Syllabus for Economics 30
Public Policy Analysis Fall 2015

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Classes: MW 15.30-17 PM
Office Hours: MW 13-14 PM
(and by appointment)

Course Description and Objective

This course investigates the role of government in the economy and aims at equipping students with tools sufficient to address a wide range of modern public policy issues. The emphasis is on microeconomic analysis of federal expenditure and taxation: the macroeconomic functions of the government are not addressed. We study the instruments by which governments intervene in the marketplace policies, and apply our tools to various real-life problems; social security, welfare spending and personal income taxation.

Prerequisites

The prerequisites for this course are Econ 1

Course Material

The required textbook for the course is:

Rosen, Harvey S. and Ted Gayer (2008), ***Public Finance***, McGraw-Hill/Irwin, 8th edition.
Recommended book: Atkinson, A.B., and J.E. Stiglitz (1987), ***Lectures on Public Economics*** (distributed chapters in class).
Recommended book: Stiglitz, J.E. (2000), ***Economics of the Public Sector.***

For background material, it may be useful to acquire an introductory and intermediate Microeconomics text.

Course Requirements and Grading

There will be several homework assignments, a midterm and a final exam. Exams will be closed book. The date of the midterm will be announced in class. I encourage you to prepare the homework attached assignments in groups of 3-4 students. The weighting for the final grade is 50% for the final and 50% for the midterm. Several make-up classes and review sessions will be held and will be announced in advance in class.

Course Outline and Suggested Readings

- Overview of Economic Activities of Government
  Stiglitz, Chapters 1-2; R&G*, Chapters 1-2.

- Review: Pareto Optimum Conditions: In Production and Consumption

- Production Possibilities Frontier. Stiglitz, pp. 67-73, R&G, Chapter 3.

- Utility Possibilities Frontier
  Intermediate Microeconomic textbook and R&G, Chapter 3.

- Social Welfare Functions (Efficiency vs. Equality): Lecture 11 in Atkinson and Stiglitz (to be distributed in class): R&G, Chapter 12.

- Externalities in Consumption and Production, R&G, Chapter 5.

- Public Goods – R&G, Chapter 4; Atkinson and Stiglitz, Chapter 15, pp. 505-518.

- Taxation: Tax Incidence – Stiglitz, Chapters 17-19.

- Issues on Optimal Taxations – R&G, Chapter 14-16.

- Personal Income Tax and its Economic Implications – Stiglitz, Chapter 22; R&G, Chapters 17-18.

*Please note: All references to the text, Rosen & Gayer, will be referred to as R&G.

EXERCISE 1

Question 1

Robinson Crusoe has the following utility function: $U = X + Y$. where X and Y are products he consumes.

He has a total of 64 available work hours per week, $(\overline{L} = 64)$.

His production function for X is as follows: $X = L_X^{0.5}$.

His production function for Y is as follows: $Y = 4L_Y^{0.5}$.

a) Find the Production Possibility Frontier for X and Y and draw it precisely.

b) Find the equilibrium values of L_x, L_y X, Y and U.

c) Explain in words the Pareto Optimum conditions you have found at equilibrium.

d) How will you answer to (a) and (b), change if there is a technological improvement in the production function of X whereby production increases fourfold for every level of L?

e) How will you answer to (b) if Robinson Crusoe has a new utility function: U=X·Y

Question 2

An economy produces two products X and Y, which are produced using two production factors. K (capital) and L (labor). The quantities of these factors are: L_0 = 160000, K_0 = 10000. The production functions are: $X = L_x^{0.25} K_x^{0.25}$, $Y = L_y^{0.25} K_y^{0.25}$.

X and Y are traded in the international market and the international price is fixed:

$P_X = P_Y = 10{,}000$. It is known that in the economy there is competition in all sectors.

a) Find and draw the contract curve.

b) Find the Production Possibility Frontier of the economy.

c) Find the quantities of X and Y that afford maximum income (welfare).

Question 3

A closed economy produces two products: Private Services X and plastic goods Y using production factors of limited quantity. Capital, $K_0 = 144$ and Labor $L_0 = 36$. The production functions are: $Y = L_Y^{0.5} K_Y^{0.5}$ and $X = L_X^{0.5} K_X^{0.5}$ The consumers' utility from the products is given by the following function: $U = X^2 Y$.

a) Draw the Edgeworth Box, the contract curve and the appropriate allocation of production factors. What is the consumer's utility level?

b) In the coming year the capital is expected to increase by 50%. The economy must continue to produce the same quantity of plastic goods as in (a) above. Find L_X, K_X expected in the coming year. What will the consumers' utility be after the increase in the amount of capital?

EXERCISE 2

Question1

An economy produces two products X and Y using the following production functions: and $X = L_X^{0.5} \cdot K_X^{0.5}$, $Y = L_Y^{0.5} \cdot K_Y^{0.5}$. The economy has 16 units of the production factor $\boldsymbol{L}$ (labor) and 144 units of the production factor $\boldsymbol{K}$ (capital).

a) Find the mathematical form and draw the graph of the production possibility frontier.

b) The economy has two consumers with the following utility functions: $U_1 = X_1^{0.5} \cdot Y_1^{0.5}, U_2 = X_2^{0.8} \cdot Y_2^{0.2}$.Find the Utility Possibility Frontier.

c) How will the economy optimally allocate the production factors and the products according to the Utilitarian Approach (U_1+ U_2)?

d) How will the economy optimally allocate the production factors and the products according to the Egalitarian Approach of Rawls $W = (Min\ (U_1 ; U_2))$?

e) How will the economy optimally allocate the production factors and the products according to the Social Welfare Function $W = U_1 \cdot U_2$?

f) How will your answers to sections (a) - (e) change if there is a technological improvement whereby a double amount of X can be produced from every combination of K_X and L_X.

Question 2

The following table includes various combinations of income (U) for Robinson Crusoe (U_1) and Man Friday (U_2). Draw the Utility Possibilities Frontier of their incomes and list the points on the graph by order of Social Welfare according to the following Social Welfare approaches:

a) Utilitarian
b) Rawls
c) $W = U_1 \cdot U_2$ (Nash)
d) Nozick for $\theta = 1$, and for $\theta = 2$
e) Extreme Equality $(\theta \to \infty)$

	A	B	C	D	E	F	G	H	I	J	K	L
U_1	3	3	4	5	6	8	7	6	5	4	3	1
U_2	0	5	4	8	8	12	20	32	50	56	60	60

Question3

A social planner claims that from the social point of view the two following allocations of wealth between two consumers are identical, i.e.,

$$W(U_1 = 110, U_2 = 120) = W(U_1 = 170, U_2 = 150)$$

a) Can this claim be accepted by the Utilitarian approach? By Rawls Approach? By intermediate approach $W = U_1 \cdot U_2$?

b) Can this approach be accepted by the Nozick approach? What is the θ and the level of W in this case? Given an accurate answer.

