Microeconomic Theory II
Preliminary Examination
June 6, 2015

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **(30 points)** A seller has two identical objects (boomerangs) to sell. There are three risk neutral potential buyers; each buyer only wants one boomerang and is indifferent between the two boomerangs (buyer i's valuation for a boomerang is v_i). The seller will use a multi-unit sealed-bid auction to sell the two boomerangs. In the multi-unit auction, the three bidders simultaneously submit bids. The two highest bidders will each receive one boomerang (in the event of a three way tie, two bidders are selected at random).

   (a) In the second-price multi-unit auction, the winning bidders pay the second highest bid (which in this auction, is the *lowest winning* bid). Prove that "bidding your valuation" is not a dominant strategy. **[5 points]**

   (b) In the third-price multi-unit auction, the winning bidders pay the third highest bid (which in this auction, is the *highest losing* bid). Is "bidding your valuation" a dominant strategy? Why or why not? **[5 points]**

   (c) In the first-price multi-unit auction, the winning bidders pay what they bid. Suppose bidder valuations are private information (with v_i known only to bidder i), with the values being determined as independent and identical draws from $[0,1]$ according to the common distribution function F, with density f.

      i. What is the interim payoff function of player i? **[5 points]**

      ii. Suppose $(\sigma_1, \sigma_2, \sigma_3)$ is a Nash equilibrium of the auction, and assume σ_i is a strictly increasing and differentiable function, for $i = 1,2,3$. Suppose moreover, that the equilibrium is symmetric, so that $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$. Describe the differential equation the strategy σ must satisfy. [In your answer, for any $i \neq j$, denote $\Pr(\min\{v_i, v_j\} \leq v)$ by $G(v)$ with density g. You do **not** need to evaluate G or g!] **[15 points]**

[Question 2 is on the next page.]

2. **(30 points)** Suppose players 1 and 2 play an infinite repetition of the prisoners' dilemma (E is effort, S is shirk, or no effort) with perfect monitoring, with stage payoffs:

	E	S
E	$4,4$	$0,6$
S	$6,0$	$2,2$

.

Both players have the same discount factor $\delta \in (0,1)$.

(a) Why is 5 the maximum average discounted payoff that either player could receive in any pure strategy subgame perfect equilibrium for any discount factor? **[5 points]**

(b) What is the lowest discount factor for which grim trigger supports EE in every period as a subgame perfect equilibrium outcome? **[5 points]**

Let δ^* denote the discount factor identified in part 2(b).

(c) Suppose $\delta = \delta^*$. Describe a subgame perfect equilibrium strategy profile that achieves the payoff 5 (from part 2(a)) for player 1. (HINT: Express the associated feasible payoff vector as a convex combination of the payoffs from pure action profiles.) **[10 points]**

Let σ^* denote the strategy profile you describe in part 2(c).

(d) Let $\hat{\sigma}$ denote the strategy profile that specifies EE in the initial period followed by σ^* and after play different from EE in the initial period is followed by permanent SS. What is the lowest discount factor for which $\hat{\sigma}$ is a subgame perfect equilibrium? Provide some intuition for the relative size of this discount factor and δ^*. **[10 points]**

[Question 3 is on the next page.]

3. **(40 points)** A public utility commission is charged with regulating a monopoly provider of electricity. The cost function of the monopoly is given by $C(q,\theta)$ where q is the quantity produced and θ is a cost shifting parameter.

The inverse demand curve for electricity is

$$p(q) \equiv \max\{1-q, 0\}.$$

Supposing there are no income effects for the good, the consumer surplus is given by,

$$V(q) = \int_0^q p(\tilde{q})d\tilde{q} - p(q)q.$$

The regulator determines whether or not the firm should operate, the quantity q the firm should produce as a function of θ, as well as (possibly) a subsidy that will be offered to the firm. The firm cannot be forced to operate—it will shut down if it makes negative profits. The regulator wishes to maximize the total of expected consumer surplus and firm profits less the subsidy.

(a) Suppose $\theta \in [0,1]$ is publicly known to both the monopolist and the commission. Further, suppose the cost function is:

$$C(q,\theta) = \begin{cases} 0 & \text{if } q = 0, \\ K + \theta q & \text{if } q > 0, \end{cases}$$

where q is the quantity produced, $K > 0$ is a publicly known fixed cost and $\theta \in [0,1]$ is the marginal cost.

What is the efficient quantity of electricity produced, and what is the subsidy offered as a function of θ and K. Describe clearly when regulator will ask the firm to operate, and when it will be shut. **[10 points]**

(b) Suppose now, instead, θ is private information, believed by the commission to be a uniform draw from $[0,1]$. Continue to assume that the cost function is as given above.

Write down carefully the regulator's problem, clearly identifying the IC and IR constraints. Do not solve the problem. **[10 points]**

(c) Now suppose instead that K is a sunk rather than a fixed cost, i.e. the monopoly must decide whether to invest in the plant *before* she learns her cost θ. Continue to assume that θ is a uniform draw from $[0,1]$, and is privately observed by the firm.

The regulator commits to a quantity the firm should produce as a function of its private information θ, and a subsidy that will be offered to the firm. The firm's operation decision is now made *ex-ante*, i.e. she must choose whether to accept the regulator's plan *before* she learns her type θ.

Again, the regulator wishes to maximize the total of expected consumer surplus and firm profits less the subsidy.

Write down carefully the regulator's problem, clearly identifying the IC and IR constraints in this case. Solve for the regulator's optimal plan. Give conditions on K under which the regulator shuts down the firm. **[20 points]**

4. **(20 points)** A seller owns one unit of a good, which she values at $c \in \{\underline{c}, \bar{c}\}$, $\underline{c} < \bar{c}$. The seller's valuation can be either with equal probability, and is privately known to the seller. A buyer may buy the unit from the seller. The buyer's valuation for the good is $\bar{v}$ if $c = \bar{c}$ and $\underline{v}$ if $c = \underline{c}$, where $\bar{v} > \bar{c}$ and $\underline{v} > \underline{c}$. The buyer thus has no private information.

   (a) Show that efficiency is consistent with the seller's and buyer's individual rationality and incentive compatibility if and only if $\bar{c} \leq \frac{1}{2}(\bar{v} + \underline{v})$. **[15 points]**

   (b) Precisely describe the bargaining environment of Myerson-Satterthwaite, and state their Impossibility Theorem. Enumerate the assumptions made in that setting which do not apply here and why. **[5 points]**