1. Bryant-Keynes-Wallace

Consider an economy consisting of overlapping generations of two-period lived agents. There is a constant population of N young agents born at each date $t \geq 1$. There is a single consumption good that is not storable. Each agent born in $t \geq 1$ is endowed with w_1 units of the consumption good when young and with w_2 units when old, where $0 < w_2 < w_1$. Each agent born at $t \geq 1$ has identical preferences

$$\log c_t^{i,t} + \log c_{t+1}^{i,t}$$

where $c_t^{i,j}$ is the consumption at time t of the agent i (where $i \in \{1, ..., N\}$) born at time j. In addition, at time 1, there are alive N old people who are endowed with a total of H_0 units of fiat money and who want to maximize their consumption of the time-1 good.

A government attempts to finance a constant level of government purchases $G_t = G > 0$ for $t \geq 1$ by printing new money. G does not yield any utility to private agents. The government's budget constraint is then:

$$G = \frac{H_t - H_{t-1}}{p_t},$$

where p_t is the price level at t, and H_t is the stock of money carried over from t to $t+1$ by agents born in t. Let

$$g = \frac{G}{N}$$

be government purchases per young person.

1. Write the aggregate resource constraint of this economy.

2. Define a sequential markets equilibrium for this economy.

3. Define a stationary sequential markets equilibrium with valued fiat money.

4. Prove that, for g sufficiently small, there exists a stationary equilibrium with valued fiat money.

5. Prove that, in general, if there exists one stationary equilibrium with value fiat money, with rate of return on money

   $$1 + r(t) = 1 + r_1,$$

   then there exists at least one other stationary equilibrium with valued money with

   $$1 + r(t) = 1 + r_2 \neq 1 + r_1,$$

6. Tell whether the equilibria described in steps 4. and 5. are Pareto optimal (given that the government needs to finance G).