How Important Are Sectoral Shocks?

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Motivation and Question

Motivation

- Most analyses of business cycles (especially since Kydland and Prescott): Fluctuations are caused by economy-wide shocks to technology, preferences, etc...
- These shocks may be built up from events at individual firms (Gabaix '11) or industries (Long and Plosser '83 and successors).

Question

- What fraction of aggregate output fluctuations come from industry-specific shocks?

Method and Main Result

Method

- Construct a multi-industry general equilibrium model.
  - Shocks to productivity and preferences, each with an industry-specific and aggregate component.
  - Each industry produces using capital, labor, and intermediate inputs.
- Estimate, via MLE:
  - Compare model's predictions on the evolution of industries' output, output prices, and intermediate input usage.
  - Infer magnitude of industry-specific and aggregate shocks, elasticities of substitution in preferences and production.

Main result: Industry-specific shocks are important; they represent **more than 60%** of aggregate volatility.

Related Literature and Contribution

Related Literature: Multi-industry real business cycle models: Long and Plosser ('83), Horvath ('98, '00), Dupor ('99), Foerster, Sarte, and Watson ('11), Acemoğlu et al. ('12, '13)

To the Long and Plosser literature (especially relative to Foerster, Sarte, Watson), I make **2 contributions**:

1. Estimate a more general sectoral production function.
    - Accommodates empirical input usage patterns.
    - Foerster et al.: Intermediate input cost shares are constant.
    - Data: St. Dev. of the growth of these cost shares = 2-3%.
2. Smaller advances:
    a. Allow for shocks to preferences.
    b. Allow for durability of consumption goods.
    c. Apply a dataset that spans the entire economy.
    d. Examine data from other countries.

Outline

1. Introduce the multi-industry general equilibrium model.
2. Describe the dataset and a pattern in the data.
3. Present the empirical results.
4. Sensitivity analysis.
5. How are the parameters identified?

1. Model

Model: Preferences

There representative consumer has preferences over consumption C_{tJ} & labor supply L_t^S.

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left\{ \left(D_{t,Agg} \cdot \sum_{J=1}^{N} D_{tJ} \cdot \xi_J \right) \times \log \left[\left(\sum_{J=1}^{N} \left(D_{tJ} \cdot \xi_J \right)^{\frac{1}{\varepsilon_D}} \left(C_{tJ} \right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right)^{\frac{\varepsilon_D}{\varepsilon_D - 1}} \right] - \frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} \left(L_t^S \right)^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} \right\}$$

Preferences are such that:

$$C_{tJ} = D_{tJ} \cdot D_{t,Agg} \cdot \xi_J \cdot \left(\frac{P_{tJ}}{P_t} \right)^{-\varepsilon_D} \cdot \frac{1}{P_t}$$

Durable goods? Derivation

Model: Production

- The production technology is a CES function of capital/labor and intermediate inputs:

$$Q_{tJ} = A_{tJ} \cdot A_{t,Agg} \cdot \left[(1-\mu_J)^{\frac{1}{\varepsilon_Q}} \left(V_{tJ} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} + \mu_J^{\frac{1}{\varepsilon_Q}} \left(M_{tJ} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} \right]^{\frac{\varepsilon_Q}{\varepsilon_Q - 1}}$$

$$V_{tJ} = \left(\frac{K_{tJ}}{\alpha_J} \right)^{\alpha_J} \cdot \left(\frac{B_{tJ} \cdot B_{t,Agg} \cdot L_{tJ}}{1-\alpha_J} \right)^{1-\alpha_J}$$

- The intermediate input bundle of sector J is a CES aggregate of the purchases from the other sectors:

$$M_{tJ} = \mathcal{C}\left[M_{t,1\to J}, M_{t,2\to J}, \ldots M_{t,N\to J}; \varepsilon_M, \Gamma^M_{I\to J} \right]$$

- The investment input bundle of sector J is a CES aggregate of the purchases from the other sectors:

$$K_{t+1,J} = (1-\delta_K) K_{tJ} + \mathcal{C}\left[X_{t,1\to J}, X_{t,2\to J}, \ldots X_{t,N\to J}; \varepsilon_X, \Gamma^X_{I\to J} \right]$$

Model: Market Clearing

- Goods market clearing conditions (one for each $I \in \{1, ..., N\}$):

$$\underbrace{Q_{tI}}_{\text{output}} = \underbrace{C_{tI}}_{\text{consumption}} + \underbrace{\sum_J X_{t,I \to J}}_{\text{investment purchases}} + \underbrace{\sum_J M_{t,I \to J}}_{\text{intermediate input purchases}}$$

- Labor market clearing condition:

$$L_t^S = \sum_J L_{tJ}$$

Model: Evolution of Exogeneous Variables

- The industry-specific components of productivity and preference shocks:

$$\log A_{t+1,J} = \rho_{Ind,A} \cdot \log A_{tJ} + \sigma_{Ind,A} \cdot \omega_{tJ}^{Ind,A} \quad \text{(factor-neutral prod.)}$$
$$\log B_{t+1,J} = \rho_{Ind,B} \cdot \log B_{tJ} + \sigma_{Ind,B} \cdot \omega_{tJ}^{Ind,B} \quad \text{(labor-aug. prod.)}$$
$$\log D_{t+1,J} = \rho_{Ind,D} \cdot \log D_{tJ} + \sigma_{Ind,D} \cdot \omega_{tJ}^{Ind,D} \quad \text{(preferences)}$$

- And the aggregate components:

$$\log A_{t+1,Agg} = \rho_{Agg,A} \cdot \log A_{t,Agg} + \sigma_{Agg,A} \cdot \omega_{t}^{Agg,A}$$
$$\log B_{t+1,Agg} = \rho_{Agg,B} \cdot \log B_{t,Agg} + \sigma_{Agg,B} \cdot \omega_{t}^{Agg,B}$$
$$\log D_{t+1,Agg} = \rho_{Agg,D} \cdot \log D_{t,Agg} + \sigma_{Agg,D} \cdot \omega_{t}^{Agg,D}$$

- ωs are i.i.d. standard normal random variables.

How are the parameters identified? (much more, later on)

- The goal of the model is to uncover the εs, σs, and ρs.

  Compare data on industries' a) sales, b) output prices, c) intermediate input purchases to their model-predicted counterparts.

- Five main ideas:
  1. Relationship between an industry's output and its output prices $\Rightarrow \varepsilon_D$.
  2. Relationship between an industry's intermediate input prices and its cost shares $\Rightarrow \varepsilon_Q$.
  3. Some cross-industry-correlation in activity is due to input-output linkages, more so the larger are μ_J and Γ_{IJ}.
  4. More cross-industry correlation in sales$\Rightarrow$ Aggregate shocks are important.
  5. More cross-industry correlation in intermediate input purchases (if $\varepsilon_Q \neq 1$) $\Rightarrow$ Aggregate shocks are important.

2. Data

I use two main data sources

- BEA: 1992 Input/Output Table & Capital Flows Table. Show Tables

- Dale Jorgenson: Annual data on industries' production, input/output prices, & inputs, from 1960 to 2005.
  1. $Y_{tJ} = \text{sales} \equiv P_{tJ} \cdot Q_{tJ}$
  2. $P_{tJ} = \text{output price}$
  3. $M_{tJ}^{share} = \text{intermediate inputs cost share} \equiv \frac{M_{tJ} \cdot P_{tJ}^{mat}}{Q_{tJ} \cdot P_{tJ}}$
  4. $P_{tJ}^{mat} = \text{price of intermediate input bundle}$
  5. $\Pi_{tJ} = P_{tJ}^{mat} \div P_{tJ}$

I make three adjustments, to align the model and the data.

