Economics 702, Dirk Krueger

1 A Pure Exchange Economy with Household Heterogeneity

Consider a stochastic pure exchange economy where the current state of the economy is described by $s_t \in S = \{s_1, s_2\}$. Event histories are denoted by s^t and the initial node s_0 is fixed. Probabilities of event histories are given by $\pi_t(s^t)$. There are 2 different types of households with equal mass normalized to 1. Households potentially differ in their endowment stream $\{e_t^i(s^t)\}$, their initial asset position a_0^i and their time discount factors $\beta_i \in (0,1)$. Preferences for each household over consumption allocations $c^i = \{c_t^i(s^t)\}$ are given by

$$u^i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} (\beta_i)^t \pi_t(s^t) U(c_t^i(s^t)).$$

where $U(.)$ is strictly increasing and strictly concave.

1. Suppose that $\pi_t(s^t)$ is Markov with transition matrix

   $$\pi(s'|s) = \begin{pmatrix} 1-\nu & \nu \\ \kappa & 1-\kappa \end{pmatrix}$$

   where $\nu, \kappa \in [0,1]$ are parameters. For which parameter combinations (ν, κ) is the associated invariant distribution Π

   (a) Unique?

   (b) Satisfy $\Pi = (0.6, 0.4)$?

2. Define an Arrow-Debreu equilibrium.

3. Households can trade a full set of Arrow securities. Define a recursive competitive equilibrium.

4. Suppose that the aggregate endowment satisfies, for all $t \geq 0$, and all s^t,

   $$e_t(s^t) = (1+g)^t$$

with $g > 0$. Preferences are characterized by $\beta_1 = \beta_2 = \beta$ as well as

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$

where $\sigma = 1$ is understood to correspond to the log-case. The individual endowments satisfy $e_t^i(s^t) > 0$ for all i, t, s^t, but no further assumptions are given, of course apart from

$$\sum_{i=1}^{2} e_t^i(s^t) = e_t(s^t) = (1+g)^t.$$

Characterize as fully as possible the Arrow Debreu equilibrium consumption allocations and the associated prices of Arrow securities in a sequential markets equilibrium.

5. How does the equilibrium risk-free interest rate depend on the growth rate of the economy g. Explain why and how your answer depends on the parameter σ. Note: a complete answer would first state what the parameter σ captures.

6. Suppose that $s_t = 1$. What is the period t state $s_t = 1$ price of a two period bond that pays 1 unit of consumption in period $t+2$ if and only if $s_{t+2} = 2$. State your answer in terms of $\beta, \sigma, g, \nu, \kappa$. Note that due to the Markov structure of the economy this price only depends on the current state s_t, and not the entire history s^t.

7. Suppose you were given the unique recursive equilibrium. How would you use it to determine the (unique) Arrow-Debreu equilibrium allocation. Your answer should start with "A recursive competitive equilibrium is given by ..." and should involve formal statements, not just a verbal description.