half of the fraction of sellers with a basket price smaller than $q^*$, i.e. $F(u_c) = G(q^*)/2$. Using these two observations and the fact that the left-hand side of (19) is equal to the left-hand side of (20), we obtain

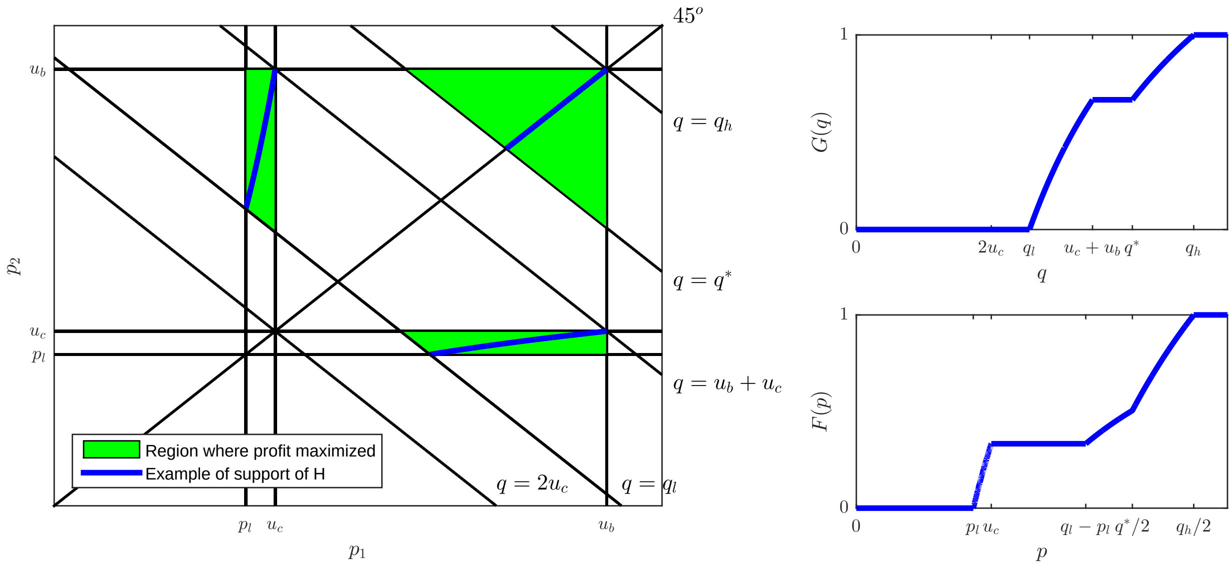
$$\begin{aligned} &\mu_b \left[\alpha + 2(1-\alpha)\left(1 - G(q^*)\right)\right](u_b + u_c) + \mu_c \left[\alpha + 2(1-\alpha)\left(1 - G(q^*)/2\right)\right] u_c \\ &= \mu_b \left\{\alpha + 2(1-\alpha)[1 - G(q^*)]\right\} q^*. \end{aligned} \tag{21}$$

Equation (21) can be solved with respect to $q^*$ to obtain

$$q^* = \frac{2\alpha(1 + u_c/u_b) + \alpha(\mu_c/\mu_b)(u_c/u_b)}{4\alpha - (2-\alpha)(\mu_c/\mu_b)(u_c/u_b)} 2u_b. \tag{22}$$

Lemma 6 implies that a seller posting any prices $(p_1, p_2)$ such that $p_2 \in (u_c, u_b]$ and $q = p_1 + p_2 \in [q_\ell, u_b + u_c]$ attains the same profit as a seller posting prices $(u_c, u_b)$, i.e.

$$\begin{aligned} &\mu_b \left[\alpha + 2(1-\alpha)\left(1 - G(q)\right)\right] q + \mu_c \left[\alpha + 2(1-\alpha)\left(1 - F(u_c)\right)\right] u_c \\ &= \mu_b \left[\alpha + 2(1-\alpha)\left(1 - G(u_b + u_c)\right)\right](u_b + u_c) + \mu_c \left[\alpha + 2(1-\alpha)\left(1 - F(u_c)\right)\right] u_c. \end{aligned} \tag{23}$$




*Notes:* This figure shows the possible range of the support of the joint distribution $H(p_1, p_2)$, the shape of the cumulative distributions $G(q)$ and an example of the shape of the cumulative distribution $F(p)$, in the Discrimination Equilibrium.

Figure 5: Discrimination Equilibrium support of $H(p_1, p_2)$ and shape of $G(q), F(p)$

Using the fact that $G(u_b + u_c) = G(q^*)$ and solving (23) with respect to $G(q)$, we find that the distribution of basket prices for $q \in [q_\ell, u_b + u_c]$ is given by

$$G(q) = G(q^*) - \frac{\alpha + 2(1-\alpha)\,(1 - G(q^*))}{2(1-\alpha)} \frac{u_b + u_c - q}{q}. \tag{24}$$

Solving the equation $G(q_\ell) = 0$ with respect to $q_\ell$, we find that the lowest price on the support of the distribution of basket prices is given by

$$q_\ell = \frac{2\alpha u_b}{2-\alpha} \frac{u_b + u_c}{q^*}. \tag{25}$$

Lemma 6 also implies that a seller posting prices $(p_1, p_2)$ such that $p_1 \in [p_\ell, u_c]$, $p_2 \in (u_c, u_b]$ and $p_1 + p_2 = q_\ell$ attains the same profit as a seller posting prices $(u_c, q_\ell - u_c)$, i.e.

$$\begin{aligned} &\mu_b\,[\alpha + 2(1-\alpha)]\,q_\ell + \mu_c\,[\alpha + 2(1-\alpha)\,(1 - F(p))]\,p \\ &= \mu_b\,[\alpha + 2(1-\alpha)]\,q_\ell + \mu_c\,[\alpha + 2(1-\alpha)\,(1 - F(u_c))]\,u_c. \end{aligned} \tag{26}$$

Using the fact that $F(u_c) = G(q^*)/2$ and solving the equation (26) with respect to $F(p)$, we find that the distribution of good-1 prices for $p \in [p_\ell, u_c]$ is given by

$$F(p) = \frac{G(q^*)}{2} - \frac{\alpha + 2(1-\alpha)\,(1 - G(q^*)/2)}{2(1-\alpha)} \frac{u_c - p}{p}. \tag{27}$$

Solving the equation $F(p_\ell) = 0$ with respect to $p_\ell$, we find that the lowest price on the support of the distribution of good-1 prices is given by

$$p_\ell = \frac{\alpha + 2(1-\alpha)\left[1 - G(q^*)/2\right]}{2-\alpha} u_c.$$

This completes the characterization of a Discrimination Equilibrium. In this type of equilibrium, there is a group of sellers who sets a basket price of $q \in [q^*, q_h]$ and the prices $p_1$ and $p_2$ in between $u_c$ and $u_b$. These sellers trade (with some probability) the basket of goods to buyers of type $b$, and never trade with buyers of type $c$. There is also a group of sellers who set a basket price of $q \in [q_\ell, u_b + u_c]$. Half of these sellers set $p_1$ below $u_c$ and $p_2$ between $u_c$ and $u_b$. These sellers trade (with some probability) the whole basket of goods to buyers of type $b$, and good 1 to buyers of type $c$. The other half of the sellers sets $p_2$ below $u_c$ and $p_1$ between $u_c$ and $u_b$. These sellers trade (with some probability) the whole basket of goods to buyers of type $b$, and good 2 to buyers of type $c$. There are no sellers who set a basket price of $q$ in the interval $(u_b + u_c, q^*)$.

The distribution of basket prices $G(q)$ is given by (17) for $q \in [q^*, q_h]$, and by (24) for $q \in [q_\ell, u_b + u_c]$. The distribution $G(q)$ is such that the seller's profit from trading the basket of goods to buyers of type $b$ is equal to $S^*$ for all $q \in [q^*, q_h]$, and it is equal to $S^* - \mu_c \left[\alpha + 2(1-\alpha)(1 - F(u_c))\right] u_c$ for all $q \in [q_\ell, u_b + u_c]$. The distribution $G(q)$ has a gap between $u_b + u_c$ and $q^*$. The gap exists because a seller with a basket price of $u_b + u_c$ trades with both buyers of type $b$ and buyers of type $c$, while a seller with a basket price greater than $u_b + u_c$ only trades with buyers of type $b$. Therefore, a seller strictly prefers setting a basket price of $u_b + u_c$ than any basket price just above $u_b + u_c$. The distribution of prices for an individual good $F(p)$ is given by (27) for $p \in [p_\ell, u_c]$. The distribution $F(p)$ is such that the seller's profit from trading the cheaper good to buyers of type $c$ is equal to $S^* - \mu_b(2-\alpha)q_\ell$ for all $p \in [p_\ell, u_c]$. The distribution $F(p)$ is not uniquely pinned down for $p \in (u_c, u_b]$. Intuitively, this is the case because a seller who charges a price of $p > u_c$ for one good only trades that good to buyers of type $b$ together with the other good.

The distribution of price vectors $H$ is not uniquely pinned down. For sellers with a basket price $q \in [q^*, q_h]$, any distribution $H$ that has support inside $R_1$ and that generates the marginal distribution of basket prices $G(q)$ in (17) is consistent with equilibrium. For example, there is an equilibrium in which, for all $q \in [q^*, q_h]$, there are $G'(q)$ sellers with a basket price of $q$ and each of them posts the prices $(q/2, q/2)$. For sellers with a basket price $q \in [q_\ell, u_b + u_c]$, any distribution $H$ that has support inside $R_2$ and that generates the marginal distribution of basket prices $G(q)$ in (24) and the marginal distribution of individual good prices $F(p)$ in (27) is consistent with equilibrium. For example, there is an equilibrium in which, for all $p \in [p_\ell, u_c]$, $2F'(p)$ sellers have a basket price of $\phi(p)$, $F'(p)$ sellers post the prices $(p, \phi(p)-p)$ and $F'(p)$ sellers post the prices $(\phi(p)-p, p)$, where

$$\phi(p) = \frac{[\alpha + 2(1-\alpha)(1-G(q^*))]\,(u_b+u_c)}{[\alpha + 2(1-\alpha)(1-G(q^*))] + 2\,[\alpha + 2(1-\alpha)(1-G(q^*)/2)]\,(u_c-p)/p}. \tag{28}$$

A Discrimination Equilibrium features price dispersion across sellers, in the sense that some sellers are on average more expensive, while some sellers are on average cheaper. This property of equilibrium follows immediately from the fact that the distribution of basket prices is non-degenerate. A Discrimination Equilibrium always features relative price dispersion, in the sense that there is variation across sellers in the price of a particular good at a particular seller relative to the average price charged by that seller. This property of equilibrium follows immediately from the fact that half of the sellers with a basket price $q \in [q_\ell, u_b+u_c]$ has a relative price for good 1 that is strictly greater than 1, while the other half of the sellers with a basket price $q \in [q_\ell, u_b+u_c]$ has a relative price for good 1 that is strictly smaller 1.

Let us briefly explain why relative price dispersion must emerge in equilibrium. Competition between sellers drives part of the distribution of basket prices to the region where $q$ is between $2u_c$ and $u_b+u_c$. A seller with a basket price between $2u_c$ and $u_b+u_c$ never finds it optimal to post the same price for both goods. Instead, the seller finds it optimal to set the price of one good below and the price of the other good above the willingness to pay of type-$c$ buyers. That is, a seller with a basket price $q$ between $2u_c$ and $u_b+u_c$ finds it optimal to follow an asymmetric pricing strategy for the two goods. However, if some sellers post a higher price for good 1 than for good 2, other sellers must post a higher price for good 2 than for good 1, or else there would be some unexploited profit opportunities. That is, the distribution of prices for the two goods must be symmetric across sellers with a basket price $q$ between $2u_c$ and $u_b+u_c$. The asymmetric pricing strategy followed by each individual seller combined with the symmetry of the price distribution across sellers implies relative price dispersion.

Sellers follow an asymmetric pricing strategy to discriminate between the two types of buyers. The difference in the willingness to pay of type-$b$ and type-$c$ buyers gives sellers a desire to price discriminate. The difference in the ability of type-$b$ buyers and of type-$c$ buyers to purchase different items in different locations gives sellers the opportunity to price discriminate. In fact, by pricing the two goods asymmetrically, a seller can charge a high average price to the high-valuation buyers who need to purchase all the items together (the buyers of type $b$), and charge a low price for one good to the low-valuation buyers who can purchase different items at different locations (the buyers of type $c$).

It is interesting to contrast the type of price discrimination described above with intertemporal price discrimination (see, e.g., Conlisk, Gerstner and Sobel 1984 and Sobel 1984 or, in a search-theoretic context, Albrecht, Postel-Vinay and Vroman 2013 and Menzio and Trachter (2015b). The key to intertemporal price discrimination is a negative correlation between a buyer's valuation and his ability to intertemporally substitute purchases. A seller can exploit this negative correlation by having occasional sales. The low valuation buyers, who are better able to substitute purchases intertemporally, will take advantage of the sales and will end up paying low prices. The high valuation buyers, who are unable to substitute purchases intertemporally, will not take advantage of the sales and will end up paying high prices. In contrast, our theory of price discrimination is based on a negative correlation between a buyer's valuation and his ability to shop in multiple stores. Moreover, while intertemporal price discrimination takes the form of time-variation in the price of the same good, our theory of price discrimination takes the form of variation in the price of different goods relative to the average store price.

