Economics 702, Dirk Krueger

1 The Stochastic Neoclassical Growth Model and Dynamic Programming

The social planner in the stochastic neoclassical growth model chooses stochastic consumption, labor and capital allocations $\{c_t, l_t, k_{t+1}\}$ to solve the following maximization problem

$$\begin{aligned} & \max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \\ & \text{s.t.} \\ c_t + k_{t+1} &= z_t k_t^{\alpha} l_t^{1-\alpha} \end{aligned}$$

with $\beta \in (0,1)$ and $\alpha \in (0,1)$ being parameters. The initial endowment of capital k_0 and the initial exogenous state z_0 is given, and households are endowed with one unit of time in every period. The technology shock z_t follows a N-state Markov chain. Let $Z = \{z_1, z_2, \ldots z_N\}$ be the state space of the Markov chain, and let $\pi(z_{t+1}|z_t)$ denote the Markov transition matrix of the chain.

1. Let $N = 3$ and suppose the Markov transition function is of the form

$$\pi = \begin{pmatrix} \rho & 1-\rho & 0 \\ 1-\rho & \rho & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   For all $\rho \in [0,1]$ determine the set of stationary distributions.

2. Formulate the problem of the social planner recursively. State clearly what the state variables and control variables are.

3. Use the first order conditions and the envelope conditions to derive the Euler equation and the intratemporal optimality condition (in recursive form).

4. Assume that the representative households owns the capital stock. Define a Recursive Competitive Equilibrium.

5. Suppose $\alpha = 0$ so that the production function is given by

$$y_t = z_t l_t.$$

   Furthermore assume that $k_0 = 0$ and that

$$U(c, l) = \log\left(c - \frac{l^{\nu}}{\nu}\right).$$

   where $\nu > 1$ is a parameter. Fully characterize the Arrow-Debreu equilibrium (you do not have to define it).

6. Now let $\alpha \in (0, 1)$ from the remainder of the question, and suppose the representative (but competitively behaving) firm owns the capital stock, makes the investment decision and maximizes the present discounted value of payments to its shareholders. The firm takes the aggregate wage function $w = w(z, K)$ and the aggregate return to capital function $r(z, K)$ as given and discounts the future with a factor $\frac{1}{1-\delta+r(z',K')}$. Current payments to shareholders are given by the difference between current output, the wage bill $w(z, K)l$ and investment i. Formulate the dynamic programming problem of the firm recursively.

7. Guess that the value function has the form

$$V(z, k, K) = \gamma_0 + (1 - \delta + r(z, K))k$$

   where γ_0 is an unknown coefficient. Verify that the value function indeed has this form, and while doing so, determine the unknown coefficient γ_0. Hint: note that the current return to capital $r(z, K)$ equals the current payment to shareholders *per unit of capital.*