1. Use growth rates of each linearly de-trend each variable.
2. Subtract off changes in overall price level from Y_{tJ}, P_{tJ}, P_{tJ}^{mat}.
3. Trim top/bottom 0.5% of each variable.

Call Δz the transformed version of variable Z.

	Δy_{tI}	Δp_{tI}	Δm_{tI}^{share}	Δp_{tI}^{mat}
Δy_{tI}	1			
Δp_{tI}	0.610*	1		
Δm_{tI}^{share}	0.107*	-0.010	1	
Δp_{tI}^{mat}	0.451*	0.745*	0.244*	1
$\Delta \pi_{tI}$	-0.516*	-0.841*	0.212*	-0.265*
SD	0.072	0.048	0.025	0.027

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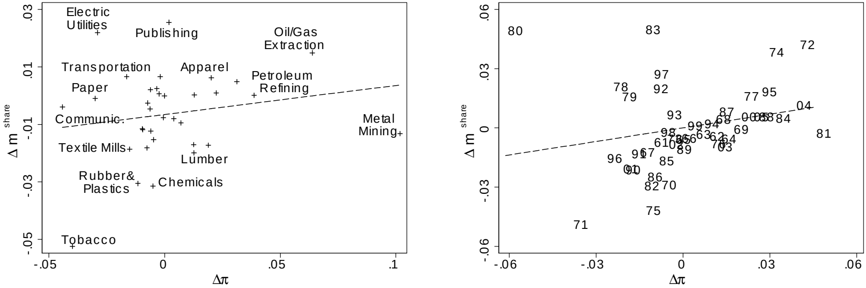
Why is ε_Q identified to be less than 1?



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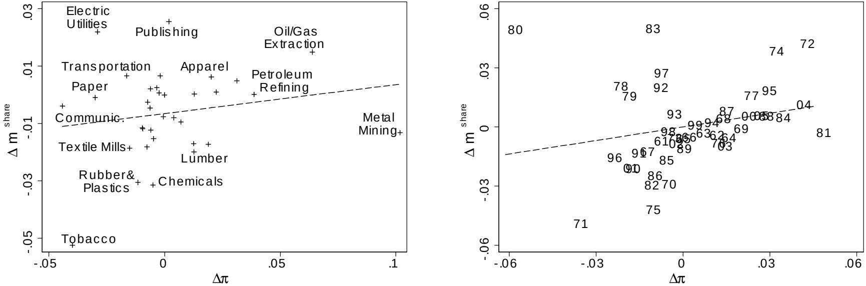

First-order condition on intermediate input purchases⇒

$$\log M_{tJ}^{share} = \log \mu_J + (1-\varepsilon_Q)\log \Pi_{tJ} + (\varepsilon_Q - 1)\log\left(A_{tJ}\cdot A_{t,Agg}\right)$$

$$\Delta m_{tJ}^{share} = (1-\varepsilon_Q)\cdot\Delta\pi_{tJ} + (\varepsilon_Q-1)\cdot\left(\Delta a_{tJ} + \Delta a_{t,Agg}\right)$$

Takeaway: Positive correlation⇒ $\varepsilon_Q < 1$.

3. Estimation and Results

I apply a mix of moment matching and MLE

- Production function and consumption shares are inferred using data from '92.
  - These parameters are informative only about the steady-state allocation/prices.
  - Data from IO Table and Capital Flows Table $\Rightarrow \Gamma^M_{I \to J}, \Gamma^K_{I \to J}$.
  - Data used to infer α_J (capital intensity), μ_J (intermediate input intensity), ξ_J (preference for good J):

Industry	s^K	s^L	s^M	Consum. Share
1. Agriculture	19.3%	23.7%	57.0%	2.2%
2. Metal Mining	20.5%	21.8%	57.7%	0.1%
...	...	...	...	...
32. Wholesale & Retail Trade	13.0%	48.1%	38.9%	11.1%
33. Finance, Insurance, R.E.	42.5%	23.5%	34.0%	16.6%
34. Personal & Bus. Services	11.0%	53.7%	35.4%	22.3%

I apply a mix of moment matching and MLE

- Production function and consumption shares are inferred using data from '92 (from previous slide).
- Other parameters (β, δ_K, ε_{LS}) taken from previous papers.
- Estimate other parameters (elasticities of substitution & dynamics of productivity and preference shocks), via MLE.
  - Δy_{tI} (output)
  - Δp_{tI} (output prices)
  - Δm_{tI}^{share} (intermediate input cost shares)
- Assume Δm_{tI}^{share} is measured with error.
  - Measurement error has both a industry-specific and aggregate component.

MLE Estimates

Specification	(1)	(2)	(3)	(4)
ε_D (preference)	0.654	1	0.587	1
ε_Q (between M and K-L)	0.046	0.020	1	1
ε_M (among intermediate inputs)	0.034	1	0.128	1
ε_X (among investment inputs)	2.870	1	2.313	1
$\sigma_{A,Ind}$ (industry factor-neutral)	0.046	0.042	0.034	0.034
$\sigma_{B,Ind}$ (industry labor-aug.)	0.110	0.110	0.000	0.000
$\sigma_{D,Ind}$ (industry preference)	0.062	0.103	0.061	0.105
$\sigma_{A,Agg}$ (agg. factor-neutral)	0.010	0.008	0.010	0.007
$\sigma_{B,Agg}$ (agg. labor-aug.)	0.040	0.040	0.001	0.015
$\sigma_{D,Agg}$ (agg. preference)	0.001	0.000	0.050	0.021
Log Likelihood	6743.0	6397.6	-94288.6	-94677.1

Robustness Checks

MLE Estimates

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Robustness Checks

Industry-specific shocks account for 60% of aggregate output volatility

Specification	(1)	(2)	(3)	(4)
Aggregate Shocks	36.9	41.4	56.6	52.0
Aggregate, Factor-Neutral Prod.	10.0	11.8	36.2	28.4
Aggregate, Labor-Augmenting Prod.	26.9	29.7	0.1	18.7
Aggregate, Demand	0.0	0.0	20.3	4.8
Industry-Specific Shocks	63.1	58.6	43.4	48.0
Industry, Factor-Neutral Prod.	21.3	17.9	37.8	35.5
Industry, Labor-Augmenting Prod.	40.2	36.5	0.0	0.0
Industry, Demand	1.7	4.1	5.6	12.5
Which Elasticities are Restricted to 1?	None	ε_D ε_M ε_X	ε_Q	All

4. Robustness Checks

Robustness Checks

- The plan for the next few slides: Sensitivity to...

  a. ... the sample period.
  b. ... the parameterization of the stochastic processes.

- If you like, we could also talk about: Sensitivity to...

  c. ... how industries are defined.
  d. ... the country.
  e. ... the treatment of trends.
  f. ... assumptions on measurement error in intermediate input purchases.
  g. ... the period length.
  h. ... the calibration of the steady state parameters.
  i. ... the trimming of outlier observations.
  j. ... the choice of shocks to include.

Robust to Time Period?

