Economics 702, Dirk Krueger

1 A Stochastic Endowment Economy

Consider a stochastic endowment economy where the current state of the economy is described by $s_t \in S = \{s_1, \ldots, s_M\}$. Event histories are denoted by s^t and the initial node s_0 is fixed. Probabilities of event histories are given by $\pi_t(s^t)$. There are I different households, and the endowment process of household i is given by $\{e_t^i(s^t)\}$. Preferences of each household over consumption allocations $c^i = \{c_t^i(s^t)\}$ are given by

$$u(c^i) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi_t(s^t) U(c_t^i(s^t)).$$

1. Define a Pareto optimal allocation.
2. Define an Arrow Debreu equilibrium.
3. State and prove the first welfare theorem. If you need to make assumptions to prove the theorem, state them clearly and indicate clearly in the proof where you used them.
4. Now consider a sequential markets equilibrium where households can trade a full set of Arrow securities. Furthermore assume that the period utility function is
$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$
where $\sigma = 1$ represents log-utility. Compute the risk-free interest rate, as a function of the fundamentals of the economy $(\beta, \sigma, \pi_t(s^t), \{e_t^i(s^t)\}_i)$.
5. Under the same assumptions as in the previous section, now in addition assume that all households are prevented from borrowing, that is, they face the constraints
$$a_{t+1}^i(s^t, s_{t+1}) \geq 0.$$

Furthermore assume that each household starts with $a_0^i = 0$ assets. Let $\sigma = 1$, $S = \{1, 2\}$, $I = 2$ and the endowment process be given by

$$\begin{aligned} e_t^1(s^t) &= \begin{cases} 3 & \text{if } s_t = 1 \\ 1 & \text{if } s_t = 2 \end{cases} \\ e_t^2(s^t) &= \begin{cases} 1 & \text{if } s_t = 1 \\ 3 & \text{if } s_t = 2 \end{cases} \end{aligned}$$

The process for $\{s_t\}$ is *iid*, and both states are equally likely (this now pertains to s_0 as well). Compute the sequential markets equilibrium (both prices and quantities). Compute the sequential markets equilibrium (both prices and quantities).

6. Is the equilibrium consumption allocation from the previous question Pareto optimal? You have to justify your answer.