

Preliminary Examination 2010
Economics 702

1 A Pure Exchange Economy and Asset Pricing (40 Points)

Consider the following two period pure exchange economy. There is no risk in the first period. In the second period one of N states is realized. Let $s \in S = \{s_1, \dots, s_N\}$ denote the realized state, and denote by $\pi(s)$ the probability of this state. There are two consumers $i = \{1, 2\}$ in this economy, with endowments of e_0^i in period 0, and $e_1^i(s)$ in period 1. They have no endowment of initial assets. Household preferences are given by

$$U(c_0^i, c_1^i(s)) = \left\{ [c_0^i]^{1-\frac{1}{\gamma}} + \beta \left[\sum_s \pi(s) [c_1^i(s)]^{1-\sigma} \right]^{\frac{1-\frac{1}{\gamma}}{1-\sigma}} \right\}^{\frac{1}{1-\frac{1}{\gamma}}}$$

where $\beta, \gamma, \sigma > 0$ are parameters. Note that this preference specification is called Epstein-Zin (or recursive) utility.

1. Suppose that $N = 1$. Determine the intertemporal elasticity of substitution.
2. Suppose that households can trade consumption goods in both periods and a one-period uncontingent bond (a claim to one unit of consumption independent of the realization of s in period 1) in period 0. Denote by q_0 the price of this bond. Define a sequential markets equilibrium.
3. Is the equilibrium allocation of such an equilibrium Pareto efficient? How does your answer depend on N ? You don't have to provide a proof, but you have to justify your answer.
4. Suppose $\sigma = \frac{1}{\gamma}$. Derive the Euler equation, relating consumption in both periods.
5. Let $N = 1$ and assume that

$$\sum_{i=1}^2 e_0^i = 1$$
$$\sum_{i=1}^2 e_1^i = 2.$$

Determine the equilibrium risk-free interest rate and discuss and interpret how it depends on the parameter γ .

2 The Neoclassical Growth Model (40 points)

Consider the neoclassical growth model. A representative firm produces output according to the production function

$$Y_t = F(K_t, N_t)$$

Capital depreciates at a constant rate $\delta \geq 0$. The aggregate resource constraint reads as

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t$$

where C_t is aggregate consumption.

There is a large number of identical households with total mass equal to 1. Each household is endowed with $k_0 = K_0$ units of capital and one unit of time in every period. The household has preferences over individual consumption streams $\{c_t\}_{t=0}^{\infty}$ representable by the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t, C_t)$$

Note that the household chooses $\{c_t\}_{t=0}^{\infty}$, but takes aggregate consumption $\{C_t\}$ as given.

1. State the social planner problem recursively. Clearly identify the state and control variables.
2. Use the first order condition and the envelope condition to derive the Euler equation of the social planner problem.
3. Define a recursive competitive equilibrium.
4. Use the first order condition and the envelope condition to derive the Euler equation that the representative household faces.
5. For the purpose of this question you can assume that there exists a unique competitive equilibrium. Is this equilibrium Pareto efficient? You don't need to provide a formal proof, but use the answers to 2. and 4. to explain your answer, and give some intuition for your answer.

3 The Overlapping Generations Model with Money (20 Points)

Consider an overlapping generations economy in which the representative consumer in each generation born in period t , $t = 1, 2, 3, \dots$, lives for two periods. She values consumption of the single nonstorable good according to

$$u(c_t^t, c_{t+1}^t) = \ln(c_t^t) + \ln(c_{t+1}^t)$$

and has strictly positive endowments of the consumption good $(c_t^t, c_{t+1}^t) = (w_1, w_2)$. Population grows at rate $n > 0$ (if the population is 1 today, it is $1 + n > 1$ tomorrow). In addition there is an initial old generation with utility function

$$u(c_1^0) = \ln(c_1^0)$$

and an endowment of the consumption good $e_1^0 = w_2$ as well as an endowment $m > 0$ of fiat money. Normalize the size of the initial old generation to 1.

1. State a necessary and sufficient condition such that this economy has a stationary monetary equilibrium in which money has positive value. Compute this equilibrium, as a function of the parameters (w_1, w_2, n) .
2. Now assume that your condition in 1. is satisfied. Is the allocation of the stationary monetary equilibrium Pareto efficient? Give the set of *all* stationary (consumption of young and old is constant over time, but can have $c_y \neq c_o$) Pareto efficient allocations.