Period	1960-2005		1960-1982		1983-2005	
ε_Q (btw. $M$and K-L)	0.046	1	0.055	1	0.063	1
Log Likelihood	6743	-94677	3146	-48566	3544	-35526
Aggregate Shocks	36.9	52.0	39.8	67.8	30.4	27.8
Factor-Neutral	10.0	28.4	11.5	34.0	5.5	1.6
Labor-Augmenting	26.9	18.7	27.8	33.8	24.9	23.3
Demand	0.0	4.8	0.5	0.0	0.0	3.0
Ind.-Specific Shocks	63.1	48.0	60.2	32.2	69.6	72.2
Factor-Neutral	21.3	35.5	22.2	26.6	24.0	45.4
Labor-Augmenting	40.2	0.0	37.1	0.0	43.7	0.0
Demand	1.7	12.5	0.9	5.6	2.0	26.8

Robust to Assumptions on the Stochastic Processes?

Reminder: The shock processes, in the benchmark specification, look like $Z \in \{A, B, D\}$:

$$\log Z_{t+1,J} = \rho_{Ind,Z} \cdot \log Z_{tJ} + \sigma_{Ind,Z} \cdot \omega_{tJ}^{Ind,Z}$$
$$\log Z_{t+1,Agg} = \rho_{Agg,Z} \cdot \log Z_{t,Agg} + \sigma_{Agg,Z} \cdot \omega_{t}^{Agg,Z}$$

Period	Benchmark		Different σs, ρs	Diff. σs, ρs + Middle Nest
ε_Q	0.046	1		
Log Likelihood	6743	-94677		
Aggregate	36.9	52.0		
Middle Nest				
Industry	63.1	48.0		

Robust to Assumptions on the Stochastic Processes?

Now the shock processes are sector-specific
$S \in \{$primary inputs, durable goods, non-durable goods, services$\}$:

$$\log Z_{t+1,J} = \rho^{S}_{Ind,Z} \cdot \log Z_{tJ} + \sigma^{S}_{Ind,Z} \cdot \omega^{Ind,Z}_{tJ}$$
$$\log Z_{t+1,Agg} = \rho^{S}_{Agg,Z} \cdot \log Z_{t,Agg} + \sigma^{S}_{Agg,Z} \cdot \omega^{Agg,Z}_{t}$$

Period	Benchmark		Different σs, ρs		Diff. σs, ρs + Middle Nest
ε_Q	0.046	1	0.027	1	
Log Likelihood	6743	-94677	7200	-94416	
Aggregate	36.9	52.0	38.2	49.5	
Middle Nest					
Industry	63.1	48.0	61.8	50.5	

Robust to Assumptions on the Stochastic Processes?

And, I add a "middle nest" stochastic processes
$S \in \{$primary inputs, durable goods, non-durable goods, services$\}$:

$$\log Z_{t+1,J} = \rho^S_{Ind,Z} \cdot \log Z_{tJ} + \sigma^S_{Ind,Z} \cdot \omega^{Ind,Z}_{tJ}$$
$$\log Z_{t+1,Agg} = \rho^S_{Agg,Z} \cdot \log Z_{t,Agg} + \sigma^S_{Agg,Z} \cdot \omega^{Agg,Z}_{t}$$
$$\log Z_{t+1,S} = \rho^S_{Mid,Z} \cdot \log Z_{tS} + \sigma^S_{Mid,Z} \cdot \omega^{Mid,Z}_{tS}$$

Period	Benchmark		Different σs, ρs		Diff. σs, ρs + Middle Nest	
ε_Q	0.046	1	0.027	1	0.075	1
Log Likelihood	6743	-94677	7200	-94416	7203	-79671
Aggregate	36.9	52.0	38.2	49.5	28.2	53.2
Middle Nest					6.0	4.9
Industry	63.1	48.0	61.8	50.5	65.7	41.9

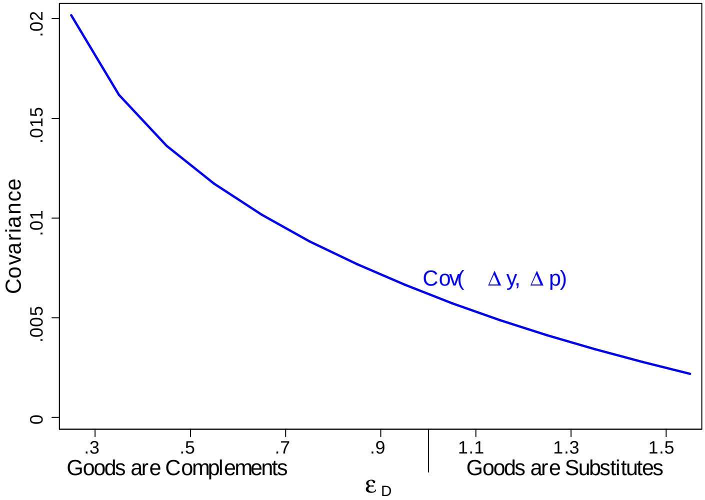
5. How Are the Parameters Identified?

How are the parameters identified?

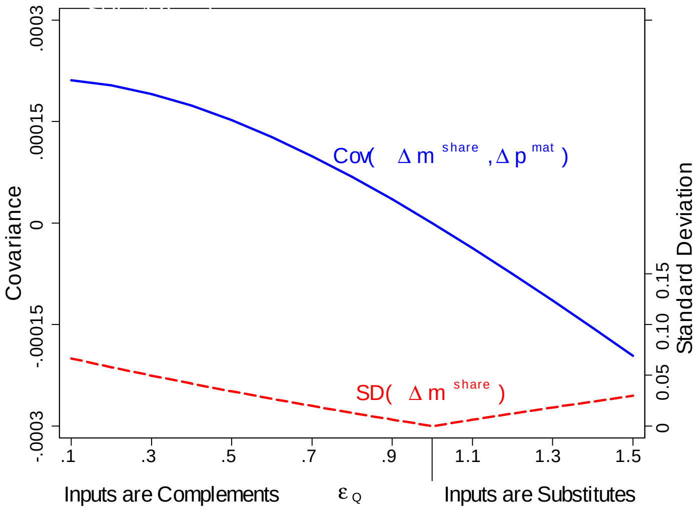
- Four results:
  1. Relationship between an industry's sales prices and its sales $\Rightarrow$ ε_D.
  2. Relationship between an industry's intermediate input prices and cost shares $\Rightarrow \varepsilon_Q$.
  3. More cross-industry correlation in sales$\Rightarrow$ Aggregate shocks are important.
  4. More cross-industry correlation in intermediate input purchases (if $\varepsilon_Q \neq 1$)$\Rightarrow$ Aggregate shocks are important.

- Two tacks:
  1. A numerical example, varying parameters around the MLE estimates.
  2. A worked-out example, using a simplified version of the model.
     Go to Simple Example

Varying ε_D, holding all other parameters fixed



Varying ε_Q, holding all other parameters fixed



Working through a simple example

- Assume:
  1. no capital ($\alpha_J = 0$)
  2. consumption goods are not durable
  3. productivity and preferences are not persistent
  4. consumption shares are identical ($\xi_J = \frac{1}{N}$)
  5. intermediate input intensities are identical ($\mu_J = \mu$)
  6. $\varepsilon_M = 0$, $\Gamma^M_{I \to J} = \frac{1}{N}$

- From Assumptions (1)-(3) :
  - The model can be solved period by period (drop t subscripts).
  - The parameters we care about (the εs, σs) are identified from the covariance matrix of the observed variables (the $\frac{P_J}{P}$s, $\frac{Y_J}{P}$s, and M_J^{share}s).

