1. Investment-Specific Technological Change with Endogenous Labor Supply

Consider the following model. There is a representative household with a utility function:

$$U = \int_0^\infty e^{-\rho t}\left(\log c - \eta\frac{l^{1+\xi}}{1+\xi}\right)dt$$

(I will drop time t in all the variables when no ambiguity occurs) where c is consumption and l is labor supply. There is no population growth.

There is a representative firm in the economy that rents capital, k, and hires labor, l, to produce a final good with technology:

$$y = k^\alpha l^{1-\alpha}$$

where $\alpha \in (0,1)$.

The household invests i such that:

$$\dot{k} = qi - \delta k$$

where δ is the depreciation rate and q is exogenous investment-specific technological change that evolves:

$$\dot{q} = q\gamma$$

where $\gamma > 1$ is parameter and $q_0 = 1$.

1. Write down the resource constraint of the economy in terms of the final good.

2. Write down the problem of the representative household and find its optimality conditions.

3. Write down the problem of the representative firm and find its optimality conditions.

4. Define an equilibrium for this economy.

5. Characterize the Balanced Growth Path (BGP) of this economy. Among other things, you need to:

    1. Find the BGP rate of growth.
    2. Show how different variables grow over time.
    3. Describe the behavior of input prices over time.

6. Describe, as far as you can, the transitional dynamics of this economy for a given k_0. Explore, in particular, the evolution of labor supply over time.