1. Search and Human Capital

Consider an economy where workers accumulate human capital over time. A worker with human capital h_t produces a final, nonstorable good y_t with the production function:

$$y_t = e^{z_t} h_t^{\alpha}$$

where $\alpha < 1$ and z_t is a stochastic process that follows $z_t = \rho z_{t-1} + \sigma \varepsilon_t$ where $0 < \rho < 1$ and $\varepsilon_t \sim \mathcal{N}(0,1)$. Workers are paid their marginal productivity.

Jobs disappear at the end of each period with probability $\delta(z_t)$ where $\delta'(z_t) < 0$. If the job is not terminated, additional human capital is accumulated with law of motion $h_{t+1} = \gamma h_t$ where $\gamma > 1$.

If the worker is unemployed, it will get an offer in period t with probability $\pi(e_t)$ where e_t is the effort devoted to search and it will not get an offer with probability $1 - \pi(e_t)$. The function $\pi(.) : [0,1] \rightarrow [0,1]$ is monotone, increasing, concave, and differentiable everywhere.

If an offer arrives, it is parameterized by a current productivity level z_t drawn from the ergodic distribution of the process for productivity described above and by a random variable b_t uniformly distributed between 0 and 1 and i.i.d. over time. A worker that accepts an offer in period t to start working in period $t+1$ has a new human capital of $h_{t+1} = b_t h_t$ (that is, a share $(1 - b_t)$ of human capital disappears for ever). If the offer does not arrive or if it is rejected, the new level of human capital will be $h_{t+1} = \mu h_t$ where $\mu < 1$. Note that the worker knows z_t (the productivity of the firm today) but not z_{t+1} (the productivity of the firm tomorrow, when it starts to work).

The utility of the worker is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{c_t - e_t\}$$

where c_t is consumption and e_t is the search effort (trivially equal to 0 when the worker is employed).

1. Set up the worker's problem as a dynamic programming problem.
2. Display the condition that describes the optimal choice of e_t.
3. Does the worker have a reservation-wage strategy? If so, show that this is the case and characterize it. If not, show that this is the case and characterize its decision rule.
4. Can it happen that a worker will accept an offer with a b_t lower than a b_t' from other offer that it would reject? If so, why? If not, why not?
5. How do your previous answers change as the worker unemployment spell becomes longer?
6. (Volatility and unemployment) Describe the effects of an increase in σ on the behavior of workers. Interpret the results.
7. (Technological depreciation and unemployment) Describe the effects of an increase in μ on the behavior of workers. Interpret the results.