EXERCISE 3

Question1

In a lake, N fishermen operate N boats such that each fisherman has his own boat. The total amount of dear fish caught, Y, depends positively on the number of fishermen in the lake and is determined according to the production function of Y lb of fish per hour is $Y = 400N^{0.5}$.

The cost of operating a boat is $40 per hour. The price of 1 lb of fish is $10.

a) Find the number of boats and fishermen that will operate under competitive equilibrium conditions.

b) An economist suggests limiting the entrance to the lake in order to improve the competitive equilibrium solution. Why? What is the optimal N?

c) What is the fee that will improve the solution, and leads to Pareto Optimum?

d) How will your answers to a) and b) change if the new cost of operation all the boats is $TC(N) = 4 \cdot N^2$?

Question 2

An economy produces honey, H, and avocado, A.

The honey producers' cost function is:

$$TC_H(H) = \frac{H^2}{100} + 40A$$

The avocado producers' cost function is:

$$TC_A = \frac{A^2}{150} - 10H$$

The honey and avocado are sold at fixed international prices, where $P_H = 80$ and $P_A = 80$.

a) When the market is competitive, how much honey and avocado will be produced? What will the income, costs, and profit of the honey and avocado producers be?

b) What are the Pareto Optimum amounts of avocado and honey and how will they be produced when producers operate when cooperating? What will their income, costs and profits be?

c) What subsidy and/or tax per unit of *A* and *H* should be set by the government in order to ensure Pareto Optimum production if each producer doesn't cooperate?

Question 3

Give one example of externalities in each of the following cases:

a) Positive externalities in consumption.

b) Positive externalities in production.

c) Negative externalities in consumption.

d) Negative externalities in production.

Do you think that in all of the above cases there is need for government intervention that corrects the inefficiency in equilibrium obtained by the market? Your answer, including the examples should not exceed 15 lines.

EXERCISE 4

Question1

a) Micronesia has 1000 inhabitants with identical incomes and tastes. Each inhabitant has a demand for *G* television hours as a function of the price *P* which is paid according to the equation: $G=30-0.1P$, where *P* is measured in dollar terms. The cost of supplying an hour of television broadcasting equals $120,000. How many hours of television broadcasting will there be in Micronesia and what is the free paid by each inhabitant?

b) 5000 new immigrants join the country. They have identical incomes and tastes, which differ from those of the original inhabitants. Each new immigrant has the following demand function: $G=100-0.5P$. How will your answer to (a) change?

c) Is the solution of item b efficient? Fair?

Question 2

An economy produces a private good *C* which is divided between a father (1) and son (2) and a public good *G* which is jointly consumer by them. A unit of *C* is produced using one unit of capital and a unit of *G* is produced using two units of capital. The economy has 1000 units of capital.

a) How many units of *C* and *G* will be produced and how many units of capital will be allocated to the production of each good?

b) How will the private product *C* be divided between C_1 and C_2?

c) What is Pareto Optimum condition that must hold in equilibrium? Does this condition hold in your answer?

Question 3

An economy has two consumers who have identical incomes and tastes with respect to the private consumption of C, and a public good G. Each consumer has \$1500. The utility of the first consumer is: $U_1 = C_1^2 \cdot G$. The utility of the second consumer is: $U_2 = C_2^2 \cdot G$. The price of the public good is P_G=4 and C_2 is measured in terms of dollar expense.

a) What is the Nash equilibrium without cooperation?

b) What is the Lindhal equilibrium with cooperation?

c) Compare the solution between a) and b).

d) Assume that cooperation between the individuals cost each of them \$200. Would they prefer to cooperate under the new conditions?

e) (Optional) What is the solution to a) (Nash Equilibrium) if individual 1 has to pay \$200 to individual 2? Who gains and who loses from it? What do you learn from your results regarding the question of inequality in income and utility?

EXERCISE 5

Question 1

"Proportional taxes and per capita taxes, which afford an equal sum of real taxes, per capita tax is always preferable". True/False

Question 2

"Progressive taxes and per capita tax which afford an equal sum of real tax, per capita tax is preferable". True/False

Question 3

"Progressive taxes may be preferable to proportional taxes which affords an equal sum of tax. However in this case there is always an alternative proportional tax which also affords an equal sum of tax and is preferable to the progressive tax." True/False

Answer these three questions using the Consumption Leisure model.

Question 4

A consumer has utility from consumption, C, and leisure, l, as follows:

$U = C^2 \cdot l$. The consumer has 24 hours per day, which he allocates to leisure, l, or work. L. The consumer earns W dollars per hour of work as well as a daily national insurance grant of $V = \$300$.

a) Find and draw the consumers labor supply curve (namely, L, as a function of W).

b) If the consumer is exempt from paying taxes, how many hours will he work? How much will he consume and what utility level will he achieve when $\overline{W} = 50$?

c) Now there is a tax on income from work only, $0<t<l$. The total amount of tax collected is: $T = t \cdot W \cdot L$. Show the Laffer curve $(T=f(t))$. What tax level, t, affords maximum income from taxes, T_0 (remember that $\overline{W} = 50$)? What will the amount of taxes collected, T_0, equal? What will the consumer's utility level be?

d) The government decides to collect the same amount of taxes, T_0, by imposing the tax on V and not on income from work. What will the consumer's utility be in this case?

e) What conclusion can be reached about the efficiency of taxes by comparing the utility of the consumer in section (b) and (c)

Question 5

A consumer has demand for alcohol, X, and cigarettes, Y as follows:

$P_X = 220 - X$, $P_Y = 180 - Y$. P_X, P_Y are the prices of alcohol and cigarettes respectively. The marginal cost of producing these two products are:

$MC_X = 70$ and $MC_Y = 80$.

a) What is the tax level per unit of product, T_X, that will maximize the government's income from taxes? What will the income from taxes be? What will the Deadweight Loss be?

b) If the government needs an income from taxes that equals 1800, what taxes per unit, t_x and t_y, will minimize the Deadweight Loss? What will be the proportion of deadweight less be relative to the tax collected from each product?

c) How will your answer to (b) change if you know that the consumption of a unit of Y and the consumption of a unit of X create damage to society worth $30?