Working through a simple example

Reminder, the production function for an industry:

$$Q_J = A_J \cdot A_{Agg} \times$$
$$\left[(1-\mu)^{\frac{1}{\varepsilon_Q}} \left(B_J \cdot B_{Agg} \cdot L_J \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} + \mu^{\frac{1}{\varepsilon_Q}} \left(\frac{1}{N} \min_I M_{I \to J} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} \right]^{\frac{\varepsilon_Q}{\varepsilon_Q - 1}}$$

Taking first-order conditions, with respect to M_{tJ}, the equilibrium intermediate input share satisfies:

$$P_J^{mat} = \frac{\partial Q_J}{\partial M_J} P_J \Rightarrow \ldots \Rightarrow$$
$$M_J^{share} = \mu \cdot (A_J \cdot A_{Agg})^{\varepsilon_Q - 1} \cdot \left(\frac{P_J^{mat}}{P_J} \right)^{1-\varepsilon_Q} \tag{1}$$

How are the parameters identified?

The cost minimization condition of each industry also implies that:

$$P_J = \frac{1}{A_J \cdot A_{Agg}} \cdot \left[(1-\mu) \cdot \left(\frac{W}{B_J \cdot B_{Agg}} \right)^{1-\varepsilon_Q} + \mu \cdot \left(P_J^{mat} \right)^{1-\varepsilon_Q} \right]^{\frac{1}{1-\varepsilon_Q}}$$

$$= \frac{1}{A_J \cdot A_{Agg}} \cdot \left[(1-\mu) \cdot \left(\frac{W}{B_J \cdot B_{Agg}} \right)^{1-\varepsilon_Q} + \mu \cdot \left(\sum_{I=1}^{N} \frac{P_I}{N} \right)^{1-\varepsilon_Q} \right]^{\frac{1}{1-\varepsilon_Q}}$$

Solving this system of equations:

$$\log \left(\frac{P_J^{mat}}{P_J} \right) \approx -\frac{1}{N} \sum_I \log \left(\frac{A_I}{A_J} \right) - (1-\mu) \log \left(\frac{B_I}{B_J} \right) \qquad (2)$$

Plug (2) into (1):

$$\log M_J^{share} = \log \mu + (\varepsilon_Q - 1) \log A_{Agg} - (\varepsilon_Q - 1)(1-\mu) \log B_J$$

From the last slide:

$$\log M_J^{share} = \log \mu + (\varepsilon_Q - 1) \log A_{Agg} - (\varepsilon_Q - 1)(1 - \mu) \log B_J \tag{3}$$

Also:

$$\log\left(\frac{P_J}{P}\right) = \frac{1}{N}\sum_I \log\left(\frac{A_I}{A_J}\right) + (1 - \mu) \log\left(\frac{B_I}{B_J}\right) \tag{4}$$

$$\begin{aligned}\log\left(\frac{Y_J}{P}\right) = \frac{1}{N}\sum_I \Bigg\{ & \frac{1 - \mu(1 - \varepsilon_Q)}{1 - \mu} \cdot [\log A_I + \log A_{Agg}] \\ & + \log B_I + \log B_{Agg} + \frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} [\log D_I + \log D_{Agg}] \\ & + (1 - \varepsilon_D(1 - \mu)) \left[\log\left(\frac{A_I}{A_J}\right) + (1 - \mu) \log\left(\frac{B_I}{B_J}\right)\right] \\ & + \log\left(\frac{1}{1 - \mu}\right) + (1 - \mu) \log\left(\frac{D_J}{D_I}\right) \Bigg\}\end{aligned} \tag{5}$$

From the last slide:

$$\log M_J^{share} = \log \mu + (\varepsilon_Q - 1) \log A_{Agg} - (\varepsilon_Q - 1)(1-\mu) \log B_J \tag{3}$$

Sensitivity of M_J^{share} of shocks is U-shaped in ε_Q.

Also:

$$\log \left(\frac{P_J}{P} \right) = \frac{1}{N} \sum_I \log \left(\frac{A_I}{A_J} \right) + (1-\mu) \log \left(\frac{B_I}{B_J} \right) \tag{4}$$

$$\begin{aligned} \log \left(\frac{Y_J}{P} \right) = \frac{1}{N} \sum_I \Bigg\{ & \frac{1-\mu(1-\varepsilon_Q)}{1-\mu} \cdot [\log A_I + \log A_{Agg}] \\ & + \log B_I + \log B_{Agg} + \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1} [\log D_I + \log D_{Agg}] \\ & + (1 - \varepsilon_D (1-\mu)) \left[\log \left(\frac{A_I}{A_J} \right) + (1-\mu) \log \left(\frac{B_I}{B_J} \right) \right] \\ & + \log \left(\frac{1}{1-\mu} \right) + (1-\mu) \log \left(\frac{D_J}{D_I} \right) \Bigg\} \end{aligned} \tag{5}$$

From the last slide:

$$\log M_J^{share} = \log \mu + (\varepsilon_Q - 1) \log A_{Agg} - (\varepsilon_Q - 1)(1-\mu) \log B_J \tag{3}$$

Sensitivity of M_J^{share} of shocks is U-shaped in ε_Q.
Also:

$$\log\left(\frac{P_J}{P}\right) = \frac{1}{N}\sum_I \log\left(\frac{A_I}{A_J}\right) + (1-\mu)\log\left(\frac{B_I}{B_J}\right) \tag{4}$$

Relative price of industry J is inversely related to A_J and B_J.

$$\log\left(\frac{Y_J}{P}\right) = \frac{1}{N}\sum_I \left\{ \frac{1-\mu(1-\varepsilon_Q)}{1-\mu} \cdot [\log A_I + \log A_{Agg}] \right. \tag{5}$$

$$+ \log B_I + \log B_{Agg} + \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1}[\log D_I + \log D_{Agg}]$$

$$+ (1-\varepsilon_D(1-\mu))\left[\log\left(\frac{A_I}{A_J}\right) + (1-\mu)\log\left(\frac{B_I}{B_J}\right)\right]$$

$$\left. + \log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log\left(\frac{D_J}{D_I}\right)\right\}$$

How are the parameters identified?

$$Cov\left(\log\left(\frac{P_J^{mat}}{P_J}\right), \log M_J^{share}\right) = (1-\mu)^2\,(1-\varepsilon_Q)\,\sigma^2_{B,Ind}$$

Result 1. Slope of the relationship between intermediate input prices and cost shares $\Rightarrow \varepsilon_Q$.

