**704 Part II**

In the following there are 3 questions for 100 points. Be as BRIEF as you can and good luck.

1. **Lucas trees with Endowments and Search**

This is a representative agent economy where each agent owns shares of trees (of which there is measure one) that produces one banana per period. In addition, agents have an endowment of $z_t < 1$ apricots which follow a Markov chain with transition matrix $\Gamma$. The agent has also one unit of time. Apricots are not traded and have to be picked up with one unit of bananas taken one unit of time. To purchase bananas, the agent have to search and find banana trees with a matching function $\phi(T, H)$ that matches trees $T$, with total amount of household searching time $H$. The agent has preferences given by

$$E\left\{\sum_{t=0}^{\infty}\beta^t\,\frac{c_t^{1-\sigma}}{1-\sigma}\right\}$$

where $c_t = \sqrt{a_t^2 + b_t^2}$ is an aggregate of apricots and bananas.

(a) (10 points) Define equilibria recursively assuming that the tree prices only depend on the current endowment of apricots.

(b) (10 points) Write a formula for an option to buy banana trees in each of the following two periods at price $p_1$.

(c) (8 points) Is it possible that some apricots are left unpicked? Explain.

(d) (7 points) Is there an alternative definition of the aggregate $c_t$ such that apricots are still liked by households, yet some are left to rot? Explain

2. **Skilled and Unskilled Workers in Growth Models**

Consider an economy with identical measures of infinitely lived skilled, unskilled, and useless agents (with total population normalized to one). They do not value leisure and have identical standard preferences over consumption, $\frac{c_t^{1-\sigma}}{1-\sigma}$ and discount the future at rate $\beta$. Production is given by

$$f(A, k, s, z\,u) = Y = A\,[\mu\, z\, s^{\sigma} + (1-\mu)(\lambda k^{\rho} + (1-\lambda)u^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1}{\sigma}}$$

where $s$ is labor of skilled workers, $u$ of unskilled workers, $k$ is capital, and the greek letters are parameters of which $\lambda$ and $\mu$ are between zero and one and $A$ and $z$ are independent Markov chains with transitions $\Gamma^A$ and $\Gamma^z$ respectively.

Closed form solutions help but are not necessary.

(a) (10 points) Solve for the steady state where the shocks are set at their unconditional means $\overline{A}$ and $\overline{z}$ and the useless agents own all the capital, while the others only have human wealth.

(b) (5 points) Compare such a steady state with the steady state of another economy (call it $B$) that differs only on the fact that the this economy $B$ has $z_B = 2\overline{z}$. Is it possible unskilled workers are worse off in economy $B$? Discuss.

(c) (5 points) Write down the equal weight social planner's problem recursively. Characterize the evolution of consumption of the three types.

(d) (10 points) Define Recursive Competitive Equilibria with complete state contingent markets. Will agents trade actively in these markets?

(e) (5 points) Imagine that all three types of agents start with equal wealth and choose equal consumption. What can you say about the distribution of wealth?

3. **Incomplete Markets and Multiple Jobs**

Imagine that there are is a measure of agents with preferences given by

$$E\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{c_t^{1-\sigma}}{1-\sigma}+\alpha(1-n_t)\right]\right\}$$

Where $1-n$ is leisure and $n$ is time spent working or looking for a job. Job search takes full time, but agents can work less than full time. Jobs are lost with probability $\delta$. Upon that happening the job becomes idle and households can choose to search or not. There are two types of potential jobs in equal measure $A$ and $B$ with $w_A > w_B$. Agents can look for $A$ or $B$ jobs in separate markets each one of them with matching function $\phi(J_i, H_i)$ where $J_i$ is the number of idle jobs of type $i \in \{A, B\}$ and $H_i$ is the measure of job searchers of type $i \in \{A, B\}$.

(a) (20 points) Write the problem of the agents assuming that they cannot quit their jobs and define stationary equilibrium. Make sure that you say something about the equilibrium condition that determines the unemployment rate and the vacancy rate of each of the two jobs.

(b) (10 points) What will change in the problem of the agents if they were able to quit?