ECON 30
MAKEUP MIDTERM EXAM
FALL 1999

Uri Spiegel

1. The production costs of firm 1 is TC_1, and firm 2 is TC_2. They are related to production X_1 of firm 1 and X_2 of firm 2, as follows:

$TC_1 = 10X_1 + X_1^2 - 20X_2$

$TC_2 = 10X_2 + 2X_2^2 + 10X_1$

The product X is sold abroad at a fixed price equal to 90.

   a. What kind of externalities do you identify in this case?
   b. What are the values of X_1 and X_2, the total revenues, costs, and profits under perfect competition equilibrium?
   c. Why is the solution to (b) not optimal?.
   d. What is the optimal solution? Compare profits between perfect equilibrium and the optimal solution.

2. "According to Utilitarian Approach, equality in utility will always be preferred to inequality in utility." True/False. Explain.
3. "Presumption of Equality requires equal sharing of income or kinds of all members in the society." True/False. Explain.
   (no more than 5 lines for each answer)
4. A certain economy has two consumers: Reuven and Simon: Reuven has utility U_R and Simon has utility V_S. The economy has 20 units of product X and 40 units of product Y. the utility functions of the two consumers are:

$U_R = 20X_R + 10Y_R$ $\qquad$ $V_S = 10X_S + 25Y_S$

   a. Show the indifference curves of the two consumers.
   b. Show the contract curve in the Edgeworth box.
   c. Find the Utility Possibility Frontier mathematically and in a graph.
   d. How would you allocate the 20 units of product X and 40 units of product Y between Reuven and Simon according to the following approaches:
      1. Utilitarian 2. Rawls 3. $W = U_R \bullet V_S$
   e. How do you change your answers to items b-d if the economy has 20 units of product X and 160 units of product Y?

GOOD LUCK!

5. A certain economy has two consumers: Reuven and Simon: Reuven has utility U_R and Simon has utility V_S. The economy has 100 units of product X and 100 units of product Y. The utility functions of the two consumers are:

$$V_S=20X_S+40Y_S \qquad U_R=10X_R+20Y_R$$

a. Show the indifference curves of the two consumers.
b. Show the contract curve in the Edgework box (explain your answer!
c. Find the Utility Possibility Frontier mathematically and draw it graphically.
d. How would you allocate the 100 units of product X and of product Y between Reuven and Simon according to the following approaches:

1. Utilitarian 2. Rawls 3. $W=U_R \bullet V_S$ 4. Nozick when $\theta=2$

e. How do you change your answers to items b-d if the utility function of Reuven is changed to the following:

$$U_R=40\ X_R+20Y_R$$

GOOD LUCK!

Uri Spiegel

ECONOMICS 31
Midterm Exam

Spring, 2000

1. In a community, there are two producers. The first producer "produces" apple, A, the second producer "produces" honey, H. The production of H depends positively on the number of workers, L_H, who are working in production of H, according to the following production function:

$$H = 100L_H - \frac{L_H^2}{2}$$

The production of apple A, depends on the number of workers who are directly involved in apple production, L_A, but also depends on the production of honey, according to the following production function:

$$A = 100L_A - \frac{L_A^2}{4} + 20H$$

   a. Can you explain the positive relationship between honey production and apple production?
   b. The labor market is a competitive market and the wage rage per worker is \$1000 (W=1000). The prices of each unit of honey and unit of apple are equal to \$100 (P_A=P_H=100). How many workers will be hired by each producer?
   c. What are the revenues costs and profits of each producer?
   d. What are the optimal number of workers from point of view of the society as a whole?
   e. What are the revenues costs and profit of each producer in the social optimum of d?
   f. Suggest (in general!) <u>several</u> ways to "adjust" the competitive equilibrium towards the social optimum: (no more than 10 lines).

2. "Presumption of Equality requires equal sharing of income or kinds of all members in a society regardless of needs and tastes." Discuss.

3. According to Harvey Rosen's book in the case of positive externalities subsidy by government is necessarily required to approach efficient solution.

4. "Not every Pareto optimum equilibrium is desirable and brings society to maximum social welfare, but any equilibrium where we reach maximum of social welfare Pareto optimum is, necessarily exists." Discuss.

5. A certain economy has two consumers: Reuven and Simon: Reuven has utility U_R and Simon has utility V_S. The economy has 100 units of product X and 100 units of product Y. The utility functions of the two consumers are:

$$V_S=20X_S+40Y_S \qquad U_R=10X_R+20Y_R$$

a. Show the indifference curves of the two consumers.
b. Show the contract curve in the Edgework box (explain your answer!
c. Find the Utility Possibility Frontier mathematically and draw it graphically.
d. How would you allocate the 100 units of product X and of product Y between Reuven and Simon according to the following approaches:

1. Utilitarian 2. Rawls 3. $W=U_R \bullet V_S$ 4. Nozick when $\theta=2$

e. How do you change your answers to items b-d if the utility function of Reuven is changed to the following:

$$U_R=40\ X_R+20Y_R$$

GOOD LUCK!

U. Spiegel

ECONOMICS 31
Midterm Exam
October, 2000

Good Luck!

1. "Competitive markets in most cases will allocate inputs to production and distribute finished goods in the most efficient manner." Discuss using equations and necessary figure(s).

2. Robinson Caruso's utility function over mangoes (m) and coconuts (c) is given by $U(m,c) = c^2 . m$. He has on his isolated island 10 mangoes and 6 coconuts.

   a. Find his initial utility level.

   The island is opened now for trade with other countries, so Caruso can trade mangoes for coconuts. According to the international prices, two coconuts can be traded for one mango, and vice versa.

   b. Find the consumption possibilities frontier of Caruso upon opening the island to international trade. Show graphically and mathematically the curve.
   c. At what point does Caruso choose to consume?
   d. How many units of export and import will be traded by Caruso?
   e. Show the Pareto optimum condition at the new consumption levels of mangoes and coconuts used by Caruso.
   f. Compare the utility levels of items (a) and (c).

3, A certain economy has two consumers: Reuven and Simon: Reuven has utility U_R and Simon has utility V_S. The economy has 100 units of product X and 100 units of product Y. the utility functions of the two consumers are:

$$V_S = 20X_S + 40Y_S \qquad U_R = 10X_R + 10Y_R$$

   a. Show the indifference curves of the two consumers.
   b. Show the contract curve in the Edgeworth box (explain your answer).
   c. Find the Utility Possibilities Frontier mathematically and draw it graphically.

d. How would you allocate the 100 units of product X and of product Y between Reuven and Simon according to the following approaches:

1. Utilitarian 2. Rawls 3. Nash: $W=U_R \cdot V_S$ 4. Nozick when $\theta = 2$

e. How do you change your answers to items b-d if the economy has now 300 units of product X and still 100 units of product Y? (Remember, X=100 and Y=300).

