1 A Stochastic Endowment Economy

Time $t = 1, 2, \ldots$ is discrete and the economy lasts forever. Let $z_t \in Z$ denote the current aggregate state of the world, where Z is a finite set with J elements. The probability of an event history $z^t = (z_1, \ldots, z_t)$ is denoted by $\pi_t(z^t)$. The aggregate state follows a Markov chain with transition matrix $\pi(z_{t+1}|z_t)$ where π_{jk} is the probability of state k tomorrow given state j today (that is, rows indicate states today, columns states tomorrow). The state z_1 at the initial date $t = 1$ is *fixed*.

The total aggregate endowment in the economy is given by $e_t(z_t)$. There are I households. The endowment $e_t^i(z_t)$ of household $i \in \{1, 2, \ldots, I\}$ at node z^t is given by

$$e_t^i(z_t) = \theta_i e_t(z_t),$$

where θ_i is the share of total endowment that a household of type i receives. The shares satisfy $\theta_i > 0$ and $\sum_{i=1}^I \theta_i = 1$. Household $i \in \{1, 2, \ldots, I\}$ starts the economy with assets (claims to period 1 consumption) a_1^i. These assets satisfy $a_1^i \neq 0$ but

$$\sum_{i=1}^{I} a_1^i = 0.$$

A consumption allocation is $\{c_t^i(z^t)\}$, where $c_t^i(z^t)$ is consumption of a household of type i at node z^t. Households have preferences over consumption streams given by

$$\sum_{t=1}^{\infty} \sum_{z^t} \beta^{t-1} \pi_t(z^t) U(c_t^i(z^t))$$

where U is the strictly concave, strictly increasing and continuously differentiable period utility function.

1. For this question only assume that Z has only two elements ($J = 2$) and assume that the Markov transition function is given by

   $$\pi = \begin{pmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{pmatrix}$$

   For *each* $\rho \in [0, 1]$ determine the set of invariant distributions associated with π.

2. Suppose households can trade a full set of one-period Arrow securities among each other. For a given initial wealth distribution $\{a_1^i\}_{i=1}^I$ define a sequential markets equilibrium.

3. Define a recursive competitive equilibrium (households can still trade a full set of Arrow securities).

4. Let the aggregate endowment be given by

   $$e_t(z_t) = z_t$$

and the period utility function be

$$U(c) = \log(c)$$

Also, for simplicity assume that the fixed initial state equals $z_1 = 1$. Characterize as fully as possible the equilibrium. You can choose whether to do this for a Sequential Markets Equilibrium, an Arrow Debreu Equilibrium (or even a Recursive Competitive Equilibrium, although this is harder and not recommended) but you have to state your choice clearly.

5. Consider a stock that, starting from a node z^t, pays out as dividend the aggregate endowment z_t at every future date and future successor node. Under the same assumptions as in the previous question determine the ex dividend price of such a stock at node z^t (in terms of the z^t consumption good).