

**Economics 706**  
**Preliminary Examination**  
**Spring 2015**  
**Professor Francis X. Diebold**

Answer all questions.

“Discuss in detail” should be understood to apply everywhere.

Good luck!

A. Consider simulating draws on a random variable  $x$  with arbitrary known density  $f(x)$ , or alternatively arbitrary known density kernel  $k(x)$ , using the simulation methods below. For each, (a) present and discuss the algorithm and its workings, and (b) compare and contrast its costs and benefits relative to each of the others.

1. Box-Mueller
2. Inverse cdf
3. Accept-Reject
4. Metropolis (Independence)
5. Metropolis (Random-Walk)

B. Consider Bayesian analysis of the linear regression model.

1. What is the natural conjugate prior distribution of  $\beta|\sigma$ , and why?
2. What is the natural conjugate prior distribution of  $\sigma|\beta$ , and why?
3. Discuss in detail the structure of the posterior distribution of  $\beta|\sigma$  when using the natural conjugate prior.
4. How would you estimate the posterior mean of  $\beta$ , as opposed to the conditional posterior mean of  $\beta|\sigma$ , using Markov chain Monte Carlo (MCMC) methods?
5. How would you estimate the uncertainty associated with your estimate of the posterior mean of  $\beta$ ?