We conclude the analysis by identifying necessary and sufficient conditions for the existence of a Discrimination Equilibrium. We find that this type of equilibrium exists if and only if

$$\frac{\mu_c}{\mu_b} > \frac{3\alpha - 2}{(2-\alpha)u_c/u_b} - 1 \tag{29}$$

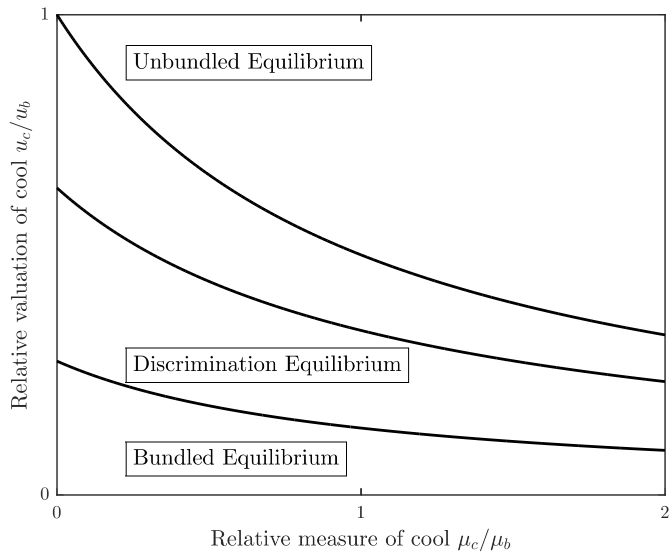
and

$$\frac{\mu_c}{\mu_b} \leq \frac{\alpha - (2-\alpha)u_c/u_b}{1 + (2-\alpha)u_c/u_b} \frac{1 + u_c/u_b}{u_c/u_b}. \tag{30}$$

Condition (29) guarantees that some sellers find it optimal to post basket prices below $u_b + u_c$. The condition is satisfied if: (i) the market is sufficiently competitive, in the sense that the fraction al of buyers who are in contact with only one seller is smaller than 2/3; or (ii) the relative number of type-$c$ buyers, $\mu_c/\mu_b$, and/or the relative willingness to pay of type-$c$ buyers, $u_c/u_b$, is large enough. Condition (30) guarantees that no seller finds it optimal to post prices below $2u_c$. The condition is satisfied if: (i) the market is not too competitive, in the sense that the fraction al of buyers who are in contact with only one seller is greater than $2(u_c/u_b)/(1 + 2(u_c/u_b))$; (ii) the relative number of type-$c$ buyers, $\mu_c/\mu_b$, and/or the relative willingness to pay of type-$c$ buyers, $u_c/u_b$, are low enough. As we shall see in the next subsection, conditions (29) and (30) define a non-empty parameter space.

The following proposition summarizes the characterization of a Bundle Equilibrium.

Figure 6: Equilibrium type depending on buyers in market




*Notes:* This figure illustrates how the type of equilibrium depends on the shares and relative valuations of the two types of buyers in the market.

**Proposition 2**: (*Discrimination Equilibrium*) (i) In a Discrimination Equilibrium, the distribution of basket prices, $G$, is continuous over the support $[q_\ell, u_b + u_c] \cup [q^*, q_h]$ and it is given by (17) for $q \in [q^*, q_h]$, and by (24) for $q \in [q_\ell, u_b + u_c]$; the distribution of prices for good $i$, $F_i$, is continuous over the interval $[p_\ell, u_c]$ and it is given by (27) for both $i = 1$ and 2; the distribution of price vectors, $H$, is not uniquely pinned down. (ii) A Discrimination Equilibrium exists if and only if (29) and (30) are satisfied.

*Proof*: Appendix B. ■

## 3.5 Other equilibria

A Bundle Equilibrium exists if and only if the relative measure of type-$c$ buyers, $\mu_c/\mu_b$, is smaller than the right-hand side of (16). The lowest curve in Figure 6 is the plot of the right-hand side of (16) in the $\{u_c/u_b, \mu_c/\mu_b\}$ space. Hence, a Bundle Equilibrium exists if and only if the parameter values lie below the bottom curve.[16] In this type of equilibrium, every seller sets a basket price $q$ strictly greater than $u_b + u_c$, and posts prices for the individual goods that are strictly greater than the willingness to pay of type-$c$ buyers and smaller than the willingness to pay of type-$b$ buyers. Every seller trades the entire basket of goods to buyers of type $b$, and trades nothing to buyers of type $c$. The equilibrium distribution of basket

[16]It is easy to verify that, when condition (16) is satisfied, the only equilibrium is a Bundle Equilibrium.

prices is the same as in the single-product model of retailing by Burdett and Judd (1983).

A Discrimination Equilibrium exists if and only if the relative measure of type-$c$ buyers, $\mu_c/\mu_b$, is greater than the right-hand side of (29) and smaller than the right-hand side of (30). Notice that the right-hand side of (29) is equal to the right-hand side of (16), which is the lowest curve in Figure 6. Also, notice that the right-hand side of (29) is strictly smaller than the right-hand side of (30), which is the middle curve in Figure 6. Hence, a Discrimination Equilibrium exists if and only if the parameter values fall in the non-empty region between the bottom and the middle curves.[17] In this type of equilibrium, there are some sellers with a basket price $q > u_b + u_c$ who post prices $p_1$ and $p_2 \in (u_c, u_b]$. These sellers trade the basket of goods to buyers of type $b$, and trade nothing to buyers of type $c$. There are also some sellers with a basket price $q \in (2u_c, u_c + u_b]$ who set the price of one good above and the price of the other good below $u_c$. These sellers trade the basket of goods to buyers of type $b$, and the cheaper good to buyers of type $c$.

If the relative measure of type-$c$ buyers is greater than the right-hand side of (30) other, more complex types of equilibria emerge. In this paper, we do not wish to analyze these equilibria in detail, but we still find it instructive to describe some of their properties. Any equilibrium is an *Unbundled Equilibrium* when and only when

$$\frac{\mu_c}{\mu_b} > \frac{1 - u_c/u_b}{u_c/u_b}. \tag{31}$$

Notice that the right-hand side of (30) is strictly smaller than the right-hand side of (31), which is the top curve in Figure 6. Hence, an equilibrium is Unbundled if and only if the parameter values fall in the region above the top curve. In this type of equilibrium, every seller sets a basket price $q$ smaller than $2u_c$, and posts prices for the individual goods that are smaller than $u_c$. Every seller trades the basket of goods to buyers of type $b$, and either the basket of goods or one of the two individual goods to buyers of type $c$.

If the relative measure of type-$c$ buyers lies between the top and the middle curves in Figure 6, the equilibrium is a combination between a Discrimination and an Unbundled Equilibrium. In this type of equilibrium, there is a first group of sellers with a basket price $q > u_b + u_c$, a second group of sellers with a basket price $q \in (2u_c, u_b + u_c]$, and a third group of sellers with a basket price $q \leq 2u_c$. Sellers in the first group trade the basket of goods to buyers of type $b$, and nothing to buyers of type $c$. Sellers in the second group trade the basket of goods to buyers of type $b$, and the cheaper good to buyers of type $c$. Sellers in the

[17]When conditions (29) and (30) are satisfied, the only equilibrium is a Discrimination Equilibrium.

third group trade the basket of goods to buyers of type $b$, and either both or either one of the individual goods to buyers of type $c$.

# 4 Calibration and Validation

In this section, we calibrate and validate our theory of relative price dispersion. In subsection 4.1, we calibrate the theory using data on the extent and sources of dispersion in the price at which the same good is sold by different sellers in the same market and week, the number of stores visited by different households and the relationship between the prices paid by a household for the same basket of goods in the same market and quarter and the number of stores from which the households shops in a given quarter. In subsection 4.2, we compare the predictions of the calibrated theory on the extent and sources of dispersion in the prices paid by different households for the same basket of goods, in the same market and quarter. We find that our theory of relative price dispersion is consistent not only with the key facts about price dispersion, but also with the main differences between the extent and sources of price dispersion across stores and price dispersion across households.

## 4.1 Calibration

For the purposes of our quantitative analysis, we need to consider a dynamic version of the static model of Section 3. We assume that the type of the buyer remains constant over time. However, a buyer may change its new network of sellers from one period to the next because, for example, he moves from one part of the city to another, he changes his job, etc... In particular, we assume that a buyer keeps the same network of sellers from one period to the next with probability $\rho$, and he samples a new network of sellers with probability $1-\rho$. Conditional on changing network, the buyer contacts one randomly selected seller with probability $\alpha$, and two randomly selected sellers with probability $1-\alpha$. Sellers post prices in every period. Since the probability that a buyer changes network is independent of his current network, the pricing problem of the sellers in every period is the same as in the static model of Section 3. Hence, in the presence of any positive menu cost, sellers find it strictly optimal to keep their prices constant from one period to the next.

When quantifying the model, we have to deal with the fact that the theory does not uniquely pin down the distribution $H$ of price vectors across sellers. As explained in Section 3, the theory uniquely pins down the distribution $G$ of basket prices and the distribution $F$ of prices of individual goods, but it does not uniquely pin down $H$. Yet, we find it natural to assume that: (i) sellers with a basket price of $q \in (u_b + u_c, 2u_b]$ set the same price for both goods; (ii) sellers with a basket price of $q \in (2u_c, u_b + u_c]$ set the prices $(\phi^{-1}(q), q - \phi^{-1}(q))$ with probability 1/2, and the prices $(q - \phi^{-1}(q), \phi^{-1}(q))$ with probability 1/2. Under conditions (i) and (ii), the equilibrium is *symmetric*, in the sense that $H(p_1, p_2) = H(p_2, p_1)$, and *rank-preserving*, in the sense that the rank of a seller in the distribution of basket prices is the same as the rank of a seller in the distribution of the lowest prices of an individual good.[18] There are other equilibria that are symmetric and rank-preserving. However, conditions (i) and (ii) select the one with the lowest relative price dispersion.[19]

The dynamic version of the model has six parameters: the measure $\mu_b$ of buyers of type $b$, the measure $\mu_c$ of buyers of type $c$, the valuation $u_b$ of buyers of type $b$, the valuation $u_c$ of buyers of type $c$, the fraction $\alpha$ of buyers that have only one seller in their network, and the probability $\rho$ that a buyers keeps the same network of sellers from one week to the next.

The equilibrium of the model depends on the ratio between the measure of type-$c$ buyers and the measure of type-$b$ buyers, but not on the two measures separately. Hence, we can normalize $\mu_b$ to 1. Similarly, the equilibrium of the model depends on the ratio between the valuation of type-$c$ buyers and the valuation of type-$b$ buyers, but not on the two valuations separately. Hence, we can normalize $u_b$ to 1. We calibrate the remaining four parameters so as to match four moments of the data. First, we target a measure of the dispersion of prices for the same good in the same market and in the same week. Second, we target a measure of the fraction of price dispersion that is due to differences in the store component of the price, and the fraction of price dispersion that is due to differences in the store-good component of the price. Third, we target a measure of the effect of shopping from an additional store on the prices paid by a household. Finally, we target a measure of the number of different stores visited by a given household.

As discussed in Section 3, we interpret the model as a theory of the persistent component of prices at different stores. For this reason, we want the model to match the variance of

[18] It is immediate to see that the equilibrium is symmetric under (i) and (ii). To see that the equilibrium is rank-preserving notice that, for sellers with $q \in [q_\ell, u_b + u_c]$, the function $\phi^{-1}(q)$ relating the price of the seller's basket to the price of the seller's cheapest good is such that $F(\phi^{-1}(q)) = G(q)/2$ and, hence, the rank of the seller in the distribution of basket prices is $G(q)$ and the rank of the seller in the distribution of the lowest price for an individual good is $2F(\phi^{-1}(q)) = G(q)$. For sellers with $q \in [q^*, 2u_b]$, the rank of the seller in the distribution of basket prices is $G(q)$ and the rank of the seller in the distribution of the lowest price for an individual good is $2F(u_c) + G(q) - G(q^*) = G(q)$.

[19] The other symmetric, rank-preserving equilibria are such that half of the sellers with a basket price of $q \in [q^*, 2u_b]$ post prices $(\psi(q), q - \psi(q))$ and the other half post prices $(q - \psi(q), \psi(q))$, with $\psi(q) \in (u_c, q/2]$. Clearly, conditions (i) and (ii) select the symmetric, rank-preserving equilibrium with the lowest amount of relative price dispersion.

prices that is caused by persistent price differences across stores. As documented in Section 2, the standard deviation of prices for the same good in the same market and in the same week is 15.3%, and the variance is 2.34%. The fraction of the variance of prices that is due to differences in the store component of prices is 15.5%. Nearly all of the differences in the store component of prices are persistent, in the sense that they are either due to differences in the fixed effect or to differences in the AR part of the store component. The fraction of the variance of prices that is due to differences in the store-good component of prices is 84.5%. Approximately 36% of the differences in the store-good component of prices are persistent and 64% are transitory. Based on these observations, the variance of prices that comes from persistent differences in prices is 1% (and the standard deviation is 10%), the fraction of this variance due to persistent differences in the store component is 34% percent, and the variance due to persistent differences in the store-good component is 66% percent.