How are the parameters identified?

$$Var\left(\log\frac{P_J}{P}\right) = \sigma^2_{A,Ind} + (1-\mu)^2\,\sigma^2_{B,Ind}$$

$$Cov\left(\log\frac{P_J}{P}, \log\frac{Y_J}{P}\right) = \left(1-\varepsilon_D\,(1-\mu)\right)\left(\sigma^2_{A,Ind} + (1-\mu)^2\,\sigma^2_{B,Ind}\right)$$

Combining these two equations:

$$\frac{\mathbb{E}\left(\log\frac{P_I}{P}\cdot\log\frac{Y_I}{P}\right)}{\mathbb{E}\left(\log\frac{P_I}{P}\right)^2} \approx \frac{Cov\left(\log\frac{P_I}{P}, \log\frac{Y_I}{P}\right)}{Var\left(\log\frac{P_I}{P}\right)} \approx 1-\varepsilon_D\cdot(1-\mu)\,.$$

Result 2. Regression coefficient of sales on prices $\Rightarrow \varepsilon_D$

How are the parameters identified?

$$Cov\left(\log\frac{Y_I}{P},\log\frac{Y_J}{P}\right)=\left(\frac{1-\mu\left(1-\varepsilon_Q\right)}{1-\mu}\right)^2\sigma^2_{A,Agg}+$$
$$\sigma^2_{B,Agg}+\left(\frac{\varepsilon_{LS}}{\varepsilon_{LS}+1}\right)^2\sigma^2_{D,Agg}$$
$$+\mathbf{1}_{I=J}\left[(1-\mu)^2\sigma^2_{D,Ind}+\left(1-\varepsilon_D\left(1-\mu\right)\right)^2\right.$$
$$\left.\times\left(\sigma^2_{A,Ind}+(1-\mu)^2\sigma^2_{B,Ind}\right)\right]$$

$$Cov\left(\log M_I^{share},\log M_J^{share}\right)=\left(\varepsilon_Q-1\right)^2\ \sigma^2_{A,Agg}+\mathbf{1}_{I=J}(\varepsilon_Q-1)^2\ \sigma^2_{B,Ind}$$

Result 3. Co-movement of $\frac{Y_I}{P}$ $\Rightarrow$Aggregate vs. Industry-specific.

Result 4. Co-movement of M_I^{share} (if $\varepsilon_Q\neq 1$)$\Rightarrow$Aggregate vs. Industry-specific.

How are the parameters identified?

$$
\begin{aligned}
Cov\left(\log\frac{Y_I}{P},\log\frac{Y_J}{P}\right) = & \left(\frac{1-\mu\left(1-\varepsilon_Q\right)}{1-\mu}\right)^2\sigma^2_{A,Agg}+ \\
& \sigma^2_{B,Agg}+\left(\frac{\varepsilon_{LS}}{\varepsilon_{LS}+1}\right)^2\sigma^2_{D,Agg} \\
& +\mathbf{1}_{I=J}\left[\left(1-\mu\right)^2\sigma^2_{D,Ind}+\left(1-\varepsilon_D\left(1-\mu\right)\right)^2\right. \\
& \left.\times\left(\sigma^2_{A,Ind}+\left(1-\mu\right)^2\sigma^2_{B,Ind}\right)\right]
\end{aligned}
$$

$$
Cov\left(\log M_I^{share},\log M_J^{share}\right)=\left(\varepsilon_Q-1\right)^2\ \sigma^2_{A,Agg}+\mathbf{1}_{I=J}\left(\varepsilon_Q-1\right)^2\ \sigma^2_{B,Ind}
$$

Result 3. Co-movement of $\frac{Y_I}{P}$ $\Rightarrow$Aggregate vs. Industry-specific.

Result 4. Co-movement of M_I^{share} (if $\varepsilon_Q\neq 1$)$\Rightarrow$Aggregate vs. Industry-specific.

How are the parameters identified?

$$Cov\left(\log\frac{Y_I}{P},\log\frac{Y_J}{P}\right)=\left(\frac{1-\mu\left(1-\varepsilon_Q\right)}{1-\mu}\right)^2\sigma^2_{A,Agg}+$$
$$\sigma^2_{B,Agg}+\left(\frac{\varepsilon_{LS}}{\varepsilon_{LS}+1}\right)^2\sigma^2_{D,Agg}$$
$$+\mathbf{1}_{I=J}\left[\left(1-\mu\right)^2\sigma^2_{D,Ind}+\left(1-\varepsilon_D\left(1-\mu\right)\right)^2\right.$$
$$\left.\times\left(\sigma^2_{A,Ind}+\left(1-\mu\right)^2\sigma^2_{B,Ind}\right)\right]$$

$$Cov\left(\log M_I^{share},\log M_J^{share}\right)=\left(\varepsilon_Q-1\right)^2\sigma^2_{A,Agg}+\mathbf{1}_{I=J}\left(\varepsilon_Q-1\right)^2\sigma^2_{B,Ind}$$

Result 3. Co-movement of $\frac{Y_I}{P}$ $\Rightarrow$Aggregate vs. Industry-specific.

Result 4. Co-movement of M_I^{share} (if $\varepsilon_Q\neq 1$)$\Rightarrow$Aggregate vs. Industry-specific.

How are the parameters identified?

$$Cov\left(\log\frac{P_I}{P},\log\frac{P_J}{P}\right)=\mathbf{1}_{I=J}\left[\sigma^2_{A,Ind}+(1-\mu)^2\,\sigma^2_{B,Ind}\right]$$

Result 5. Volatility of industry-specific prices ⇒ industry-specific productivity shocks

How are the parameters identified?

$$Cov\left(\log\frac{Y_I}{P},\log\frac{Y_J}{P}\right)=\left(\frac{1-\mu\left(1-\varepsilon_Q\right)}{1-\mu}\right)^2\sigma^2_{A,Agg}+$$
$$\sigma^2_{B,Agg}+\left(\frac{\varepsilon_{LS}}{\varepsilon_{LS}+1}\right)^2\sigma^2_{D,Agg}$$
$$+\mathbf{1}_{I=J}\left[\left(1-\mu\right)^2\sigma^2_{D,Ind}+\left(1-\varepsilon_D\left(1-\mu\right)\right)^2\right.$$
$$\left.\times\left(\sigma^2_{A,Ind}+\left(1-\mu\right)^2\sigma^2_{B,Ind}\right)\right]$$

Result 6. Covariance of $\frac{Y_I}{P}$s$\Rightarrow$ Volatility of industry-specific and aggregate preference shocks.

Appendix Slides

Robust to Period Length?

Period Length	1 year		2 years	
ε_Q (between M and K-L)	0.046	1	0.031	1
Log Likelihood	6743.0	-94677.1	2486.8	-20266.3
Aggregate Shocks	36.9	52.0	44.2	55.0
Factor-Neutral Prod.	10.0	28.4	12.8	26.7
Labor-Augmenting Prod.	26.9	18.7	31.4	28.1
Demand	0.0	4.8	0.0	0.2
Industry-Specific Shocks	63.1	48.0	55.8	45.0
Factor-Neutral Prod.	21.3	35.5	19.9	35.2
Labor-Augmenting Prod.	40.2	0.0	34.6	0.0
Demand	1.7	12.5	1.2	9.8

Go Back, Robustness, Table of Contents

Robust to Calibration of ξ_J, α_J, μ_J, and $\Gamma^M_{I\to J}$?

Period Length	Original (1992)		Alternative (1972)	
ε_Q (between M and K-L)	0.046	1	0.056	1
Log Likelihood	6743.0	-94677.1	5368.2	-96087.4
Aggregate Shocks	36.9	52.0	25.4	63.4
Factor-Neutral Prod.	10.0	28.4	9.7	0.0
Labor-Augmenting Prod.	26.9	18.7	15.7	9.1
Demand	0.0	4.8	0.0	54.4
Industry-Specific Shocks	63.1	48.0	74.6	36.6
Factor-Neutral Prod.	21.3	35.5	16.7	3.5
Labor-Augmenting Prod.	40.2	0.0	23.8	0.0
Demand	1.7	12.5	34.0	33.0

Go Back, Robustness, Table of Contents

Robust to Cut-off?

