

Consider a representative agent who maximizes his lifetime utility as given by

$$\sum_{t=0}^{\infty} \beta^t c_t,$$

subject to

$$c_t + k_{t+1} - k_t = y_t,$$

where c_t , k_t , and y_t represent period- t consumption capital and output. The agent inelastically supplies s units of skilled labor and u units of unskilled labor for production. Final output can be produced by one of following two technologies:

$$y_t = \bar{\phi}[\theta k_t^\rho + (1 - \theta)s^\rho]^{\alpha/\rho} u^{(1-\alpha)},$$

or

$$y_t = \psi_t[\theta k_t^\rho + (1 - \theta)s^\rho]^{\alpha/\rho} u^{1-\alpha}.$$

The first production function represents a primitive technology, the second a more advanced technology. Capital and skilled labor are complementary in production in the sense that $\rho < 0$. At each point in time the agent is free to choose which technology he wants to use. Now, let ψ_t follow a law of motion of the form: $\psi_t = \bar{\psi}(1 - 1/g^t)$ with $\bar{\psi} > \bar{\phi}$ and where $g > 1$. Your analysis will be graded upon *both* the technical ability you display as well as the economic intuition you demonstrate.

1. Formulate and solve the representative agent's choice problem *using a dynamic programming formulation*. *Hint*: Define a value function $V(\cdot)$ assuming that the agent uses the first technology in current period but is free to choose the one he wants next period. Likewise, define the value function $W(\cdot)$ when he uses the second technology in the first period but is free to choose the one he wants next period.
2. Characterize the transitional dynamics for the economy under study. Specifically, analyze how income and the skill premium (the ratio of skilled to unskilled labor earnings per unit of work) behave over time. How are the dynamics affected by g ? Explain how this could be a model of an industrial revolution.
3. Discuss and analyze extensions to this framework.