1 Optimal Insurance and Effort with an Outside Option

Parts of this problem are novel and challenging, but the first part is pretty standard. So, be careful to get the first part right and to think a bit in doing the second two.

Consider the following 3 period model with enforcement issues. Time is discrete and indexed by $t = 0, 1, 2$. The first period, $t = 0$, is the ex ante contracting period. There is a single risk-neutral principal who wants to insure an agent against his stochastic productivity opportunities in periods $t = 1$ and $t = 2$. In these periods the agent receives a stochastic probability draw of $\theta_t \in \Theta$ where the probability of an individual draw is $\Pi(\theta_t|\theta_{t-1})$ and the initial seed is θ_0. His output is given by $\theta_t l_t$. Also in these periods the agent draws a continuation utility from the set $u_t \in U$. These draws are i.i.d. with probability $P(u_t)$.

At the beginning of periods 1 and 2 the agent draws his productivity level θ_t and his outside option u_t. These draws are public information and the agent cannot be prevented from taking his outside option if he wants to. If the agent does not choose the outside option his continuation payoff in period t is given by

$$V_t = u(\theta_t l_t + \tau_t) - v(\theta_t l_t) + \beta E V_{t+1},$$

where τ_t is the state-contingent transfer under the insurance contract. If he does choose the outside option, then his payoff is simply $V_t = u_t$ and the game is over (so to speak). If he has not chosen his outside option through period 2, then he automatically receives the outside option drawn in period $t = 3$.

A) Assume initially that it is never efficient to have the agent take his outside option before period 3. What are the elements of the contract? Write down the efficient contracting problem. If the outside option does not bind, what does the efficient outcome look like? If the outside option binds in a period how does this effect the optimal choices under the contract?

B) Now assume that it may be efficient to have the agent take the outside option for certain realizations of θ_t and u_t. Let $I_t(u_t, \theta_t) = 1$ denote not taking the outside option and $I_t(u_t, \theta_t) = 0$ denote taking it. Write down the revised contracting problem.

C) Try and argue that $I_t(u_t, \theta_t)$ will satisfy a simple cut-off rule: if $u_t > \bar{u}_t(\theta_t)$ then efficient contract will specify that the agent takes the outside option and if $u_t < \bar{u}_t(\theta_t)$ then it will specify that he will not. What do you think you would need to assume about $\Pi(\theta_t|\theta_{t-1})$ to argue that the efficient contract would also have a simple cut-off structure with respect to θ_t.