In order to obtain a measure of the effect of shopping at more stores on the prices paid by households, we use the Kilts Nielsen Consumer Panel (henceforth, KNCP). The KNCP tracks the shopping behavior of approximately 50,000 households over the period 2004-2009. Households are drawn from 54 geographically dispersed markets, known as Scantrack markets, each of which roughly corresponds to a Metropolitan Statistical Area. Demographic data on panelists are collected at the time of entry into the panel, and updated annually through a written survey. Panelists provide information about each of their shopping trips using a Universal Product Code (UPC) scanning device. More specifically, when a panelist returns from a shopping trip, he uses the device to enter details about the trip, including the date and the store where the purchases were made. The panelist then scans the barcode of the purchased good and enters the number of units purchased. The price of the good is recorded either automatically or manually depending on whether the store where the good was purchased is covered by Nielsen or not.

We follow the methodology in Aguiar and Hurst (2007) to compute a price index for every household in the KNCP. Specifically, we define the price index of household $i$ in market $m$ and quarter $t$ as the ratio between the dollar amount that the household paid to purchase its basket of goods and the amount that the household would have spent had it paid, for each good in its basket, the average price of that good in market $m$ and quarter $t$. We refer the reader to Kaplan and Menzio (2015) for further details about the construction of price indexes and for the sample selection criteria.

In Panel (a) of Table 5 we report the results of a regression of the price index of household $i$ in quarter $t$ on the log (or the level) of the ratio between the number of different stores

Table 5: Regression of household price indexes on indicators of multi-stop shopping

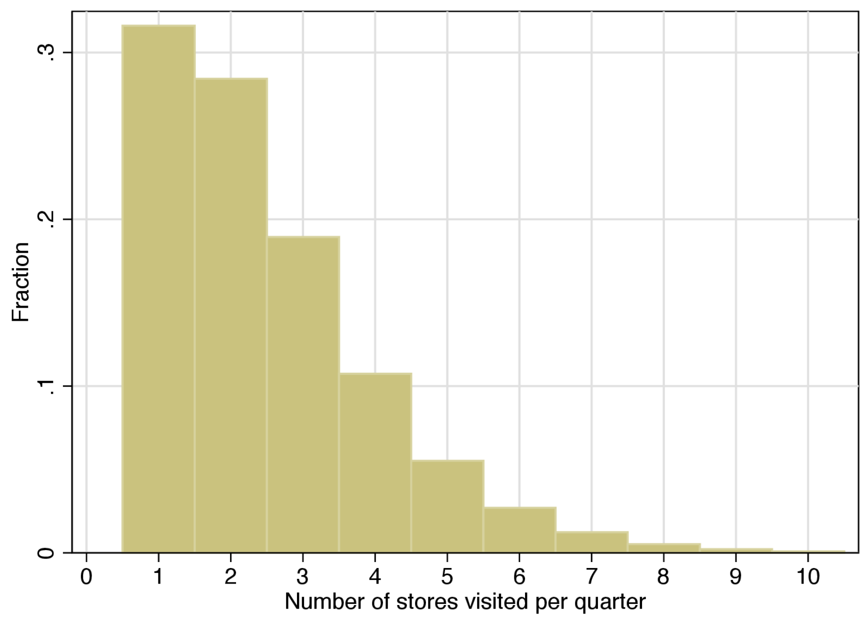
| (a) | Log | Log | Level | Level |
|---|---|---|---|---|
| Stores/Expenditure | -0.01124** | -0.01291** | -0.09099** | -0.10365** |
| | (0.00041) | (0.00041) | (0.00363) | (0.00349) |
| FE | No | Yes | No | Yes |
| $R^2$ | 0.01776 | 0.0074 | 0.02041 | 0.01523 |
| (b) | Log | Log | Level | Level |
| Stores | -0.03424** | -0.01577** | -0.01196** | -0.00367** |
| | (0.00072) | (0.00051) | (0.00027) | (0.00018) |
| Expenditure | 0.00964** | 0.01261** | 0.00008** | 0.00007** |
| | (0.00041) | (0.00044) | (0.00000) | (0.00000) |
| FE | No | Yes | No | Yes |
| $R^2$ | 0.02875 | 0.00753 | 0.02707 | 0.00235 |

*Notes:* This table presents results for regressions of household price indexes on indicators of multi-stop shopping: the average number of different stores visited per dollar spent in the quarter (Panel A), and number of different stores visited per quarter, conditioning on dollars spent per quarter (Panel B). The level models have expenditures in levels, and Log models have expenditures in logs. In all regressions: N=880104, clusters=78758.

in which household $i$ shopped during quarter $t$ and the dollar expenditures of household $i$ in quarter $t$. In all specifications, we control for household size, the age and education of household members and on market dummies. In some specifications, we control for household fixed effects, while in other specifications we do not. Column 1 reports the result of the regression on the log of stores-per-dollar without household fixed effects. It shows that a household that visits twice as many stores per dollar spent has a price index that is 1.12% lower. Column 2 reports the results of the regression on the log of stores-per-dollar with household fixed effects. It shows that, in quarters when a household visits twice as many stores per dollar spent, it enjoys a 1.29% lower price index. Columns 3 and 4 report the results of the regressions on the level of stores-per-dollar when household fixed effects are respectively excluded and included. For all specifications, the regression coefficient on stores per dollar is negative and significant. In Panel (b) we carry out the same regression using the log (or the level) of the number of different stores in which the households shopped during quarter $t$. We find that the regression coefficients on the number of stores is always negative and significant. Since our model has only limited heterogeneity across buyers, we choose as a calibration target the regression coefficient on the log of stores-per-dollar.

In Figure 7, we display the distribution of the number of stores visited in a quarter by households in KNCP. Approximately 32% of households do all of their shopping in a quarter at one store. Approximately 28% of households do all of their shopping in a quarter at two stores. Only 40% of households shop from more than two stores in a quarter. As a calibration target for the model, we choose the average number of stores, 2.48, from which a household shops in a quarter.

Figure 7: Number of different stores visited by households




*Notes:* This figure displays a histogram of the number of different stores visited by households within a quarter.

We calibrate the parameters of the model by minimizing the sum of the absolute value of the percentage deviation between each of the targeted moments and its counterpart in the model. Table 6 reports the targeted moments, the value of these moments in the calibrated model, and the calibrated value of the parameters.

## 4.2 Validation

Our theory of shopping and pricing in the retail market—albeit rather stylized—matches the extent and sources of dispersion in the price of the same good across different sellers. Indeed, the calibrated model matches the standard deviation of prices posted by different

Table 6: Calibration

| Targets | Data | Model |
|---|---|---|
| Standard deviation of prices | 10% | 10% |
| Share of variance of prices due to store component | 34% | 37% |
| Average number stores visited | 2.48 | 2.41 |
| Regression coefficient of price index on number stores | -1.3% | -1.9% |
| Additional moments | | |
| Standard deviation of price index | 7.8% | 6.6% |
| Share of variance of price index due to store component | 55% | 71% |

| Parameter | $\mu_c/\mu_b$ | $u_c/u_b$ | $\alpha$ | $\rho$ |
|---|---|---|---|---|
| Value | 0.68 | 0.041 | 0.89 | 0.87 |

sellers for the same good, in the same market and in the same week. Also, the calibrated model matches quite well the fraction of the variance of prices posted by different sellers that is due to differences in the store component of the price (37% in the model, 34% in the data), and the fraction that is due to differences in the store-good component of the price (63% in the model, 66% in the data). The theory's explanation for this decomposition is simple. There is dispersion in the store component of prices for the same reasons as in Burdett and Judd (1983). Specifically, as some buyers have only one seller in their network and other buyers have multiple sellers in their network, sellers must post different basket prices in equilibrium. There is dispersion in the store-good component of prices (i.e. relative price dispersion) because of price discrimination. Specifically, as high-valuation buyers need to purchase all the goods from the same retailer and low-valuation buyers can purchase different goods from different retailers, an individual seller want to set asymmetric prices for the two goods. And, in equilibrium, for every seller posting a higher price for the first good than for the second, there must be another seller doing the opposite.

Our theory matches the negative relationship between the price index of a household and the number of stores from which the household shops during a quarter (normalized by the household's expenditures). This is intuitive. According to the model, households who shop from more sellers are more likely to be buyers who are willing to go through the stores in their network to purchase different goods at the lowest price. Households who shop from fewer stores are more likely to be buyers who need to purchase everything from one of the stores in their network. Because of relative price dispersion, i.e. because equally expensive stores posts different prices, a buyer who is willing to purchase different goods at different stores in his network can pay less from the same basket of goods than a buyer who needs to purchase everything from just one of the stores in his network. Thus, households who shop from more stores tend to have a lower price index.

The theory makes other predictions about the dispersion in the prices paid by different households for the same basket of goods. Before reviewing these predictions, we look at the extent and sources of households' price index dispersion in the KNCP data. We normalize the price $p_{ijmt}$ at which good $j$ is traded in transaction $i$ taking place in market $m$ during quarter $t$ by the average price of that good in that market and in that quarter. We then decompose the price $p_{ijmt}$ into a store component, a store-good component, and a transaction component. The store component is defined as the average of the (normalized) price of the goods sold by the store $s_i$ where transaction $i$ took place. The store-good component is defined as the difference between the average of the (normalized) price at which good $j$ is sold at store $s_i$ during quarter $t$ and the store component. The transaction component is defined as the difference between the (normalized) price at which good $j$ is sold in transaction $i$ and the average of the (normalized) price at which good $j$ is sold at store $s_i$ during quarter $t$ and the store component. We refer the reader to Kaplan and Menzio (2015) for details about this decomposition. For our purposes, it is important to notice that, roughly speaking, the store component and the store-good components of pijmt are "persistent" parts of the price, as they are measured as quarterly averages. The transaction component is a "transitory" part of the price, as it is measured as a deviation from a quarterly average.

We can decompose the households' price indexes using the decomposition of the prices $p_{ijmt}$. Specifically, we express the price index of a household as the sum of a store component, a store-good component and a transaction component. The store component of the price index is the (expenditure-weighted) average of the store component of all household transactions. The store-good component of the price index is the (expenditure-weighted) average of the store-good component of all household transactions. The transaction component of the price index is the (expenditure-weighted) average of the transaction component of all household transactions. Then, we decompose the variance of the households' price indexes in market m and quarter t into the variance of the store component, the variance of the store-good component, the variance of the transaction component, and two covariance terms.

The average standard deviation of price indexes is 9%, and the average variance is 0.81%. The fraction of the variance due to differences in the store component is 42%, the fraction due to differences in the store-good component is 58%, the fraction due to differences in the transitory store-good component is 19%. The covariance between the store component and the store-good component is $-24\%$, while the covariance between the store component and the transaction component is 5%. Since ours is a theory of the persistent component of prices, we need to purge the price indexes from the transaction component. After doing so, we find that the average standard deviation of price indexes is 7.8%, and the average variance is 0.61%. The fraction of the variance due to differences in the store component is 55%, the fraction due to differences in the persistent store-good component (plus the covariance term) is 45%.

Two features of price index dispersion are remarkable. First, the standard deviation of price indexes (7.8%) is smaller than the standard deviation of prices (10%). Second, the fraction of the variance of price indexes due to differences in the store component (55%) is higher than the fraction of the variance of prices due to differences in the store component (35%). Conversely, the fraction of the variance of price indexes due to differences in the store-good component (together with the covariance term) is smaller than the variance of prices that is due to differences in the persistent part of the store-good component (45% vs 65%).

Our theory accounts surprisingly well for both features of price index dispersion, even though it was not designed or calibrated to do so. First, in the calibrated model, the standard deviation of price indexes is 6.6% while the standard deviation of prices is 10%. Thus, the model can account, both qualitatively and quantitatively, for the fact that the dispersion of prices paid by households for the same basket of goods is lower than the dispersion of prices posted by sellers for the same good. Second, in the calibrated model, the fraction of the variance of price indexes due to differences in the store component is 71%, while the fraction of the variance of prices due to differences in the store component is 37%. Conversely, the fraction of the variance of price indexes due to the store-good component (together with the covariance) is 29%, while the fraction of the variance of prices due to the persistent part of store-good component is 63%. Thus, the model can account qualitatively for the fact that differences in the store component are a larger source of dispersion of prices paid by households for the same basket of goods than they are a source of dispersion of prices posted by sellers for the same good.

The explanation for these phenomena provided by our theory is simple. Some sellers post a relatively high price for the first good and a relatively low price for the second good. Other, equally expensive sellers do the opposite, as they post a relatively low price for the first good and a relatively high price for the second good. The variation across stores in the store-good component of prices contributes for a large fraction of the overall variance of posted prices. Now, let us turn to the buyers. Recall that buyers of type $b$ have to purchase both goods from the same seller. In the price index of this type of buyers, the variation in the store-good component of prices washes out as one of the prices they pay has a positive store-good component and the other has a negative store-good component. For this reason, the dispersion in price indexes across buyers is smaller than the dispersion in prices across sellers. Also for this reason, the variation in the store component of prices is more important for the dispersion of price indexes across buyers than for the dispersion of prices across sellers.