4. The following table includes various combinations of utilities for Robinson Caruso (U_1) and man Friday (U_2). Rank these points by order of Social Welfare according to the following Social Welfare functions:

a. Utilitarian
b. Rawls
c. Nash
d. Nozick for $\theta = 2$
e. Extreme Equality $\theta \to \infty$

Table

	A	B	C	D	E	F
U_1	2	3	2	16	4	4
U_2	4	3	8	4	4	2

5. "According to Nozick's approach, equal sharing of either income or goods between two individuals is always preferred to any other distribution". Discuss this statement according to the principles you have studied in the course.

Dr. U. Spiegel

ECONOMICS 31
1st midterm makeup exam
October 31, 2000

GOOD LUCK!

1. "According to the utilitarian social welfare function, society is indifferent to an increase of one orange for a rich individual and a decrease of one orange for a poor individual." Discuss.

2. "If the production functions of goods X and Y are both produced by two inputs and have the characteristics of constant return to scale, then the production possibility frontier is straight line." Discuss.

3. Under what conditions are allocation of utilities between two individuals by Nozick differ from Rawls approach?

4. An economy produces two products X and Y according to the following production functions:

$$X = L_X^{1/4} \cdot K_X^{1/4} \; ; \; Y = L_Y^{1/4} \cdot K_Y^{1/4}$$

The economy has 32 units of the production factor L, and 512 units of production factor K.

a. Find the production possibility frontier
b. The economy has two consumers with the following utility functions

$$U_1 = X_1 + Y_1 \qquad U_2 = 4(X_2 + Y_2)$$

Find the Utility Possibility Frontier

c. How will the economy optimally allocate the production and the products according to the utilitarian approach, Rawlsian approach, and the Intermediate approach?

5. The following table includes various combinations of utilities for Robinson Caruso (U_1) and man Friday (U_2). Rank these points by order of Social Welfare according to the following Social Welfare functions:

   a. Utilitarian
   b. Rawls
   c. Nash
   d. Nozick for $\theta = 2$
   e. Extreme Equality $\theta \to \infty$

Table

	A	B	C	D	E	F	G	H
U_1	2	3	2	16	4	4	4	12
U_2	4	3	8	4	4	2	15	3

Econ 31 Midterm
Fall 2003

Uri Spiegel

1)An Economic Planner claims that the following utility distribution are equal in social terms:
$W(U_1=110, U_2=120) = W(U_1=160, U_2=130) = \overline{W}$

a) This planner "believes" in the Nozick approach. Discuss why.

b) What is the value of the parameter θ according to the economic planner.

c) What is the value of $\overline{W}$?

d) What will be the values of $W(U_1=110, U_2=120)$ and of $W(U_1=160, U_2=130)$ according to the other approaches you have studied in the course?

2)Two individuals consume Grapefruit and Kiwi and gain units of vitamin C according to the following values

-Individual 1 gains 10 units from each Grapefruit and 20 units from each Kiwi
-Individual 2 gains 50 units from each Grapefruit and 5 units from each Kiwi

Assuming the total quantities of Grapefruit is equal to 100 and the total quantity of Kiwi is equal to 50, find the following:

a) The contract curve
b) The utility function of individuals 1 and 2
c) The U.P.F.
d) How would you allocate the Grapefruit and Kiwi between the two individuals according to the following social welfare functions:
   i) Utilitarian
   ii) Rawlsian
   iii) Nash
e) What are the welfare values (according to the three approaches)

Optional (10% extra credit)

f) A new soft drink can be produced from Grapefruit and Kiwi and the technology is such that from each unit of Kiwi and **(or)** 2 units of grapefruit, a bottle of soft drink can be produced. As a result, each bottle of the drink supplied to each individual 25 **(or 50)** units of vitamin C. Is it worthwhile to produce the soft drink or to continue to consume grapefruit and Kiwi as a regular fruit if we want to maximize the social welfare function according to the 3 approaches mentioned above? **Hint**: Find the new utilities of individuals and compare the new UPF to the old one.

Answer 3 questions out of Questions (3) – (6) , and Question (7).

3) "All Pareto Optimum allocations of utility between individuals are desirable according to the utilitarian approach but not according to the other approaches."

True/False/Uncertain? Discuss Briefly

4) "All Pareto Optimum conditions exist under perfect competitive equilibrium. Therefore competitive equilibrium is socially desirable."

True/False/Uncertain? Discuss Briefly

5) Discuss the circumstances under which Nozick and Rawls equilibria are the same or different.

6) Discuss the circumstances under which Utilitarian and Nash equilibria are the same or different.

7) Explain according to Harvey Rosen's book, and what you have studied in the course why income egalitarianism (equality) is not as desirable as when special commodities are equally distributed (also known as commodity egalitarianism).

Econ 30 Midterm for 2011 Fall
November 14, 2011
Professor Uri Spiegel

The exam is 75 minutes. You are allowed to use formula sheet and calculator.

1. In an economy, there are six different allocations of income (six scenarioes) between individual 1 and 2, respectively that are available,

Scenario	income of individual 1	income of individual 2
A	2	1
B	3	3
C	6	8
D	6	10
E	5	15
F	2	17

a. Combine the Income Possibilities Frontier of the 6 income allocations.

b. Five social planners who represent the five approaches of social selfare functions are asked to rank the scenarioes. Help them to rank all the six scenarioes based on each approach, assuming that the planner whose name is Nozick claims to the same welfare for scenarioes B and C (hint. find $\hat{\theta}$)

c. An economist suggests that we take the scenario where the total income is at maximum and tax the rich while subsidizing the poor by transfer payment. By doing so, the income of the poor is increased by 3 dollars, but it costs 2-dollar reduction in the total income. Who may argue for Pareto improvement among the five planners? Who opposes this income re-distribution? explain WHY!

2. In "Ideal" country, live 100 individuals who are employed as 100 workers (full employment), L and 200 machines, K, are allocated between production of two goods X and Y, according to the following production functions:

$$\begin{aligned} X &= 10L_x + 5K_x \\ Y &= 5L_y + 20K_y \end{aligned}$$

a. Using the Edgeworth Box, draw the iso-quant curves and contract curve precisely.

b. Plot the Production Possibilities Frontier

c. In this economy, all individuals are the 100 workers who enjoy X and Y in the same way and the representative worker/ individual benefits from X and Y according to the following utility function.

$$U = X + Y$$

How do you allocate L and K between good X and Y, and What will be the total utility level of each individual at "Ideal"?

d. A social planner found that by trading with some other country, he can send 20 individuals (workers) from "Ideal" abroad and in return, the economy," Ideal" loses 40 machines. Is it a good deal for "Ideal"? explain!!

