Preliminary Examination

Econ 702-Macroeconomics
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Instructions: There is one question, divided in several subquestions. The number in brackets represents the number of points awarded for answering correctly the each subquestion. Total available points are 120. If the description of the environment seems incomplete to you, explain why, make the assumptions that you deem necessary to proceed and continue. Good Luck!

Neutrality of a One-Time Monetary Injection? Consider a version of the OLG model in which population grows over time. Specifically, generation t has measure $G_t = \gamma^t$, with $\gamma \geq 1$ and $G_0 = 1$, and endowment $(e_{1t}, e_{2t}) = (e_1, e_2)$, with $e_1 > 0$ and $e_2 > 0$. Each member of generation t has preferences described by the utility function $u(c_{t1}, c_{t2}) = \log c_{t1} + \log c_{t2}$. There is a quantity of fiat money M, allocated to the old of generation 0.

A. (35) **No monetary injection**. Here, we want to solve for the monetary equilibrium when the quantity of money remains constant at M forever.

1. (5) Define a Recursive Competitive Equilibrium.

2. (5) Consider a monetary RCE, i.e. an equilibrium in which the price of money q_t is always strictly positive. State the problem of the household of generation t. Derive an Eurler Equation for the household's consumption and a no-arbitrage condition relating the real interest rate R_t to the growth rate of the price of money q_{t+1}/q_t.

3. (5) Using the market clearing condition for money and consumption and the household's budget constraint, show that savings s_t are zero and solve for consumption c_{t1} and c_{t2} as functions of q_t and q_{t+1}.

4. (5) Using the findings in (3) and the definition of real balances $z_t = q_t M/\gamma^t$, write the household's Euler Equation as a difference equation for z_t and z_{t+1}.

5. (10) Express the Euler Equation as a function $z_{t+1} = f(z_t)$. Plot $f(z_t)$ together with the 45 degree line. Find conditions under which there exists a stationary monetary equilibrium (i.e. an equilibrium with z_t constant) and solve for the value z_S of real balances in such an equilibrium.

6. (5) Solve for real balances z_t, price of money q_t, price of goods $1/q_t$ and consumption $c_{t,1}$, $c_{t-1,2}$ in the stationary monetary equilibrium.

B. (20) **Unannounced monetary injection**. Now, we want to study the effect of an unannounced monetary injection at date T. The size of the monetary injection is $\Delta M = (\alpha - 1)M$, with $\alpha > 1$. The monetary injection is implemented as a lump-sum transfer to the old of generation $T-1$. The monetary injection is unanticipated by the households of generations $t = 0, 1, ...T-1$, i.e. they expect the quantity of money to remain constant and they expect the economy to remain in the stationary monetary equilibrium.

1. (2.5) Write down the Euler Equation for households of generation $t \geq T$.

2. (5) Using market clearing in the money and goods market and the definition of real balances $z_t = q_t \alpha M/\gamma^t$, express the Euler Equation as a function $z_{t+1} = f_+(z_t)$.

3. (2.5) Plot $f_+(z_t)$ together with the 45 degree line. Solve for the value z_S^+ of real balances in the stationary monetary equilibrium.

4. (5) Assuming $z_t = z_S^+$ for all $t \geq T$, compute real balances, price of money, price of goods and consumption at dates $t \geq T$. Compare these objects with their counterpart at dates $t \geq T$ in the economy without intervention (part A.6). Explain your findings.

5. (5) Are the equilibrium outcomes at dates $t = 1, ...T-1$ in the economy with an unannounced monetary injection different from the equilibrium outcomes at dates $t = 1, ...T-1$ in the economy without intervention (part A.6)? Explain your findings.

C. (65) **Announced monetary injection**. Lastly, we want to study the effect of a monetary injection that takes place at date T and is announced at date 0. The size of the monetary injection is $\Delta M = (\alpha - 1)M$, with $\alpha > 1$. The monetary injection is implemented as a lump-sum transfer to the old of generation $T-1$.

1. (2.5) Solve for the Euler Equation of households of generation $t = 1, 2, ...T-1$.

2. (5) Using market clearing in the money and goods market and the definition of real balances $z_t = q_t M/\gamma^t$, express the Euler Equation as a function $z_{t+1} = f_-(z_t)$.

3. (5) Solve for the Euler Equation of households of generation $T-1$.

4. (5) Using market clearing in the money and goods market and the definition of real balances $z_{T-1} = q_{T-1} M/\gamma^{T-1}$ and $z_T = q_T \alpha M/\gamma^T$ express the Euler Equation as a function $z_T = f_0(z_{T-1})$.

5. (2.5) Solve for the Euler Equation of households of generation $t \geq T$.

6. (5) Using market clearing in the money and goods market and the definition of real balances $z_t = q_t \alpha M/\gamma^t$, express the Euler Equation as a function $z_{t+1} = f_+(z_t)$.

7. (5) Plot $f_+(z_t)$ and find the value z_S^+ of real balances in a stationary monetary equilibrium.

8. (10) Plot $f_0(z_t)$ and $f_-(z_T)$. Assuming $z_t = z_S^+$ for all $t \geq T$, use this plot to trace back the time-series of real balances z_{T-1}, z_{T-2},.....

9. (10) How does the path of z_t differ in the case in which the monetary injection is announced (part C7 and C8) and in the case in which the monetary injection is unannounced (part B4 and B5)? Explain your findings.

10. (7.5) How does the path of the price of money q_t differ in the case in which the monetary injection is announced (part C7 and C8) and in the case in which the monetary injection is unannounced (part B4 and B5)? Explain your findings.

11. (7.5) How does the path of consumption differ in the case in which the monetary injection is announced (part C7 and C8) and in the case in which the monetary injection is unannounced (part B4 and B5)? Explain your findings.