Econ 705 : Preliminary Examination

Instructions:

(1) All the answers should be written legibly.

(2) In giving answers, try to provide conditions, if needed, which justify the answers.

(3) Even when you are not able to delineate precise conditions, try to provide a sketch of derivations and solutions as clearly as possible.

(1)(a-d) (40 Points) Suppose that one has i.i.d. observations of $\{(Y_{1i}, Y_{2i}, X_{1i}, X_{2i})\}_{i=1}^{n}$ which are assumed to have been generated from the following regression models:

$$
\begin{aligned}
Y_{1i} &= \beta_{10} + X_{1i}\beta_1 + u_{1i} \text{ and} \\
Y_{2i} &= \beta_{20} + X_{2i}\beta_2 + u_{2i}.
\end{aligned}
$$

Here X_{1i} and X_{2i} are (scalar) random variables such that $Var(X_{1i}) \in (0, \infty)$ and $Var(X_{2i}) \in (0, \infty)$. We assume that the conditional distribution of (u_{1i}, u_{2i}) given (X_{1i}, X_{2i}) is a joint normal distribution with mean $\gamma = [\gamma_1, \gamma_2]'$ and the variance $\sigma^2 I$, where I is a 2×2 identity matrix. The quantities $\beta_{10}, \beta_{20}, \beta_1, \beta_2, \gamma_1, \gamma_2$ and σ^2 are unknown parameters.

(a) Show that both β_1 and β_2 are identified in this set-up.

(b) Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be least squares estimators separately obtained from each regression model. Show that $\hat{\beta}_1$ is consistent.

(c) Derive the asymptotic distribution of $\hat{\beta}_1 - \hat{\beta}_2$.

(d) Suppose that one is interested in testing:

$$
\begin{aligned}
H_0 &: \beta_1 = \beta_2 \text{ against} \\
H_1 &: \beta_1 \neq \beta_2.
\end{aligned}
$$

Using the result in (c), construct an appropriate Wald test and explain how one can obtain an asymptotic 5% critical value for the test.

(2)(a-f) (60 Points) A researcher is interested in estimating the effect of a job training program. Let X_i denote the i-th worker's education and D_i the indicator of participation in the program. As for the participation in the program, the researcher considers estimating the following model:

$$D_i = \begin{cases} 1 \text{ if } 0.5 + X_i\beta > \varepsilon_i \\ 0 \text{ if } 0.5 + X_i\beta \leq \varepsilon_i \end{cases},$$

where the conditional distribution of ε_i given X_i is $N(0,1)$. Assume that $\{(D_i, X_i, \varepsilon_i)\}_{i=1}^n$ are i.i.d. and only $\{(D_i, X_i)\}_{i=1}^n$ are observed. The quantity β denotes the unknown parameter.

(a) Write the log-likelihood function for β, and derive the asymptotic distribution of the MLE $\hat{\beta}$ for β.

(b) One is interested in estimating the conditional choice probability: $P\{D_i = 1|X_i = x\}$ for a fixed number x. Using the estimator in (a), provide an estimator of this conditional choice probability and derive its asymptotic distribution.

Furthermore, the researcher observes the outcome variable Y_i which is the earnings of the i-th worker and specified as follows:

$$Y_i = \gamma_0 + D_i\gamma_1 + v_i$$

where v_i is an unobserved random variable such that the conditional distribution of v_i given (X_i, ε_i) satisfies

$$v_i \mid X_i, \varepsilon_i \sim N(0,1).$$

We assume that the random vectors $\{(Y_i, D_i, X_i, \varepsilon_i, v_i)\}_{i=1}^n$ are i.i.d. and only $\{(Y_i, D_i, X_i)\}_{i=1}^n$ are observed. The quantities γ_0 and γ_1 denote unknown parameters.

(c) Provide a verbal interpretation of the parameter γ_1. (HINT: Use the conditional mean function of Y_i given D_i.)

(d) Find at least three moment conditions for identification of γ_0 and γ_1.

(NOTE: Questions (e)-(f) are on the next page.)

(e) Suppose that consistent estimators of γ_0 and γ_1 are given. Using these estimators, suggest an estimated optimal weighting matrix for the GMM estimation of γ_0 and γ_1 based on the moment conditions in (d), and define the GMM estimators of γ_0 and γ_1 using the estimated optimal weighting matrix.

(f) Is it possible to test whether the moment conditions in (d) are satisfied? If so, explain how you would test it. (You need to discuss how you would obtain critical values.)