Economics 706 Preliminary Examination
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Provide detailed discussion throughout. Good luck!

Consider a dynamic factor model (*N*-variable 2-factor) for approximating the dynamics of real macroeconomic activity, as pioneered by Geweke, Sargent and Sims, and significantly extended by Stock and Watson, among others:

$$y_t = \mu + \lambda f_t + \varepsilon_t \quad (1)$$

$$f_t = \phi_1 f_{t-1} + \phi_2 f_{t-2} + \eta_t , \quad (2)$$

where y is a vector of real activity indicators, μ is a vector of constants, λ is a matrix of factor loadings, ϕ_1 and ϕ_2 are matrices of coefficients, f is a vector of latent common factors evolving independently of each other, and all stochastic shocks (i.e., the elements of ε and η) are contemporaneously and serially uncorrelated Gaussian white noise with zero mean and unit variance.

(A) Under what conditions is the vector f covariance stationary? Under what conditions is y covariance stationary? From this point onward, assume that those conditions are satisfied unless explicitly stated otherwise.

(B) Fully characterize the spectrum of f in both rectangular (co-spectrum and quadrature spectrum) and polar (gain, coherence, phase shift) forms. Do the same for the spectrum of y. Do the spectra of f and y have the same qualitative properties?

(C) Cast the system in state space form and display its measurement and transition equations. Temporarily assuming known system parameters, consider optimal 1-step-ahead prediction of y (i.e., calculation of $y_{T+1,T}$) using (1) Kalman filter methods, and (2) Wiener-Kolmogorov methods. Are the two predictors the same? Why or why not? (Hint: Think about the steady state of the Kalman filter.)

(D) Show how to evaluate the Gaussian log likelihood of the model via a prediction-error decomposition in conjunction with the Kalman filter applied to the state-space representation.

(E) Show how to obtain maximum-likelihood estimates of the model parameters using the EM algorithm. What are the large-sample properties of this maximum-likelihood estimator? After estimating the model, how would you obtain an optimal estimate of the state vector? What is meant by "optimal"?

(F) Show how to perform a Bayesian estimation of model using Carter-Kohn multi-move Gibbs sampling. What key additional assumption(s) is(are) required relative to classical maximum-likelihood estimation?