To summarize, our simple theory of pricing and shopping in the retail market provides an explanation for the extent and sources of dispersion in the prices posted by different sellers for the same good, for the buyers' return from shopping at an additional store, and for the extent and sources of dispersion in the prices paid by different households for the same basket of goods. Thus, our theory responds to the call in Kaplan and Menzio (2015) for a model of the retail market that is consistent not only with the key facts about price dispersion, but also with the key differences between the extent and sources of price and price index dispersion.

# 5 Conclusions

In this paper, we used the KNRS to measure the extent and sources of price dispersion, i.e. the dispersion in the price at which the same good is sold by different stores in the same market and in the same week. We found that a significant fraction of price dispersion is due to the fact that stores that are, on average, equally expensive choose to set persistently different prices for the same good. We labelled this phenomenon *relative price dispersion*. We then developed a theory of relative price dispersion in the context of the canonical model of price dispersion of Burdett and Judd (1983). According to our theory, relative price dispersion is an equilibrium manifestation of the sellers' attempt to discriminate between different buyers. In particular, an individual seller finds it optimal to charge asymmetric prices for different goods to discriminate between high-valuation buyers who need to purchase everything in the same location, and low-valuation buyers who are willing to purchase different goods at different locations. In equilibrium, for every seller that charges a relatively high price for one good, there must be another seller that is equally expensive on average but charges a relatively low price for the same good. We showed that our theory of relative price dispersion can not only account for the extent and sources of price dispersion, but also for the extent and sources of variation in the prices paid by different households for the same basket of goods. Thus our theory offers a response to the call in Kaplan and Menzio (2015) for a model of the retail market that is consistent not only with the key facts about price dispersion, but also with the key facts about the dispersion of price indexes across households.

Several extensions of our theory seem worthwhile. On the descriptive side, it would be interesting to combine our model of the retail market with a model of temporary price reductions (as Sobel 1984 or Aguirregabiria 1999). The resulting model would offer a truly comprehensive theory of price dispersion, in which price dispersion occurs because some stores are cheap and some are expensive, because equally expensive stores have different average prices for the same good, and because the same store has a different price for the same good on different days. As in this paper, the resulting theory could be tested using data on the dispersion in prices paid by different households. On the normative side, it would be interesting to use the calibrated model to measure the extent of inefficiency in the retail market, identify which policies might be welfare improving, and which ones might exacerbate inefficiencies. Finally, it might be worthwhile extending the model presented in this paper to include a richer pattern of heterogeneity among buyers (i.e., more than two types of buyers), a richer set of goods (i.e., more than two goods), to endogenize the buyers' network of sellers, the buyers' decision to shop from multiple sellers or not, and the sellers' entry decision.

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# Appendix

## A Empirical Appendix

Table 7: Share of UPCs across Product Groups: Baseline

| Product Group | Percent |
|---|---|
| YOGURT | 10.7 |
| CARBONATED BEVERAGES | 9.3 |
| FRESH PRODUCE | 6 |
| BREAD AND BAKED GOODS | 5.4 |
| PIZZA/SNACKS/HORS DOEURVES-FRZN | 4.4 |
| MILK | 3.6 |
| VEGETABLES - CANNED | 3.4 |
| SOFT DRINKS-NON-CARBONATED | 3.3 |
| SOUP | 3.3 |
| CANDY | 3.2 |
| CEREAL | 3 |
| FRESH MEAT | 3 |
| SNACKS | 3 |
| CHEESE | 2.9 |
| PAPER PRODUCTS | 2.8 |
| BREAKFAST FOOD | 2.3 |
| CRACKERS | 2.1 |
| DRESSINGS/SALADS/PREP FOODS-DELI | 1.8 |
| PREPARED FOOD-DRY MIXES | 1.8 |
| PASTA | 1.7 |
| EGGS | 1.6 |
| JUICE, DRINKS - CANNED, BOTTLED | 1.6 |
| COOKIES | 1.3 |
| COT CHEESE, SOUR CREAM, TOPPINGS | 1.3 |
| BUTTER AND MARGARINE | 1.2 |
| PREPARED FOODS-FROZEN | 1.2 |
| CONDIMENTS, GRAVIES, AND SAUCES | 1.1 |
| PREPARED FOOD-READY-TO-SERVE | 1.1 |
| VEGETABLES-FROZEN | 1.1 |
| PACKAGED MEATS-DELI | 1 |
| GUM | 0.8 |
| SEAFOOD - CANNED | 0.7 |
| TOBACCO AND ACCESSORIES | 0.6 |
| DOUGH PRODUCTS | 0.5 |
| FRUIT - DRIED | 0.5 |
| JUICES, DRINKS-FROZEN | 0.5 |
| SALAD DRESSINGS, MAYO, TOPPINGS | 0.5 |
| SUGAR, SWEETENERS | 0.5 |
| BOOKS AND MAGAZINES | 0.4 |
| DESSERTS, GELATINS, SYRUP | 0.4 |
| ICE CREAM, NOVELTIES | 0.4 |
| ORAL HYGIENE | 0.4 |
| BAKING SUPPLIES | 0.3 |
| BREAKFAST FOODS-FROZEN | 0.3 |
| FRESHENERS AND DEODORIZERS | 0.3 |
| ICE | 0.3 |
| JAMS, JELLIES, SPREADS | 0.3 |
| BABY FOOD | 0.2 |
| BAKING MIXES | 0.2 |
| DESSERTS/FRUITS/TOPPINGS-FROZEN | 0.2 |
| FRUIT - CANNED | 0.2 |
| HOUSEHOLD CLEANERS | 0.2 |
| KITCHEN GADGETS | 0.2 |
| PACKAGED MILK AND MODIFIERS | 0.2 |
| WRAPPING MATERIALS AND BAGS | 0.2 |
| AUTOMOTIVE | 0.1 |
| COFFEE | 0.1 |
| DISPOSABLE DIAPERS | 0.1 |
| FLOUR | 0.1 |
| HARDWARE, TOOLS | 0.1 |
| HOUSEHOLD SUPPLIES | 0.1 |
| LIGHT BULBS, ELECTRIC GOODS | 0.1 |
| PICKLES, OLIVES, AND RELISH | 0.1 |
| SHORTENING, OIL | 0.1 |
| SNACKS, SPREADS, DIPS-DAIRY | 0.1 |
| SPICES, SEASONING, EXTRACTS | 0.1 |
| UNPREP MEAT/POULTRY/SEAFOOD-FRZN | 0.1 |

Table 8: Share of UPCs across Departments

| | Baseline | UPC Alt | UPC National | Coca-Cola | Unilever | Low durab. | High durab. | Low price | High price |
|---|---|---|---|---|---|---|---|---|---|
| ALCOHOLIC BEVERAGES | 0% | 0% | 6% | 0% | 0% | 0% | 13% | 0% | 0% |
| DAIRY | 22% | 7% | 12% | 13% | 2% | 35% | 0% | 25% | 11% |
| DELI | 2% | 1% | 2% | 0% | 0% | 0% | 0% | 1% | 3% |
| DRY GROCERY | 52% | 49% | 53% | 87% | 12% | 4% | 0% | 58% | 49% |
| FRESH PRODUCE | 6% | 1% | 5% | 0% | 0% | 9% | 0% | 4% | 6% |
| FROZEN FOODS | 9% | 12% | 4% | 0% | 8% | 32% | 0% | 7% | 11% |
| GENERAL MERCHANDISE | 1% | 4% | 2% | 0% | 1% | 0% | 82% | 1% | 1% |
| HEALTH AND BEAUTY | 0% | 12% | 2% | 0% | 62% | 0% | 4% | 0% | 1% |
| MEAT | 4% | 3% | 3% | 0% | 0% | 20% | 0% | 1% | 10% |
| NON-FOOD | 4% | 11% | 11% | 0% | 14% | 0% | 0% | 3% | 8% |
| TOTAL | 1000 | 1000 | 1463 | 3608 | 10917 | 12301 | 32989 | 430 | 315 |

Table 9: Parameter estimates

| | Store component | | | | |
|---|---|---|---|---|---|
| Models | $\rho^y$ | $\theta_{y,1}$ | $Var(\alpha^y)$ | $Var(\eta^y)$ | $Var(\varepsilon^y)$ |
| Baseline | 0.982613573 | 0 | 0.002793056 | 2.46E-05 | 0.000116634 |
| State | 0.983490975 | 0.038776 | 0.002663621 | 2.25E-05 | 0.000130635 |
| County | 0.991166659 | 0.217979 | 0.001667647 | 3.04E-05 | 0.000214859 |
| $N_1 = 50$ | 0.987028926 | 0.086678 | 0.005004708 | 1.47E-05 | 0.00028866 |
| $N_1 = 500$ | 0.990499883 | 0 | 0.002280235 | 1.32E-05 | 0.000110802 |
| $N_2 = 25$ | 0.983067304 | 0 | 0.002687951 | 2.39E-05 | 0.000112249 |
| $N_2 = 100$ | 0.983882217 | 0.006013 | 0.003159055 | 2.40E-05 | 0.000131709 |
| Quant weighted | 0.98573063 | 0.154354 | 0.00103578 | 8.43E-06 | 0.000346942 |
| UPC alt | 0.989435059 | 0.039211 | 0.002554465 | 8.57E-06 | 4.07E-05 |
| UPC national | 0.868614266 | 0 | 0.005151485 | 2.49E-05 | 0.000119115 |
| Low price | 0.980758409 | 0.131075 | 0.00555965 | 2.16E-05 | 0.000576657 |
| High price | 0.980959918 | 0.06653 | 0.003284731 | 8.60E-06 | 0.000645019 |
| Low durability | 0.962261334 | 0 | 0.003505668 | 2.45E-05 | 0.000160791 |
| High durability | 0.985078092 | 0.408077 | 0.000928235 | 2.73E-05 | 0.000614076 |
| Unilever | 0.984127103 | 0.07472 | 0.002913701 | 1.28E-05 | 0.001244086 |
| Coca-cola | 0.995571178 | 0.0427 | 0.000230254 | 4.10E-05 | 0.000892025 |

| | Store-good component | | | | |
|---|---|---|---|---|---|
| Models | $\rho^z$ | $\theta_{z,1}$ | $Var(\alpha^z)$ | $Var(\eta^z)$ | $Var(\varepsilon^z)$ |
| Baseline | 0.965159943 | 0.025611 | 0.003273669 | 0.000262241 | 0.012660499 |
| State | 0.965000278 | 0.042116 | 0.003592662 | 0.00027266 | 0.013102838 |
| County | 0.964487279 | 0.034764 | 0.00267737 | 0.000245359 | 0.012897066 |
| $N_1 = 50$ | 0.969583498 | 0.03929 | 0.004748385 | 0.000264688 | 0.015791481 |
| $N_1 = 500$ | 0.964557595 | 0.016314 | 0.003151334 | 0.000252595 | 0.012829199 |
| $N_2 = 25$ | 0.965167539 | 0.026807 | 0.003236118 | 0.000258127 | 0.012361954 |
| $N_2 = 100$ | 0.964220785 | 0.032336 | 0.003191119 | 0.000266847 | 0.012973907 |
| Identity weights | 0.96724252 | 0.030042 | 0.003211072 | 0.000246603 | 0.012723392 |
| MA(5) | 0.970311168 | 1 | 0.003151381 | 0.000213223 | 0.006327444 |
| MA(10) | 0.980000019 | 0.115018 | 0.003150801 | 0.000204187 | 0.006453931 |
| Skewed MA(1) | 0.965159943 | 0.025611 | 0.003273669 | 0.000262241 | 0.012660499 |
| Quant weighted | 0.966668046 | 0.046261 | 0.004811795 | 0.000285567 | 0.015285712 |
| UPC alt | 0.967386315 | 0.241865 | 0.003009445 | 0.000216338 | 0.006292217 |
| UPC national | 0.967495404 | 0.073928 | 0.004770164 | 0.000269885 | 0.013976282 |
| Low price | 0.968290143 | 0.058805 | 0.005416631 | 0.000350237 | 0.014827101 |
| High price | 0.97228473 | 0 | 0.002602982 | 0.000133687 | 0.01691263 |
| Low durability | 0.966802325 | 0.056946 | 0.002670222 | 0.000216992 | 0.010632362 |
| High durability | 0.954017962 | 0.071649 | 0.001431966 | 0.000294292 | 0.005906032 |
| Unilever | 0.973958285 | 0.151937 | 0.002962665 | 0.000187877 | 0.01006858 |
| Coca-cola | 0.952179616 | 0.004329 | 0.002045257 | 0.000280396 | 0.01120506 |

| | $\rho^z$ | $Var(\alpha^z)$ | $Var(\eta^z)$ | sale prob | discount |
|---|---|---|---|---|---|
| Sales | 0.966006219 | 0.003260204 | 0.000251805 | 0.046535283 | -0.520418332 |

*Notes:* The baseline model is estimated on a baseline sample of UPCs using data for the Minneapolis-St Paul designated market area. The following rows present results for alternative specifications discussed in the text: defining the market as the state or county, alternative cutoffs for constructing the samples, quantity weighting in constructing the store components, the alternative selections of goods “UPC alt” and “UPC nationwide,” the low- and high-price samples, the low- and high-durability samples, and the Unilever and Coca-cola samples. The top table considers the store component and the bottom the store-good component. For the latter we further investigate an alternative GMM weighting matrix (identity weighting), as well as alternative specifications for the transitory variation: MA(5), MA(10), MA(1) with skewness in disturbances, and an explicit model of sales.