Cutoff	0.25%		0.5%		1.0%	
ε_Q (btw. $M$and K-L)	0.049	1	0.046	1	0.043	1
Log Likelihood	6493	-19032	6743	-94677	6980	-11496
Aggregate Shocks	34.6	46.1	36.9	52.0	35.4	55.2
Factor-Neutral	8.5	39.5	10.0	28.4	6.1	39.4
Labor-Augmenting	26.2	3.2	26.9	18.7	29.3	6.5
Demand	0.0	3.3	0.0	4.8	0.0	9.3
Ind.-Specific Shocks	65.4	53.9	63.1	48.0	64.6	44.8
Factor-Neutral	21.9	39.6	21.3	35.5	22.0	32.8
Labor-Augmenting	41.1	0.0	40.2	0.0	40.5	0.0
Demand	2.4	14.3	1.7	12.5	2.0	12.0

Go Back, Robustness, Table of Contents

Robust to Industry Classification?

Industry Classification	Original: 34 industries		Coarse: 8 industries	
ε_Q (between M and K-L)	0.046	1	0.020	1
Log Likelihood	6743.0	-94677.1	1975.3	-8106.8
Aggregate Shocks	36.9	52.0	29.3	35.2
Factor-Neutral Prod.	10.0	28.4	0.0	20.7
Labor-Augmenting Prod.	26.9	18.7	28.9	5.5
Demand	0.0	4.8	0.5	9.1
Industry-Specific Shocks	63.1	48.0	70.7	64.8
Factor-Neutral Prod.	21.3	35.5	38.9	46.1
Labor-Augmenting Prod.	40.2	0.0	33.8	11.4
Demand	1.7	12.5	1.0	7.3

Go Back, Robustness, Table of Contents

Robust to Country? (1)

Country	Denmark		Netherlands		Spain	
ε_Q (btw. M and K-L)	0.036	1	0.148	1	0.042	1
Log Likelihood	2415	-52262	3814	-12633	2183	-11458
Aggregate Shocks	4.5	10.6	27.0	41.1	5.0	42.0
Factor-Neutral	3.6	0.0	14.4	39.5	5.0	33.9
Labor-Augmenting	0.0	0.0	12.3	0.0	0.0	0.9
Demand	0.8	10.6	0.3	1.6	0.0	7.2
Ind.-Specific Shocks	95.5	89.4	73.0	58.9	95.0	58.0
Factor-Neutral	18.7	16.3	13.8	9.3	10.5	7.3
Labor-Augmenting	29.9	42.5	22.4	33.5	25.0	21.5
Demand	46.9	30.6	36.8	16.2	59.5	29.3

Go Back, Robustness, Table of Contents

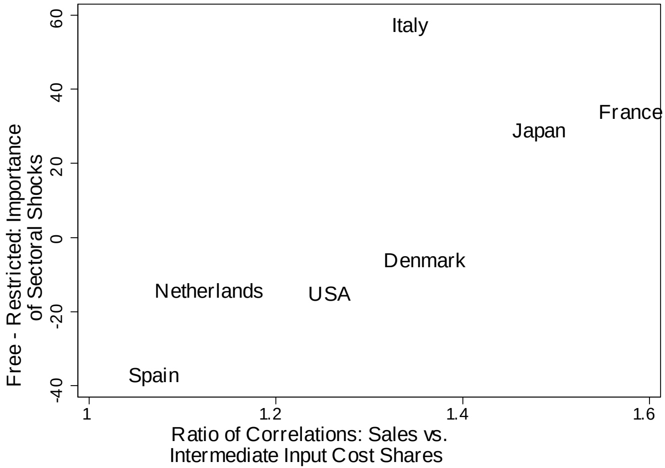
Robust to Country? (2)

Country	France		Italy		Japan	
ε_Q (btw. M and K-L)	0.118	1	0.068	1	0.027	1
Log Likelihood	1716	-10578	2568	-27190	1766	-17110
Aggregate Shocks	44.4	7.7	91.6	34.4	37.3	8.2
Factor-Neutral	44.4	0.0	67.6	0.0	1.3	4.0
Labor-Augmenting	0.0	6.6	22.7	11.5	35.9	0.1
Demand	0.0	1.1	1.3	22.9	0.1	4.1
Ind.-Specific Shocks	55.6	92.3	8.4	65.6	62.7	91.8
Factor-Neutral	5.1	6.0	1.5	7.3	3.8	7.8
Labor-Augmenting	14.4	18.8	3.5	25.1	9.5	14.9
Demand	36.1	67.5	3.4	33.3	49.3	69.1

Go Back, Robustness, Table of Contents

Robust to Country? (3)

Why does $\varepsilon_Q \neq 1$ lead to higher estimates for some countries, and lower estimates for others?



How to deal with trends in the data?

- In the benchmark specification, I linearly de-trend each data series.
- Two concerns:
  1. De-trending removes potentially useful variation.
  2. Parameter estimates may be sensitive to the de-trending procedure (Canova '13).
- Ways to address these concerns:
  1. Try different de-trending procedures (next slide).
  2. Include trends, permanent shocks, and stationary shocks in the model.

Robust to De-trending Procedure?

De-trending Procedure	Linear		Linear + Break at 1983		HP Filter	
ε_Q (btw. $M$and K-L)	0.046	1	0.050	1	0.039	1
Log Likelihood	6743	-94677	6887	-83620	8230	-45668
Aggregate Shocks	36.9	52.0	36.8	52.1	23.5	60.7
Factor-Neutral	10.0	28.4	9.1	28.6	10.1	0.0
Labor-Augmenting	26.9	18.7	27.7	18.7	13.4	52.4
Demand	0.0	4.8	0.0	4.8	0.0	8.3
Ind.-Specific Shocks	63.1	48.0	63.1	47.9	76.4	39.3
Factor-Neutral	21.3	35.5	21.3	35.9	27.4	33.6
Labor-Augmenting	40.2	0.0	40.6	0.0	48.5	0.0
Demand	1.7	12.5	1.2	12.0	0.6	5.7

Go Back, Robustness, Table of Contents

Robust to Measurement Error?

$\sigma_{M,Ind} =$	0.2%	0.1%	0.4%	0.2%
$\sigma_{M,Agg} =$	0.2%	0.1%	0.2%	0.4%
ε_Q (between M and K-L)	0.046	0.045	0.046	0.047
Log Likelihood	6743.0	6493.3	6758.0	6748.1
Aggregate Shocks	36.9	35.8	37.6	31.1
Factor-Neutral Prod.	10.0	10.6	10.2	8.5
Labor-Augmenting Prod.	26.9	25.1	27.4	22.6
Demand	0.0	0.0	0.0	0.0
Industry-Specific Shocks	63.1	64.2	62.4	68.9
Factor-Neutral Prod.	21.3	21.6	21.4	23.4
Labor-Augmenting Prod.	40.2	41.0	39.3	43.6
Demand	1.7	1.7	1.7	1.8

Go Back, Robustness, Table of Contents

Other Estimates of ε_D and ε_Q

- ε_Q (between intermediate inputs and capital/labor)
  - Bruno ('84): 0.3
  - Rotemberg and Woodford ('96): 0.7
- ε_D (preference elasticity)
  - Ngai and Pissarides ('07), and Acemoğlu and Guerrieri ('08): <1.
  - Not appropriate: Broda and Weinstein ('06) or Foster, Haltiwanger, and Syverson ('08): $\gg 1$.

Go Back

Robust to Choice of ε_{LS}?

