Department of Economics
University of Pennsylvania

Preliminary Examination
Microeconomic Theory I

Exam Date: August 7, 2017

Instructions

This exam has four questions and is worth 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly.

Use WORDS – NOT math only!

Good luck!

1. (25 pts) Suppose a locally nonsatiated utility function $u : \mathbb{R}^n_+ \to \mathbb{R}$ gives rise to a demand function $\mathbf{x}(\mathbf{p}, y) = (x_1(\mathbf{p}, y), \ldots, x_n(\mathbf{p}, y))$ defined on $\mathbb{R}^{n+1}_{++}$ ($\mathbf{p}$ is the price vector, y is income). Assume it and any other functions you use to answer this question are twice continuously differentiable.

   (a) (8 pts) State four properties this demand function must satisfy.

   (b) (17 pts) For each property you listed in (a), sketch a proof of why it must be satisfied.

2. (25 pts) Mr. 1 has a complete and transitive preference ordering $\succeq_1$ over monetary lotteries that is monotone in the following sense: $\delta_x \succ_1 \delta_y$ for all $x > y$, where δ_x and δ_y are the degenerate lotteries that put probability one on the amounts x and y, respectively. Assume Mr. 1 is strictly risk averse, and let $\tilde{x}$ be a given non-degenerate monetary lottery.

   (a) (5 pts) Let c_1 be Mr. 1's certainty equivalent for $\tilde{x}$. What is the relationship between c_1 and $\mathbb{E}\tilde{x}$? Prove your answer.

   Now assume Mr. 1 satisfies the Expected Utility Hypothesis, and his Bernoulli utility function is u_1. Ms. 2 similarly has a Bernoulli utility function u_2. Both functions are twice differentiable, with $u_1'(x) > 0$ and $u_2'(x) > 0$ for all $x \in \mathbb{R}$. Let $A_i(x)$ denote the coefficient of absolute risk aversion of u_i, and assume $A_1(x) > A_2(x)$ for all $x \in \mathbb{R}$.

   (b) (10 pts) Show that there exists a strictly concave increasing function $h : u_2(\mathbb{R}) \to \mathbb{R}$ such that $u_1 = h \circ u_2$.

   (c) (10 pts) Show that $c_1 < c_2$, where c_1 and c_2 are their certainty equivalents for $\tilde{x}$.

3. (25 pts) Three farmers, $i = 1, 2, 3$, grow corn along a river that is subject to flooding and is protected by a dyke that protects the adjacent land from flooding when the river gets too high. In the absence of flooding, each farmer's crop will be 100 units of corn. There has been very heavy rain and it is known that the dyke will be breached tomorrow (date $t = 1$), flooding exactly one of the farms and ruining its crop. There are thus three states of the world tomorrow, $s = 1, 2, 3$: in state s the farm of farmer $i = s$ is flooded. For each farmer $i = 1, 2, 3$, let ω^i denote his initial endowment vector of state-contingent crop, so that

$$\omega^1 = (0, 100, 100), \quad \omega^2 = (100, 0, 100), \quad \omega^3 = (100.100, 0).$$

   Today (date $t = 0$) the farmers arrange for how the the corn that is harvested tomorrow (date $t = 1$) will be shared in each state. The utility function of farmer i is

$$U^i(x^i) = \Sigma_{s=1}^3 \pi_s^i u^i(x_s^i),$$

   where x_s^i is his consumption of corn in state s, and π_s^i is his belief probability that state s will occur. Assume u^i is continuously differentiable, strictly concave, and strictly increasing.

   (a) (8 pts) Suppose the farmers agree that the state probabilities are $(\pi_1, \pi_2, \pi_3) \gg \mathbf{0}$. Show that in any interior Pareto efficient allocation, farmer 1 will consume the same amount of corn regardless of whose farm is flooded.

For the remaining parts, assume instead that each farmer is sure that his farm will not be flooded, and believes that it is equally likely that the other two farms will be flooded: $\pi_i^i = 0$ for each i and $\pi_s^i = 1/2$ for $s \neq i$.

(b) (8 pts) Prove that if (x^{1*}, x^{2*}, x^{3*}) is a Pareto efficient allocation, then $x_i^{i*} = 0$ for all i.

(c) (9 pts) Find a competitive equilibrium price vector (p_1, p_2, p_3), where p_s is the price at date 0 for contingent corn to be consumed at date 1 in state s.

4. (25 points) Walrasian equilibrium with production.

(a) (8 pts) State precisely the definition of a Walrasian equilibrium for an economy with production.

(b) (8 pts) Given standard assumptions on preferences and interior endowments, what conditions on the production technology are sufficient for a Walrasian equilibrium with production to exist? (Little if any credit will be given for trivial conditions such as "the production set is empty".)

(c) (9 pts) Give an example in which one of the conditions on the technology you gave in (b) is not satisfied, and a Walrasian equilibrium nonetheless exists. A graphic example carefully done is sufficient.