1. An Endogenous Growth Model with Expiring Patents

Consider the following model. There is a representative household with a utility function:

$$U(0) = \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} dt$$

Labor is $L = 1$ and there is no population growth.

There is a perfectly competitive final good producer with technology:

$$Y(t) = \frac{1}{\alpha} \left(\int_0^{N(t)} x(v,t)^\alpha \, dv \right) L^{1-\alpha}$$

where $x(v,t)$ is an input, fully depreciated in production and sold at price $p(v,t)$, and $N(t)$ is the total number of existing varieties.

Each input is produced either by the monopolist that invent it, if the patent that was awarded to the inventor is still in force, or by a competitive producer that sells at marginal cost. Patents are awarded when the new invention is developed and expire with a Poisson rate i. The production of inputs is at (constant) marginal cost ψ.

New varieties are developed according to the innovation function:

$$\dot{N}(t) = \eta Z(t)$$

where $Z(t)$ is investment in R&D and with some initial $N(0)$. There is free entry into the innovation activity. The initial amount of varieties with unexpired patents is $N_1(0)$ and the initial amount of varieties with expired patents is $N_2(0)$.

1. Define an equilibrium for this economy.

2. Characterize the Balanced Growth Path (BGP) of this economy. Among other things, you need to:

   1. Find the price of each variety.

   2. Write the Hamilton-Bellman-Jacobi equation of the monopolist.

   3. Find an expression that relates the growth rate of the economy along the BGP with the Poisson rate i of patent expiration.

3. Are there transitional dynamics in this economy? Justify your answer as a function of $N_1(0)$ and $N_2(0)$.

4. Solve the social planner's problem. Compare with the solution above.

5. Imagine a benevolent government that can pick the Poisson rate i but that, otherwise, has to respect the equilibrium allocations and prices generated by the market (this is often known as a Ramsey government). Discuss how that benevolent government that should pick i. Compare with the solution of the social planner's problem above.