Microeconomic Theory II
Preliminary Examination
August 7, 2017

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. Sheila moves first and chooses either H or L. Bruce receives a signal, h or l, about Sheila's behavior. The distribution of the signal is $\Pr(h|H) = p$, $\Pr(l|H) = 1 - p$, $\Pr(h|L) = q$, $\Pr(l|L) = 1 - q$, with $p > \frac{1}{2} > q$. After observing the signal, Bruce chooses either A or B. Payoffs are:

Action profile	Sheila's payoff	Bruce's payoff
HA	5	2
HB	2	1
LA	6	1
LB	4	2

   (a) Suppose $p = 1$ and $q = 0$. Note that for these values of p and q, the signals are *perfectly* informative about Sheila's actions (i.e., this is equivalent to Burce observing Sheila's actions). What is the normal form of this game? What are the pure strategy Nash equilibria? **[10 points]**

   (b) Again for the parameter values $p = 1$ and $q = 0$, what is the result of deleting weakly dominated strategies? Describe the extensive form game of perfect information that has the normal form obtained in part (a), and apply backward induction. **[5 points]**

   (c) Suppose $p, q \in (0, 1)$ (that is, $0 < p, q < 1$). What is the normal form of this game? What are the pure strategy Nash equilibria? What happens to the pure strategy equilibria as $p \to 1$ and $q \to 0$? Explain why backward induction cannot be applied to the game when both p and q are strictly between 0 and 1. **[15 points]**

[Question 2 is on the next page.]

2. A firm with a *single* vacancy is considering hiring one of two identically productive workers. Suppose the value of the output produced by either worker is s.

   (a) Suppose that in the first period, the firm chooses a worker to bargain with, and then play proceeds as an alternating offer bargaining game with the firm making the initial offer (the other worker is excluded from the negotiations, and so receives a payoff of 0). The workers and firm discount the future with possibly different discount factors, δ_1, δ_2, and $\delta_F \in (0,1)$. Assume $\delta_1 > \delta_2$. Describe the subgame perfect equilibria of the game. **[20 points]**

   (b) Suppose now that the government, concerned that a firm can exclude a worker by simply choosing to negotiate with the other worker, passes a law that gives the workers "equal access" to the firm as follows: Each worker i simultaneously announces an opening offer of a wage $w_i \leq s$. The firm, on the basis of the offered wages, can decide to accept one of the offers, in which case that worker receives the wage, the firm receives the remaining surplus, and the other worker receives zero. Only if the firm rejects both offers, can the firm then chose a worker to bargain with (the other worker is then permanently excluded from further negotiations, and receives 0). The game now proceeds (as in part (a)) to an alternating offer bargaining game with the firm making the initial offer in the period after the rejection. Describe the subgame perfect equilibria of the game. Do the workers benefit from this law? **[10 points]**

3. An entrepreneur is contemplating selling all or part of his startup to outside investors. The profits from the startup are risky and the entrepreneur is risk averse. The entrepreneur's preferences over $x \in [0,1]$, the fraction of the startup the entrepreneur retains, and p, the price "per share" paid by the outside investors, are given by

$$u(x,\theta,p) = \theta x - x^2 + p(1-x),$$

where $\theta > 1$ is the value of the startup (i.e., expected profits). The quadratic term reflects the entrepreneur 's risk aversion. The outside investors are risk neutral, and so the payoff to an outside investor of paying p per share for $1-x$ of the startup is then

$$\theta(1-x) - p(1-x).$$

There are at least two outside investors, and the price is determined by a first price sealed bid auction: The entrepreneur first chooses the fraction of the firm to sell, $1-x$; the outside investors then bid, with the $1-x$ fraction going to the highest bidder (ties are broken with a coin flip). **Important convention:** The outside investors submit bids in "price per share" p, so the amount paid is $p(1-x)$.

   (a) Suppose θ is public information. What fraction of the startup will the entrepreneur sell, and how much will he receive for it? **[5 points]**

   (b) Suppose now θ is privately known to the entrepreneur. The outside investors have common beliefs, assigning probability $\alpha \in (0,1)$ to $\theta = \theta_1 > 1$ and probability $1-\alpha$ to $\theta = \theta_2 > \theta_1$. Suppose $\theta_2 - \theta_1 > 2$. Characterize the separating perfect Bayesian equilibria. Are there any other perfect Bayesian equilibria? **[15 points]**

   (c) Maintaining the assumption that θ is privately known to the entrepreneur, suppose now that the outside investors' beliefs over θ have support $[\theta_1, \theta_2]$, so that there is a continuum of possible values for θ. What is the initial value problem (differential equation plus initial condition) characterizing separating perfect Bayesian equilibria? **DO NOT ATTEMPT TO SOLVE IT.** **[10 points]**

[Question 4 is on the next page.]

4. (A mechanism design perspective on startup funding.) As in Question 3, there is an entrepreneur with a start-up who would like to obtain external funding. Unlike the previous question, though, we now consider a monopoly provider of investment funds (call it the bank). The bank proposes a mechanism to the entrepreneur to determine the terms of any funding (p, x), $x \in [0,1]$, provided.

   (a) We begin, as in Question 3(a) by assuming the parameter $\theta > 1$ is public information. What is the optimal take-it-or-leave-it offer from the bank? **[5 points]**

   As in Question 3(b), we now assume the parameter θ is privately known to the entrepreneur, and so not known by the bank. The bank assigns probability $\alpha \in (0,1)$ to $\theta = \theta_1 > 1$ and probability $1-\alpha$ to $\theta = \theta_2 > \theta_1$. Suppose $\theta_2 - \theta_1 > 2$.

   (b) State the revelation principle and explain its importance in the current setting. **[10 points]**

   (c) Describe the optimization problem the bank must solve in order to determine its optimal direct mechanism (in other words, state the objective function and all the constraints) **[5 points]**

   (d) Prove that in any direct mechanism satisfying the constraints from part (c), the share retained by θ_2 is at least as large as that retained by θ_1. **[5 points]**

   (e) One of the constraints in part (c) is redundant. Which one and why? **[5 points]**