Economics 706 Preliminary Examination

F.X. Diebold

Good luck!

Consider the stochastic process,

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \ , \quad \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2). \qquad (1)$$

a. Cast the process in state-space form, and give the condition for its covariance stationarity in terms of the state-space representation. How would you obtain and interpret maximum likelihood parameter estimates using the state-space representation, assuming correct specification?

b. Compute and discuss the process' spectral density function. What shapes can it take, and under what conditions? Can it have an internal peak? Two internal peaks? How is the variance of the sample mean of the process related to its spectrum at frequency zero?

c. Provide a detailed characterization of the process via its Wold representation. What is the relationship between its Wold representation and spectral density function? Do the innovations associated with the process' Wold representation necessarily have constant conditional variance? Is the innovation conditional variance necessarily smaller than the unconditional variance? Why or why not? Are the innovations conditionally Gaussian? Unconditionally Gaussian? Unconditionally symmetric? Unconditionally leptokurtic? Covariance stationary? Strictly stationary?

From now on, suppose that:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \ , \quad \varepsilon_t | \Omega_{t-1} \sim N(0, h_t) \ , \quad h_t = 20 + .13\ \varepsilon_{t-1}^2 + .85\ h_{t-1}. \qquad (2)$$

d. Do the innovations associated with the process' Wold representation necessarily have constant conditional variance? Is the innovation conditional variance necessarily smaller than the unconditional variance? Why or why not? Are the innovations uncorrelated? Independent? Conditionally Gaussian? Unconditionally Gaussian? Unconditionally symmetric? Unconditionally leptokurtic? Covariance stationary? Strictly stationary?

e. Suppose now that you fit model (2) to data by MLE, but you're not certain that it's correctly specified. How would you assess the adequacy of the specification? You may want to consider, among other things:

(a) Neglected conditional mean dynamics
(b) Neglected conditional variance dynamics
(c) Adequacy of the normality assumption
(d) Structural change.