Cole's Problem:

Consider a version of our Ramsey model with two types of agents indexed by $i = 1, 2$. The problem of a type i household can be written as

$$\max \sum_t \beta^t u^i(c_t^i, l_t^i) \ \text{ subject to}$$

$$\sum_t p_t\{(1-\tau_{l,t}^i)w_t l_t^i + (1-\tau_{k,t}^i)q_t k_t^i\} = \sum_t p_t \left\{c_t^i + k_{t+1}^i - (1-\delta)k_t^i\right\},$$

$$k_t^i \geq 0 \text{ and } k_0^i, b_0^i, r_0, \tau_{k,0}^i, \tau_{l,0}^i \text{ given.}$$

Household consumption is given by c_t^i, labor by l_t^i, the rental rates by w_t and q_t, and capital holdings by k_t^i. The individual's labor and capital tax rates are given by $\tau_{k,t}^i$ and $\tau_{l,t}^i$ respectively. The household's utility function u^i satisfies all of our usual assumptions. The resource constraint is given by

$$\sum_i \left[c_t^i + k_{t+1}^i - (1-\delta)k_t^i\right] + g_t = F(\sum_i k_t^i, \sum_i l_t^i),$$

where F is an aggregate production function that also satisfies all the usual assumptions. The government's objective is given by

$$\sum_t \beta^t \left[\sum_i \omega_i u(c_t^i, l_t^i)\right]$$

and its budget constraint is

$$\sum_t p_t\{\tau_{l,t}^i w_t l_t^i + \tau_{k,t}^i q_t k_t^i\} = \sum_t p_t g_t.$$

A) Construct the set of conditions on allocation variables which characterize the set of allocations that can be supported as a competitive equilibrium with a feasible government policy.

B) Set up the government's optimal policy problem and derive the first-order conditions that characterize an optimal allocation. If the economy coverages to a steady state, does the capital tax go to zero?

C) To what extent are your conclusions about the steady state capital tax dependent upon the fact that we allowed for type-specific taxes?

D) If $\omega_1 > \omega_2$ (but the types were otherwise identical) can you say anything about how this would affect the government's optimal policy choices?