3. Randy (R) and Simon (S) benefit from eating Grapefruit (G) and kiwi (K) since they consume vitamin C according to the following function:

$$C_R = 20(K_R + G_R), \text{ and } C_s = 60(K_S + G_S)$$

a. Assuming that the harvest of Grapefruit and Kiwi is $\bar{G}$=120 and $\bar{K}$=180. How do you allocate the two fruits between Randy and Simon, based on the three social welfare functions you have studied, Utilitarian, Rawlsian, Nash?

b. Technological improvement allows us to take the same amount of G and K such that we can squeeze a new drink named "Grawi", GR, according to the following function :

$$GR = 10 \cdot \min[K, G]$$

Randy and Simon immediately stop eating Grapefruit or Kiwi and totally are addicted to the new "tasty" drink according to the following function:

$$C_R = 8GR_R, \text{ and } C_S = 8GR_S$$

Does the new drink indicate Pareto improvement? If so, according to all 3 approaches?

ECON 30
FINAL EXAM MAKEUP
DR. SPIEGEL

January 21, 2000

Instructions:

- **Answer all sections of this test. This is a 120 minute exam; You have 10 minutes for review.**
- **Write all answers in the blue books provided. Show all work. Use diagrams where appropriate and label all diagrams carefully.**
- **Write your name and your Instructor's name in every blue book that you use.**
- **This exam is given under the rules of Penn's Honor System.**
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1. A consumer has utility from consumption, C, and leisure, ℓ, as follows:

   $$U = C \cdot \ell + 72\ell$$

   The consumer's wage ($)W per hour of work and his daily earning is from his working hours only.

   a. Calculate and draw the consumer labor supply (namely, L as a function of W)

   b. If the consumer's Wage per hour is W=80 and W is exempt from paying taxes, how many hours will he work? How much will he consume? And what utility level will be achieved?

   c. Now there is a tax on income from work only (proportional tax t where $0<t<1$). The total amount of tax collected is $T = t \cdot W \cdot L$. If still W=80, find the Laffer equation/curve. What is the tax rate t, that guarantees the government tax revenue of $30?

   d. What is the tax rate t* that leads to maximum tax revenues? What are the tax revenues?

   e. Is this proportional taxes efficient?

2.

   a. "Progressive tax is always more efficient and more equitable tax than proportional tax". Discuss (no more than 7 lines).

   b. Explain why the Lindhal solution is more efficient, and why the expenditure on public good is larger than Nash solution in the context of public good provision (no more than 7 lines).

c. "Public good cannot be produced and financed efficiently because of the free rider problem." Discuss (no more than 7 lines).

d. "A government which must finance a recent large expense over a certain period, will prefer to do so by imposing a high tax on few products over that period. It will avoid placing the burden just on our present generation which is created by imposing high taxes at the time of the expense." Based on what you have learned in the course, explain whether this statement is true, partially true, or false (no more than 10 lines).

3. A consumer has demand for good X and good Y as follows:

$$P_X = 260 - X \qquad P_Y = 200 - 2Y$$

The marginal cost of producing these two products are equal to 80.

If the government needs an income from tax, equals (to) 1000, what tax per unit t_X and t_Y will minimize the Deadweight Loss?

4. An economy has two consumers who have identical incomes and tastes with respect to the private consumption, C, and public good, G. Each consumer has $1200, and the utilities of consumer 1 and 2 are as follows:

$$U_1 = C_1^2 \bullet G \qquad U_2 = C_2^2 \bullet G$$

The price of the public good is $P_G = 6$ and C_1 and C_2 measured in terms of dollar expenses

a. What is the Nash equilibrium (without cooperation) in terms of C_i, G, U_i?
b. What is the Lindhal equilibrium (with cooperation) in terms of C_i, G, U_i?
c. If cooperation cost for each individual is $100, would they prefer to cooperate or choose the Nash equilibrium?

If you have difficulty in calculating, give a general answer that will be partially credited?

GOOD LUCK!

ECON 31
FINAL EXAM
DR. SPIEGEL

May 4, 2000

Instructions:

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GOOD LUCK!

1. A consumer has utility from consumption, C, and leisure, ℓ, as follows:

$$U=C^2 \bullet \ell$$

The consumer's wage (\$)W per hour of work and his daily earning from his working hours is W(24-ℓ). In addition, he earns every day \$V from interest payments.

a. Calculate and draw the consumer labor supply (namely, L as a function of W and V).

b. If the consumer's Wage per hour is W=50 and V=100, W is exempt from paying taxes, how many hours will he work? How much will he consume? And what utility level will be achieved?

c. Now there is a tax on income from work only (proportional tax t where $0<t<1$). The total amount of tax collected is $T = t \bullet W \bullet L$. If still W=50, find the Laffer equation/curve. What is the tax rate t, that guarantees the government tax revenue of \$20?

d. What is the tax rate t* that leads to maximum tax revenues? What are the tax revenues?

2.

a. "Lumpsum tax is always more efficient and more equitable tax than progressive tax". Discuss (no more than 7 lines).

b. "An increasing marginal tax rate is a good and fair tax system as always it reduces income inequality." Discuss. (no more than 7 lines).

3. Discuss briefly and give examples to the following terms:

   a. horizontal equality
   b. consumer tax burden
   c. externalities in consumption

4. An economy produces public good G, where the marginal production cost is $20 per unit. Two individuals 1 and 2 are sharing the benefit and the cost of the public good supply. Their demand curves are as follows:

   d_1 of individual 1 is: $P_1 = \frac{425}{G}$ for $G \geq 1$

   d_2 of individual 2 is: $P_2 = 100 - G$

   a. How many units of the public good will be supplied to the two individuals?
   b. What is the price that each individual pays per each unit of the public good?
   c. Find the consumer surplus of each individual at equilibrium. Who benefits more from the public good?
   d. After technological improvement, the production cost of the public good is reduced by 50%. Answer (a) and (b) after this improvement.
   e. Who benefits more from the technological improvement?

5. An economy produces two products H and A. The cost function of H is:

   $TC(H) = 8H^2A$

   The cost function of A is:

   $TC(A) = A^3$

   The prices of the two products are constant where $P_H = 64$ and $P_A = 108$

   a. Find the competitive equilibrium of H and A in terms of quantities revenues costs and profits.
   b. What are the Pareto quantities revenues costs and profits?

6. Two commodities X and Y are imported at a fixed price of 20 per unit ($P_X = P_Y = 20$). The demand curves for the two goods are:

$P_X = 200 - X$ $\qquad$ $P_Y = 140 - 2Y$

   a. The government decides to maximize tax revenues. What are the values of tax per unit for each commodity?
   b. What are the tax revenues from commodity X and from commodity Y?
   c. Does the Ramsey rule hold in this case?