Table 10: Share of UPCs across Product Groups: Alternative

| Product Group | Percent |
|---|---|
| CANDY | 5.31 |
| PREPARED FOODS-FROZEN | 3.5 |
| SNACKS | 3.2 |
| JUICE, DRINKS - CANNED, BOTTLED | 2.8 |
| PAPER PRODUCTS | 2.8 |
| PACKAGED MEATS-DELI | 2.6 |
| BREAD AND BAKED GOODS | 2.3 |
| CONDIMENTS, GRAVIES, AND SAUCES | 2.3 |
| SOUP | 2.3 |
| ICE CREAM, NOVELTIES | 2.2 |
| PET FOOD | 2.2 |
| DEODORANT | 2.1 |
| CHEESE | 2 |
| CARBONATED BEVERAGES | 1.9 |
| PIZZA/SNACKS/HORS DOEURVES-FRZN | 1.9 |
| BABY FOOD | 1.8 |
| COOKIES | 1.8 |
| VEGETABLES-FROZEN | 1.8 |
| MEDICATIONS/REMEDIES/HEALTH AI | 1.7 |
| BREAKFAST FOOD | 1.6 |
| CEREAL | 1.6 |
| DESSERTS, GELATINS, SYRUP | 1.6 |
| DETERGENTS | 1.6 |
| PREPARED FOOD-DRY MIXES | 1.6 |
| SALAD DRESSINGS, MAYO, TOPPINGS | 1.6 |
| VEGETABLES - CANNED | 1.6 |
| CRACKERS | 1.5 |
| FRUIT - DRIED | 1.5 |
| SANITARY PROTECTION | 1.5 |
| DRESSINGS/SALADS/PREP FOODS-DELI | 1.4 |
| YOGURT | 1.4 |
| MILK | 1.3 |
| ORAL HYGIENE | 1.3 |
| PREPARED FOOD-READY-TO-SERVE | 1.3 |
| COFFEE | 1.2 |
| FRESH PRODUCE | 1.2 |
| LAUNDRY SUPPLIES | 1.1 |
| BAKING SUPPLIES | 1 |
| DISPOSABLE DIAPERS | 1 |
| BAKING MIXES | 0.9 |
| FIRST AID | 0.9 |
| SHAVING NEEDS | 0.9 |
| SKIN CARE PREPARATIONS | 0.9 |
| STATIONERY, SCHOOL SUPPLIES | 0.9 |
| GUM | 0.8 |
| NUTS | 0.8 |
| PERSONAL SOAP AND BATH ADDITIV | 0.8 |
| SOFT DRINKS-NON-CARBONATED | 0.8 |
| BAKED GOODS-FROZEN | 0.7 |
| COUGH AND COLD REMEDIES | 0.7 |
| FRESHENERS AND DEODORIZERS | 0.7 |
| FRUIT - CANNED | 0.7 |
| HARDWARE, TOOLS | 0.7 |
| HOUSEHOLD CLEANERS | 0.7 |
| HOUSEHOLD SUPPLIES | 0.7 |
| LIGHT BULBS, ELECTRIC GOODS | 0.7 |
| TEA | 0.7 |
| FRESH MEAT | 0.6 |
| HAIR CARE | 0.6 |
| TOBACCO AND ACCESSORIES | 0.6 |
| WRAPPING MATERIALS AND BAGS | 0.6 |
| COT CHEESE, SOUR CREAM, TOPPINGS | 0.5 |
| DOUGH PRODUCTS | 0.5 |
| GLASSWARE, TABLEWARE | 0.5 |
| JAMS, JELLIES, SPREADS | 0.5 |
| VITAMINS | 0.5 |
| DESSERTS/FRUITS/TOPPINGS-FROZEN | 0.4 |
| JUICES, DRINKS-FROZEN | 0.4 |
| SNACKS, SPREADS, DIPS-DAIRY | 0.4 |
| SPICES, SEASONING, EXTRACTS | 0.4 |
| UNPREP MEAT/POULTRY/SEAFOOD-FRZN | 0.4 |
| BREAKFAST FOODS-FROZEN | 0.3 |
| BUTTER AND MARGARINE | 0.3 |
| GROOMING AIDS | 0.3 |
| PASTA | 0.3 |
| PICKLES, OLIVES, AND RELISH | 0.3 |
| PUDDING, DESSERTS-DAIRY | 0.3 |
| SEAFOOD - CANNED | 0.3 |
| VEGETABLES AND GRAINS - DRIED | 0.3 |
| BATTERIES AND FLASHLIGHTS | 0.2 |
| BOOKS AND MAGAZINES | 0.2 |
| COOKWARE | 0.2 |
| PACKAGED MILK AND MODIFIERS | 0.2 |
| PET CARE | 0.2 |
| AUTOMOTIVE | 0.1 |
| ELECTRONICS, RECORDS, TAPES | 0.1 |
| FEMININE HYGIENE | 0.1 |
| FLOUR | 0.1 |
| GRT CARDS/PARTY NEEDS/NOVELTIE | 0.1 |
| KITCHEN GADGETS | 0.1 |
| SEASONAL | 0.1 |
| SHOE CARE | 0.1 |
| SHORTENING, OIL | 0.1 |
| SUGAR, SWEETENERS | 0.1 |
| TABLE SYRUPS, MOLASSES | 0.1 |
| WINE | 0.1 |

Table 11: Share of UPCs across Product Groups: National

| Product Group | Percent |
|---|---|
| CARBONATED BEVERAGES | 7.59 |
| FRESH PRODUCE | 5.06 |
| JUICE, DRINKS - CANNED, BOTTLED | 4.65 |
| CANDY | 4.58 |
| PAPER PRODUCTS | 4.38 |
| BEER | 4.24 |
| SNACKS | 3.76 |
| SOFT DRINKS-NON-CARBONATED | 3.69 |
| BREAD AND BAKED GOODS | 3.63 |
| YOGURT | 3.63 |
| CEREAL | 3.49 |
| TOBACCO AND ACCESSORIES | 3.49 |
| PACKAGED MEATS-DELI | 2.6 |
| PET FOOD | 2.6 |
| MILK | 2.53 |
| DRESSINGS/SALADS/PREP FOODS-DELI | 2.26 |
| SOUP | 2.26 |
| CHEESE | 2.05 |
| VEGETABLES - CANNED | 1.92 |
| CONDIMENTS, GRAVIES, AND SAUCES | 1.71 |
| DETERGENTS | 1.57 |
| PREPARED FOODS-FROZEN | 1.37 |
| BABY FOOD | 1.3 |
| BUTTER AND MARGARINE | 1.23 |
| CRACKERS | 1.09 |
| GUM | 1.09 |
| LIQUOR | 1.09 |
| PREPARED FOOD-READY-TO-SERVE | 1.09 |
| EGGS | 1.03 |
| MEDICATIONS/REMEDIES/HEALTH AI | 0.96 |
| PACKAGED MILK AND MODIFIERS | 0.96 |
| PIZZA/SNACKS/HORS DOEURVES-FRZN | 0.96 |
| PREPARED FOOD-DRY MIXES | 0.89 |
| COT CHEESE, SOUR CREAM, TOPPINGS | 0.82 |
| SEAFOOD - CANNED | 0.82 |
| BREAKFAST FOOD | 0.62 |
| COFFEE | 0.62 |
| COOKIES | 0.62 |
| BOOKS AND MAGAZINES | 0.55 |
| DISPOSABLE DIAPERS | 0.55 |
| TEA | 0.55 |
| VEGETABLES-FROZEN | 0.55 |
| WINE | 0.55 |
| ICE CREAM, NOVELTIES | 0.48 |
| PASTA | 0.48 |
| SALAD DRESSINGS, MAYO, TOPPINGS | 0.48 |
| SUGAR, SWEETENERS | 0.48 |
| COUGH AND COLD REMEDIES | 0.41 |
| FRUIT - CANNED | 0.41 |
| UNPREP MEAT/POULTRY/SEAFOOD-FRZN | 0.41 |
| BATTERIES AND FLASHLIGHTS | 0.34 |
| FRESH MEAT | 0.34 |
| JAMS, JELLIES, SPREADS | 0.34 |
| SHAVING NEEDS | 0.34 |
| WRAPPING MATERIALS AND BAGS | 0.34 |
| BAKING SUPPLIES | 0.27 |
| DESSERTS, GELATINS, SYRUP | 0.27 |
| HOUSEWARES, APPLIANCES | 0.27 |
| ICE | 0.27 |
| LAUNDRY SUPPLIES | 0.27 |
| SHORTENING, OIL | 0.27 |
| BAKING MIXES | 0.21 |
| BREAKFAST FOODS-FROZEN | 0.21 |
| DOUGH PRODUCTS | 0.21 |
| ELECTRONICS, RECORDS, TAPES | 0.21 |
| HOUSEHOLD SUPPLIES | 0.21 |
| PICKLES, OLIVES, AND RELISH | 0.21 |
| CHARCOAL, LOGS, ACCESSORIES | 0.14 |
| DESSERTS/FRUITS/TOPPINGS-FROZEN | 0.14 |
| NUTS | 0.14 |
| ORAL HYGIENE | 0.14 |
| SPICES, SEASONING, EXTRACTS | 0.14 |
| DIET AIDS | 0.07 |
| FLOUR | 0.07 |
| FRESHENERS AND DEODORIZERS | 0.07 |
| FRUIT - DRIED | 0.07 |
| HOUSEHOLD CLEANERS | 0.07 |
| PHOTOGRAPHIC SUPPLIES | 0.07 |
| SNACKS, SPREADS, DIPS-DAIRY | 0.07 |
| STATIONERY, SCHOOL SUPPLIES | 0.07 |
| VITAMINS | 0.07 |

Table 12: Share of UPCs across Product Groups: Low Price

| Product Group | Percent |
|---|---|
| YOGURT | 21.63 |
| CARBONATED BEVERAGES | 10.23 |
| VEGETABLES - CANNED | 7.67 |
| SOUP | 6.51 |
| SOFT DRINKS-NON-CARBONATED | 6.05 |
| CANDY | 5.58 |
| BREAD AND BAKED GOODS | 4.42 |
| FRESH PRODUCE | 4.19 |
| PASTA | 3.72 |
| PREPARED FOOD-DRY MIXES | 3.02 |
| PIZZA/SNACKS/HORS DOEURVES-FRZN | 2.33 |
| VEGETABLES-FROZEN | 2.09 |
| GUM | 1.86 |
| PAPER PRODUCTS | 1.86 |
| PREPARED FOOD-READY-TO-SERVE | 1.86 |
| EGGS | 1.63 |
| SEAFOOD - CANNED | 1.63 |
| JUICE, DRINKS - CANNED, BOTTLED | 1.16 |
| CHEESE | 0.93 |
| CONDIMENTS, GRAVIES, AND SAUCES | 0.93 |
| JUICES, DRINKS-FROZEN | 0.93 |
| PREPARED FOODS-FROZEN | 0.93 |
| DRESSINGS/SALADS/PREP FOODS-DELI | 0.7 |
| FRESHENERS AND DEODORIZERS | 0.7 |
| ICE | 0.7 |
| PACKAGED MEATS-DELI | 0.7 |
| BAKING MIXES | 0.47 |
| COT CHEESE, SOUR CREAM, TOPPINGS | 0.47 |
| DESSERTS/FRUITS/TOPPINGS-FROZEN | 0.47 |
| FRUIT - CANNED | 0.47 |
| KITCHEN GADGETS | 0.47 |
| ORAL HYGIENE | 0.47 |
| SUGAR, SWEETENERS | 0.47 |
| BAKING SUPPLIES | 0.23 |
| BUTTER AND MARGARINE | 0.23 |
| COOKIES | 0.23 |
| CRACKERS | 0.23 |
| DESSERTS, GELATINS, SYRUP | 0.23 |
| HARDWARE, TOOLS | 0.23 |
| LIGHT BULBS, ELECTRIC GOODS | 0.23 |
| MILK | 0.23 |
| PACKAGED MILK AND MODIFIERS | 0.23 |
| PICKLES, OLIVES, AND RELISH | 0.23 |
| SNACKS | 0.23 |
| SPICES, SEASONING, EXTRACTS | 0.23 |

