

Consider the following model of a two-period economy. There is a representative agent whose lifetime utility is given by

$$\alpha \ln(c) + (1 - \alpha) \ln(1 - l) + \beta \alpha \ln c', \text{ with } 0 < \alpha, \beta < 1,$$

where c and c' are consumption when young and old and l is his labor supply when young. In the first period the representative worker can produce output, o , according to

$$o = wl,$$

where w is a constant. There is a storage technology that both private citizens and the government can use for savings in the first period that yields a gross return of $r > 1$ in the second period.

1. *Government provided social security.* The government taxes output in the first period at the rate τ . It saves using the storage technology the proceeds from taxing individuals and distributes them in the second period in the form of old age pensions in the *lump-sum* amount, p' . Assume that individuals still do some savings on their own.
 - (a) Formulate and solve the representative agent's decision problem.
 - (b) What is the equilibrium solution for labor supply? How do taxes affect labor supply?

2. *Mandated private accounts.* The government requires individuals to save the fraction τ of their first-period income in private accounts, which are invested in the storage technology. Each individual receives in the second period the balance of their *own* private account plus interest as a pension. Assume that individuals still do some savings on their own.
 - (a) Formulate and solve the representative agent's decision problem.
 - (b) What is the equilibrium solution for labor supply? How do taxes affect labor supply? Which system has the highest level of output and pensions?