ECON 31
FINAL EXAM MAKEUP
DR. SPIEGEL

January, 2001

Instructions:

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Good Luck!

1. Based on your textbook and your lectures in class, discuss the issue of inefficient direct and indirect taxes.

2. "Free Rider problem will lead to inefficient supply of the public good." Discuss the statement mentioning the main conditions under which free rider occurs.

3. "Imposition of excise tax on an inferior good is more efficient but more regressive than on luxury good." Discuss.

4. The demand for good X is: $X = 100 \cdot P^{-2}$
The demand for good Y is: $Y = 200 \cdot P^{-2}$

"Using the Ramsey rule if the marginal cost of X and the marginal cost of Y are equal, we can conclude that the tax per unit of good X is twice as much as the tax per unit on good Y." Discuss.

5. The family (father and son) income of $100 is allocated between two goods; G that all family members consume together (public good) and C separately (private good). The prices of unit G is $2 while the unit of C costs $1. The father in the family decides to provide a given utility to his son of 200 when the son is indifferent between C and G as both provide him a marginal utility of 4 units of utility from each additional unit consumption of either G or C.
The utility of the father is $U_1 = C_1 \bullet G$
   a. Find C of the son C of the father and G
   b. Show the pareto optimum condition at equilibrium
   c. How do you change your answer to (a) if the income of family increases to $150?

6. Consumer's income is \$200. He buys X and Y, price per unit are: $P_X = 10$, $P_Y = 5$ while his utility function is $U = X \bullet Y$.

a. Find X, Y and U of equilibrium.
b. The government has to finance a project that costs \$20 and decides to collect \$20 from the consumer. The government imposes excise tax on X find t_X (tax per unit on X) X, Y and U.
c. Instead the government impose Lump sum tax what will be X, Y and U?
d. The comparison of your result of (b) and (c) shows that excise tax are inefficient explain why?

7. An economy produces avocado, A, and honey, H. The production functions of avocado and honey are:

$$A = 100 \bullet L_A - \frac{L_A^2}{2} + 10H \qquad L_A \text{ - No. of workers producing avocado}$$

$$H = 10 \bullet L_H \qquad L_H \text{ - No. of workers producing honey}$$

The avocado and honey are sold at fixed international prices where:

$$P_A = 1 \text{ and } P_H = 5$$

In the economy Total Number of Workers $L = 120$ and each work costs $2(P_L = 2)$

a. When the market is competitive
(1) How much of the honey and avocado will be produced?
(2) How many workers work in the avocado and honey production?
(3) What are the revenue costs and profit?

b. Repeat on your answers of (a) under Pareto Optimum conditions

ECON 30

FINAL EXAM

URI SPIEGEL

DECEMBER 16, 2010

Instructions:

- Answer all sections of this test. This is a 120 minute exam: You have 10 minutes for review.
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- This exam is given under the rules of Penn's Honor System.
- All bluebooks, blank or filled, must be handed in at the end of this exam. No blue books may be taken from the room.

GOOD LUCK!

1. Based on your textbook and your lectures in class, discuss the issue of "Tax Fairness".
2. "Free Rider problem will lead to zero supply of the public good". Discuss the statement mentioning the main conditions under which free rider occurs.
3. "Imposition of excise tax on a necessary good is more effective but more regressive than on luxury good". Discuss.
4. The demand good X is: $X = 100 - 2P_x$

   The demand good Y is: $Y = 200 - P_y$

   "Using the Ramsey rule, as the marginal production cost of Y is constant and is twice as much as the constant marginal cost production of X we can conclude that the tax per unit of good X should be equal to the tax per unit on good Y." Discuss.
5. An economy has two consumers who have identical incomes and tastes with respect to the private consumption, C, and public good, G. Each consumer has income of \$2400 and the utilities of consumer 1 and 2 are as follows:

$$U_1 = C_1 \bullet G^3 \qquad U_2 = C_2 \bullet G^3$$

The price of the public good is $P_G = 8$ dollars and C_1 and C_2 measured in terms of dollar expenses.

a. What is the Nash equilibrium (with cooperation) in terms of C_i, G, U_i?

b. What is the Lindhal equilibrium (with cooperation) in terms of C_i, G, U_i?

c. If cooperation cost for each individual is \$400, would they prefer to cooperate or choose the Nash equilibrium?

If you have difficulty in calculating, give a general answer that will be partially credited.

6. A consumer has utility from consumption, C, and leisure, l, as follows:

$$U = C^2 \bullet l$$

The consumer's wage (\$)W per hour of work and his daily earning is from his working hours only. In addition, the daily income of V is from other income resources.

a. Calculate and draw the consumer labor supply (namely, L as a function of W and V)

b. If the consumer's wage per hour is W=60 and V=120 where W and V are measured in \$ terms and W is exempt from paying taxes, how many hours will he work? How much will he consumer? And what utility level will be achieved?

c. Now there is a tax on income from work only (proportional tax t where $0<t<1$). The total amount of tax collected is $T = t \bullet W \bullet L$. If still W=60, find the Laffer equation/curve. What are the tax revenues when $t = 0.1$?

d. What is the tax rate t^* that leads to maximum tax revenues? What are the tax revenues T^*?

e. The government impose Lump-sum tax of T^* that you found in item (d) above. What is the utility U^{**} that the consumer achieves in this case?

7. Discuss very briefly five ways that a policy maker can correct inefficiency of competitive equilibrium in the case of negative externalities.

Econ 030 Final Exam

Dr. Uriel Speigel - December 16, 2013

Name: _____

1. In Modern Land, there are two individuals, individual i and individual j, and two securities, a private security C, and a public security G. For individual i, the public and private securities are perfect substitutes, i.e., each unit of C_i can be substituted for one unit of the public good. However, for individual j, the security level he gains, S_j is of the following form:

$$S_j = C_j \times G$$

In this economy each unit of private security C requires one workers, while each unit of the public security G requires two worker.

In the economy, there are 200 total workers.

(a) Define the security function of individual i:

$$S_i = \_\_\_\_\_$$

(b) Define the PPF between securities C and G

(c) Assuming the Samuelson model, where individual j is the leader and individual i is the follower who is guaranteed $S_i = 100$, find the consumption possibilities frontier of S_j.

(d) Find the equilibrium values of S_i, S_j, C_i, C_j, C, G.