	$\varepsilon_{LS} = 1$		$\varepsilon_{LS} = 2$	
ε_Q (between M and K-L)	0.046	1	0.046	1
Log Likelihood	6743.0	-94677.1	6735.3	-16209.9
Aggregate Shocks	36.9	52.0	32.6	38.8
Factor-Neutral Prod.	10.0	28.4	20.3	6.3
Labor-Augmenting Prod.	26.9	18.7	23.7	31.4
Demand	0.0	4.8	8.9	1.1
Industry-Specific Shocks	63.1	48.0	67.4	61.2
Factor-Neutral Prod.	21.3	35.5	19.9	45.3
Labor-Augmenting Prod.	40.2	0.0	34.6	0.0
Demand	1.7	12.5	1.2	15.9

Go Back, Robustness, Table of Contents

Robust to Choice of Shocks?

- Alter the capital accumulation condition of each industry to:

$$K_{t+1,J} = (1-\delta_K)\cdot K_{tJ} \\ + \iota_{tJ}\cdot\iota_{t,Agg}\cdot\mathcal{C}\left[X_{t,1\to J}, X_{t,2\to J}, \ldots X_{t,N\to J}; \varepsilon_X, \Gamma^X_{I\to J}\right]$$

- In one-sector analyses, shocks to the ιs explain a substantial fraction of output variation (Fisher '06, Justiniano, Primiceri, and Tambalotti '10)

Go Back, Robustness, Table of Contents

Robust to Inclusion of Investment Shocks?

	Benchmark		Investment Shocks	
ε_Q (btw. $M$and K-L)	0.046	1	0.052	1
Log Likelihood	6743	-94677	6766	-94629
Aggregate Shocks	36.9	52.0	37.1	68.7
Factor-Neutral	10.0	28.4	12.4	8.3
Labor-Augmenting	26.9	18.7	24.7	0.0
Demand	0.0	4.8	0.0	0.0
Investment			0.0	60.5
Ind.-Specific Shocks	63.1	48.0	62.9	31.3
Factor-Neutral	21.3	35.5	15.9	10.0
Labor-Augmenting	40.2	0.0	23.0	2.0
Demand	1.7	12.5	1.5	0.0
Investment			22.4	19.3

Go Back, Robustness, Table of Contents

Robust to Choice of Shocks?

	Benchmark		Investment Shocks		No Labor-Aug. Shocks	
ε_Q (btw. $M$and K-L)	0.046	1	0.052	1	0.754	1
Log Likelihood	6743	-94677	6766	-94629	-88401	-94677
Aggregate Shocks	36.9	52.0	37.1	68.7	69.5	48.3
Factor-Neutral	10.0	28.4	12.4	8.3	43.2	48.3
Labor-Augmenting	26.9	18.7	24.7	0.0		
Demand	0.0	4.8	0.0	0.0	26.3	0.0
Investment			0.0	60.5		
Ind.-Specific Shocks	63.1	48.0	62.9	31.3	30.5	51.7
Factor-Neutral	21.3	35.5	15.9	10.0	6.9	38.4
Labor-Augmenting	40.2	0.0	23.0	2.0		
Demand	1.7	12.5	1.5	0.0	23.5	13.2
Investment			22.4	19.3		

Go Back, Robustness, Table of Contents

Why do the results change so drastically when I set $\sigma_B = 0$?

- Reminder, from the simple example:

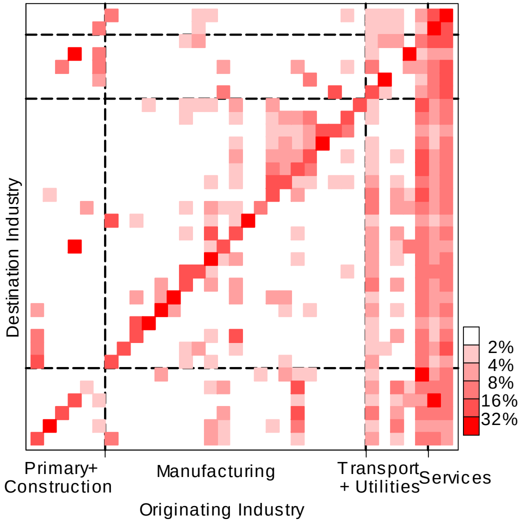
$$Cov\left(\log M_I^{share}, \log M_J^{share}\right) = \left(\varepsilon_Q - 1\right)^2 \sigma_{A,Agg}^2 + + \mathbf{1}_{I=J} \left(\varepsilon_Q - 1\right)^2 \sigma$$

- When $\sigma_{B,Ind}^2 = 0$, only common, factor-neutral productivity shocks can explain volatile intermediate input cost shares.

- Because the movements in intermediate input cost shares are so uncorrelated, the likelihood drops substantially.

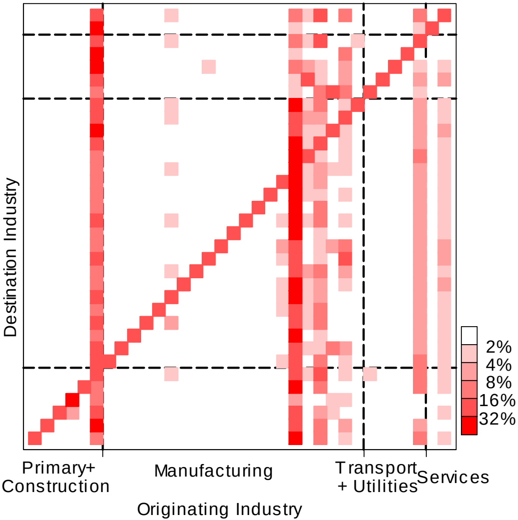
Go Back, Robustness, Table of Contents

There are substantial flows of intermediate inputs, across industries



Go Back

A small number of industries produce most of the investment goods



Go Back

Model: Preferences (With Durable Goods)

Preferences are described by:

$$U = \sum_{t=0}^{\infty} \beta^t \left\{ \left(D_{t,Agg} \cdot \sum_{J=1}^{N} D_{tJ} \cdot \xi_J \right) \times \right.$$
$$\left. \log \left[\left(\sum_{J=1}^{N} \left(D_{tJ} \cdot \xi_J \right)^{\frac{1}{\varepsilon_D}} \left(\delta_{C_J} \cdot C_{tJ} \right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right)^{\frac{\varepsilon_D}{\varepsilon_D - 1}} \right] - \frac{\varepsilon^{LS}}{\varepsilon^{LS} + 1} \left(L_t^S \right)^{\frac{\varepsilon^{LS}+1}{\varepsilon^{LS}}} \right\}$$

Here, C_{tJ} is the stock of the durable good J, is a durable. The stock evolves according to:

$$C_{tJ} = C_{t-1,J} \cdot (1 - \delta_{C_J}) + \tilde{C}_{tJ}$$

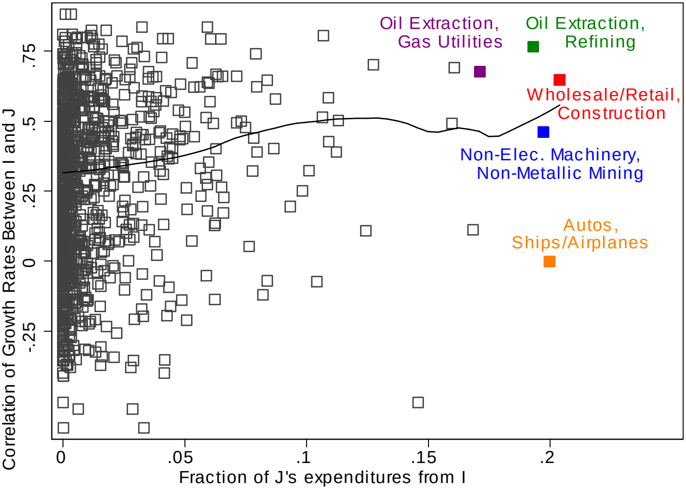
Go Back

Model: Preferences (With Durable Goods)