Table 13: Share of UPCs across Product Groups: High Price

| Product Group | Percent |
|---|---|
| CARBONATED BEVERAGES | 13.65 |
| PIZZA/SNACKS/HORS DOEURVES-FRZN | 8.89 |
| FRESH MEAT | 8.57 |
| CEREAL | 6.35 |
| SNACKS | 6.35 |
| FRESH PRODUCE | 6.03 |
| PAPER PRODUCTS | 5.08 |
| BREAD AND BAKED GOODS | 4.44 |
| MILK | 4.44 |
| BREAKFAST FOOD | 3.17 |
| CRACKERS | 3.17 |
| COOKIES | 2.54 |
| DRESSINGS/SALADS/PREP FOODS-DELI | 2.54 |
| JUICE, DRINKS - CANNED, BOTTLED | 2.54 |
| SOFT DRINKS-NON-CARBONATED | 2.22 |
| TOBACCO AND ACCESSORIES | 1.9 |
| BUTTER AND MARGARINE | 1.59 |
| CANDY | 1.59 |
| COT CHEESE, SOUR CREAM, TOPPINGS | 1.59 |
| PACKAGED MEATS-DELI | 1.59 |
| CHEESE | 1.27 |
| EGGS | 1.27 |
| BOOKS AND MAGAZINES | 0.95 |
| ICE CREAM, NOVELTIES | 0.95 |
| SALAD DRESSINGS, MAYO, TOPPINGS | 0.95 |
| BABY FOOD | 0.63 |
| CONDIMENTS, GRAVIES, AND SAUCES | 0.63 |
| ORAL HYGIENE | 0.63 |
| PREPARED FOODS-FROZEN | 0.63 |
| YOGURT | 0.63 |
| BREAKFAST FOODS-FROZEN | 0.32 |
| COFFEE | 0.32 |
| DISPOSABLE DIAPERS | 0.32 |
| HOUSEHOLD CLEANERS | 0.32 |
| HOUSEHOLD SUPPLIES | 0.32 |
| SNACKS, SPREADS, DIPS-DAIRY | 0.32 |
| SUGAR, SWEETENERS | 0.32 |
| UNPREP MEAT/POULTRY/SEAFOOD-FRZN | 0.32 |
| VEGETABLES-FROZEN | 0.32 |
| WRAPPING MATERIALS AND BAGS | 0.32 |

Table 14: Share of UPCs across Product Groups: Coca-cola

| Product Group | Percent |
|---|---|
| CARBONATED BEVERAGES | 56.26 |
| JUICE, DRINKS - CANNED, BOTTLED | 18.4 |
| YOGURT | 12.89 |
| SOFT DRINKS-NON-CARBONATED | 12.2 |
| COFFEE | 0.06 |
| JUICES, DRINKS-FROZEN | 0.06 |
| VITAMINS | 0.06 |
| BUTTER AND MARGARINE | 0.03 |
| CHEESE | 0.03 |
| GLASSWARE, TABLEWARE | 0.03 |

Table 15: Share of UPCs across Product Groups: Unilever

| Product Group | Percent |
|---|---|
| HAIR CARE | 31.83 |
| PERSONAL SOAP AND BATH ADDITIV | 13.55 |
| DEODORANT | 11.87 |
| SKIN CARE PREPARATIONS | 9.51 |
| ICE CREAM, NOVELTIES | 7.82 |
| TEA | 5.25 |
| MEN'S TOILETRIES | 2.78 |
| SOUP | 2.39 |
| FRAGRANCES - WOMEN | 2.1 |
| GROOMING AIDS | 1.95 |
| YOGURT | 1.39 |
| SALAD DRESSINGS, MAYO, TOPPINGS | 1.36 |
| PREPARED FOOD-DRY MIXES | 1.22 |
| CONDIMENTS, GRAVIES, AND SAUCES | 1.03 |
| BUTTER AND MARGARINE | 0.98 |
| FIRST AID | 0.95 |
| SHAVING NEEDS | 0.67 |
| PAPER PRODUCTS | 0.62 |
| HOUSEWARES, APPLIANCES | 0.58 |
| DESSERTS/FRUITS/TOPPINGS-FROZEN | 0.45 |
| MEDICATIONS/REMEDIES/HEALTH AI | 0.35 |
| SPICES, SEASONING, EXTRACTS | 0.32 |
| COSMETICS | 0.23 |
| SANITARY PROTECTION | 0.08 |
| SHORTENING, OIL | 0.08 |
| SNACKS | 0.08 |
| SOFT DRINKS-NON-CARBONATED | 0.08 |
| JUICE, DRINKS - CANNED, BOTTLED | 0.07 |
| DRESSINGS/SALADS/PREP FOODS-DELI | 0.06 |
| DESSERTS, GELATINS, SYRUP | 0.06 |
| ETHNIC HABA | 0.04 |
| PREPARED FOOD-READY-TO-SERVE | 0.04 |
| BAKING SUPPLIES | 0.03 |
| VEGETABLES - CANNED | 0.03 |
| BABY NEEDS | 0.02 |
| BAKING MIXES | 0.02 |
| CANDY | 0.02 |
| AUTOMOTIVE | 0.01 |
| FRESHENERS AND DEODORIZERS | 0.01 |
| HOUSEHOLD CLEANERS | 0.01 |
| HOUSEHOLD SUPPLIES | 0.01 |
| JUICES, DRINKS-FROZEN | 0.01 |
| PACKAGED MILK AND MODIFIERS | 0.01 |
| SEWING NOTIONS | 0.01 |
| SNACKS, SPREADS, DIPS-DAIRY | 0.01 |
| VEGETABLES AND GRAINS - DRIED | 0.01 |

Table 16: Share of UPCs across Product Groups: Low Durability

| Product Group | Percent |
|---|---|
| ICE CREAM, NOVELTIES | 31.52 |
| PACKAGED MEATS-DELI | 20.29 |
| YOGURT | 18.56 |
| FRESH PRODUCE | 9.28 |
| MILK | 8.59 |
| CHEESE | 4.11 |
| BABY FOOD | 3.16 |
| COT CHEESE, SOUR CREAM, TOPPINGS | 2.15 |
| EGGS | 1.78 |
| BAKING SUPPLIES | 0.28 |
| DESSERTS/FRUITS/TOPPINGS-FROZEN | 0.09 |
| SNACKS, SPREADS, DIPS-DAIRY | 0.02 |
| SOFT DRINKS-NON-CARBONATED | 0.02 |
| CONDIMENTS, GRAVIES, AND SAUCES | 0.02 |
| DISPOSABLE DIAPERS | 0.02 |
| PAPER PRODUCTS | 0.02 |
| PET FOOD | 0.02 |
| PREPARED FOOD-READY-TO-SERVE | 0.02 |
| VITAMINS | 0.02 |
| CARBONATED BEVERAGES | 0.01 |
| CEREAL | 0.01 |
| GROOMING AIDS | 0.01 |
| HOUSEHOLD SUPPLIES | 0.01 |
| JAMS, JELLIES, SPREADS | 0.01 |

Table 17: Share of UPCs across Product Groups: High Durability

| Product Group | Percent |
|---|---|
| KITCHEN GADGETS | 35.83 |
| LIGHT BULBS, ELECTRIC GOODS | 18.04 |
| HOUSEWARES, APPLIANCES | 13.32 |
| LIQUOR | 13.01 |
| STATIONERY, SCHOOL SUPPLIES | 10.08 |
| PHOTOGRAPHIC SUPPLIES | 4.73 |
| BABY NEEDS | 4.4 |
| CHARCOAL, LOGS, ACCESSORIES | 0.4 |
| WINE | 0.12 |
| BEER | 0.03 |
| BATTERIES AND FLASHLIGHTS | 0.01 |
| JUICE, DRINKS - CANNED, BOTTLED | 0.01 |
| SOFT DRINKS-NON-CARBONATED | 0.01 |
| COOKWARE | 0 |
| DOUGH PRODUCTS | 0 |
| GRT CARDS/PARTY NEEDS/NOVELTIES | 0 |
| TOBACCO AND ACCESSORIES | 0 |

# B Proofs for Section 3

## B.1 Proof of Lemma 1

(i) First, consider a seller posting prices $(p_1, p_2)$ with $p_1 > u_b$ and $p_2 > u_b$. The seller's profit is zero, as there are no buyers willing to purchase a good at a price strictly greater than $u_b$. If the seller instead posts the prices $(p_1', p_2')$ with $p_1' = p_2' = u_b$, it sells both goods to all captive buyers of type $b$ and it attains a profit of at least $2\mu_b \alpha u_b > 0$. Therefore, a seller never finds it optimal to post prices $(p_1, p_2)$ such that $p_1 > u_b$ and $p_2 > u_b$.

Consider a seller posting prices $(p_1, p_2)$ with $p_1 > u_b$ and $p_2 \in (u_c, u_b]$. The seller attains a profit of

$$S(p_1, p_2) = \mu_b \left\{ \alpha + 2(1-\alpha) \left[ 1 - \hat{G}(u_b + p_2) + \hat{\nu}(u_b + p_2)/2 \right] \right\} p_2. \tag{32}$$

The expression above is easy to understand. The seller is in the network of $\mu_b \alpha$ captive buyers of type $b$. A captive buyer of type $b$ purchases good 2 from the seller with probability 1. The seller is also in the network of $2\mu_b(1-\alpha)$ non-captive buyers of type $b$. A non-captive buyer of type $b$ purchases good 2 from the seller with probability $1 - \hat{G}(u_b + p_2) + \hat{\nu}(u_b + p_2)/2$, where $1 - \hat{G}(u_b + p_2)$ denotes the fraction of sellers that charge prices $(p_1', p_2')$ such that $\min\{p_1', u_b\} + \min\{p_2', u_b\} > u_b + p_2$, and $\hat{\nu}(u_b + p_2)$ is the measure of sellers that charge prices $(p_1', p_2')$ such that $\min\{p_1', u_b\} + \min\{p_2', u_b\} = u_b + p_2$. The seller is in the network of $\mu_c \alpha$ captive buyers of type $c$ and of $2\mu_c(1-\alpha)$ non-captive buyers of type $c$, but it does not trade with any of them as both of its prices are greater than $u_c$. If the seller instead posts the prices $(p_1', p_2')$ with $p_1' = u_b$ and $p_2' = p_2$, it attains a profit of

$$\begin{aligned} S(p_1', p_2') &= \mu_b \left\{ \alpha + 2(1-\alpha) \left[ 1 - \hat{G}(u_b + p_2) + \hat{\nu}(u_b + p_2)/2 \right] \right\} (u_b + p_2) \\ &> S(p_1, p_2). \end{aligned} \tag{33}$$

The strict inequality in (33) follows from the fact that, by lowering the price of good 1 to $u_b$, the seller trades both good 1 and good 2 to its customers of type $b$. Hence, a seller never finds it optimal to post prices $(p_1, p_2)$ with $p_1 > u_b$ and $p_2 \in (u_c, u_b]$. For the same reason, a seller never finds it optimal to post prices $(p_1, p_2)$ with $p_1 \in (u_c, u_b]$ and $p_2 > u_b$.

Finally, consider a seller posting prices $(p_1, p_2)$ with $p_1 > u_b$ and $p_2 \in [0, u_c]$. The seller attains a profit of

$$\begin{aligned} S(p_1, p_2) &= \mu_b \left\{ \alpha + 2(1-\alpha) \left[ 1 - \hat{G}(u_b + p_2) + \hat{\nu}(u_b + p_2)/2 \right] \right\} p_2 \\ &\quad + \mu_c \left\{ \alpha + 2(1-\alpha) \left[ 1 - F_2(p_2) + \lambda_2(p_2)/2 \right] \right\} p_2. \end{aligned} \tag{34}$$

The expression in (34) is the same as in (32) with the addition of the term in the second line. This term represents the profit that the seller makes from trading good 2 to buyers of type $c$. If the seller instead posts the prices $(p_1', p_2')$ with $p_1' = u_b$ and $p_2' = p_2$, it attains a profit of

$$\begin{aligned} S(p_1', p_2') &= \mu_b \left\{ \alpha + 2(1-\alpha) \left[ 1 - \hat{G}(u_b + p_2) + \hat{\nu}(u_b + p_2)/2 \right] \right\} (u_b + p_2) \\ &+ \mu_c \left\{ \alpha + 2(1-\alpha) \left[ 1 - F_2(p_2) - \lambda_2(p_2)/2 \right] \right\} p_2 \\ &> S(p_1, p_2) \end{aligned} \tag{35}$$

The strict inequality in (35) follows from the fact that, by lowering the price of good 1 to $u_b$, the seller trades both goods 1 and good 2 to its type $b$ customers. Hence, a seller never finds it optimal to post prices $(p_1, p_2)$ such that $p_1 > u_b$ and $p_2 \in [0, u_c]$. For the very same reason, a seller never finds it optimal to post prices $(p_1, p_2)$ such that $p_1 \in [0, u_c]$ and $p_2 > u_b$.