(e) Show that in equilibrium, Pareto Optimality holds (Hint: $MRS_i + MRS_j = RPT$)

2. Show graphically, and discuss in a few sentences the idea of how the Lindahl solution with cooperative behavior leads to a Pareto improvement when compared to the Nash solution with non-cooperative behavior in the case of Public Goods supply. (Hint: Remember "Substitution Effect")

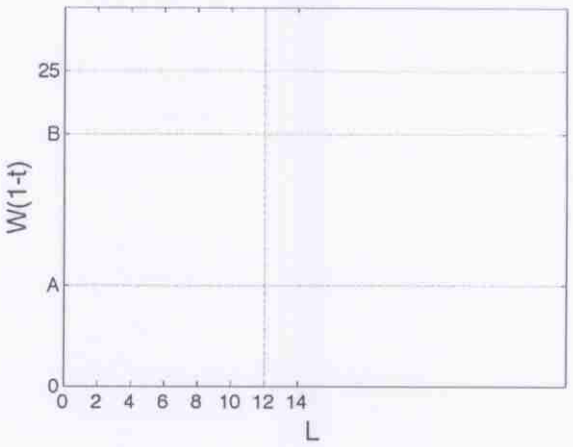
3. The supply curve for daily working hours of a certain employee, S_L, is as follows:

$$S_L: \quad W(1-t) = 8 + L \qquad \text{for } 0 < L < 12$$

Where $W(1-t)$ is dollars in net wage per hour.

When the net wage per hour is more than \$20, the worker does not change his behavior and supplies only 12 hours of daily work (rigid supply curve/vertical supply curve).

(a) Plot the supply curve on the graph below and label the necessary points on the y-axis.

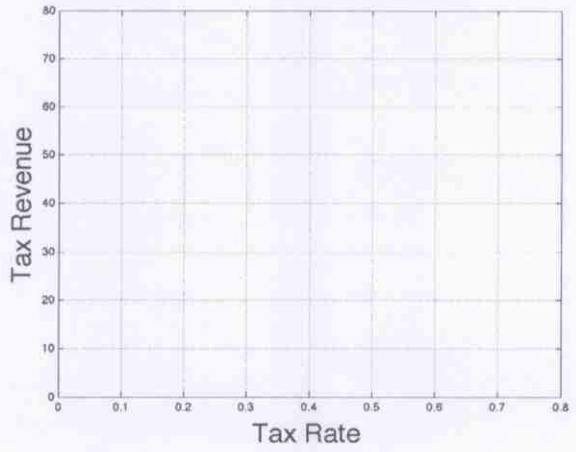


(b) At which ("minimum") net wage rate does the worker no longer work? (Point A)
At which ("maximum") net wage rate does the worker stop responding to the wage rate changes? (Point B)

(c) The gross wage rate an employee offers is \$25. Calculate the number of hours worked and gross and net income by filling out the following table for different tax rates. Note that $\bar{W}$ is the *gross* wage rate paid by the employer, t is the tax rate, L is the labor supply, $\bar{W} \cdot L$ is the gross income, and $T = t \cdot \bar{W} \cdot L$ is the tax revenue.

t	$\bar{W}(1-t)$	L	$\bar{W} \cdot L$	T	Net Income
0	25	12	300	0	
0.1					
0.2					
0.3					
0.4					
0.5					
0.6					
0.7	7.5				
0.8	5			0	

(d) Plot the Laffer Curve on the below figure, and indicate the tax rate at which Tax Revenue is maximized.



4. (a) Briefly discuss the issue of Horizontal Equality that is raised by Martin Feldstein regarding the connection between income-loving and leisure-loving customers.

   (b) Is it OK to tax food in order to finance a new project of infrastructure?

5. Consider the situation with two different goods, Beer (B) and Whiskey (W), with respective demand functions, D_B, D_W. The production cost of each bottle is \$2.

   The demand functions are as follows:

$$D_B: \ P_B = 10 - 0.2B, \qquad D_W: \ P_W = 20 - 2W$$

   (a) According to Ramsey, which item should be taxed at the higher rate?

   (b) If the target is to maximize tax revenues, and you are only allowed to tax one item, which item would you suggest to tax? (Hint: Compare the tax revenue of both). Give a specific number to prove your answer. General answers will be considered only for partial credit.

6. Why are the main free highways more congested, while turnpikes are less? How can a policy of "certain subsidies" on mass transportation and trains help to resolve the congestion? Discuss briefly what we mean by "certain subsidies, and demonstrate graphically."

7. An economy produces Apples (A) and Honey (H). The production functions for each good are:

$$A = 20L_A - \frac{L_A^2}{2} + 5H$$
$$H = 16\sqrt{L_H}$$

   Where L_A are the number of workers producing apples, and L_H are the number of people producing Honey.

   Both products are sold internationally at fixed prices:

$$P_A = 4 \quad \text{and} \quad P_H = 10$$

   The price of labor for both goods is $P_L = 20$.

   (a) When the market is competitive,

      i. How much Honey and Apples will be produced?

      ii. How many workers go into Honey and Apple production?

      iii. What are the revenues, costs and profits?

   (b) Repeat your answers to (a) under the Pareto Optimal Conditions. If you have difficulty with the calculations, demonstrate the answer graphically for partial credit.

Good Luck!!

Econ 030 Make Up Final Exam

Dr. Uriel Speigel - January 12 , 2015

Name: _____

1. In Modern Land, there are two individuals, individual i and individual j, and two securities, a private security C, and a public security G. For individual i, the public and private securities are perfect substitutes, i.e., each unit of C_i can be substituted for one unit of the public good. However, for individual j, the security level he gains, S_j is of the following form:

$$S_j = (C_j)^2 \times G$$

In this economy each unit of private security C requires one workers, while each unit of the public security G requires two worker.

In the economy, there are 200 total workers.

(a) Define the security function of individual i:

$$S_i = \_\_\_\_\_$$

(b) Define the PPF between securities C and G

(c) Assuming the Samuelson model, where individual j is the leader and individual i is the follower who is guaranteed $S_i = 200$, find the consumption possibilities frontier of S_j.

(d) Find the equilibrium values of S_i, S_j, C_i, C_j, C, G.

(e) Show that in equilibrium, Pareto Optimality holds (Hint: $MRS_i + MRS_j = RPT$)

2. Show graphically, and discuss in a few sentences the idea of how the Lindahl solution with cooperative behavior leads to a Pareto improvement when compared to the Nash solution with non-cooperative behavior in the case of Public Goods supply. (Hint: Remember "Substitution Effect")

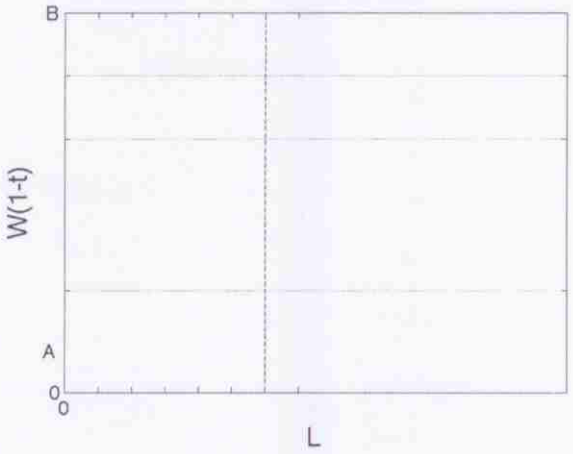
3. The supply curve for daily working hours of a certain employee, S_L, is as follows:

$$S_L: \quad W(1-t) = 10 + L \qquad \text{for } 0 < L < 10$$

Where $W(1-t)$ is dollars in net wage per hour.