Name	Depreciation Rate
Construction	2.1%
Lumber and Wood Products	11.8%
Furniture and Fixtures	11.8%
Stone, Clay, and Glass Products	16.5%
Primary Metals	16.5%
Fabricated Metal Products	16.5%
Non-Electrical Machinery	16.5%
Electrical Machinery	17.0%
Motor Vehicles	35.3%
Other Transportation Equipment	16.5%
Instruments	16.7%
Miscellaneous Manufacturing	16.2%

Go Back

Industries in input-output relationships co-move more strongly



Equilibrium Definition

For a given set of initial conditions, a perfectly competitive equilibrium consists of shock vectors $\left\{\omega_{tJ}^{Ind,A}, \omega_{t}^{Agg,A}, \omega_{tJ}^{Ind,B}, \omega_{t}^{Agg,B}, \omega_{tJ}^{Ind,D}, \omega_{t}^{Agg,D}\right\}_{t=0}^{\infty}$, price vectors $\left\{W_t,\ P_{tJ},\ P_{tJ}^{mat},\ P_{tJ}^{inv}\right\}_{t=0}^{\infty}$, and quantity vectors $\left\{L_t^S,\ C_{tJ},\ Q_{tJ},\ M_{tJ},\ L_{tJ},\ X_{tJ}\right\}_{t=0}^{\infty}$ such that:

1. The representative consumer chooses C_{tJ} and L_t^S to maximize expected utility.
2. Each industry chooses L_{tJ}, X_{tJ}, and M_{tJ} to maximize expected profits.
3. The capital stocks, durable goods stocks evolve as described in other slides.
4. The demand and productivity stochastic processes evolve as described in other slides.
5. The labor market and N goods markets clear.

Write the Lagrangian of the social planner:

$$\mathcal{L} = \mathbb{E}_0 \sum_t \beta^t \left\{ D_{t,Agg} \cdot \left(\sum_J \xi_J \cdot D_{tJ} \right) \times \right.$$

$$\log \left[\left[\sum_J (\xi_J \cdot D_{tJ})^{\frac{1}{\varepsilon_D}} (C_{tJ})^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right]^{\frac{\varepsilon_D}{\varepsilon_D - 1}} \right] - \left(\sum_J L_{tJ} \right)^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}}$$

$$+ \sum_J P_{tJ}^{inv} \left[X_{tJ} + (1 - \delta_K) K_{tJ} - K_{t+1,J} \right]$$

$$\left. + \sum_J P_{tJ} \left[Q_{tJ} - C_{tJ} - X_{tJ} - \sum_I M_{t,J \to I} \right] \right\}$$

Take first-order conditions with respect to C_{tJ} :

$$P_{tJ} = D_{t,Agg} \cdot \left(\sum_J \xi_J \cdot D_{tJ} \right) \cdot (D_{tJ} \cdot \xi_J)^{\frac{1}{\varepsilon_D}} (C_{tJ})^{-\frac{1}{\varepsilon_D}}$$

$$\times \left(\sum_I (D_{tI} \cdot \xi_I)^{\frac{1}{\varepsilon_D}} (C_{tI})^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right)^{-1}$$

Re-arrange:

$$C_{tJ} = (P_{tJ})^{-\varepsilon_D} \cdot \xi_J \cdot D_{tJ} \cdot \left[D_{t,Agg} \cdot \left(\sum_I \xi_I \cdot D_{tI} \right) \right]^{\varepsilon_D}$$

$$\times \left[\sum_J (\xi_I \cdot D_{tI})^{\frac{1}{\varepsilon_D}} (C_{tI})^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right]^{-\varepsilon_D} \qquad (6)$$

Note also that:

$$C_{tI} = (P_{tI})^{-\varepsilon_D} \cdot \xi_I \cdot D_{tI} \cdot \left[D_{t,Agg} \cdot \left(\sum_{K=1}^{N} \xi_K \cdot D_{tK} \right) \right]^{\varepsilon_D}$$

$$\times \left[\sum_{K=1}^{N} (\xi_K \cdot D_{tK})^{\frac{1}{\varepsilon_D}} (C_{tK})^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right]^{-\varepsilon_D}$$

$$(C_{tI})^{\frac{\varepsilon_D - 1}{\varepsilon_D}} = (P_{tI})^{1-\varepsilon_D} \cdot \xi_I \cdot D_{tI} \cdot \left[D_{t,Agg} \cdot \left(\sum_K \xi_K \cdot D_{tK} \right) \right]^{\varepsilon_D - 1}$$

$$\times \left[\sum_K (\xi_K \cdot D_{tK})^{\frac{1}{\varepsilon_D}} (C_{tK})^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right]^{1-\varepsilon_D} (\xi_I \cdot D_{tI})^{-\frac{1}{\varepsilon_D}} \qquad (7)$$

Plugging Equation (7) into Equation (6):

$$C_{tJ} = (P_{tJ})^{-\varepsilon_D} \cdot \left[D_{t,Agg} \cdot \left(\sum_J \xi_I \cdot D_{tJ} \right) \right]^{\varepsilon_D} \cdot \xi_J \cdot D_{tJ} \cdot$$

$$\times \left(\sum_I (P_{tI})^{1-\varepsilon_D} \cdot \xi_I \cdot D_{tI} \right)^{-1} \cdot \left[D_{t,Agg} \cdot \left(\sum_I \xi_K \cdot D_{tK} \right) \right]^{1-\varepsilon_D}$$

$$= D_{t,Agg} \cdot \xi_J \cdot D_{tJ} \cdot (P_{tJ})^{-\varepsilon_D} \cdot \left(\sum_I \frac{\xi_I \cdot D_{tI}}{\sum_K \xi_K \cdot D_{tK}} (P_{tI})^{1-\varepsilon_D} \right)^{-1}$$

Defining the aggregate price level as:

$$P_t \equiv \left(\sum_I \frac{\xi_I \cdot D_{tI}}{\sum_K \xi_K \cdot D_{tK}} (P_{tI})^{1-\varepsilon_D} \right)^{\frac{1}{1-\varepsilon_D}}$$

We get that:

$$C_{tJ} = D_{t,Agg} \cdot \xi_J \cdot D_{tJ} \cdot (P_{tJ})^{-\varepsilon_D} \cdot (P_t)^{\varepsilon_D - 1} .$$

Conclusion

- Main result: Industry-specific shocks are important (account for $\frac{3}{5}$ths of aggregate volatility)
  - Positive correlation between intermediate inputs and intermediate input prices $\Rightarrow \varepsilon_Q$ is small.
  - Movements in intermediate input cost shares are uncorrelated $\Rightarrow$ industry-specific shocks are important.
- Other studies on the sources of business cycles:
  - monetary policy shocks
  - news about future economic activity
  - uncertainty about future productivity
- Possible avenue for future work: Re-examine these sources of variation with the understanding that they may come from the micro level.

Table of Contents