Microeconomic Theory II
Preliminary Examination
August 8, 2016

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **(30 points)** Suppose the government (through its agency, the Internal Revenue Service—IRS) wants to ensure compliance with the tax code. A taxpayer, in filling out a tax return, has a choice, be truthful (T) or lie (L). The IRS can audit the return to determine whether the taxpayer was truthful on his return(A), or can decide not to audit (N) . The taxpayer's payoffs are given by $u_t(T,A)=2$, $u_t(T,N)=3$, $u_t(L,A)=-2$, and $u_t(L,N)=4$. The government's payoffs are given by $u_g(T,A)=-1$, $u_g(T,N)=1$, $u_g(L,A)=0$, and $u_g(L,N)=-2$. (These payoffs capture the idea that the taxpayer has the natural ranking $LN \succ TN \succ TA \succ LA$, while the IRS has the natural ranking $TN \succ LA \succ TA \succ LN$.)

   Suppose the taxpayer does not know if he will be audited when filling out the return, and the IRS cannot condition the audit decision on specifics in the return. In other words, the taxpayer and IRS are effectively making their decisions simultaneously.

   (a) What is the normal form of this game? **[5 points]**

   (b) What are the Nash equilibria of this game? **[5 points]**

   Suppose now the IRS can commit to a probability of auditing the return *before* the taxpayer fills out his return, and that the taxpayer *observes* this probability. (This commitment may involve the early hiring of IRS auditors, for example.)

   (c) What is the normal form of this game? **[5 points]**

   (d) What is the backward induction solution of this game? Is it unique? **[10 points]**

   (e) Will an announcement by the IRS of intended auditing probabilities serve the same function as the commitment? **[5 points]**

[Question 2 is on the next page.]

2. **(30 points)** Consider the following infinitely repeated game with three players. In each period, player 1 is simultaneously playing a prisoners' dilemma with player 2 and another prisoners' dilemma with player 3 (so that within the stage game, all players simultaneously choose actions). The stage games are given by

player 2

player 1		E_2	S_2
	E_1	$3,3$	$-1,4$
	S_1	$4,-1$	$0,0$

player 3

player 1		E_3	S_3
	E_1	$2,2$	$-1,4$
	S_1	$4,-2$	$0,0$

(Note that the two games are *not* the same!) All players have a common discount factor $\delta \in (0,1)$. Player 1's payoffs are the *sum* of payoffs in the two games.

Suppose there is a technological restriction that forces player 1 to choose the same action in the two different games (i.e., player 1 plays E_1 against player 2 if and only if he does so against player 3).

(a) Suppose there is perfect monitoring of all players' past actions. Describe the "grim-trigger" strategy profile that induces the outcome path in which $E_1E_2E_3$ is played in every period, and which is subgame perfect for large δ. What is the smallest value of δ for which the profile is a subgame perfect equilibrium? [Remember to provide support for your answer.] **[5 points]**

Suppose now that while player 1 observes the past actions of players 2 and 3, players 2 and 3 only observe the past actions of player 1. Players 2 and 3 do not observe each others past actions. Grim trigger is now: player 1 plays E_1 in the first period and then as long as $E_1E_2E_3$ always played, otherwise play S_1; player i plays E_i in the first period and then as long as i observes only E_1E_i, otherwise play S_i, $i = 2,3$.

(b) For what values of δ is the grim trigger profile just described a Nash equilibrium? Why is every Nash equilibrium of this game subgame perfect? **[5 points]**

(c) Suppose δ is such that the grim trigger profile is a Nash equilibrium.

i. Give an intuitive description of the specific restrictions that sequential rationality imposes on the grim trigger profile.. **[5 points]**

ii. Prove that the grim trigger profile is not sequentially rational for large δ by showing that one of the players has a profitable one shot deviation. **[5 points]**

iii. For what values of δ is the profile sequentially rational? **[5 points]**

(d) Suppose now there is no technological restriction: player 1 is free to choose different actions in the two prisoners' dilemmas. Let a_1^j denote the action a_1 played by player 1 in the game with player j, $j = 2,3$. Maintain the observability assumption introduced just before part 2(b). Describe a grim trigger like subgame perfect strategy profile that supports $E_1^2E_1^3E_2E_3$ in every period for large δ. Explain why your answer differs from your analysis in part 2(c). **[5 points]**