(ii) Given part (i), it follows immediately that, if in equilibrium, a seller posts prices $(p_1, p_2)$ with $p_1 + p_2 > u_b + u_c$, it must be the case that $p_1 \in (u_c, u_b]$ and $p_2 \in (u_c, u_b]$. ■

## B.2 Proof of Lemma 3

(i) On the way to a contradiction suppose there is an equilibrium where $G$ has a mass point at $q_0$, i.e. $\nu(q_0) > 0$. First, notice that no seller finds it optimal to post $p_1 = p_2 = 0$, and hence the mass point cannot be at $q_0 = 0$. Second, notice that if, in equilibrium, a seller posts $(p_1, p_2)$ with $p_1 + p_2 = q_0$, it must be posting $p_1 \in [0, u_b]$ and $p_2 \in [0, u_b]$. Therefore, this seller attains a profit of

$$\begin{aligned} S(p_1, p_2) &= \mu_b \left\{ \alpha + 2(1-\alpha) \left[ 1 - G(q_0) + \nu(q_0)/2 \right] \right\} q_0 \\ &+ \textstyle\sum_{i=1}^{2} \mu_c \left\{ \alpha + 2(1-\alpha) \left[ 1 - F_i(p_i) + \lambda_i(p_i)/2 \right] \right\} \mathbf{1}[p_i \le u_c] p_i, \end{aligned} \tag{36}$$

where $\mathbf{1}[p_i \le u_c]$ is the indicator function that takes the value 1 if $p_i \le u_c$ and 0 otherwise. Suppose that the seller deviates and posts prices $(p_1', p_2')$ with $0 \le p_1' = p_1 - \epsilon_1$, $0 \le p_2' = p_2 - \epsilon_2$, $\epsilon = \epsilon_1 + \epsilon_2$, where $\epsilon_1 \ge 0$, $\epsilon_2 \ge 0$ and $\epsilon > 0$ all arbitrarily small. Then, the seller attains a profit of

$$\begin{aligned} S(p_1', p_2') &= \mu_b \left\{ \alpha + 2(1-\alpha) \left[ 1 - G(q_0 - \epsilon) \right] \right\} (q_0 - \epsilon) \\ &+ \textstyle\sum_{i=1}^{2} \mu_c \left\{ \alpha + 2(1-\alpha) \left[ 1 - F_i(p_i - \epsilon_i) + \lambda_i(p_i - \epsilon_i)/2 \right] \right\} \mathbf{1}[p_i \le u_c](p_i - \epsilon_i) \\ &> S(p_1, p_2), \end{aligned} \tag{37}$$

where the inequality follows from the fact that $G(q_0) - \nu(q_0)/2 - G(q_0 - \epsilon) \ge \nu(q_0)/2$ while $\epsilon$, $\epsilon_1$ and $\epsilon_2$ are all arbitrarily small. Since $S(p_1', p_2') > S(p_1, p_2)$, there cannot be a mass point at $q_0$.

(ii) On the way to a contradiction, suppose there is an equilibrium where $F_1$ has a mass point at $p_{1,0} \in (0, u_c]$. If in equilibrium a seller posts the price $p_1 = p_{1,0}$ for the first good, it must post a price $p_2 \in [0, u_b]$ for the second good. This seller attains a profit of

$$\begin{aligned} S(p_1,p_2) \quad &= \mu_b \left\{\alpha + 2(1-\alpha)[1 - G(p_1+p_2)]\right\}(p_1+p_2) \\ &+ \textstyle\sum_{i=1}^{2} \mu_c \left\{\alpha + 2(1-\alpha)\left[1 - F_i(p_i) + \lambda_i(p_i)/2\right]\right\} \mathbf{1}[p_i \le u_c] p_i. \end{aligned} \tag{38}$$

If the seller deviates and posts prices $(p_1', p_2')$ with $p_1' = p_{1,0} - \epsilon$, $p_2' = p_2$, for $\epsilon > 0$ arbitrarily small, it attains a profit of

$$\begin{aligned} S(p_1',p_2') \quad &= \mu_b \left\{\alpha + 2(1-\alpha)[1 - G(p_{1,0}+p_2-\epsilon)]\right\}(p_{1,0}+p_2-\epsilon) \\ &+ \mu_c \left\{\alpha + 2(1-\alpha)\left[1 - F_1(p_{1,0}-\epsilon)\right]\right\}(p_{1,0}-\epsilon) \\ &+ \mu_c \left\{\alpha + 2(1-\alpha)\left[1 - F_2(p_2) + \lambda_2(p_2)/2\right]\right\} \mathbf{1}[p_2 \le u_c] p_i \\ &\ge S(p_1,p_2), \end{aligned} \tag{39}$$

where the first inequality follows from the fact that the $F_1(p_{1,0}) - \lambda_1(p_{1,0})/2 - F_1(p_{1,0}-\epsilon) \ge \lambda_1(p_{1,0})/2$ and $\epsilon$ is arbitrarily small. Since $S(p_1', p_2') > S(p_1, p_2)$, there cannot be a mass point at $p_{1,0}$. ■

## B.3 Proof of Proposition 1

We established part (i) in the main text. We have also established that, given the marginal distribution $G$ in (14), a seller attains a profit of $S^*$ for all $(p_1, p_2) \in R_1$ such that $p_1 + p_2 \in [q_\ell, q_h]$. In order to complete the proof of part (ii), all we need to do is find a condition under which a seller cannot attain a profit strictly greater than $S^*$ by posting some off-equilibrium prices.

In Lemma 1, we proved that a seller never finds it optimal to post prices $(p_1, p_2)$ with either $p_1$, $p_2$ or both $p_1$ and $p_2$ strictly greater than $u_b$. If the seller posts prices $(p_1, p_2) \in R_1$ with $p_1 + p_2 \in (u_b + u_c, q_\ell)$, it attains a profit of

$$\begin{aligned} S_1(p_1+p_2) \quad &= \mu_b \left\{\alpha + 2(1-\alpha)\right\}(p_1+p_2) \\ &< \mu_b \left\{\alpha + 2(1-\alpha)\right\} q_\ell = S^*, \end{aligned} \tag{40}$$

where the first line makes use of the fact that $G(p_1 + p_2) = 0$ and the second line makes use of the fact that $p_1 + p_2 < q_\ell$. Hence, a seller does not find it optimal to deviate from the equilibrium and post prices $(p_1, p_2) \in R_1$ with $p_1 + p_2 \in (u_b + u_c, q_\ell)$.

In Lemma 2, we proved that a seller never finds it optimal to post prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, u_b + u_c]$ and both $p_1$ and $p_2$ greater than $u_c$ and smaller than $u_b$. If the seller posts prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, u_b + u_c]$, $p_1 \in [0, u_c]$ and $p_2 \in (u_c, u_b]$, it attains a profit of

$$\begin{aligned} S(p_1, p_2) &= \mu_b \{\alpha + 2(1-\alpha)\} (p_1 + p_2) + \mu_c \{\alpha + 2(1-\alpha)\} p_1 \\ &\leq \mu_b \{\alpha + 2(1-\alpha)\} (u_c + u_b) + \mu_c \{\alpha + 2(1-\alpha)\} u_c \\ &= S(u_c, u_b), \end{aligned} \tag{41}$$

where the first line makes use of $G(p_1 + p_2) = 0$ and $F(p_1) = 0$, and the second line makes use of $G(u_c + u_b) = 0$, $F(u_c) = 0$, $p_1 + p_2 \leq u_b + u_c$ and $p_1 \leq u_c$. The equilibrium profit $S^*$ is greater than $S(u_c, u_b)$ if and only if

$$\frac{\mu_c}{\mu_b} \leq \frac{3\alpha - 2}{(2-\alpha)u_c/u_b} - 1. \tag{42}$$

Hence, if and only if (42) holds a seller does not find it optimal to deviate from the equilibrium and post any prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, u_b + u_c]$, $p_1 \in [0, u_c]$ and $p_2 \in (u_c, u_b]$. Similarly, condition (42) guarantees that a seller does not find it optimal to deviate from the equilibrium and post any prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, u_b + u_c]$, $p_1 \in (u_c, u_b]$ and $p_2 \in [0, u_c]$.

If the seller posts prices $(p_1, p_2)$ with $p_1 \in [0, u_c]$ and $p_2 \in [0, u_c]$, it attains a profit of

$$\begin{aligned} S(p_1, p_2) &= (\mu_b + \mu_c) \{\alpha + 2(1-\alpha)\} (p_1 + p_2) \\ &\leq (\mu_b + \mu_c) \{\alpha + 2(1-\alpha)\} 2u_c \\ &= S(u_c, u_c), \end{aligned} \tag{43}$$

where the first line makes use of $G(p_1 + p_2) = 0$ and $F_i(p_i) = 0$ for $i = \{1, 2\}$, and the second line makes use of $G(2u_c) = 0$, $F_i(u_c) = 0$ for $i = \{1, 2\}$, and $p_1 + p_2 \leq 2u_c$. The equilibrium profit $S^*$ is greater than $S(u_c, u_c)$ if and only if

$$\frac{\mu_c}{\mu_b} \leq \frac{\alpha}{(2-\alpha)u_c/u_b} - 1. \tag{44}$$

Hence, if and only if (44) holds a seller does not find it optimal to deviate from the equilibrium and post any prices $(p_1, p_2)$ with $p_1 \in [0, u_c]$ and $p_2 \in [0, u_c]$. Finally, notice that if condition (42) holds, so does condition (44). Therefore, a seller does not want to deviate from the equilibrium if and only if condition (42), which is the same as condition (16), is satisfied. ■

## B.4 Proof of Lemma 5

It is easy to verify that in a Discrimination Equilibrium, $F_i(p)$ does not have a mass point at $p = 0$ for $i = \{1, 2\}$. Hence, $F_i(0) = 0$ and $F_i(p)$ is continuous over the interval $[0, u_c]$ for

$i = \{1, 2\}$. If $F_1 \neq F_2$ for some $p \in [0, u_c]$, there exist prices $p'$ and $p''$, with $0 \leq p' < p'' \leq u_c$, for which $F_1(p') = F_2(p')$ and either $F_1(p) > F_2(p)$ or $F_1(p) < F_2(p)$ for all $p \in (p', p'')$. Without loss in generality, assume $F_1(p) > F_2(p)$ for all $p \in (p', p'')$. Any price $p \in (p', p'')$ for good 1 is posted by a seller in region $R_2$, i.e. a seller with a basket price of $q \in (2u_c, u_b + u_c]$ and a price for good 2 of $p_2 \in (u_c, u_b]$. This seller attains a profit of $S_{21}(q, p)$. However, if the seller inverts the prices of goods 1 and 2, it attains a profit of $S_{22}(q, p)$ which is strictly greater than $S_{21}(q, p)$ because $F_1(p) > F_2(p)$. In turn, $S_{21}(q, p) < S_{22}(q, p)$ implies that in equilibrium there are no sellers posting a price $p \in (p', p'')$ for good 1, i.e. $F_1(p) = F_1(p')$ for all $p \in (p', p'')$. Since $F_1(p) = F_1(p')$ for all $p \in (p', p'')$, $F_2(p) \geq F_2(p')$ for all $p \in (p', p'')$ and $F_1(p') = F_2(p')$, it follows that $F_2(p) \geq F_1(p)$ for all $p \in (p', p'')$. We have thus reached a contradiction. ■

## B.5 Proof of Lemma 6

We need to prove that the seller's profit $S_2(q, p)$ is constant for all $(q, p)$ such that $q \in [q_\ell, u_b + u_c]$ and $p \in [p_\ell, u_c]$. To this aim, it is useful to write the seller's profit $S_2(q, p)$ as

$$S_2(q, p) = B(q) + C(p), \tag{45}$$

where $B(q)$ and $C(p)$ are defined as

$$\begin{aligned} B(q) &= \mu_b \left[\alpha + 2(1-\alpha)(1 - G(q))\right] q, \\ C(p) &= \mu_c \left[\alpha + 2(1-\alpha)(1 - F(p))\right] p. \end{aligned}$$

In words, $B(q)$ is the profit that the seller attains from trades with the buyers of type $b$, and $A(p)$ is the profit that the seller attains from trades with the buyers of type $c$.

First, we show that the distribution $G(q)$ has full support over the interval $[q_\ell, u_b + u_c]$. On the way to a contradiction, suppose that $G(q)$ is such that $G'(q_0) > 0$ and $G(q_0) = G(q_1)$ for some $q_0$ and $q_1$ with $q_\ell < q_0 < q_1 \leq u_b + u_c$. Since $G(q) = G(q_0)$ for all $q \in [q_0, q_1]$, the function $B(q)$ is strictly increasing in $q$ for all $q \in [q_0, q_1]$. That is, the seller's profit from buyers of type $b$ is strictly increasing in $q$ for all $q$ between $q_0$ and $q_1$.

Now, consider a seller with a basket price of $\hat{q} \in (q_0, q_1)$. This seller may post any price $p$ for the cheaper good between $\hat{q} - u_b$ and $u_c$. In fact, for $p < \hat{q} - u_b$, the price of the more expensive good would be greater than $u_b$, which is never optimal. For $p > u_c$, the seller would be posting a price outside of the $R_2$ region, which is also never optimal. Hence, the profit of this seller is

$$S_2^*(\hat{q}) = B(\hat{q}) + \max_{p \in [\hat{q} - u_b, u_c]} C(p). \tag{46a}$$

Next, consider a seller with a basket price of $q_0$. This seller may post any price $p$ for the cheaper good between $q_0 - u_b$ and $u_c$. Hence, the profit of this seller is

$$S_2^*(q_0) = B(q_0) + \max_{p \in [q_0 - u_b, u_c]} C(p). \tag{47}$$

Since $S_2^*(q_0) \geq S_2^*(\hat{q})$ and $B(q_0) < B(\hat{q})$, it must be that $\max_{p \in [q_0 - u_b, u_c]} C(p)$ is strictly greater than $\max_{p \in [\hat{q} - u_b, u_c]} C(p)$. In turn, this implies that $\max_{p \in [q_0 - u_b, \hat{q} - u_b)} C(p)$ is strictly greater than $\max_{p \in [\hat{q} - u_b, u_c]} C(p)$. Since $\hat{q}$ was chosen arbitrarily in the interval $(q_0, q_1)$, $C(q_0 - u_b)$ is strictly greater than $C(p)$ for all $p \in (q_0 - u_b, u_c]$.