When the net wage per hour is more than \$25, the worker does not change his behavior and supplies only 15 hours of daily work (rigid supply curve/vertical supply curve).

(a) Plot the supply curve on the graph below and label the necessary points on the y-axis.

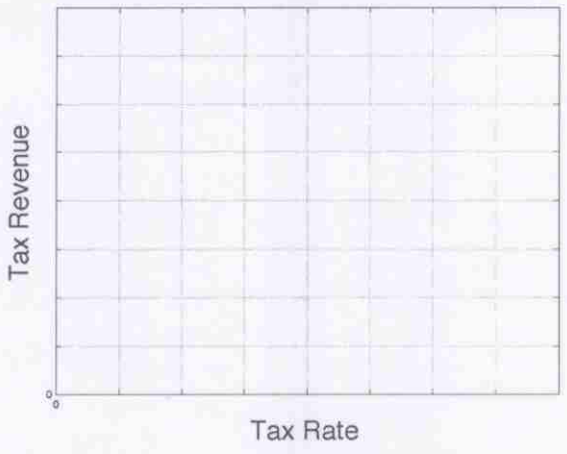


(b) At which ("minimum") net wage rate does the worker no longer work? (Point A)
At which ("maximum") net wage rate does the worker stop responding to the wage rate changes? (Point B)

(c) The gross wage rate an employee offers is \$30. Calculate the number of hours worked and gross and net income by filling out the following table for different tax rates. Note that $\bar{W}$ is the *gross* wage rate paid by the employer, t is the tax rate, L is the labor supply, $\bar{W} \cdot L$ is the gross income, and $T = t \cdot \bar{W} \cdot L$ is the tax revenue.

t	$\bar{W}(1-t)$	L	$\bar{W} \cdot L$	$TaxRevenue$	Net Income
0	30	15	450	0	
0.1					
0.2					
0.3					
0.4					
0.5					
0.6					
0.66	10			0	

(d) Plot the Laffer Curve on the below figure, and indicate the tax rate at which Tax Revenue is maximized.



4. (a) Briefly discuss the issue of Horizontal Equality that is raised by Martin Feldstein regarding the connection between income-loving and leisure-loving customers.

   (b) Is it OK to tax food in order to finance a new project of infrastructure?

5. Consider the situation with two different goods, Beer (B) and Whiskey (W), with respective demand functions, D_B, D_W. The production cost of each bottle is \$2.

   The demand functions are as follows:

$$D_B: \ P_B = 15 - 0.2B, \qquad\qquad D_W: \ P_W = 25 - 2W$$

   (a) According to Ramsey, which item should be taxed at the higher rate?

   (b) If the target is to maximize tax revenues, and you are only allowed to tax one item, which item would you suggest to tax? (Hint: Compare the tax revenue of both). Give a specific number to prove your answer. General answers will be considered only for partial credit.

6. Why are the main free highways more congested, while turnpikes are less? How can a policy of "certain subsidies" on mass transportation and trains help to resolve the congestion? Discuss briefly what we mean by "certain subsidies, and demonstrate graphically."

Good Luck!!

Formula Sheet

Derivation:

$$Y = \frac{f(x)}{g(x)} \qquad \frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

If demand: A-BQ=P
And Supply (MC): C=P

Then: $\frac{tx}{Ax-Cx-tx} = \frac{ty}{Ay-Cy-ty}$

Quadratic Formula:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\frac{MU_1}{MU_c} = w(1-t)$

$\frac{E_x}{P_x} \cdot \frac{t_x}{MC_x} = \frac{E_y}{P_y} \cdot \frac{t_y}{MC_y}$

Ramsey Rule:

$$\frac{L_x}{T_x} = \frac{L_y}{T_y}$$

Nash:

$I = C_1 + P_g (G-G_2)$
$I = C_2 + P_g (G-G_1)$

Lindhal:

$I = C_1 + (P_g/2)G$
$I = C_2 + (P_g/2)G$

Samuelon:

$U_1 = f(C_1, G)$
$U_2 = f(C_2, G)$
s.t: PPF C=f(G)
$MRS_1 + MRS_2 = RPT$ (unless there is a corner solution)

Laffer Curve:

T=twL, L(w,t)

Budget Constraint for Income-leisure Model:

$C = w(1-t)[T-l] + V$
$L = T - l$

Tax burden on consumer: $\Delta P = \frac{\alpha t}{\alpha+\beta}$

Tax burden on producer: $t - \Delta P = \frac{\beta t}{\alpha+\beta}$

Where α is slope is supply curve, and β is slope of the supply curve.

$L_X = \frac{\Delta x * tx}{2}$ $\quad T_X X + t_y Y = T$ $\quad P_X = MC_X + tx$

If: Demand: $A-BQ=P$
Supply(MC): $C=P$
Then: $t_x/(A_x-C_x-t_x)=t_y/(A_y-C_y-t_y)$ will minimize $DWL=L_x+L_y$

Ramsey Rule: $L_x/T_x=L_y/T_y$

Nash: $I=C_1+P_g(G-G_2)$
$I=C_2+P_g(G-G_1)$

Lindhal: $I=C_1+(P_g/2)G$
$I=C_2+(P_g/2)G$

Samuelson: $U_1=f(C_1,G)$
$U_2=g(C_2,G)$
s.t. PPF $C=f(G)$
$MRS_1+MRS_2=RPT$ unless there is corner solution

Laffer Curve: $T=twL$, $L(w,t)$

Budget constraint for Income-Leisure model: $C=w(1-t)[T-l]+V$

If: Demand: $A-BQ=P$
Supply: $C+EQ=P$
Then: Consumer Burden: $B/(B+E)$
Producer Burden: $E/(B+E)$

Random Stuff:

If: $f(x)=g(x)/h(x)$ Then: $f'(x)=g'(x)h(x)-g(x)h'(x)/(h(x))^2$

$MU_1/MU_c=w(1-t)$

$$X_{12}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$