3. **(30 points)** Suppose that the payoff to a firm from hiring a worker of type θ with education e at wage w is

$$f(e,\theta) - w = 3e\theta - w.$$

The utility of a worker of type θ with education e receiving a wage w is

$$w - c(e,\theta) = w - \frac{e^3}{\theta}.$$

The worker's ability is privately known by the worker. There are at least two firms. The worker (knowing his ability) first chooses an education level $e \in \mathbb{R}_+$; firms then compete for the worker by simultaneously announcing a wage; finally the worker chooses a firm. Treat the wage determination as in class, a function $w : \mathbb{R}_+ \to \mathbb{R}_+$ determining wage as a function of eduction.

Suppose the support of the firms' prior beliefs ρ on θ is $\Theta = \{\theta_L, \theta_H\}$ where $\theta_L = 1$ and $\theta_H = 3$.

(a) What is the full information efficient education level for each type of worker? **[5 points]**

(b) Is there a perfect Bayesian equilibrium in which both types of worker choose their full information eduction level? Be sure to verify that all the incentive constraints are satisfied. **[5 points]**

(c) Suppose e_L is the education level undertaken by θ_L, while e_H is the education level taken by θ_H in a separating Perfect Bayesian Equilibrium. What can you conclude about e_H and e_L? Be as precise as possible. **[10 points]**

(d) Suppose the firms' prior beliefs ρ are that the worker has type θ_H with probability $\frac{2}{3}$, and type θ_L with probability $\frac{1}{3}$. Is there a pooling Perfect Bayesian equilibrium in this setting? If yes, describe a pooling PBE and argue that it is one. If not, why not? **[10 points]**

[Question 4 is on the next page.]

4. **(30 points)** Mussa Ltd is faced with a single buyer, Mr. Rosen. It is known that Mr. Rosen has constant marginal utility θ for its product. In particular, if Mr. Rosen has a marginal utility of θ, buys q units of the product, and pays p, his net utility is:

$$u(q,p,\theta) = \theta q - p.$$

Mr. Rosen's outside option is normalized to 0.

Mussa Ltd has a cost function $c(q) = \frac{1}{2}cq^2$, so its total profit if it sells q units for p is:

$$\pi(p,q) = p - \frac{1}{2}cq^2.$$

(a) Suppose Mussa Ltd knows Mr. Rosen's marginal utility θ. Describe, as a function of θ, the quantity $q(\theta)$ and price $p(\theta)$ it offers Mr. Rosen to maximize its profit. **[5 points]**

(b) Suppose now that Mussa Ltd does not know Mr. Rosen's marginal utility, and instead believes it to be drawn from the uniform distribution over $[0,1]$.

Due to complexity issues, Mussa Ltd offers a menu of 2 bundles: a bundle (q_1, p_1) intended for Mr. Rosen if his type $\theta \in [\theta_1, \theta_2)$, and a bundle (q_2, p_2) if his type is in $[\theta_2, 1]$, where $0 < \theta_1 < \theta_2 < 1$. The values of θ_1 and θ_2 are fixed (e.g., provided by the marketing department); you only need to supply the optimal bundles given the values of θ_1 and θ_2.

Describe carefully Mussa Ltd's expected profit maximization problem (i.e., what bundles (q_1, p_1) and (q_2, p_2) to offer) when he would like $[\theta_1, \theta_2)$ to purchase bundle 1, $[\theta_2, 1]$ to purchase bundle 2, and $[0, \theta_1)$ to purchase nothing. (**Hint:** the relevant IC/IR constraints here are that the intended bundle purchase should be optimal for those types among the available bundles and the option to purchase nothing.)

Solve for the profit maximizing bundle given θ_1, θ_2. **REMEMBER θ_1 AND θ_2 ARE FIXED.** You may assume that $\theta_1 + \theta_2 > 1$. **[25 points]**