A seller with a basket price of $q \leq q_0$ does not find it optimal to choose a price $p \in (q_0 - u_b, q_1 - u_b]$ for the cheaper good, as $C(q_0 - u_b) > C(p)$ for all $p \in (q_0 - u_b, q_1 - u_b]$. A seller with a basket price of $q \geq q_1$ does not find it optimal to post a price $p \in (q_0 - u_b, q_1 - u_b]$ for the cheaper good, as this would imply its prices are outside the $R_2$ region. Finally, there are no sellers with a basket price of $q \in (q_0, q_1)$, as $G(q_0) = G(q_1)$. These observations imply that the distribution $F(p)$ has a gap between $q_0 - u_b$ and $q_1 - u_b$. In turn, this implies that the function $C(p)$ is strictly increasing in $p$ for all $p \in [q_0 - u_b, q_1 - u_b]$, which contradicts $C(q_0 - u_b) > C(p)$ for all $p \in (q_0 - u_b, u_c]$.

Second, we show that the distribution $F(p)$ has full support over the interval $[p_\ell, u_c]$. On the way to a contradiction, suppose that $F(p)$ is such that $F'(p_0) > 0$ and $F(p_0) = F(p_1)$ for some $p_0$ and $p_1$ such that $p_\ell < p_0 < p_1 \leq u_c$. Since $F(p) = F(p_0)$ for all $p \in [p_0, p_1]$, the function $C(p)$ is strictly increasing in $p$ for all $p \in [p_0, p_1]$. By continuity of $C(p)$, there is an interval $[p_{-1}, p_0]$ such that $C(p) < C(p_1)$ for all $p \in [p_{-1}, p_0]$. A seller with a basket prices of $q \leq u_b + p_0$ does not find it optimal to post a price $p \in [p_{-1}, p_0]$ for the cheaper good, as $C(p_1) > C(p)$ for all $p \in [p_{-1}, p_0]$. A seller with a basket price of $q > u_b + p_0$ does not find it optimal to post a price $p \in [p_{-1}, p_0]$ for the cheaper good, as this would imply that its prices are outside the $R_2$ region. Therefore, there are no sellers posting $p \in [p_{-1}, p_0]$, which contradicts $F'(p_0) > 0$.

Finally, we show that $C(p)$ is constant for all $p \in [p_\ell, u_c]$. First, suppose that $C(p)$ is strictly increasing over some interval $(p_0, p_1) \subset [p_\ell, u_c]$. If that is the case, a seller with a basket price of $q \leq u_b + p_1$ does not find it optimal to post a price $p \in (p_0, p_1)$ for the cheaper good, as it can post the price $p_1$ instead and attain a higher profit. Similarly, a seller with a basket price of $q > u_b + p_1$ cannot post a price $p \in (p_0, p_1)$, as this would imply that its prices are outside of the $R_2$ region. Hence the distribution $F$ has a gap between $p_0$ and $p_1$. However, we have established that the distribution $F$ has full support over the interval $[p_\ell, u_c]$. Therefore, $C(p)$ must be weakly decreasing for all $p \in [p_\ell, u_c]$.

Now, suppose that $C(p)$ is strictly decreasing over the interval $p \in [p_\ell, u_c]$. In this case, a seller with a basket price $q \in [q, u_b + u_c]$ chooses the lowest possible price $p$ for the cheaper good, i.e. $u_b + p$. Hence, $F(p) = G(u_b + p)/2$ for all $p \in [\underline{p}, u_c]$. Moreover $F(p)$ is such that $B(u_b + p) + C(p) = S^*$ for all $p \in [p_\ell, u_c]$. After solving this equal profit condition with respect to $F(p)$, we find that $C(p)$ is strictly increasing in $p$ over the interval $[p_\ell, u_c]$, which contradicts the assumption that $C(p)$ is strictly decreasing. The same argument can be applied to rule out the case in which $C(p)$ is strictly decreasing over some interval $(p_0, p_1) \subset [p_\ell, u_c]$. Therefore, $C(p)$ must be weakly increasing for all $p \in [p_\ell, u_c]$. Since $C(p)$ is both weakly decreasing and weakly increasing, it must be constant for all $p \in [p_\ell, u_c]$. In turn, this implies that $B(q)$ must be constant for all $q \in [q_\ell, u_b + u_c]$. ■

## B.6 Proof of Proposition 2

We established part (i) in the main text. Here we prove part (ii). To this aim, we need to show that there exists a joint distribution $H$ that generates the marginals $G$ and $F$ specified in part (i), and such that, on every point on the support of $H$, the profit of the seller is maximized.

We begin the analysis by identifying the region where the profit of the seller are maximized. In Lemma 1, we proved that a seller never finds it optimal to post prices $(p_1, p_2)$ with either $p_1$, $p_2$ or both $p_1$ and $p_2$ strictly greater than $u_b$. It is also straightforward to show that a seller never finds it optimal to post prices $(p_1, p_2)$ with either $p_1$, $p_2$ or both strictly smaller than $p_\ell$. Therefore, we only need to check the seller's profit associated to prices $(p_1, p_2) \in [p_\ell, u_b] \times [p_\ell, u_b]$.

First, we compute the seller's profit for prices $(p_1, p_2)$ with $p_1 + p_2 \in (u_b + u_c, q_h]$. If the seller posts prices $(p_1 + p_2)$ with $p_1 + p_2 \in [q^*, q_h]$, it attains a profit of $S^*$, as guaranteed by the construction of $G$. If the seller posts prices $(p_1, p_2)$ with $p_1 + p_2 \in (u_b + u_c, q^*)$, it attains a profit of

$$\begin{aligned} S_1(p_1 + p_2) &= \mu_b \{\alpha + 2(1-\alpha)[1 - G(p_1 + p_2)]\} (p_1 + p_2) \\ &< \mu_b \{\alpha + 2(1-\alpha)[1 - G(q^*)]\} q^* = S^*, \end{aligned} \tag{48}$$

where the second line uses the fact that $G(p_1 + p_2) = G(q^*)$ for $p_1 + p_2 \in (u_b + u_c, q^*)$.

Second, we compute the seller's profit for prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, u_b + u_c]$. In Lemma 2, we showed that the seller never finds it optimal to post prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, u_b + u_c]$, $p_1 \in (u_c, u_b]$ and $p_2 \in (u_c, u_b]$. If the seller posts prices $(p_1, p_2)$ with $p_1 + p_2 \in (q_\ell, u_b + u_c]$, $p_1 \in (u_c, u_b]$ and $p_2 \in [\underline{p}, u_c]$, it attains a profit of $S^*$, as guaranteed by Lemma 6. Similarly, if the seller posts prices $(p_1, p_2)$ with $p_1+p_2 \in (q_\ell, u_b+u_c]$, $p_1 \in (u_c, u_b]$ and $p_2 \in (u_c, u_b]$, it attains a profit of $S^*$. If the seller posts prices $(p_1, p_2)$ with $p_1 + p_2 \in (2u_c, q_\ell)$, $p_1 \in [p_\ell, u_b]$ and $p_2 \in (u_c, u_b]$, it attains a profit of

$$\begin{aligned} S_2(p_1 + p_2, p_1) &= \mu_b \{\alpha + 2(1-\alpha)\} (p_1 + p_2) + \mu_c \{\alpha + 2(1-\alpha)[1 - F(p_1)]\} p_1 \\ &\leq \mu_b \{\alpha + 2(1-\alpha)\} q_\ell + \mu_c \{\alpha + 2(1-\alpha)[1 - F(p_1)]\} p_1 = S^*. \end{aligned} \tag{49}$$

The first line uses the fact that the seller trades both goods with all of its potential customers of type $b$. The second line uses the fact that the seller would also trade both goods with all of its potential customers of type $b$ at the basket price $q_\ell$, and the last line uses Lemma 6. Similarly, $S_2(p_1+p_2, p_2) \leq S^*$ for all $(p_1, p_2)$ with $p_1+p_2 \in (2u_c, q_\ell)$, $p_1 \in (u_c, u_b]$, $p_2 \in [\underline{p}, u_c]$.

Third, we compute the seller's profit for prices $(p_1, p_2) \in [p_\ell, u_c] \times [p_\ell, u_c]$. If the seller posts such prices, it attains a profit of

$$\begin{aligned} S(p_1, p_2) &= \mu_b \{\alpha + 2(1-\alpha)\} (p_1 + p_2) + \textstyle\sum_{i=1}^{2} \mu_c \{\alpha + 2(1-\alpha)[1 - F(p_i)]\} p_i \\ &\leq \mu_b \{\alpha + 2(1-\alpha)\} 2u_c + \mu_c \{\alpha + 2(1-\alpha)[1 - F(u_c)]\} 2u_c \\ &= S(u_c, u_c). \end{aligned} \tag{50}$$

The first line uses the fact that the seller trades both goods to all its potential customers of type $b$, it trades good 1 to $\mu_c \{\alpha + 2(1-\alpha)[1 - F(p_1)]\}$ buyers of type $c$, and it trades good 2 to $\mu_c \{\alpha + 2(1-\alpha)[1 - F(p_2)]\}$ buyers of type $c$. The second line uses the fact that the seller would also trade both goods to all its potential customers of type $b$ at the prices $(u_c, u_c)$, and that the profit that the seller makes off of buyers of type $c$ by posting the price $p_i \in [p_\ell, u_c]$ for good $i = \{1, 2\}$ is the same it would make by posting the price $u_c$ instead.

If and only if $S(u_c, u_c) \leq S^*$, the highest profit that the seller can attain is $S^*$. Using the fact that $S_2(u_b + u_c, u_c) = S^*$, we can write the condition $S(u_c, u_c) \leq S^*$ as

$$\begin{aligned} &\mu_b \{\alpha + 2(1-\alpha)\} 2u_c + \mu_c \{\alpha + 2(1-\alpha)[1 - G(q^*)/2]\} 2u_c \\ &\leq \mu_b \{\alpha + 2(1-\alpha)[1 - G(q^*)]\} (u_b + u_c) + \mu_c \{\alpha + 2(1-\alpha)[1 - G(q^*)/2]\} u_c. \end{aligned} \tag{51}$$

After substituting out $G(q^*)$, we can write the inequality above as (30).

The functions $G$ and $F$ are proper distribution functions if and only if

$$\frac{\mu_c}{\mu_b} > \frac{3\alpha - 2}{(2-\alpha)u_c/u_b} - 1, \tag{52}$$

and

$$\frac{\mu_c}{\mu_b} < \frac{\alpha(1 - u_c/u_b)}{u_c/u_b}. \tag{53a}$$

Condition (52) is necessary and sufficient for $G(q^*) > 0$, and it is condition (29). Condition (53a) is necessary and sufficient for $q^* < 2u_b$, and it holds whenever condition (30) is satisfied. If and only if (52) and (53a) are satisfied, $G$ and $F$ are proper distribution functions. That is, the interval $[q_\ell, u_b + u_c]$ is non-empty and, over this interval, $G(q)$ is strictly increasing in $q$, and such that $G(q_\ell) = 0$ and $G(u_b + u_c) = G(q^*)$, where $G(q^*) \in (0, 1)$. The interval $[q^*, q_h]$ is non-empty and, over this interval, $G(q)$ is strictly increasing in $q$, and such that $G(q^*) = G(u_b + u_c)$ and $G(q_h) = 1$. Similarly, the interval $[p_\ell, u_c]$ is non-empty and, over this interval, $F(p)$ is strictly increasing in $p$ and such that $F(p_\ell) = 0$ and $F(u_c) = G(q^*)/2 \in (0, 1)$.

In the main text, we established that there exists a joint distribution $H$ that generates the marginal $F$ for $p \in [p_\ell, u_c]$ and the marginal $G$ for $q \in [q_\ell, u_b + u_c]$ and that has support over the region of prices $(p_1, p_2)$ such that $p_1 + p_2 \in [q_\ell, u_b + u_c]$, and $p_1 \in [p_\ell, u_c]$, $p_2 \in (u_c, u_b]$ or $p_1 \in (u_c, u_b]$, $p_2 \in [p_\ell, u_c]$. Over this region, the seller's profit is $S^*$. Moreover, we established that there exists a joint distribution $H$ that generates the marginal $G$ for $q \in [q^*, q_h]$ and that has support over the region of prices $(p_1, p_2)$ such that $p_1 + p_2 \in [q^*, q_h]$ and $p_1 = p_2$. Over this region, the seller's profit is $S^*$. ■